Analytical fit of distribution function of neutrino in the supernova outer part

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Core-collapse supernova

- Supernova (SN) matter is opaque for neutrinos $\Rightarrow$ neutrino interaction with SN matter is an important ingredient of core-collapse supernova models
- Description of neutrino propagation in SNs is required a self-consistent solution of hydrodynamic and neutrino transport equations
- Boltzmann equation for non-equilibrium neutrino distribution function in SN matter is solved only numerically $\Rightarrow$ it makes difficulties for use of results obtained to other problems
- It is useful to find an analytical approximation of results of numerical simulation

Neutrino distribution function in SN

- Neutrino propagation in SN is close to spherically symmetric $\Rightarrow$ local non-equilibrium distribution function $\mathcal{F}_\nu$ of neutrinos can be approximated by function $f_\nu(r,\theta,\omega)$ which depends on there variables: r is distance from SN center, θ is angle between neutrino momentum and radial direction of SN, ω is neutrino energy
- Approximation $f_\nu(r,\theta,\omega)$ in SN conditions could be factorized as

$$f_\nu(r,\theta,\omega) \approx N(r)\ \Phi(r,\theta)\ F(r,\omega),$$

- Neutrino energy distribution $F(r,\omega)$ is well investigated [1, 2]
- Neutrino angular distribution $\varPhi(r,\theta)$ can be approximated by simple one-parametric Gaussian-like function [3]

Used numerical data

In our analysis we use of the PROMETHEUS-VERTEX code [4],which represent the result of self-consistent solution of 1D hydrodynamic and neutrino transport equations.

Proposed analytical fit

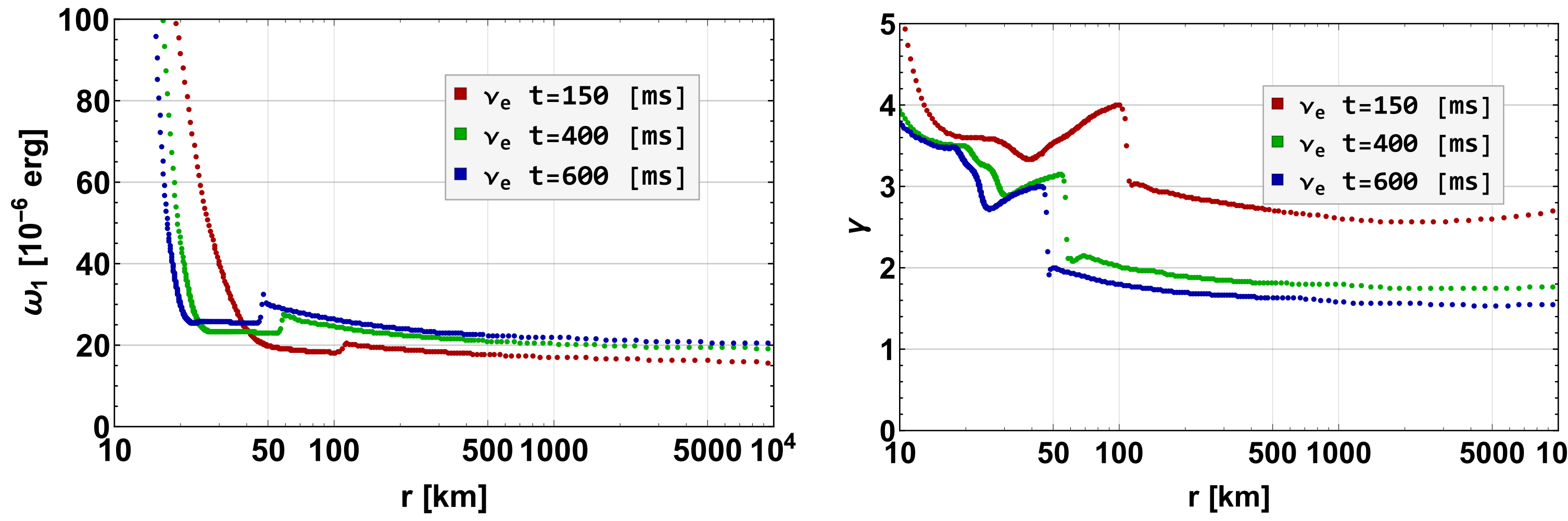
We consider analytical approximation of f_ν, which based on following angular $\varPhi(r,\theta)$ [3] and energy $F(r,\omega)$ [2] neutrino distribution:

$$f_\nu(r,y,\omega) = N\ \exp\left(-\frac{y}{\sqrt{2}\,\sigma}\right)^2 \left(\frac{\omega}{\omega_1}\right)^{\gamma-3} \exp\left(-\frac{\gamma\,\omega}{\omega_1}\right),$$

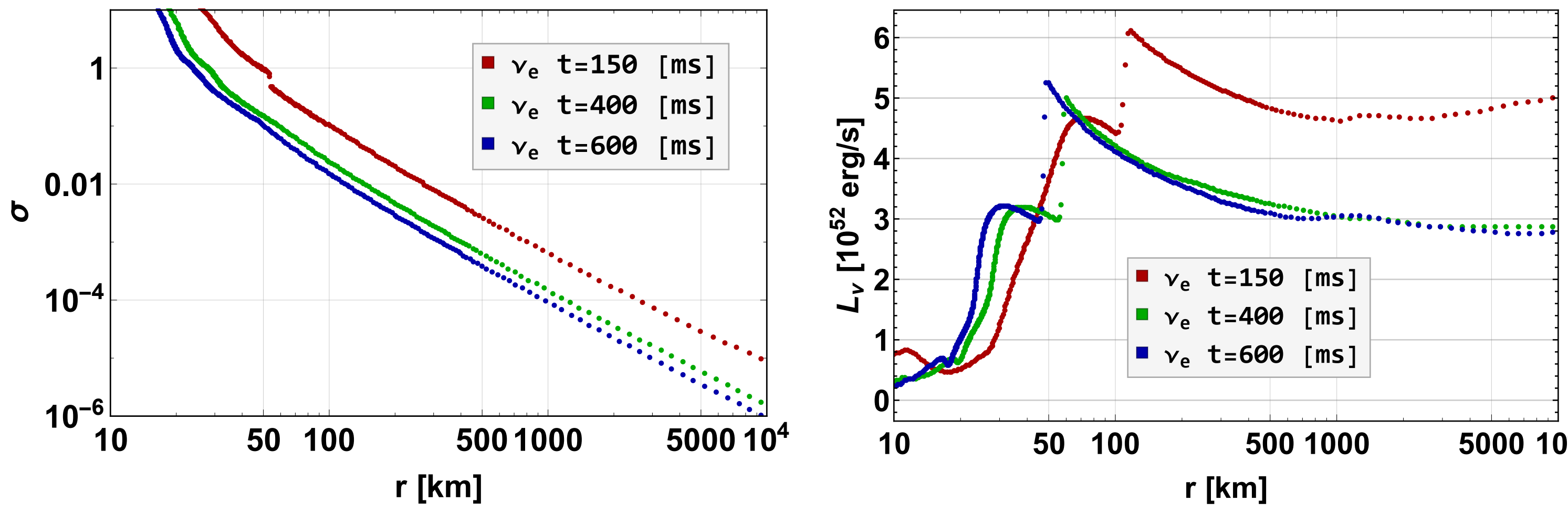
where $y = 1-\cos\theta$, σ describes the width of angular neutrino distribution, ω_1 is the neutrino mean energy and γ represents the amount of spectral "pinching". All parameters N, σ, ω_1 and γ are defining approximation of f_ν. These parameters were obtained from numerical data as functions of distant r and time after bounce t.

Analysis of parameters in the supernova outer part

The analysis of the spectral approximation parameters shows that ω_1 and γ are practically independent on the distance in the supernova outer part.



Behavior of the angular distribution parameter in the SN outer part shows that it obeys the quadratic law with good accuracy: $\sigma(r) \simeq (R/r)^2$, where R is proportional of protoneutron star radius.



Remaining normalized coefficient N could be expressed as

$$N \approx \sqrt{2\pi}\, c^2\, \hbar^3 \frac{\gamma^\gamma}{\Gamma(\gamma)}\, \frac{L_\nu}{R^2\,\omega_1^4},$$

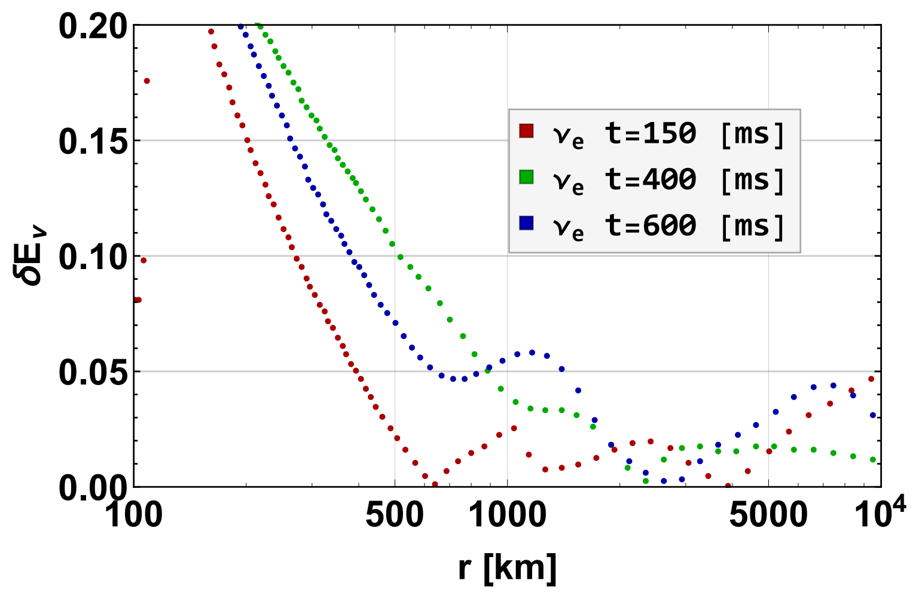
where we take into account that $\sigma(r) \ll 1$ in outer part of SN. Here $L_\nu(r)$ is a local neutrino luminosity. Note, that this quantity is approximately constant in considered SN region. So, the dependence of neutrino distribution function on distance can be presented explicitly:

$$f_\nu(r,y,\omega) \approx \sqrt{2\pi}\, \frac{c^2\,\hbar^3\,\widetilde{\gamma}^{\widetilde{\gamma}}\,\widetilde{L_\nu}}{\Gamma(\widetilde{\gamma})\,R^2\,\widetilde{\omega_1}^{\,4}}\ \exp\left(-\frac{y\,r^2}{\sqrt{2}\,R^2}\right)^2 \left(\frac{\omega}{\widetilde{\omega_1}}\right)^{\widetilde{\gamma}-3} \exp\left(-\frac{\widetilde{\gamma}\,\omega}{\widetilde{\omega_1}}\right),$$

where $\widetilde{\omega_1}$, $\widetilde{\gamma}$ and $\widetilde{L_\nu}$ are the values of the corresponding quantities at large distances r, where they become equal to constants.

Verification of the proposed analytical fit

The accuracy of the proposed analytical fit is checked using the neutrino energy density. We represent value of $\delta E_\nu = |E_\nu - E_\nu^{(d)}|/E_\nu^{(d)}$, where E_ν is the neutrino energy density obtained from the fit and $E_\nu^{(d)}$ is the neutrino energy density calculated using the numerical data. We check the accuracy of our fit at fixed values of parameters:



$\widetilde{\omega_1}^{(150)} = 1.64\times10^{-5}$ ergs, $\widetilde{\gamma}^{(150)} = 2.59$,
$\widetilde{L_\nu}^{(150)} = 4.76 \times 10^{52}$ ergs/s;
$\widetilde{\omega_1}^{(400)} = 1.97\times10^{-5}$ ergs, $\widetilde{\gamma}^{(400)} = 1.76$,
$\widetilde{L_\nu}^{(400)} = 2.92 \times 10^{52}$ ergs/s;
$\widetilde{\omega_1}^{(600)} = 2.10\times10^{-5}$ ergs, $\widetilde{\gamma}^{(600)} = 1.55$,
$\widetilde{L_\nu}^{(600)} = 2.88 \times 10^{52}$ ergs/s

and $R^{(150)} = 25.39$ km, $R^{(400)} = 12.07$ km, $R^{(600)} = 9.53$ km. Similar results were obtained for other neutrino flavors.

Conclusion

A simple analytical approximation of the nonequilibrium neutrino distribution function is presented, which is applicable for the outer part of the supernova. It is shown that it can be represented as an explicit function of the distance from the SN center. In this case, it is determined by 4 time-dependent parameters. The average neutrino energy and neutrino luminosity weakly depend on time, and the main influence on the distribution function is exerted by the parameters R and γ, which are determinated the width of angular and spectral distribution. It is shown that the accuracy of the proposed approximation in the outer part of the SN doesn't exceed $5-10$ % in dependence of value of time after a bounce.

References

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