Valentin Kuzmin

MPD TPC alignment

Theoretical basis
Practical part
Accuracy study
Conclusion

Introduction

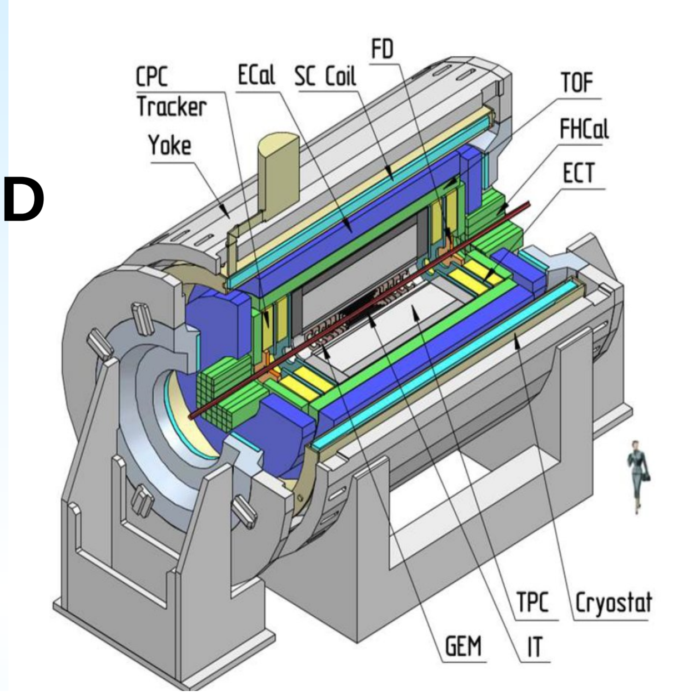
Many experimental results of high-energy physics are obtained by comparing the predictions of theoretical models simulated for a specific detector with the experimental distributions obtained. Track detectors in modern experiments provide basic information about charged particles born in collisions.

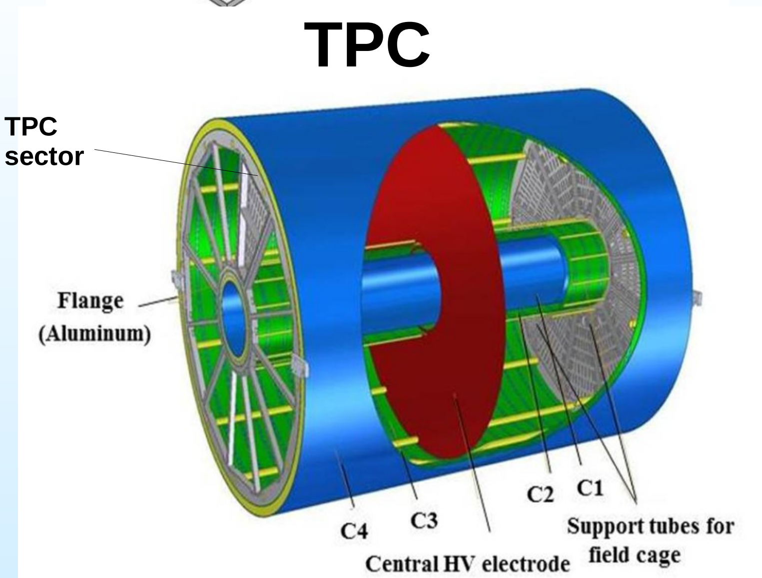
In order to achieve high resolution of particle momentum and reduce systematic errors, it is necessary to know the exact location of the detector parts. A partial solution to this problem can be achieved by creating special optical/laser systems for monitoring the position of detector parts during the experiment. The TPC has a laser system, but it is designed exclusively for monitoring the properties of the gas inside the TPC. We investigate the possibility of using it to adjust the sensitive elements of the track detector.

The ultimate alignment precision, however, is achieved by using the fitted tracks themselves.

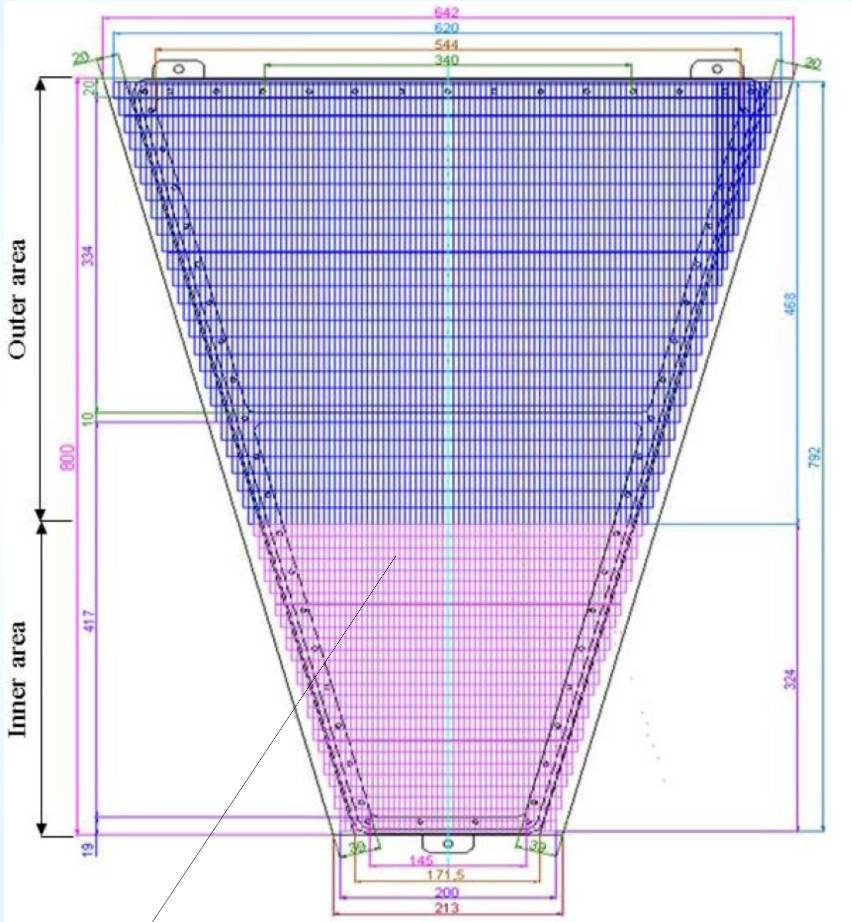
TPC alignment

MPD





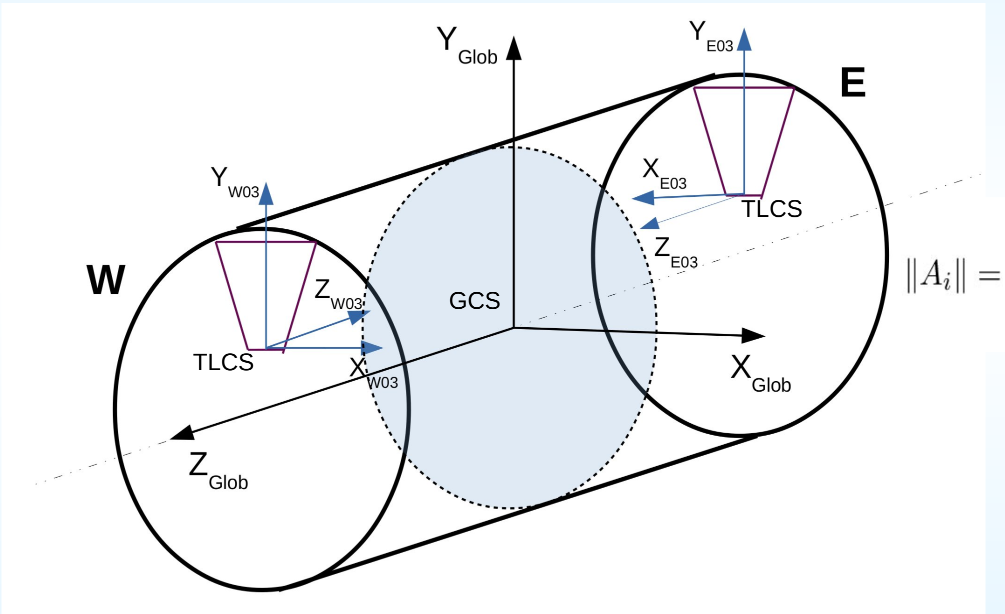
TPC sector



4074 sensitive elements that fix the projection of the track on the sector

TPC alignment

Global Coordinate System of the TPC (GCS), Theoretical Local Coordinate System of the sector (TLCS) and Local Coordinate System of the sector (LCS)



$$X_g = S_i^{tl} + \left\| T_i^{-1} \right\| X_{tl}$$ TLCS → GCS

$$X_{tl} = S_i^{A} + \left\| A_i^{-1} \right\| X_{l}$$ LCS → TLCS

S^{tl}, T – constants, $S^A(x_0, y_0, z_0)$, $A(\alpha, \beta, \gamma)$

$$\|A_i\| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma_i) & \sin(\gamma_i) \\ 0 & -\sin(\gamma_i) & \cos(\gamma_i) \end{pmatrix} \times \begin{pmatrix} \cos(\beta_i) & 0 & -\sin(\beta_i) \\ 0 & 1 & 0 \\ \sin(\beta_i) & 0 & \cos(\beta_i) \end{pmatrix} \times \begin{pmatrix} \cos(\alpha_i) & \sin(\alpha_i) & 0 \\ -\sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left\| R_i^{-1} \right\| = \left\| T_i^{-1} \right\| \left\| A_i^{-1} \right\|$$ LCS → TLCS

The position of sector *i* is determined by the 6 parameters p_{i1}, p_{i2}, p_{i3}, p_{i4}, p_{i5}, p_{i6}, which in the alignment problem are called global, and they need to be found for each sector.

Thus, the position of each track hit $h(p_i)$ is a function of 6 variables.

TPC alignment

The parameters of the charged particle are found by fitting the experimentally found hits of the track by its mathematical model

Line

$$\begin{pmatrix} x = q_1 + \sin(q_4)\cos(q_5)t \\ y = q_2 + \sin(q_4)\sin(q_5)t \\ z = q_3 + \cos(q_4)t \end{pmatrix}$$

Helix

$$\begin{pmatrix} x = q_1 + q_4\cos(q_5 + t) \\ y = q_2 + q_4\sin(q_5 + t) \\ z = q_3 + q_6 t \end{pmatrix}$$

$$\chi^2 = F(\overline{q}, \overline{p}) = \sum_{events}^{all} \sum_{i}^{track} \frac{(\overline{r}_i(\overline{p}_k) - T_i(\overline{q}))^2}{\sigma^2}$$

The minimum of F gives the true values of the global parameters

$$\begin{cases} \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}} + \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{q}^2}\triangle\boldsymbol{q} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial\boldsymbol{q}\partial\boldsymbol{p}}\triangle\boldsymbol{p} = 0 \\ \frac{\partial F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial\boldsymbol{q}\partial\boldsymbol{p}}\triangle\boldsymbol{q} + \frac{\partial^2 F(\boldsymbol{p}_0,\boldsymbol{q}_0)}{\partial \boldsymbol{p}^2}\triangle\boldsymbol{p} = 0 \end{cases}$$

$$\sigma^2 \frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{p}^2} = 2\left(\frac{\partial^2 \boldsymbol{h}}{\partial \boldsymbol{p}^2}(\boldsymbol{h}-\boldsymbol{T}) + \left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}\right)^2\right)$$

$$\sigma^2 \frac{\partial F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{p}} = \frac{\partial(\boldsymbol{h}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{q}))^2}{\partial \boldsymbol{p}} = 2\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}(\boldsymbol{h}-\boldsymbol{T})$$

$$\sigma^2 \frac{\partial F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{q}} = 2\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{q}}(\boldsymbol{T}-\boldsymbol{h})$$

$$\frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial \boldsymbol{q}^2} = 2\left(\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{q}^2}(\boldsymbol{T}-\boldsymbol{h}) + \frac{\partial \boldsymbol{T}}{\partial q_i}\frac{\partial \boldsymbol{T}}{\partial q_j}\right)$$

$$\sigma^2 \frac{\partial^2 F(\boldsymbol{p},\boldsymbol{q})}{\partial\boldsymbol{q}\partial\boldsymbol{p}} = -2\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{q}}\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}}$$

$$\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{p}} = \left\|T^{-1}\right\| \frac{\partial}{\partial \boldsymbol{p}}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} + \frac{\partial\left\|R^{-1}\right\|}{\partial \boldsymbol{p}}\boldsymbol{h}_s = \begin{pmatrix} T^{-1}_{11} & T^{-1}_{12} & T^{-1}_{13} & \sum_j \frac{\partial R^{-1}_{1j}}{\partial p_4}h_{s_j} & \sum_j \frac{\partial R^{-1}_{1j}}{\partial p_5}h_{s_j} & \sum_j \frac{\partial R^{-1}_{1j}}{\partial p_6}h_{s_j} \\ T^{-1}_{21} & T^{-1}_{22} & T^{-1}_{23} & \sum_j \frac{\partial R^{-1}_{2j}}{\partial p_4}h_{s_j} & \sum_j \frac{\partial R^{-1}_{2j}}{\partial p_5}h_{s_j} & \sum_j \frac{\partial R^{-1}_{2j}}{\partial p_6}h_{s_j} \\ T^{-1}_{31} & T^{-1}_{32} & T^{-1}_{33} & \sum_j \frac{\partial R^{-1}_{3j}}{\partial p_4}h_{s_j} & \sum_j \frac{\partial R^{-1}_{3j}}{\partial p_5}h_{s_j} & \sum_j \frac{\partial R^{-1}_{3j}}{\partial p_6}h_{s_j} \end{pmatrix}$$

line

$$\frac{\partial \boldsymbol{T}(\boldsymbol{q})}{\partial \boldsymbol{q}} = \begin{pmatrix} 1 & 0 & 0 & \cos(q_4)\cos(q_5)t & -\sin(q_4)\sin(q_5)t \\ 0 & 1 & 0 & \cos(q_4)\sin(q_5)t & \sin(q_4)\cos(q_5)t \\ 0 & 0 & 1 & -\sin(q_4)t & 0 \end{pmatrix}$$

helix

$$\left\|\frac{\partial \boldsymbol{T}(\boldsymbol{q})}{\partial \boldsymbol{q}}\right\| = \begin{pmatrix} 1 & 0 & 0 & \cos(q_5+t) & -q_4\sin(q_5+t) & 0 \\ 0 & 1 & 0 & \sin(q_5+t) & q_4\cos(q_5+t) & 0 \\ 0 & 0 & 1 & 0 & 0 & t \end{pmatrix}$$

A detailed description of the theoretical basis of the alignment task for MPD TPC is here: ***http://mpdroot-forum.jinr.ru/download/file.php?id=1***

Mini Monte-Carlo for TPC

MSU

To study the accuracy of measuring the alignment of the device, the simplified simulation of the TPC MPD reaction was used:

1.A charged particle leaves a d_{track} width (8mm*) strip on the surface of the sector.

2. The center of the strip corresponds to the projection of the track along the electric field on the plane of the sector.

3.The amplitude of the pad signal is proportional to the area of its coverage by the band and the final value is played according to Gaussian.

4.The time of arrival of the signal is proportional to the distance along the electric field from the particle to the plane of the sector.It is played according to Gaussian.

5.Adjacent pads of the same row with a signal above the threshold form a cluster, the local coordinates of which are the weighted sum of the coordinates of individual pads. According to these coordinates, the coordinates of the hit are calculated in the global coordinate system of the detector.

6. The global hits are fitted by the mathematical model of the track.

7. According to the results of the fit, function $F(\mathbf{p},\mathbf{q})$ and its derivatives are calculated.

Simulated variants:

1.Muons from the IP in the detector magnetic field.

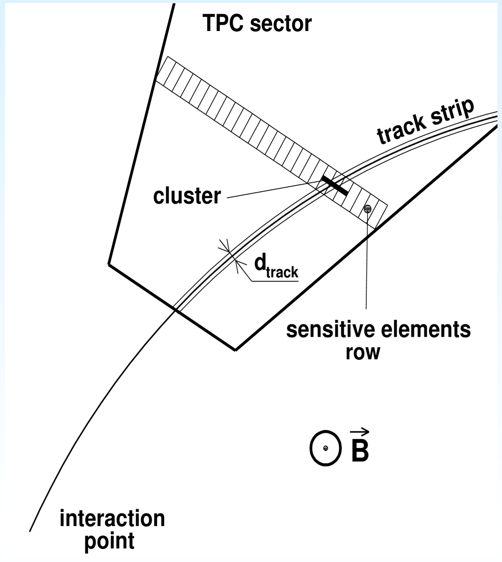
2. Cosmic rays without a magnetic field in the detector.

3. The beams of the TPC laser system.

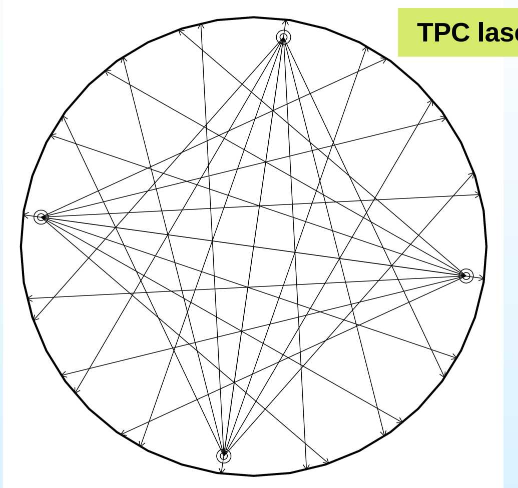
----

* V. Kolesnikov a , A. Mudrokh a , V. Vasendina and A. Zinchenko, «*Towards a Realistic Monte Carlo Simulation of the MPD Detector at NICA*», Physics of Particles and Nuclei Letters, 2019, Vol. 16, No. 1, pp. 6–15.

track simulation



TPC laser ystem.



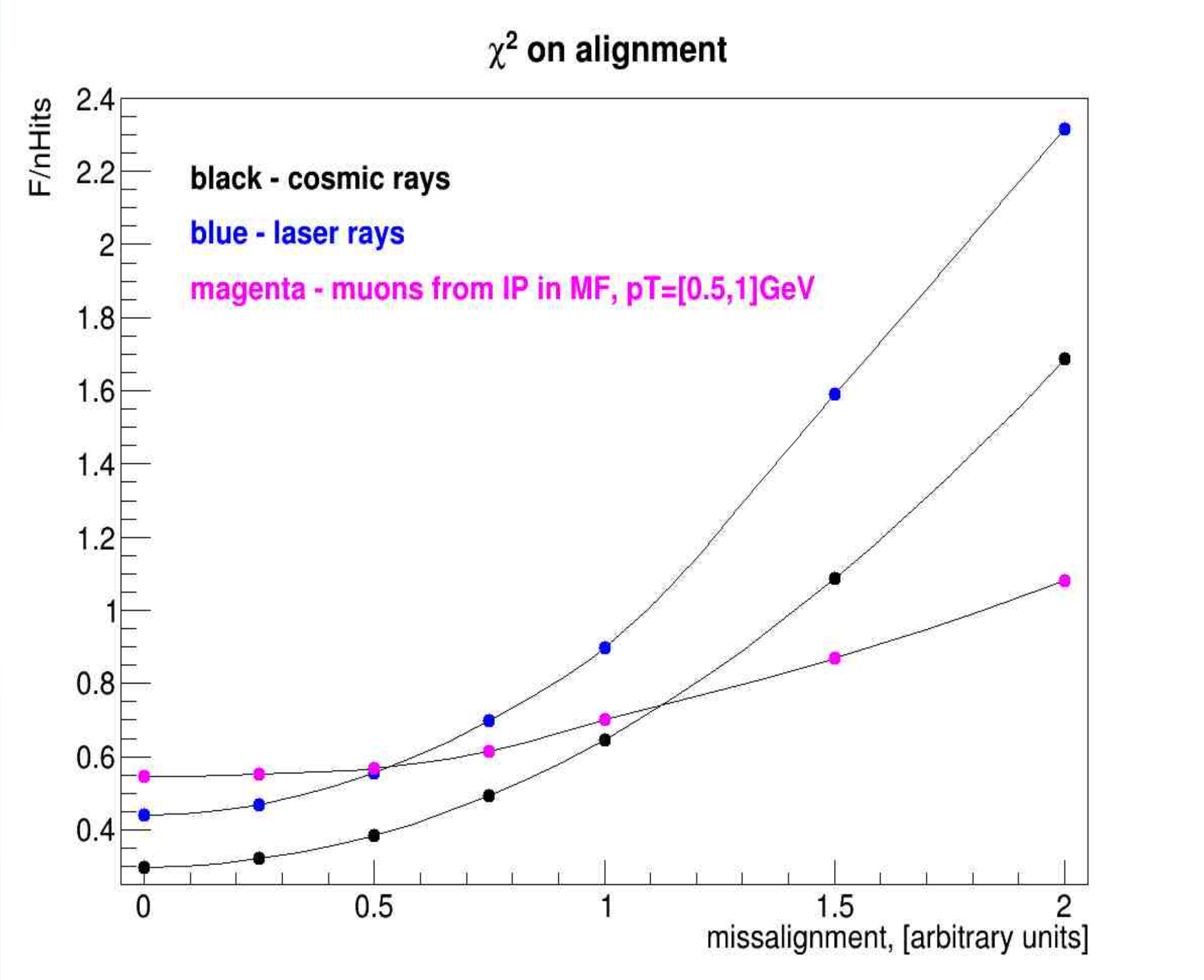
On 4 planes perpendicular to Z inside the TPC chamber from 4 points, there are 7 rays, the projections of which intersect all sectors.

χ^2 track simulation in TPC

MSU

What does 1 correspond to on the abscissa scale?
This is the next alignment:

1. The shifts of the centers of local coordinate systems relative to the theoretical ones are distributed uniformly randomly in the interval [-1,+1] cm, and random deviations of the Euler angles are in the interval [-1,+1] deg.
2. Artificial tracks will be simulated for the above alignment.
3. Reconstruction of the track parameters is performed for theoretical alignment of the detector with zero shifts and Euler angles, i.e. for incorrect alignment



The gradient for muons in a magnetic field is much smaller than for cosmic muons or beams of the TPC laser system

The accuracy of the alignment calculation by muons in the events from the collision of nuclei in the detector will be lower than in the case of cosmic rays or by the rays of the TPC laser system.

Alignment accuracy simulation

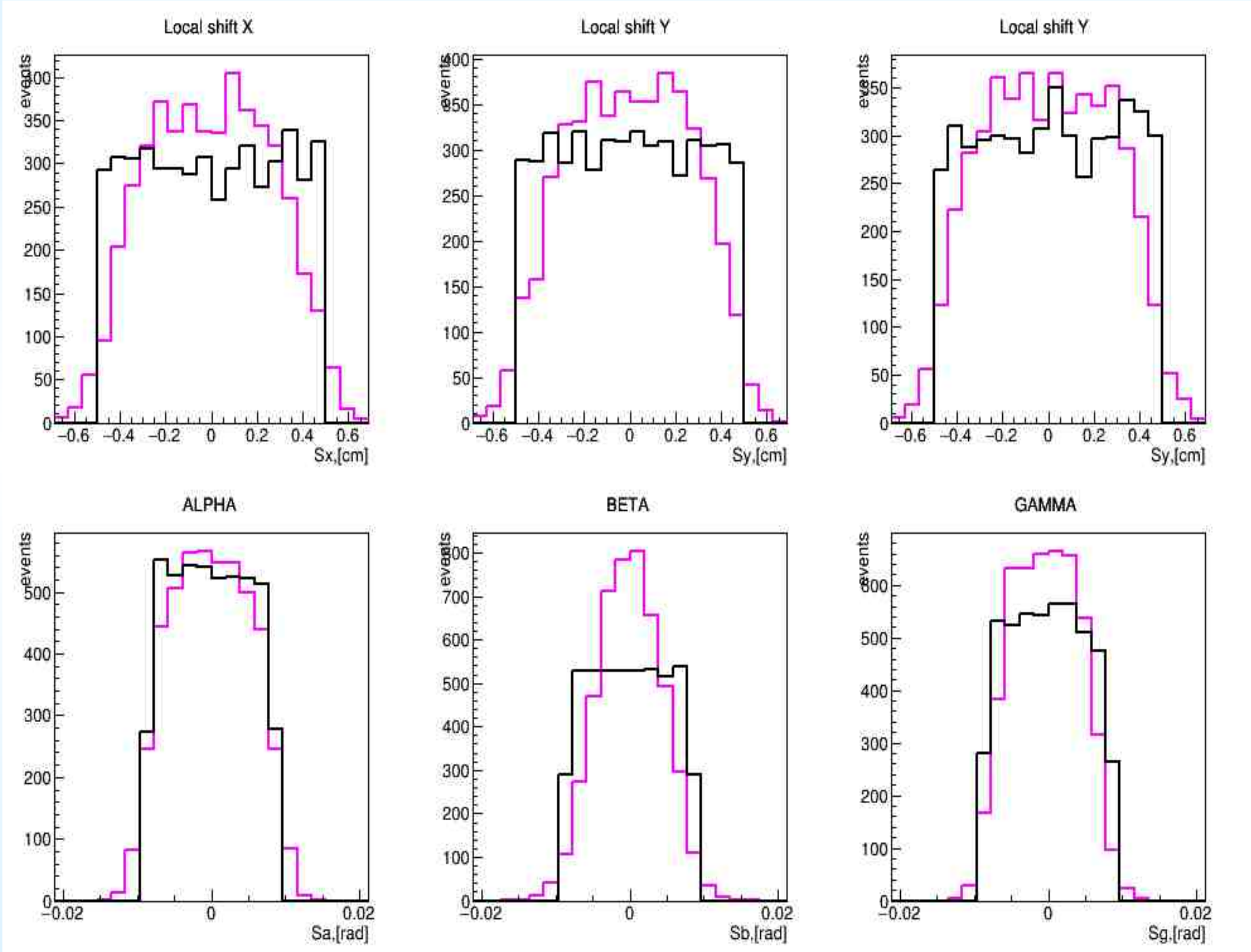
The model experiment consists in the simulation tracks for alignment with deviations from the zero alignment and reconstruction of tracks using zero alignment. Using this sample of tracks hits we find real alignment.

400 such model experiments were conducted for sets of 10,000 tracks from cosmic and experimental muons and the TPC laser system rays.

The black histograms show the distributions of the input global parameters.
The magenta histograms are the distributions of these values after
the adjustment procedure (the case of cosmic rays).

Discrepancies at the ends of the intervals due to the accuracy of determining the adjustment parameters.

Global parameters



cosmic rays case

χ^2 track simulation in TPC

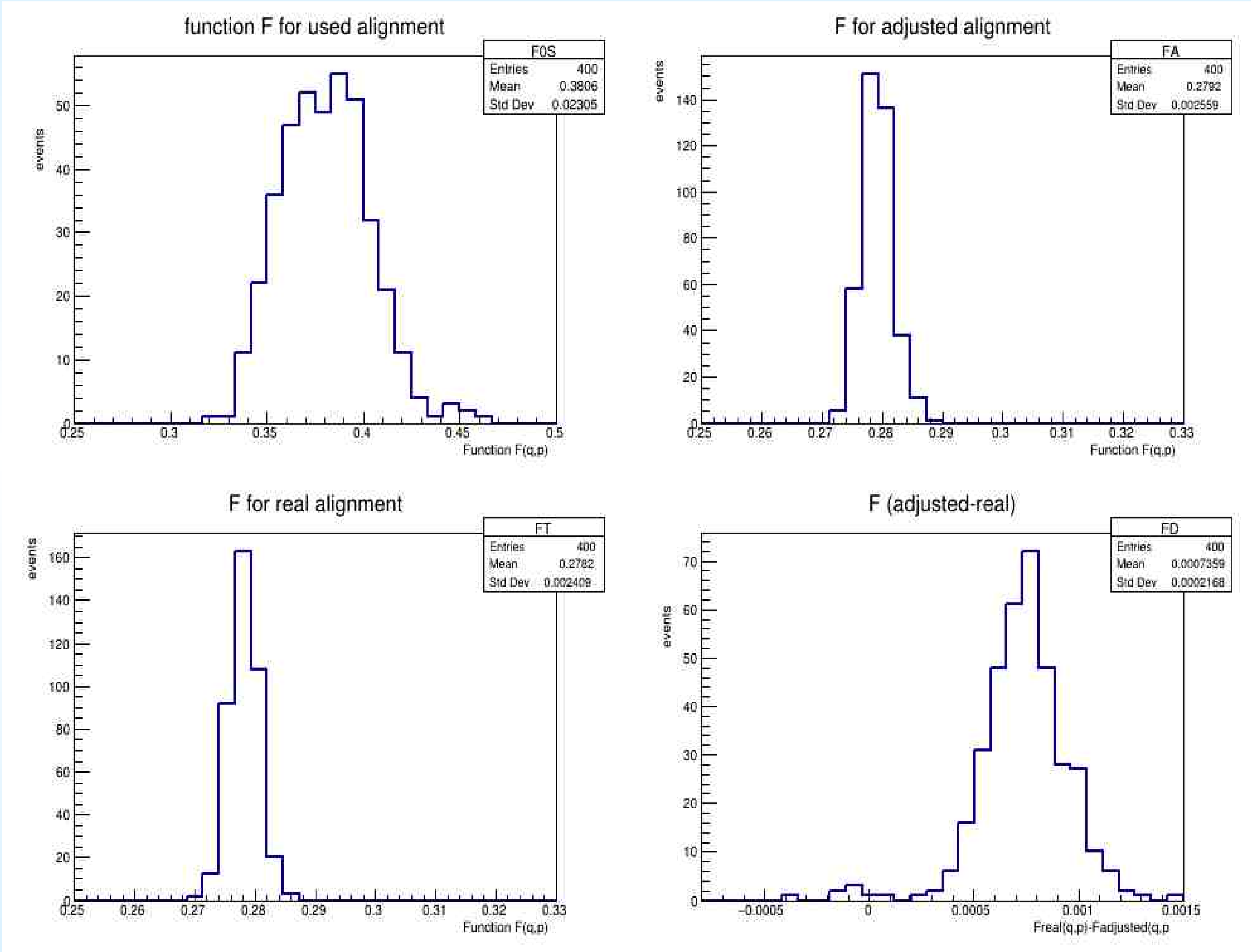
$$\chi^2 = F(p,q)/N_{hits}$$

"F for used alignment" is results of random alignment when for tracks reconstruction theoretical “zero” alignment was used.

"F for adjusted alignment" is final results of finding the alignment,
i.e. after minimizing the function F.

"F for real alignment" is simulated (not recovered) values
of the function F.

"F(adjucted - real)" is the difference between F-values of found and simulated alignments.



cosmic rays case

χ^2 track simulation in TPC

MSU

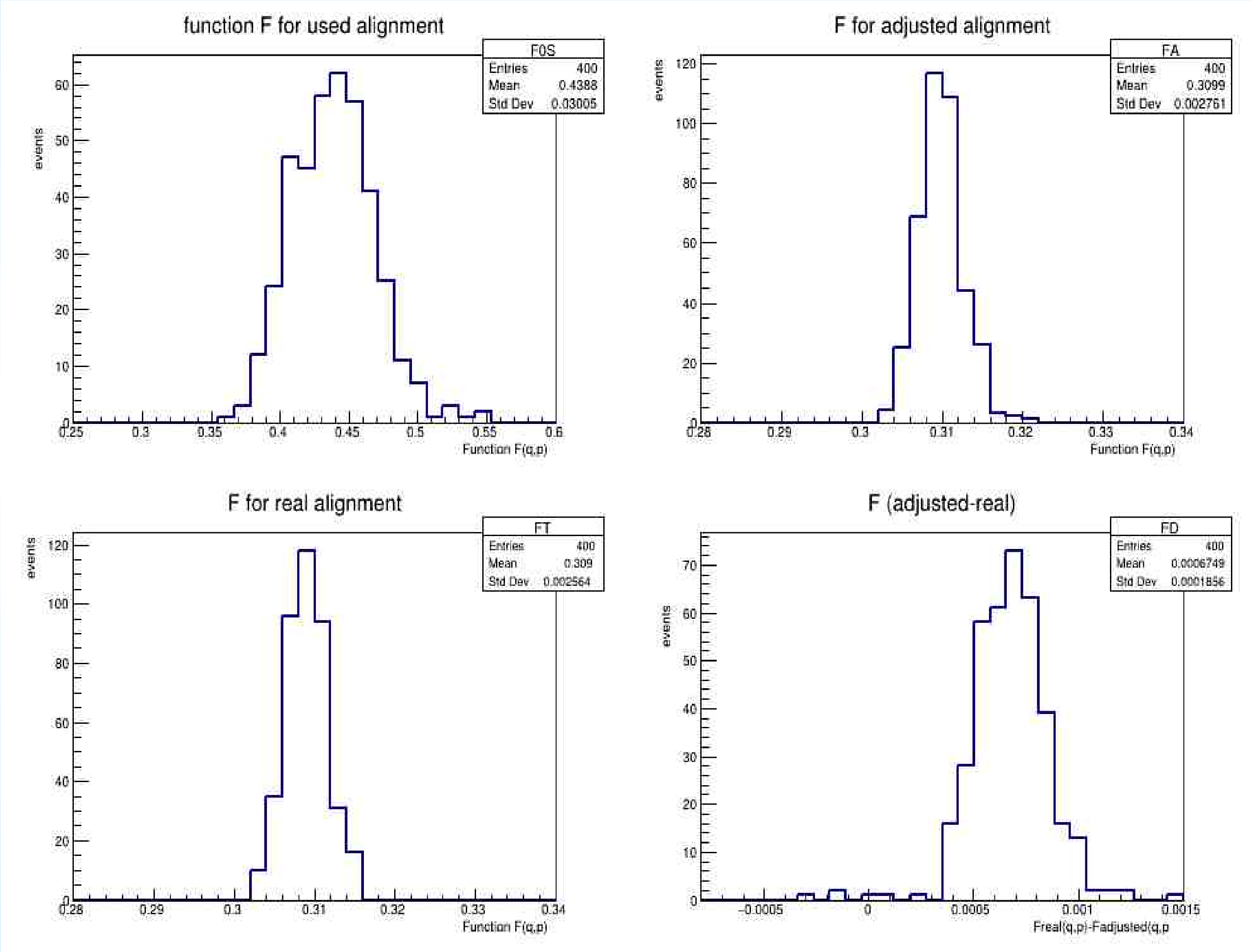
$$\chi^2 = F(p,q)/N_{hits}$$

If we compare these results with the distributions for cosmic muons we conclude:

1. Average χ^2 values are shifted a bit right.

2. The width of input and recovered distributions is very close.

3. The values of χ^2 for recovered alignment are systematically greater than the real values in both cases.



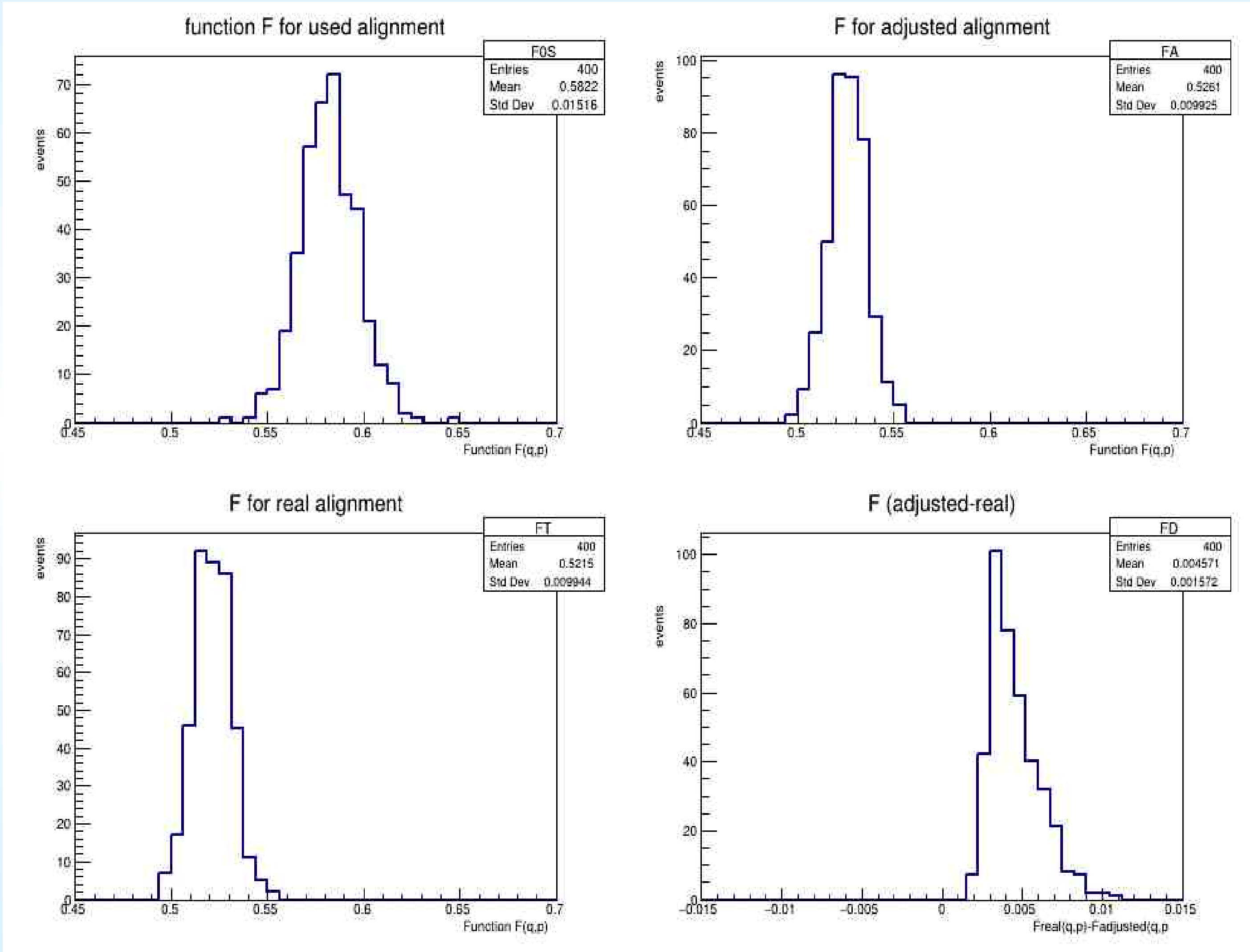
laser rays case

χ^2 track simulation in TPC

$$\chi^2 = F(p,q)/N_{hits}$$

The distributions for muons in a magnetic field have the same properties as the distributions in the two previous cases for straight tracks.

There is a significant difference in the width of the original and restored distributions, which is almost 4 times wider than for straight tracks



Muons in the magnetic field

Alignment accuracy

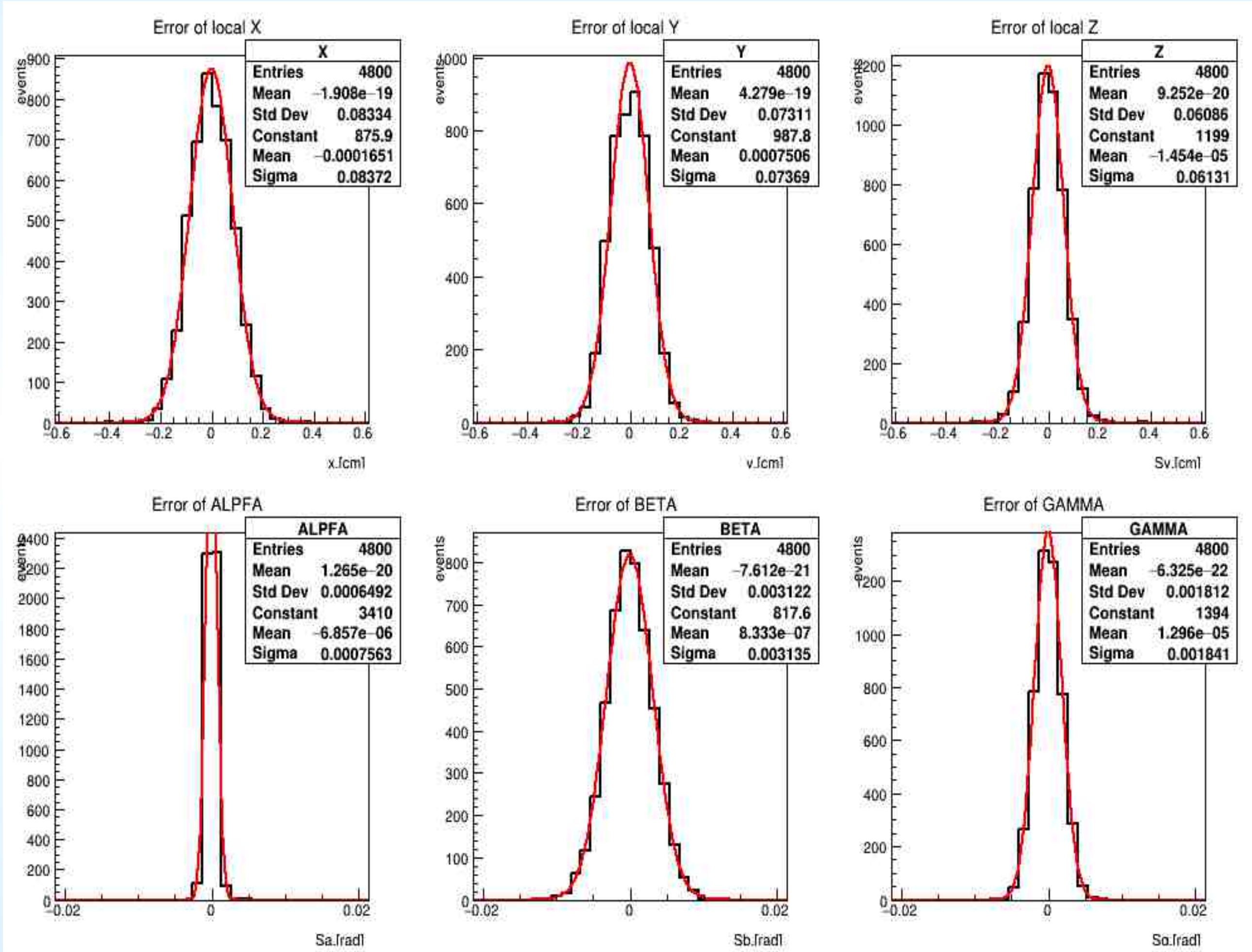
MSU

(simulated - adjusted) alignment parameters

The problem of finding the alignment is determined up to the simultaneous identical shift of the centers of local coordinate systems and simultaneous rotation of sectors around the global Z axis. In both cases, the minimum F does not change.

The nature of this condition lies in the equations for F, which do not change when the variable is shifted by a constant value.

In order to exclude the accumulation of this type of shifts during the process of minimizing, the average shifts of variables X,Y, Z and the average Euler angles were fixed.



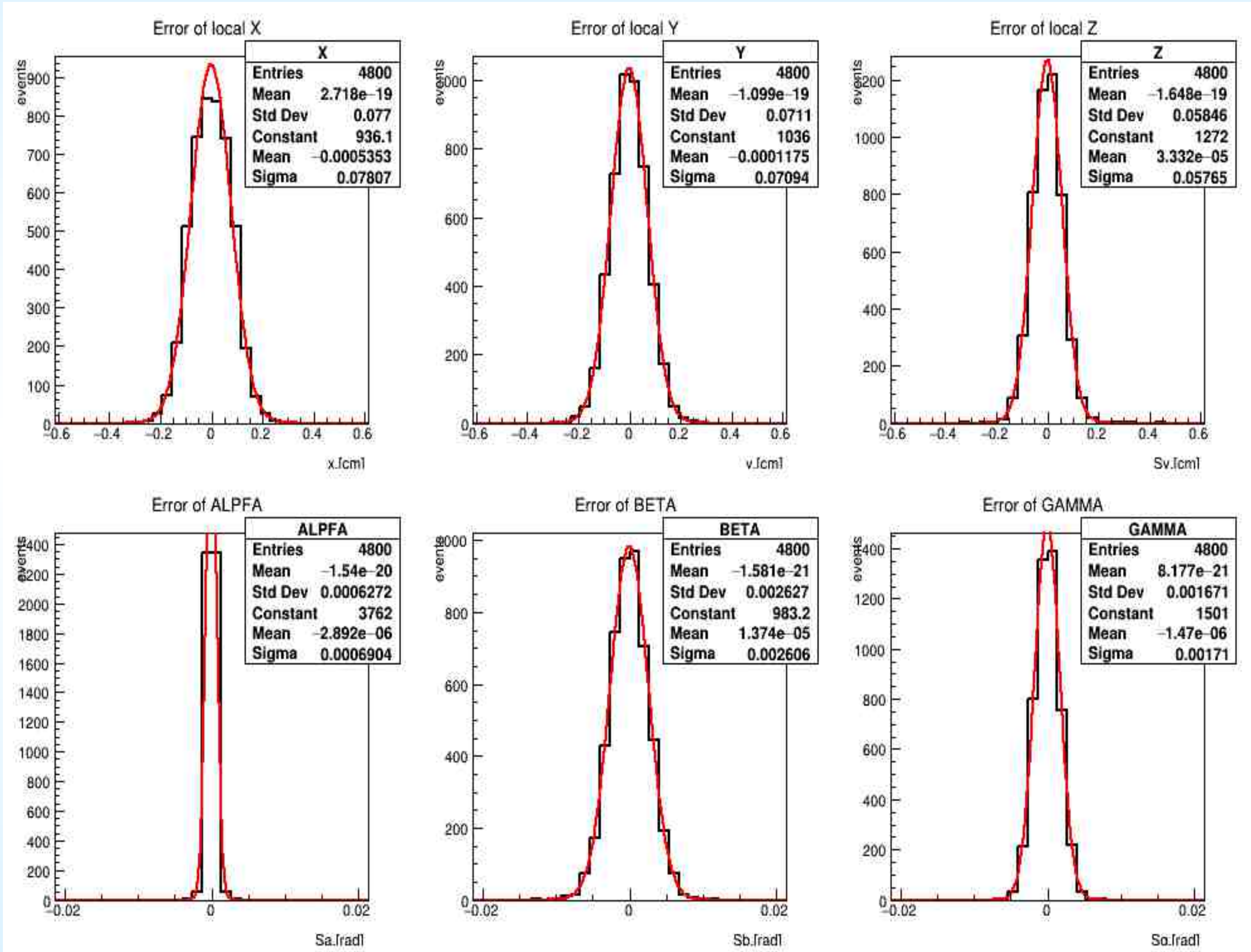
cosmic rays

Alignment accuracy

MSU

(simulated - adjusted) alignment parameters

The alignment accuracy by laser rays is very close to the results of cosmic rays and is about 750 microns for shifts and about 7 minutes for Euler angles BETA and GAMMA.
The accuracy of the ALPHA angle is 2 minutes because of the greater sensitivity of *F* on this angle.



laser rays

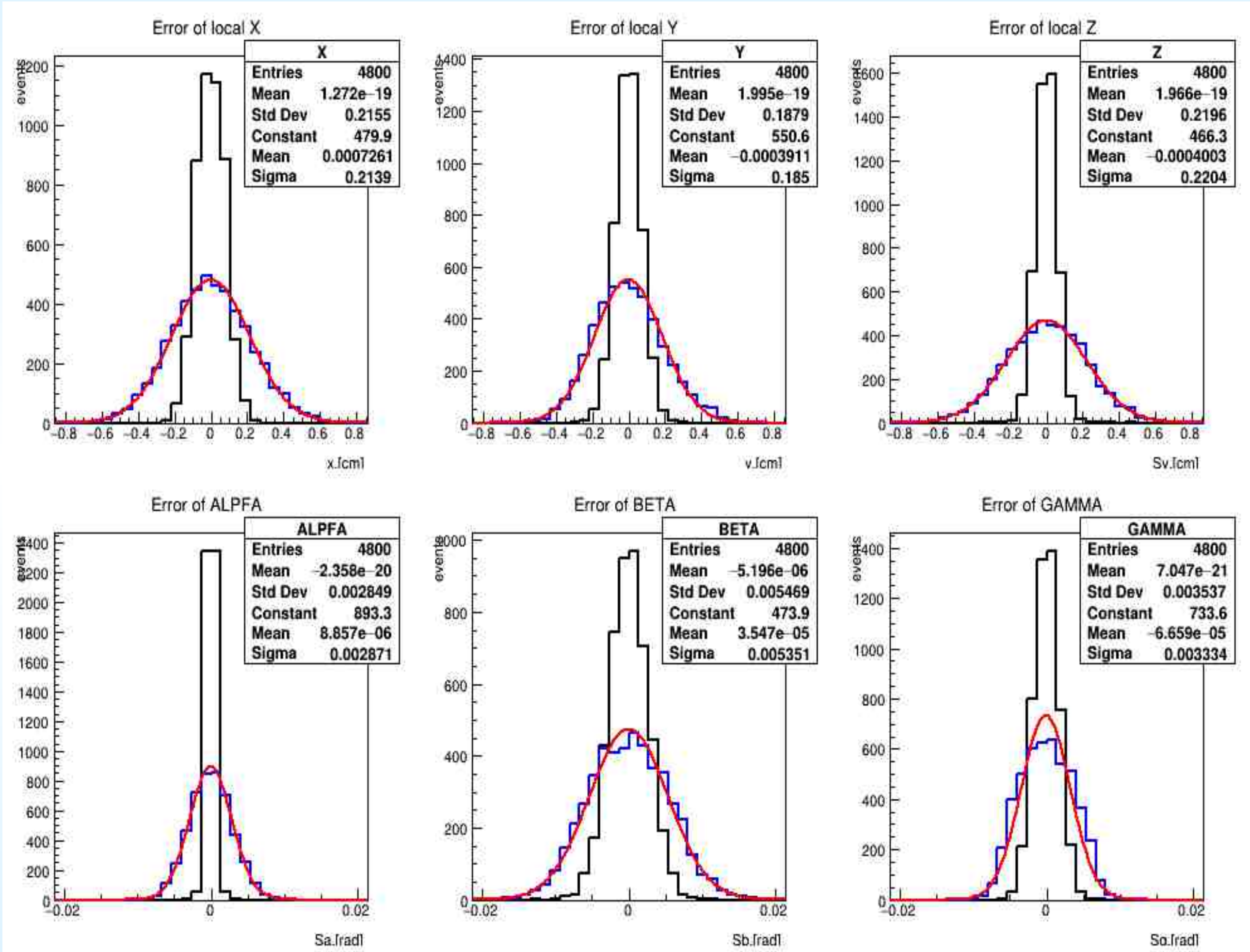
Alignment accuracy

MSU

The alignment accuracy by muons in a magnetic field produced in the IP (blue histograms) is several times worse than that by straight tracks.

For comparison, the results for laser beams are shown in black.

(simulated - adjusted) alignment parameters



Muons in the magnetic field & laser rays

Alignment accuracy

The equations for the minimum of *F* are not an exact condition. They are of first order relative increments of the global variables.

We should use an interactive procedure and when we have to stop it?

We used condition: $\varepsilon = \dfrac{F_i - F_{i+1}}{F_i} < 10^{-4}$

The smoothness of the function near the minimum depends on the magnitude of the track statistics. How many tracks do we need to use in order not to lose accuracy?

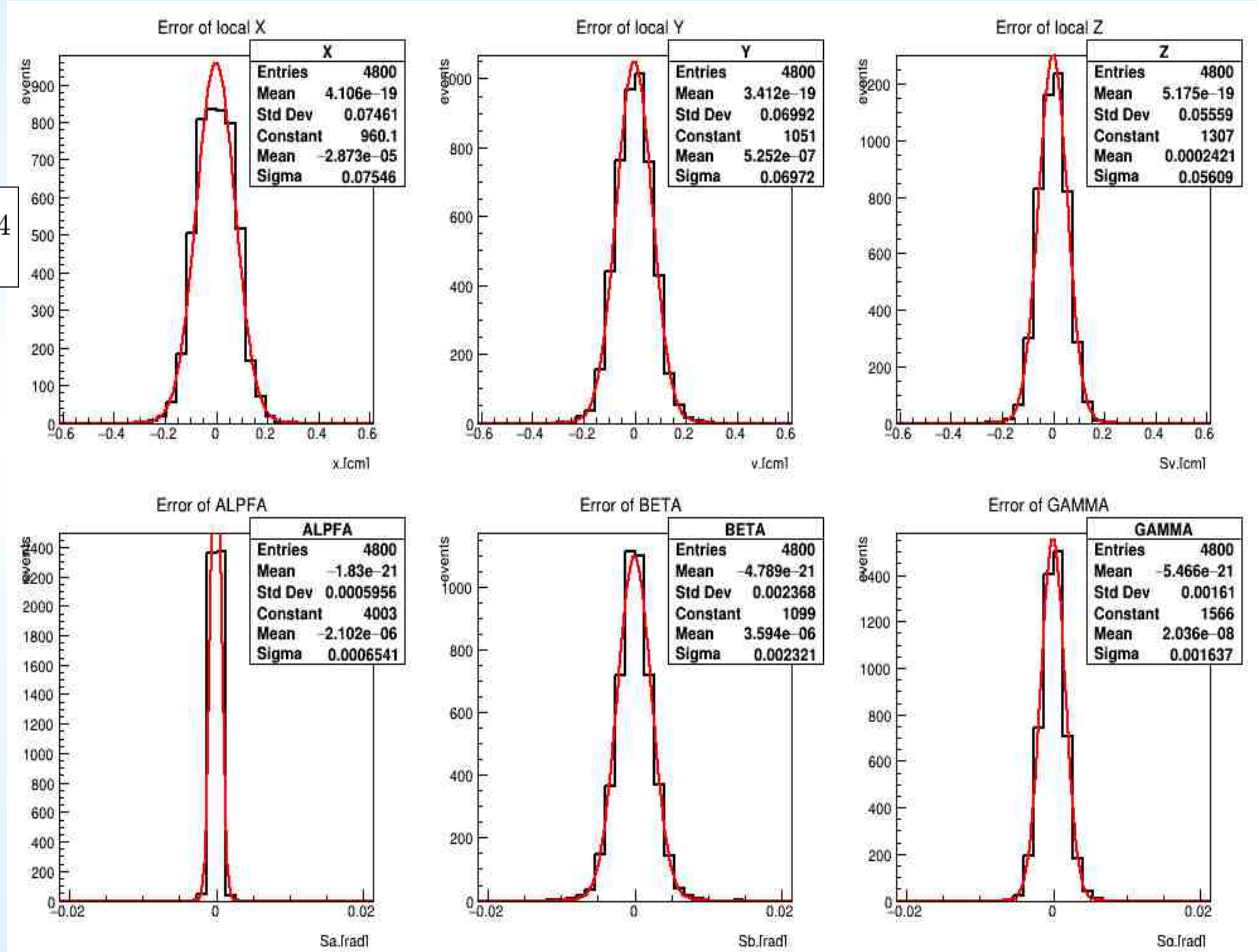
We used N_{traks}=10000

$\varepsilon=2\cdot10^{-5}$ & N_{traks}=50000 do not increase the accuracy of global parameters

It is enough to use:

$\varepsilon=10^{-4}$ & N_{traks}=10000

(simulated - adjusted) alignment parameters



laser rays (50000 tracks per experiment) & $\varepsilon=2\cdot10^{-5}$

Conclusions

- A theoretical basis has been created for determining the MPD TPC alignment using its experimental data.
- As part of the *mpdroot* computer system, a simplified simulation of the detector response with subsequent reconstruction of tracks in the detector was created to study the accuracy of finding the TPC alignment.
- The possibility of using the TPC laser system for detector alignment has been studied. This system was originally designed only to monitor the properties of the gas. It is shown that it can be used for permanent TPC alignment monitoring.
- The accuracy of finding the alignment for three types of events is investigated: cosmic rays without a magnetic field in the detector, laser system rays and muons in the magnetic field of the detector from the interaction point. The accuracy in the first two cases is approximately the same. It is ~700 microns for the shift of the origin of the sector and 7 angular minutes (0.0023rad) for Euler angles. The accuracy in the case of muons born in collisions of nuclei is several times worse.
- The proposed method of finding global alignment parameters can be applied to any track detector consisting of separate parts with sensitive elements rigidly fixed on them, for example, silicon vertex detectors, in particular the MPD Inner Tracking System.