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Measurement of angular coefficients in $W \rightarrow \ell \nu$ events in low- μ pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector

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This note presents a measurement of angular coefficients and differential cross section as functions of transverse momentum and rapidity of W bosons. The measurement is performed in the $W \rightarrow e \nu$ and $W \rightarrow \mu \nu$ decay channels. The analysis uses the low pileup dataset collected with the ATLAS detector at 13 TeV in Run 2. The measurement is performed in the full solid angle and in the region of the W boson, defined as $|y_W| < 3.6$.

16 **ChangeLog for version 0.3 → 0.4 in CDS (XX/02/2023)**

- 17 • Answered to open CDS comments from Lorenzo.
- 18 • Added yields table to the Section [6](#).
- 19 • Update MJ shape estimation formula (Eq. [49](#)).
- 20 • Added sliced 2D control plots to the Appendix [F](#).

21 To do list:

- 22 • Update all control and MJ plots with new results obtained using new MJ shape estimation Eq. [49](#).
- 23 • Improve acceptance correction method by use of the coarse bins instead of inclusive SR.
- 24 • MJ acc. correction functions and MJ systematics breakdown plots.

25 **ChangeLog for version 0.0 → 0.3 in CDS (30/09/2022)**

- 26 • Answers to open CDS comments from Camilla and Frank.
- 27 • Polishing and restructuring text.

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97 1 Introduction

98 In this note we present the measurement of the polar and azimuthal distributions of the $\ell\nu$ pairs produced
99 from W decays in the centre-of-mass frame of the W boson using low- μ data collected from pp collisions
100 at $\sqrt{s} = 13$ TeV in the ATLAS detector at the Large Hadron Collider.

101 A detailed understanding of the Drell-Yan production mechanism for gauge bosons $p_1 + p_2 \rightarrow V + X$,
 102 where p_1 and p_2 are the incoming partons and $V = \{W, Z\}$, can be obtained by measuring the angular
 103 distributions of the leptonic decay products $\ell^\pm + \nu/\ell^\mp$ of a gauge boson. This is done by defining an event
 104 plane with beam and the gauge boson momentum direction as a reference frame, which provides detailed
 105 information of the lepton-hadronic correlation effects using the leptons produced by the decay of the gauge
 106 bosons. These correlations are described using nine hadronic “structure” functions and can be calculated
 107 using perturbative QCD for the parton model. A more detailed description of the theoretical formalism can
 108 be found in [1–4].

109 Without lepton selection requirements the angular distributions are determined by the vector boson
 110 polarization. However, the angular distributions become dominated by selection requirements when they
 111 are applied. These polarization effects can be recovered by comparing data to the ratio of MC distributions
 112 to those obtained using isotropic gauge boson decays. With enough data statistics a two-dimensional fit on
 113 the angular distributions can be used to extract the eight coefficients that describe them as discussed in
 114 Section 1.1.

115 The most recent measurement of these angular coefficients was performed by the LHCb Collaboration
 116 with $\sqrt{s} = 13$ TeV data for $Z \rightarrow \mu\mu$ events and extracted four of the eight coefficients as a function of p_T^Z
 117 [5]. All eight coefficients were measured as a function of p_T^Z and y^Z by the ATLAS Collaboration with
 118 $\sqrt{s} = 8$ TeV data for the Z boson using $Z \rightarrow \ell\ell$ where $\ell = \{e, \mu\}$ events [6] with the use one coefficient they
 119 were able to extract the weak mixing angle, $\sin^2 \theta_W$ [7]. Other measurements of the Z boson coefficients
 120 have also been done previously by the CMS Collaboration who extracted five of the eight coefficients as a
 121 function of p_T^Z at $\sqrt{s} = 8$ TeV with just $Z \rightarrow \mu\mu$ events [8] and four of the eight coefficients as a function
 122 of p_T^Z by the CDF Collaboration at $\sqrt{s} = 1.96$ TeV with $Z \rightarrow ee$ events [9].

123 Only two of these angular coefficients for the W boson as a function of p_T^W have been measured previously
 124 by CDF Collaboration at $\sqrt{s} = 1.96$ TeV with $W^\pm \rightarrow \ell^\pm \nu$ events [10]. However, other measurements
 125 of the W boson polarization have measured previously by the ATLAS Collaboration with $\sqrt{s} = 7$ TeV
 126 data [11] and by the CMS Collaboration with $\sqrt{s} = 7, 13$ TeV data [12, 13]. These polarization fraction
 127 measurements are related to two coefficients and can be used to probe their values.

128 The angular coefficients for the W boson are important for modeling QCD production and have impact
 129 on other measurements like the W boson mass. Currently the modeling of these coefficients is done by
 130 extrapolating from the coefficients of the Z boson which increases the uncertainty on m^W measurements.
 131 The ATLAS Collaboration measurement of m^W at $\sqrt{s} = 7$ TeV reported an uncertainty from the modeling
 132 of the angular coefficients of $\delta p_T^\ell = 5.8$ MeV and $\delta m_T = 5.3$ MeV on these distributions used to extract m^W
 133 [14]. The LHCb Collaboration reported a systematic uncertainty of $\delta m^W = 10$ MeV from the modeling on
 134 their measurement of m^W at $\sqrt{s} = 13$ TeV [15]. One of the criticisms of the most recent measurement of
 135 m^W by the CDF Collaboration at $\sqrt{s} = 1.96$ TeV is that they didn’t include an uncertainty relating to the
 136 modeling of these angular coefficients [16].

137 1.1 Differential Cross-section

138 The differential cross-section, at all orders of QCD for non-zero transverse momentum, for the process
 139 $p_1 + p_2 \rightarrow W + X \rightarrow \ell\nu + X$ has a general form as seen in Equation 1. Where p_T^W , m^W , and y^W are the
 140 transverse momentum, mass, and rapidity in the laboratory frame and θ and ϕ are the polar and azimuthal
 141 angles of the leptons in the W boson rest frame. Rapidity is calculated using $y = \frac{1}{2} \ln [(E + p_z)/(E - p_z)]$
 142 where E is the energy of the W boson and p_z is the momentum along the z -axis in the coordinate system.

143 Equation 1 is a result of the W boson having spin 1, and the lepton-neutrino pair having spin 1/2 and is
 144 further discussed in [1, 3].

$$\begin{aligned} \frac{d\sigma}{dp_T^W dm^W dy^W d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^W dm^W dy^W} & \left\{ \left(1 + \cos^2\theta\right) + \frac{1}{2}A_0 \left(1 - 3\cos^2\theta\right) \right. \\ & + A_1 \sin 2\theta \cos\phi + \frac{1}{2}A_2 \sin^2\theta \cos 2\phi \\ & + A_3 \sin\theta \cos\phi + A_4 \cos\theta + A_5 \sin^2\theta \sin 2\phi \\ & \left. + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi \right\}. \end{aligned} \quad (1)$$

145 The differential unpolarized cross section, denoted σ^{U+L} , has been factored out resulting in the coefficients
 146 $A_0 - A_7$ which are ratios of the helicity dependent cross-sections relative to the unpolarized cross-section.
 147 These coefficients are dependent on the W boson's kinematics and vanish when p_T^W is zero, except for A_4
 148 which is present for all orders of QCD and is what generates the forward-backward asymmetry in $\cos\theta$.
 149 Integrating Equation 1 over the full lepton kinematic phase space, $\cos\theta$ and ϕ , results in the A_i coefficients
 150 vanishing which results in the triple differential unpolarized cross-section:

$$\int_1^1 d\cos\theta \int_0^{2\pi} d\phi \frac{d\sigma}{dp_T^W dm^W dy^W d\cos\theta d\phi} = \frac{d\sigma^{U+L}}{dp_T^W dm^W dy^W} \quad (2)$$

151 Integrating Equation 1 over ϕ results in:

$$\frac{d\sigma}{dp_T^W dm^W dy^W d\cos\theta} = \frac{3}{8} \frac{d\sigma^{U+L}}{dp_T^W dm^W dy^W} \left\{ \left(1 + \cos^2\theta\right) + \frac{1}{2}A_0 \left(1 - 3\cos^2\theta\right) + A_4 \cos\theta \right\} \quad (3)$$

152 Integrating Equation 1 over $\cos\theta$ results in:

$$\frac{d\sigma}{dp_T^W dm^W dy^W d\phi} = \frac{1}{2\pi} \frac{d\sigma^{U+L}}{dp_T^W dm^W dy^W} \left\{ 1 + \frac{1}{4}A_2 \cos 2\phi + \frac{3\pi}{16}A_3 \cos\phi + \frac{1}{2}A_5 \sin 2\phi + \frac{3\pi}{16}A_7 \sin\phi \right\} \quad (4)$$

153 Integrating over either angle results in the loss of information of A_1 and A_6 , which both require the full
 154 two-dimensional angular phase space.

155 1.2 Collins-Soper Frame

156 The Drell-Yan angular coefficients are reference frame dependent because they are dependent on the W
 157 boson kinematics. For this analysis the Collins-Soper (CS) frame [17] is chosen for direct comparison
 158 to the coefficients of the Z boson. The CS frame is defined using the rest frame of the W boson and the
 159 direction of the protons in the W boson rest frame. To define the z-axis the momentum of one proton and
 160 the inverse of the momentum of the second is used to create a bisecting angle, α as seen in Figure 1, and
 161 the sign is chosen to be in the same direction as the W boson's momentum in the laboratory frame. The
 162 y-axis is then chosen to be normal to the proton-proton plane and the x-axis is chosen to be orthogonal to
 163 complete a right-handed Cartesian coordinate system.

164 The direction of the leptons in the CS frame are defined by two angles, θ_{CS} and ϕ_{CS} . The angle θ_{CS} is
 165 defined as the angle between the momentum of the negatively charged lepton and the z-axis. In the case of
 166 W^+ there is no negatively charged lepton and instead the direction of the neutrino is used. The formula used
 167 to calculate $\cos \theta_{CS}$ can be expressed using the kinematics of the leptons in the laboratory frame [18]

$$\cos \theta_{CS} = \frac{p_z^{\ell\nu}}{|p_z^{\ell\nu}|} \frac{2}{m^W \sqrt{m^W{}^2 + p_T^{\ell\nu 2}}} (P_1^+ P_2^- - P_1^- P_2^+), \quad (5)$$

168 with

$$P_i^\pm = \frac{1}{\sqrt{2}} (E_i \pm p_{z,i}). \quad (6)$$

169 E_i and $p_{z,i}$ are the energy and longitudinal momentum of the lepton, respectively, such that $i = 1$ for the
 170 lepton and $i = 2$ for the anti-lepton. $p_z^{\ell\nu}$ is the longitudinal momentum of the dilepton system. The angle
 171 ϕ_{CS} is chosen to be the angle between the lepton plane and the xz-plane and must be calculated in the CS
 172 frame.

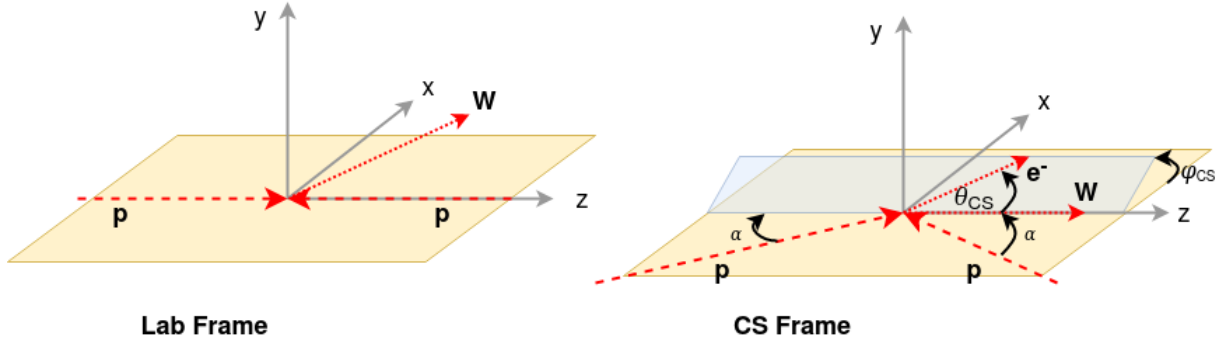


Figure 1: On the left is the lab frame with protons coming from opposite directions resulting in the W boson, in an arbitrary direction. On the right is the Collin-Soper frame.

173 1.3 Moments Method

174 A method to extract the coefficients from generated Monte Carlo events was proposed in [2] and was used
 175 in the analysis [6]. It is possible to isolate each coefficient while still remaining as functions of p_T^W and y^W .
 176 This is done by taking moments of the distributions with respect the appropriate product of trigonometric
 177 polynomials where the moment can be defined as

$$\langle P_i(\theta, \phi) \rangle = \frac{\int d\sigma(p_T, m, y, \theta, \phi) P_i(\theta, \phi) d \cos \theta d\phi}{\int d\sigma(p_T, m, y, \theta, \phi) d \cos \theta d\phi}. \quad (7)$$

178 For each coefficient this results in

$$\begin{aligned}
\langle \frac{1}{2} (1 - 3 \cos^2 \theta) \rangle &= \frac{3}{20} \left(A_0 - \frac{2}{3} \right); & \langle \sin 2\theta \cos \phi \rangle &= \frac{1}{5} A_1; \\
\langle \sin^2 \theta \cos 2\phi \rangle &= \frac{1}{10} A_2; & \langle \sin \theta \cos \phi \rangle &= \frac{1}{4} A_3; \\
\langle \cos \theta \rangle &= \frac{1}{4} A_4; & \langle \sin^2 \theta \sin 2\phi \rangle &= \frac{1}{5} A_5; \\
\langle \sin 2\theta \sin \phi \rangle &= \frac{1}{5} A_6; & \langle \sin \theta \sin \phi \rangle &= \frac{1}{4} A_7.
\end{aligned} \tag{8}$$

179 The underlying mechanic of taking the moment to isolate each coefficient is that it exploits the orthogonality
180 of the spherical harmonics that the trigonometric polynomials are related to as in Table 1. The definition of
181 the moment in Equation 7 requires the integration of the full lepton phase space for the orthogonality to
182 hold so it can only be applied to MC samples.

A_i	P_i	$Y_l^m(\theta, \phi)$	Coupling	Non-Zero
A_0	$1/2 (1 - 3 \cos^2 \theta)$	Y_2^0		$O(\alpha_S^1)$
A_1	$\sin 2\theta \cos \phi$	$(Y_2^{-1} - Y_2^1)$	$(v_\ell^2 + a_\ell^2) (v_q^2 + a_q^2)$	$O(\alpha_S^1)$
A_2	$1/2 \sin^2 \theta \cos 2\phi$	$(Y_2^{-2} + Y_2^2)$		$O(\alpha_S^1)$
A_3	$\sin \theta \cos \phi$	$(Y_1^{-1} - Y_1^1)$	$v_\ell a_\ell v_q a_q$	$O(\alpha_S^1)$
A_4	$\cos \theta$	Y_1^0		$O(\alpha_S^0)$
A_5	$\sin^2 \theta \sin 2\phi$	$(Y_2^{-2} - Y_2^2)$	$(v_\ell^2 + a_\ell^2) (v_q a_q)$	$O(\alpha_S^2)$
A_6	$\sin 2\theta \sin \phi$	$(Y_2^{-1} + Y_2^1)$		$O(\alpha_S^2)$
A_7	$\sin \theta \sin \phi$	$(Y_1^{-1} + Y_1^1)$	$(v_\ell a_\ell) (v_q^2 + a_q^2)$	$O(\alpha_S^2)$

Table 1: Summary of Drell-Yan angular coefficients, polynomials coupled to each coefficient, how the polynomials relate to the spherical harmonics, the vector and axial-vector couplings, and order at which they become non-zero.

183 1.4 Measurement Goals

184 The goal of this analysis is to measure four (A_0 , A_2 , A_3 , and A_4) of the eight of the coefficients in
185 Equation 1 as well as the differential cross-section. The coefficients A_1 and $A_5 - A_7$ are out of the scope of
186 this measurement due the statistical sensitivity required as found in the sensitivity studies described in
187 Appendix A. These coefficients will still be extracted at the same time but are expected to give inconclusive
188 results. The coefficients are to be measured as a function of p_T^W and independently as function of $-y^W$ —
189 for W^- and W^+ in both an $e^- \bar{\nu}$ ($e^+ \nu$) and $\mu^- \bar{\nu}$ ($\mu^+ \nu$) channel.

190 2 Theoretical Predictions

191 *This chapter will contain the various theoretical predictions to be compared against once they are compiled*
 192 *and the theoretical formalism. We plan to compare the measured differential distributions to fixed order*
 193 *predictions up to order α_S^3 in the perturbative expansion of the strong-coupling constant [19], which*
 194 *is next-to-next-to-leading order (NNLO) for the transverse-momentum distribution and next-to-next-to-*
 195 *next-to-leading order (N3LO) for the rapidity distribution. We also plan to include comparisons of the*
 196 *differential distributions with POWHEG+PYTHIA8 and DYTurbo.*

197 2.1 Theoretical Formalism

198 The lepton angular distribution in the gauge boson rest frame is determined by the gauge boson polarization.
 199 The general structure of the angular distribution is given by nine helicity cross-sections corresponding
 200 to the nine helicity density matrix elements for the gauge boson [1]. In the LO parton subprocesses
 201 ($q\bar{q} \rightarrow V \rightarrow \ell\ell'$) only the transverse polarization gauge boson contributes to the helicity density matrix.
 202 At higher order, in general, all three polarizations of the gauge boson contribute. At $\mathcal{O}(\alpha_S)$ in perturbative
 203 QCD the angular distribution is described by six helicity cross-sections, which are functions of the
 204 transverse momentum and rapidity of the gauge boson. At the $\mathcal{O}(\alpha_S^2)$ contribution from additional three
 205 helicity cross-sections is non-zero. Following convention and notations of [2], let us briefly recall that the
 206 lepton-hadron correlations are described by the contraction of the lepton tensor $L_{\mu\nu}$ with the hadron tensor
 207 at the parton level $H^{\mu\nu}$, where $L_{\mu\nu}$ acts as an analyzer of the structure of $H^{\mu\nu}$ which carries the effective
 208 information on the polarization of the gauge boson produced in the interaction. The angular dependence
 209 can be extracted introducing helicity cross-sections corresponding to the non-zero combinations of the
 210 polarization density matrix elements

$$H_{mm'} = \epsilon_\mu^*(m) H^{\mu\nu} \epsilon_\nu(m') \quad (9)$$

211 where $m, m' = +1, 0, -1$ and

$$\epsilon_\mu(\pm 1) = \frac{1}{\sqrt{2}} (0, \pm 1, -i, 0), \quad \epsilon_\mu(0) = \frac{1}{\sqrt{2}} (0, 0, 0, 1) \quad (10)$$

212 are the polarization vectors for the gauge boson defined with respect to the chosen gauge boson rest frame.
 213 The angular dependence of the differential cross-section can be written as

$$\frac{d\sigma}{dp_T^W dm^W dy^W d\cos\theta d\phi} = \frac{3}{16\pi} \sum_{\alpha \in M} g_\alpha(\theta, \phi) \frac{d\sigma^\alpha}{dp_T^W dm^W dy^W}, \quad M = \{U+L, L, T, I, P, A, 7, 8, 9\} \quad (11)$$

214 where the g_α represent the second-order harmonic polynomials multiplied by normalization constants,
 215 $\cos\theta$ and ϕ are the polar and azimuthal decay angles of the leptons in the gauge boson reference frame,
 216 and $d\sigma^\alpha$ denote helicity cross sections corresponding to nine helicity matrix elements.

$$\begin{aligned}
\sigma^{U+L} &\sim H_{00} + H_{++} + H_{--} \\
\sigma^L &\sim H_{00} \\
\sigma^T &\sim 1/2 (H_{+-} + H_{-+}) \\
\sigma^I &\sim 1/4 (H_{+0} + H_{0+} - H_{-0} - H_{0-}) \\
\sigma^P &\sim H_{++} + H_{--} \\
\sigma^A &\sim 1/4 (H_{+0} + H_{0+} + H_{-0} + H_{0-}) \\
\sigma^7 &\sim -i/2 (H_{+-} - H_{-+}) \\
\sigma^8 &\sim -i/4 (H_{+0} - H_{0+} + H_{-0} - H_{0-}) \\
\sigma^9 &\sim -i/4 (H_{+0} - H_{0+} - H_{-0} + H_{0-})
\end{aligned} \tag{12}$$

217 The unpolarized differential cross section is denoted by σ^{U+L} , where $\sigma^{L,T,I,P,A}$ characterize polarization of
218 the gauge boson, respectively cross-section for longitudinally polarized gauge boson, transverse interference
219 cross-section, transverse-longitudinal interference cross-section, etc. [1]. The cross-sections are related to
220 the vector v_x and axial-vector a_x coupling coefficients of $x = \{\ell, q\}$ to the W boson in the following way
221 [1]:

$$\begin{aligned}
\sigma^{U+L,L,T,I} &\sim (v_\ell^2 + a_\ell^2) (v_q^2 + a_q^2). \\
\sigma^{P,A} &\sim v_\ell a_\ell v_q a_q. \\
\sigma^{7,8} &\sim (v_\ell^2 + a_\ell^2) (v_q a_q). \\
\sigma^9 &\sim (v_\ell a_\ell) (v_q^2 + a_q^2).
\end{aligned} \tag{13}$$

222 The $\sigma^{U+L,L,T,I,9}$ cross-sections receive contributions from the parity conserving (p.c.) part of the hadron
223 tensors, the remaining four $\sigma^{P,A,7,8}$ are proportional to the parity violating (p.v.) part of $H^{\mu\nu}$, i.e.
224 they change sign under parity (P) transformation. Since the angular polynomials $g_{P,A,9}(\theta, \phi)$ change sign
225 too, angular distributions involving $\sigma^{U+L,L,T,I,P,A}$ are parity conserving.

226 The standard polynomials, $g_\alpha(\theta, \phi)$, are:

$$\begin{aligned}
g_{U+L} &= 1 + \cos^2 \theta; & g_L &= 1 - 3 \cos^2 \theta; & g_T &= 2 \sin^2 \theta \cos 2\phi; \\
g_I &= 2\sqrt{2} \sin 2\theta \cos \phi; & g_P &= 2 \cos \theta; & g_A &= 4\sqrt{2} \sin \theta \cos \phi; \\
g_7 &= 2 \sin^2 \theta \sin 2\phi; & g_8 &= 2\sqrt{2} \sin 2\theta \sin \phi; & g_9 &= 4\sqrt{2} \sin \theta \sin \phi.
\end{aligned} \tag{14}$$

227 The helicity cross sections $d\sigma^\alpha$ are obtained by convoluting the partonic cross sections of partons with
228 momenta $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$

$$\frac{d\sigma^\alpha}{dp_1^W dm^W dy^W} = \sigma_{a,b} \int x_1 dx_2 f_1^{h1}(x_1, \mu_F^2) f_2^{h2}(x_2, \mu_F^2) s \frac{d\hat{\sigma}_{ab}^\alpha}{dt du}(p_1, p_2, \alpha_S(\mu_R^2)) \tag{15}$$

229 where μ_F and μ_R denote respectively factorization and renormalization scale. The standard notation is then
230 to factorize out unpolarized cross-section and present formula of Equation 11 as expansion into standard
231 polynomials and dimensionless angular coefficients representing ratios of helicity cross-sections.

$$\begin{aligned}
A_0 &= 2d\sigma^L/d\sigma^{U+L}; & A_1 &= 2\sqrt{2}d\sigma^I/d\sigma^{U+L}; & A_2 &= 4d\sigma^T/d\sigma^{U+L}; \\
A_3 &= 4\sqrt{2}d\sigma^A/d\sigma^{U+L}; & A_4 &= 2d\sigma^P/d\sigma^{U+L}; & A_5 &= 2d\sigma^7/d\sigma^{U+L}; \\
A_6 &= 2\sqrt{2}d\sigma^8/d\sigma^{U+L}; & A_7 &= 4\sqrt{2}d\sigma^9/d\sigma^{U+L}.
\end{aligned} \tag{16}$$

232 This conveniently rewrites the equation into the form of Equation 1 or more compactly as

$$\frac{d\sigma}{dp_T^W dm^W dy^W d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^W dm^W dy^W} \left\{ \left(1 + \cos^2\theta\right) + \sum_{i=0}^7 A_i \left(p_T^W, y^W, m^W\right) P_i(\cos\theta, \phi) \right\}. \tag{17}$$

233 In the limit of zero transverse momenta all coefficients except A_4 vanish. Another important thing to note
234 is that because of the (p_T^W, y^W) dependence on the A_i coefficients depend strongly on the choice of the
235 z-axis. The coefficients $A_0, A_2,$ and A_3 are increasing functions of p_T^W and the deviations from lowest order
236 expectation ($A_0 = A_2 = 0$) are quite large at modest value of p_T^W , i.e. $p_T^W = 20 - 50$ GeV. These coefficients
237 are exactly equal at LO [20] but is not true at NLO where corrections produce deviations of less than 10%
238 [2]. The deviation of A_1 from zero is much smaller even at large p_T^W . A_4 contributes the most at small p_T^W
239 and decreases with as p_T^W increases. A_3 and A_4 contribute more for the W boson compared to the Z boson
240 due to the fact that A_3 and A_4 are proportional to $v_\ell a_\ell v_q a_q$ which means that in the case of the Z boson
241 they are proportional to the weak mixing angle and are suppressed.

242 The theoretical uncertainties due to the choice of the factorization and renormalization scales are very
243 small for the cross section ratios A_i . Also, most of the uncertainties of the structure functions and of the
244 choice of the factorization scheme cancels in the ratio.

245 3 Measurement Methodology

246 This chapter will cover the measurement methodology using a log-likelihood to extract the angular
 247 coefficients. A more detailed description can also be found in the internal note [21]. The methodology
 248 often expresses the measurement in terms of p_T^W but the same methodology is used for extracting the
 249 coefficients in y^W .

250 3.1 Likelihood Method

251 A likelihood is built from the nominal templates and the varied templates reflecting the systematic
 252 uncertainties. A set of nuisance parameters (NPs) $\theta = \{\beta, \gamma\}$ is used to interpolate between them which
 253 are constrained by auxiliary probability density functions.

254 The first category, β , are the NPs representing experimental and theoretical uncertainties. Each β^m in
 255 the set $\beta = \{\beta^1, \dots, \beta^M\}$ are constrained by unit Gaussian probability density functions $G(0|\beta^m, 1)$ and
 256 interpolate between nominal and varied templates. They are defined by having a nominal value of zero and
 257 $\beta^m = \pm 1$ corresponding to $\pm 1\sigma$ for the considered systematic uncertainty.

258 The second category, γ , are the NPs that handle systematic uncertainties from the limited size of the MC
 259 samples. For each $\cos \theta_{CS}, \phi_{CS}$ reco bin n there is a γ^n in the set $\gamma = \{\gamma^1, \dots, \gamma^{N_{\text{bins}}}\}$, where $N_{\text{bins}} = 8 \times 11$
 260 is the number of $\cos \theta_{CS}, \phi_{CS}$ bins in each reco kinematic (p_T^W, y^W) bin. γ^n has a nominal value of one
 261 and normalizes the expected events in bin n of the templates. They are constrained by Poisson probability
 262 density functions, $P(N_{\text{eff}}^n | \gamma^n N_{\text{eff}}^n)$, where N_{eff}^n is the effective number of MC events in bin n . Effective is
 263 used because of the non-uniform event weights used for to properly scale the MC to account for things like
 264 detector inefficiencies. The expected number of events in each bin n can be written as:

$$N_{\text{exp}}^n(A, \sigma, \theta) = \left\{ \sum_{j=0}^{N_{\text{bins}}^{\text{ana}}} \sigma_j \times L \left[T_{8,j}^n(\beta) + \sum_{i=0}^7 A_{ij} \times T_{ij}^n(\beta) \right] \right\} \times \gamma^n + \sum_B^{\text{bkgds}} T_B^n(\beta) \quad (18)$$

265 where:

- 266 • A_{ij} : Coefficient parameter A_i for observation bin j
- 267 • A : Set of all A_{ij}
- 268 • σ_j : Signal cross-section parameter
- 269 • σ : Set of all σ_j
- 270 • θ : Set of all NPs
- 271 • L : Integrated luminosity constant
- 272 • β : Set of all Gaussian-constrained NPs
- 273 • γ^n : Poisson-constrained NP
- 274 • T_{ij} : P_i template
- 275 • T_B : Background templates.

276 Summation over index j takes into account the contribution for all truth kinematic bins at generator levels
 277 in each reconstructed kinematic bin to account for migration between bins as explained in Section 7.1. Far
 278 away bins have very little contribution and are pruned if the expected number of events is smaller than
 279 10^{-4} in order to improve CPU and memory requirements.

280 The likelihood is the product of Poisson probabilities across all N_{bins} and of auxiliary constraints for each
 281 NP β_m :

$$\mathcal{L}(A, \sigma, \theta | N_{\text{obs}}) = \prod_n^{N_{\text{bins}}} \left\{ P(N_{\text{obs}} | N_{\text{exp}}^n(A, \sigma, \theta)) P(N_{\text{eff}}^n | \gamma^n N_{\text{eff}}^n) \right\} \times \prod_m^M G(0 | \beta^m, 1) \quad (19)$$

282 3.2 Template Folding

283 We define a set of observables in truth space t with underlying function $f(t)$. Different detector effects
 284 like limited acceptance, reconstruction efficiency, and resolution will smear our observables t to have
 285 reconstructed values r with distribution $g(r)$. The probability to observe a reconstructed r given truth
 286 values t is expressed as $p(r|t)$. This probability can be convoluted with the truth distribution to get the
 287 reconstructed distribution:

$$g(r) = \int f(t) p(r|t) dt \quad (20)$$

288 Another way to express this is that the truth distribution $f(t)$ is folded through the convolution to produce
 289 $g(r)$.

290 Likewise, the polynomials in the differential cross-section decomposition can be folded in this manner
 291 to describe the reconstructed distributions. In this case the set of truth and reconstructed variables are
 292 $t = (\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}}, p_{\text{T}}^{\text{W,Truth}})$ and $r = (\cos \theta_{\text{CS}}^{\text{Reco}}, \phi_{\text{CS}}^{\text{Reco}}, p_{\text{T}}^{\text{W,Reco}})$ respectively. The folded polynomial
 293 $\tilde{P}_{ij} \left(r | p_{\text{T}}^{\text{W,Truth}} \in (\Delta p_{\text{T}}^{\text{W,Truth}})_j \right)$ is restricted to the finite bin width, $(\Delta p_{\text{T}}^{\text{W,Truth}})_j$, and can be expressed
 294 as:

$$\tilde{P}_{ij} \left(r | p_{\text{T}}^{\text{W,Truth}} \in (\Delta p_{\text{T}}^{\text{W,Truth}})_j \right) = \int_{(\Delta p_{\text{T}}^{\text{W,Truth}})_j} \int_{\cos \theta_{\text{CS}}^{\text{Truth}}} \int_{\phi_{\text{CS}}^{\text{Truth}}} P_i(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}}) p(r|t) dt. \quad (21)$$

295 The folded polynomials acquire a $p_{\text{T}}^{\text{W,Reco}}$ dependence from the migrations in this observable and it is
 296 important to distinguish this from $p_{\text{T}}^{\text{W,Truth}}$. The \tilde{P}_{ij} are a function of $p_{\text{T}}^{\text{W,Reco}}$ but are categorized in bins
 297 of $p_{\text{T}}^{\text{W,Truth}}$ by index j . It is also important to note that \tilde{P}_{ij} are continuous versions of the templates T_{ij}
 298 in Equation 18 and not used in this analysis but to transition the concepts to the actual folded templates
 299 used.

300 There is no analytical expressions for $p(r|t)$ so MC is used to approximate them. This is done by
 301 calculating the reference coefficients, A_{ij}^{ref} , with the moments method described in Section 1.3. These
 302 reference coefficients fully describe the 2D angular PDF, $f_j(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}})$ in the full phase space in
 303 each $p_{\text{T}}^{\text{W,Truth}}$ bin j :

$$f_j \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right) = \sigma_j \left\{ P_8 \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right) + \sum_{i=0}^7 A_{ij}^{\text{ref}} P_i \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right) \right\}. \quad (22)$$

304 Where σ_j is the differential cross-section in the $p_{\text{T}}^{\text{W,Truth}}$ bin j . For an MC event with weight w_{event} and
 305 corresponding truth and reconstructed variables t and r , respectively, we can estimate $p(r|t)$ with:

$$p^{\text{MC}}(r|t) = \frac{w_{\text{event}}(r, t)}{f_j \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right)}. \quad (23)$$

306 Here w_{event} is the produce of generator weights and scale factor weights used in the analysis. Division by
 307 f_j removes strong generator dependence on the coefficients used to build the templates assuming the MC
 308 follows the proper decomposition giving closure to unity. The final templates T_{ij} can then be expressed as
 309 the sum over all events within each bin j :

$$T_{ij} = \sum_{\text{event} \in \Delta_j} \frac{P_i \left(\cos \theta_{\text{CS}}^{\text{Reco}}, \phi_{\text{CS}}^{\text{Reco}} \right) w_{\text{event}}(r, t)}{\sigma_j \left\{ P_8 \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right) + \sum_{i=0}^7 A_{ij}^{\text{ref}} P_i \left(\cos \theta_{\text{CS}}^{\text{Truth}}, \phi_{\text{CS}}^{\text{Truth}} \right) \right\}}, \quad (24)$$

310 where Δ_j is the observable bins for the event and can be expressed as $\Delta_j = \left(\Delta p_{\text{T}}^{\text{W,Truth}} \right)_j, \left(\Delta p_{\text{T}}^{\text{W,Reco}} \right)_j,$
 311 $\left(\Delta \cos \theta_{\text{CS}}^{\text{Reco}} \right)_j,$ and $\left(\Delta \phi_{\text{CS}}^{\text{Reco}} \right)_j$. Examples of the templates for $p_{\text{T}}^{\text{W,Truth}}$ bins can be seen in Figures 2, 3,
 312 and 4.

313 For the templates there are 11 bins in $\cos \theta_{\text{CS}}$ in the range $[-1, 1.75]$ and 8 bins in ϕ_{CS} in the range $[0,$
 314 $2\pi]$. The $\cos \theta_{\text{CS}}$ range is extended past 8 bins in the range of $[-1, 1]$ with three additional $\cos \theta_{\text{CS}}$ bins
 315 with bin boundaries $[1, 1.25, 1.5, 1.75]$ in order to separate events where there is no real solution for p_z^{Y} .
 316 This is done to not distort the shape of $\cos \theta_{\text{CS}}$ which is discussed in more detail in Section 4.4. The first
 317 bin is intentionally left empty in order to prevent bins from either solution region from merging if there
 318 are not enough events. The other two bins are for events that are either forward $\cos \theta_{\text{CS}} > 0$ or backward
 319 $\cos \theta_{\text{CS}} < 0$ and allow us to retain the ϕ_{CS} shape information for these events. The templates are weighted
 320 by the polynomials as seen in Equation 24 which means that bins will have negative values in some bins.

321 To understand how the shape of the angular distributions evolve with $p_{\text{T}}^{\text{W,Truth}}$, the polynomial templates are
 322 projected onto their relevant 1D axis. For A_0 and A_4 this is integrating over ϕ_{CS} as they are only dependent
 323 on $\cos \theta_{\text{CS}}$ as seen in Figures 5 and 6, while for A_2 and A_3 this is integrating over $\cos \theta_{\text{CS}}$ as seen in Figures
 324 7 and 8. This doesn't work for A_1 as it depends on $\cos \theta_{\text{CS}}$ and ϕ_{CS} as discussed in Section 1.1

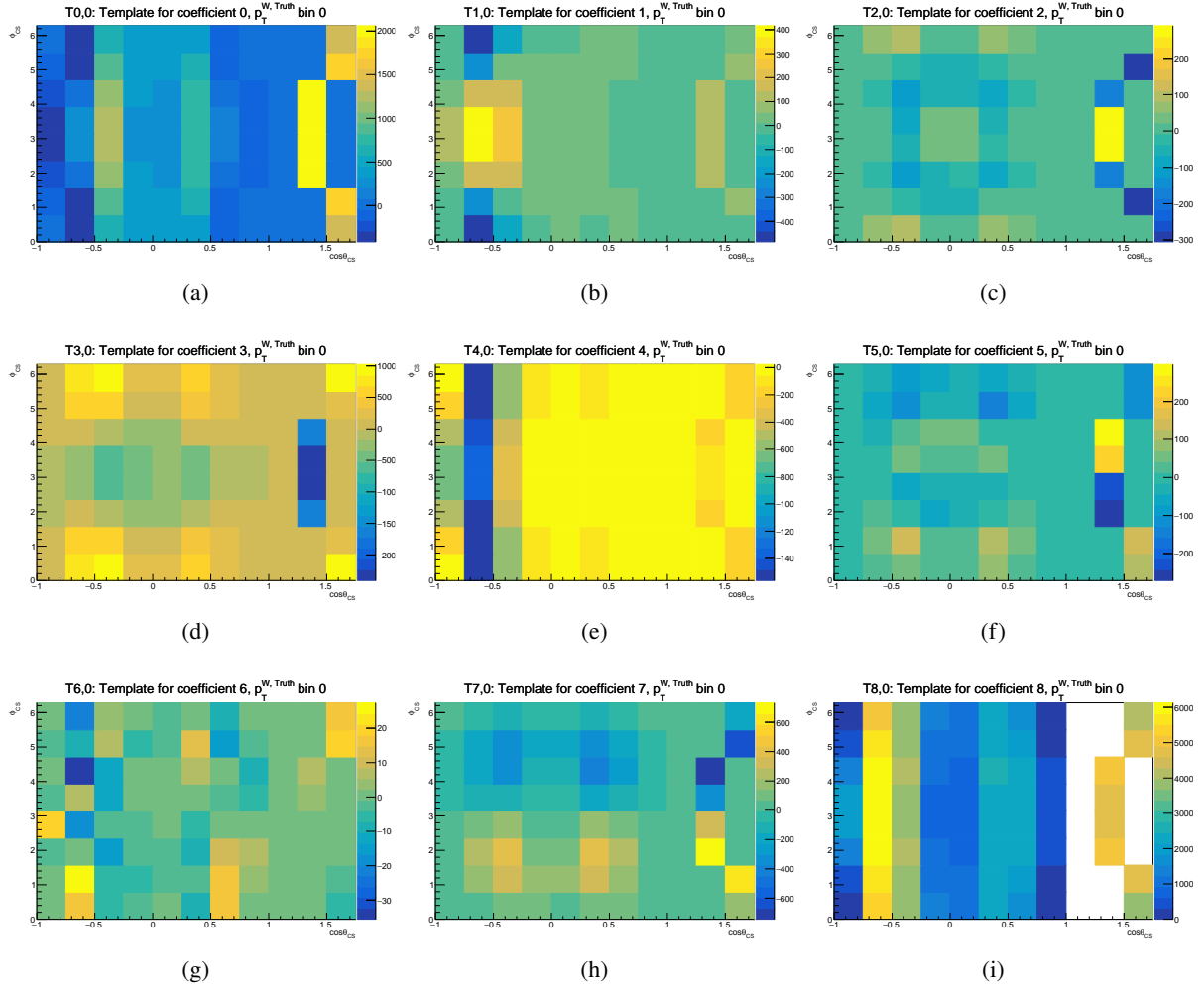


Figure 2: Folded polynomial templates projected onto the $(\cos\theta_{CS}^{\text{Reco}}, \phi_{CS}^{\text{Reco}})$ plane for $p_T^{W, \text{Truth}}$ bin 0 for $W^- \rightarrow e^- \bar{\nu}$. The templates are weighted by the polynomials resulting in negative values in some bins.

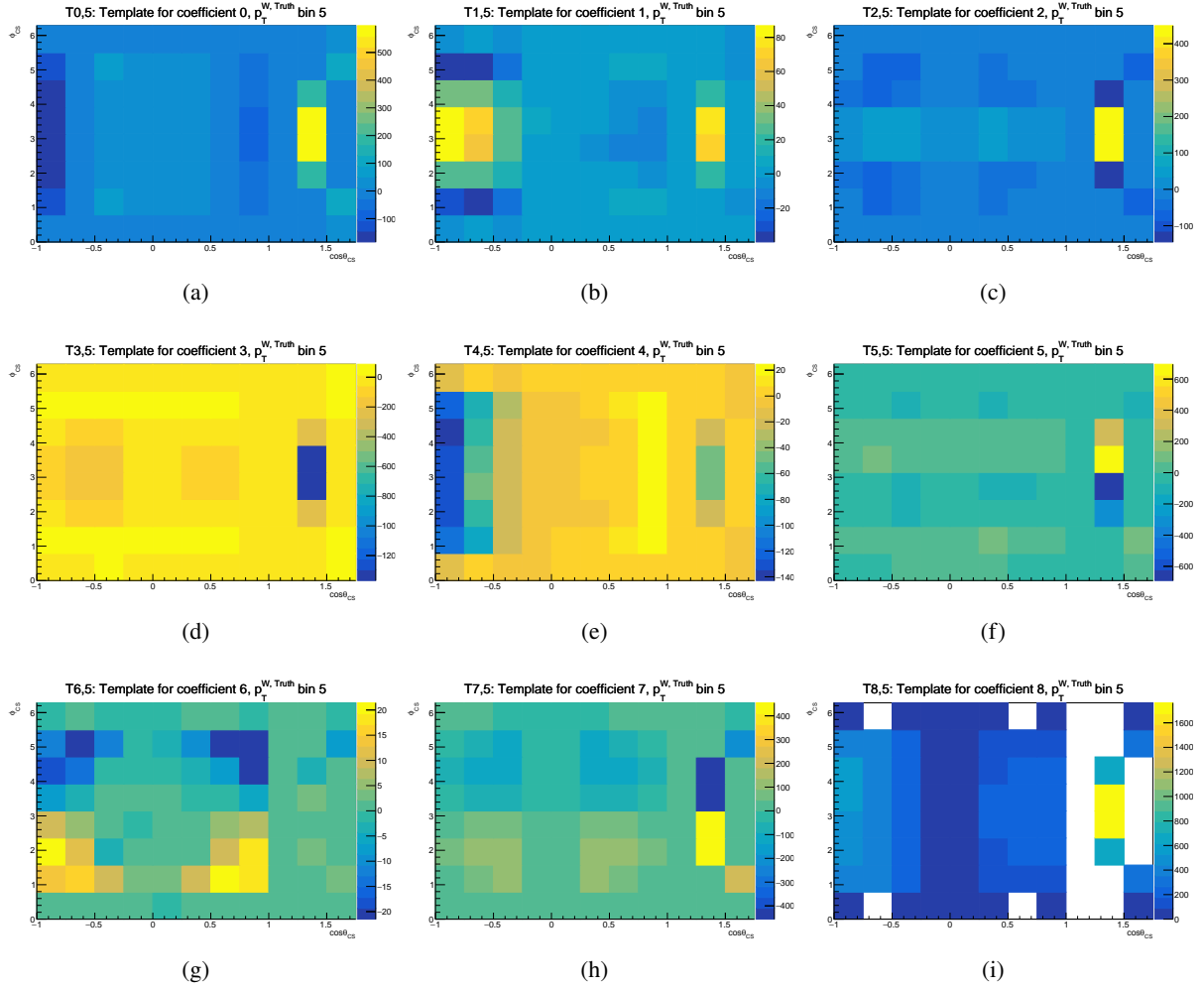


Figure 3: Folded polynomial templates projected onto the $(\cos \theta_{CS}^{\text{Reco}}, \phi_{CS}^{\text{Reco}})$ plane for $p_T^{W, \text{Truth}}$ bin 5 for $W^- \rightarrow e^- \bar{\nu}$. The templates are weighted by the polynomials resulting in negative values in some bins.

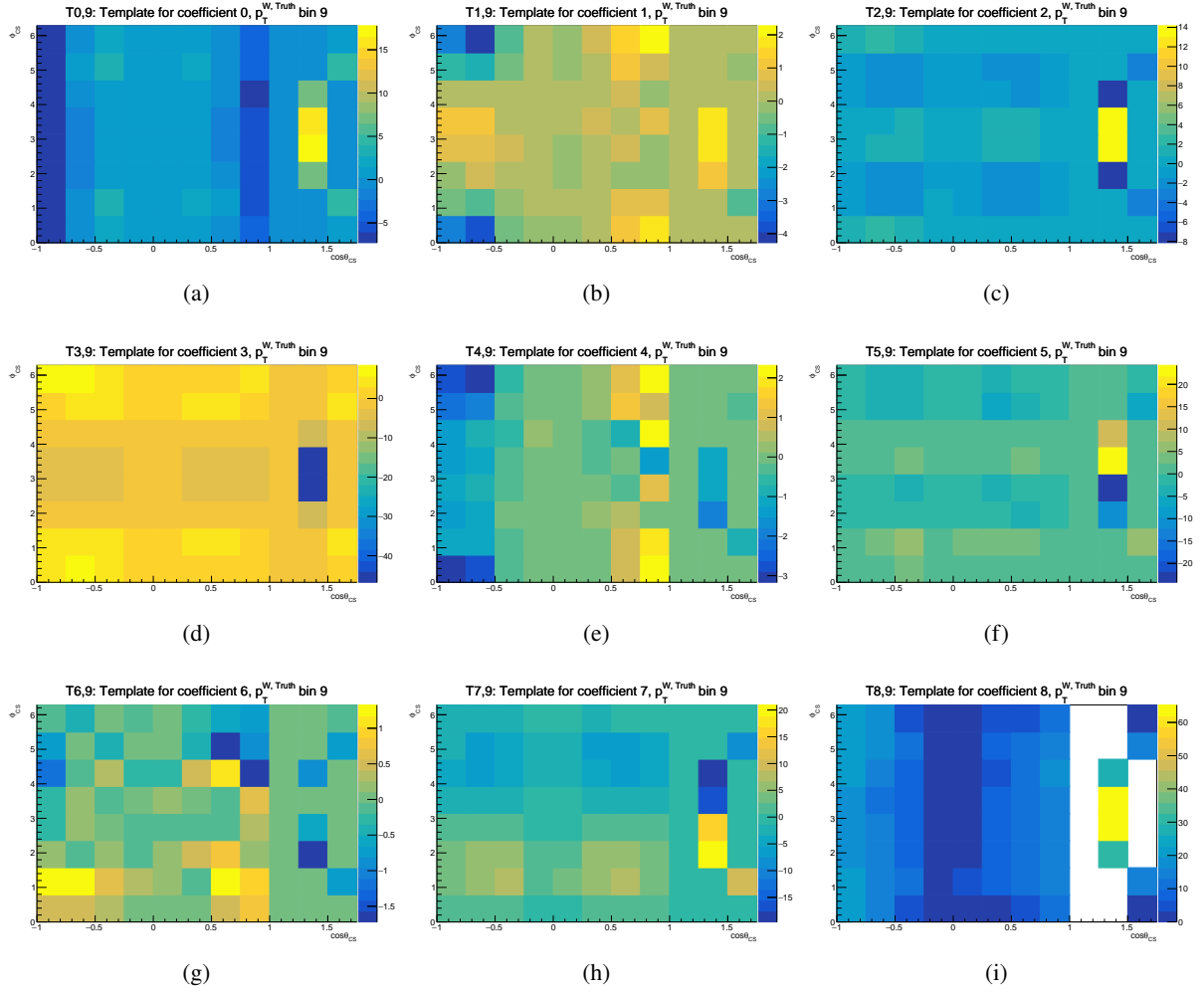


Figure 4: Folded polynomial templates projected onto the $(\cos \theta_{\text{CS}}^{\text{Reco}}, \phi_{\text{CS}}^{\text{Reco}})$ plane for $p_T^{W, \text{Truth}}$ bin 9 for $W^- \rightarrow e^- \bar{\nu}$. The templates are weighted by the polynomials resulting in negative values in some bins.

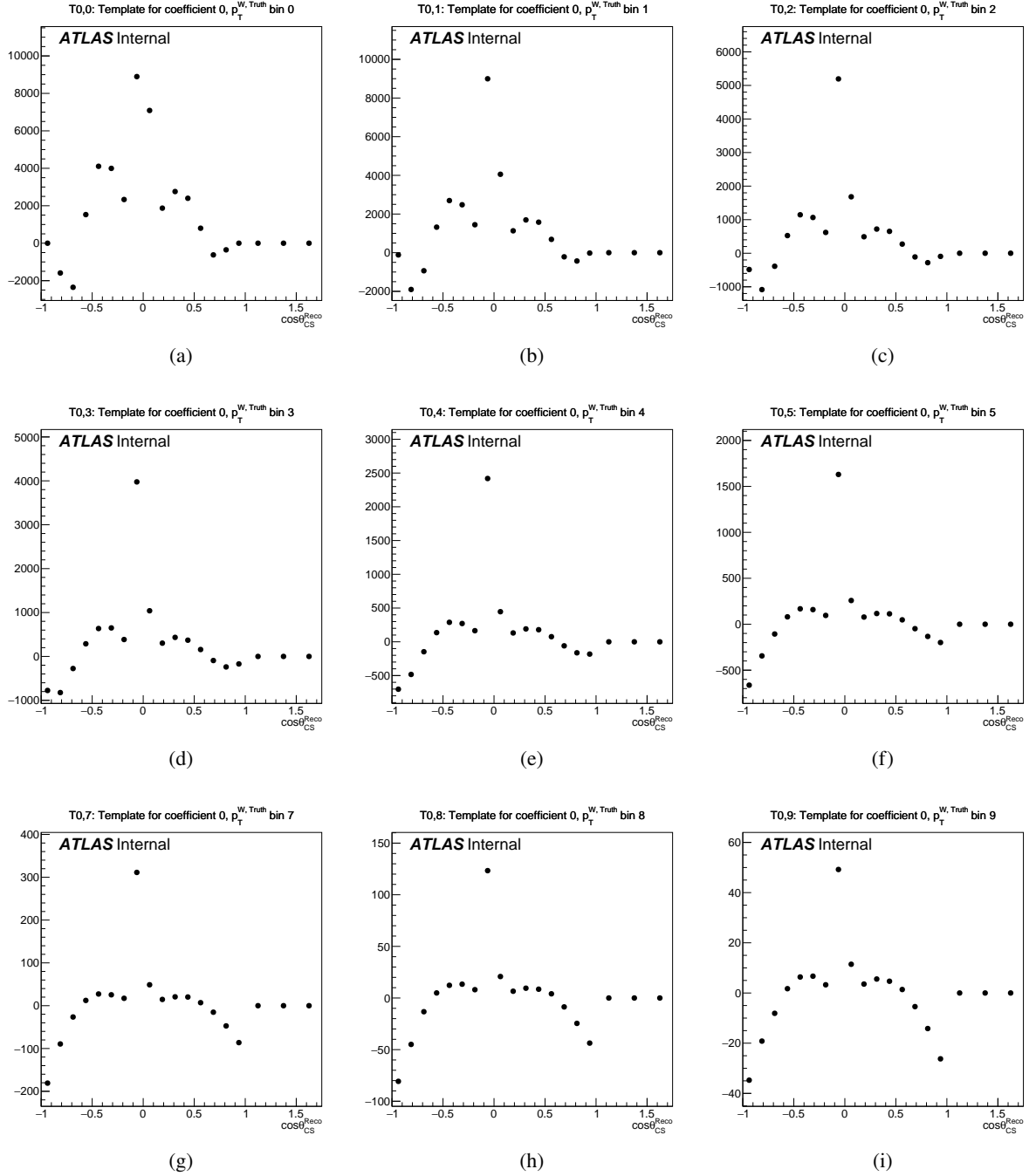


Figure 5: 1D Projections of folded polynomial templates onto $\cos\theta_{CS}^{\text{Reco}}$ for T_{0j} for each $p_T^{W, \text{Truth}}$ bin for $W^- \rightarrow e^- \bar{\nu}$.

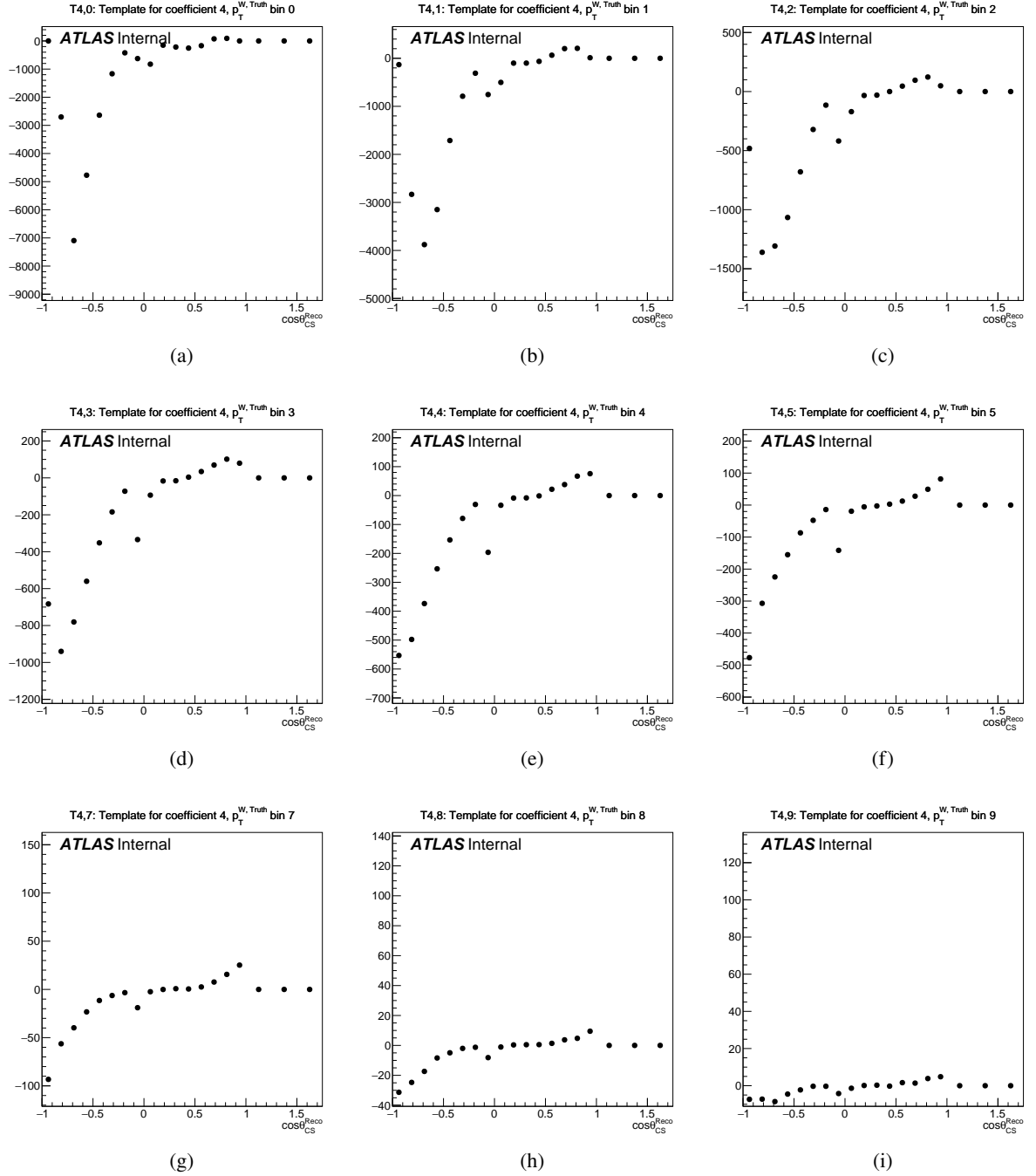


Figure 6: 1D Projections of folded polynomial templates onto $\cos\theta_{CS}^{\text{Reco}}$ for T_{4j} for each $p_T^{W, \text{Truth}}$ bin for $W^- \rightarrow e^- \bar{\nu}$.

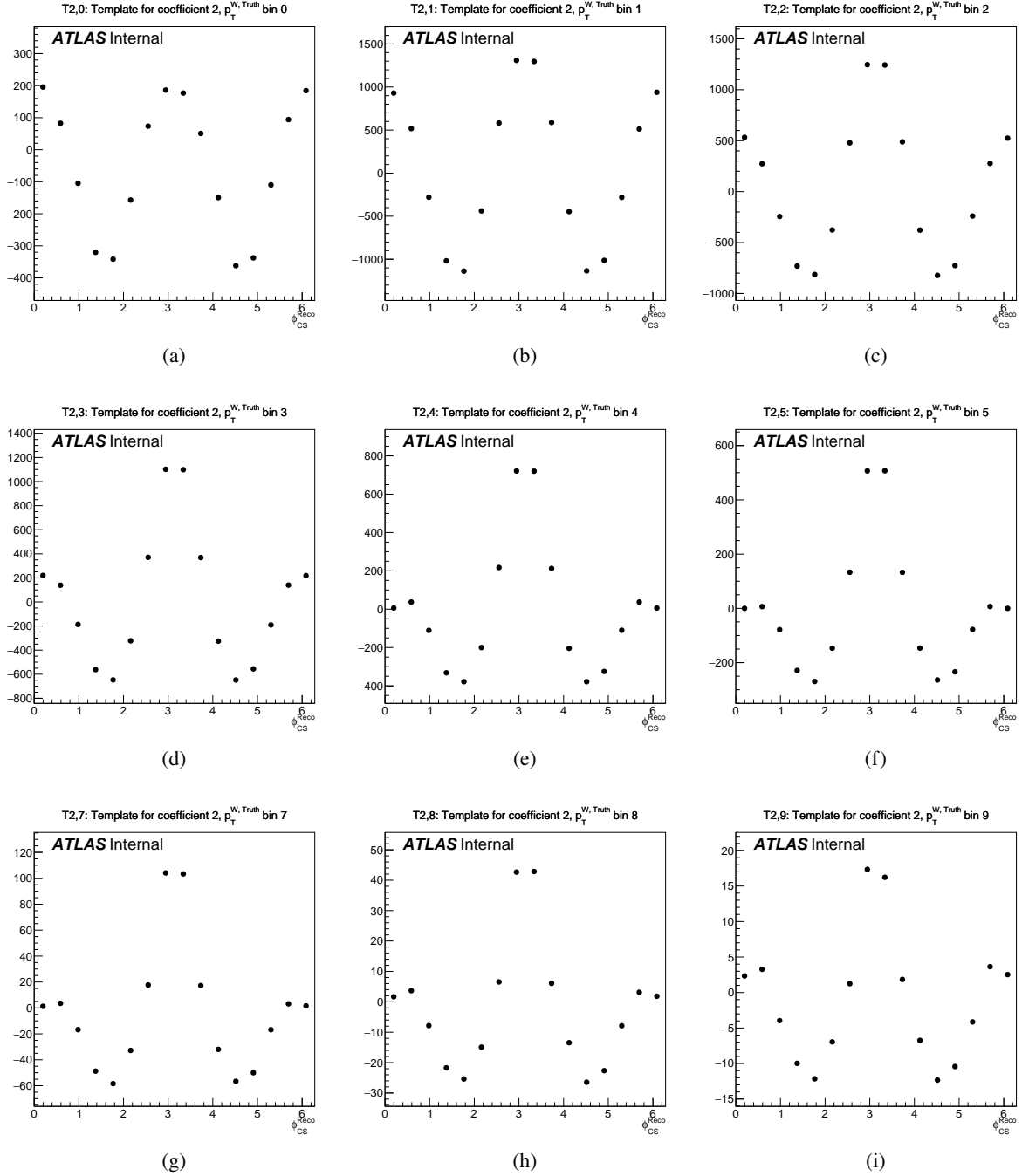


Figure 7: 1D Projections of folded polynomial templates onto ϕ_{CS}^{Reco} for T_{2j} for each $p_T^{W, \text{Truth}}$ bin for $W^- \rightarrow e^- \bar{\nu}$.

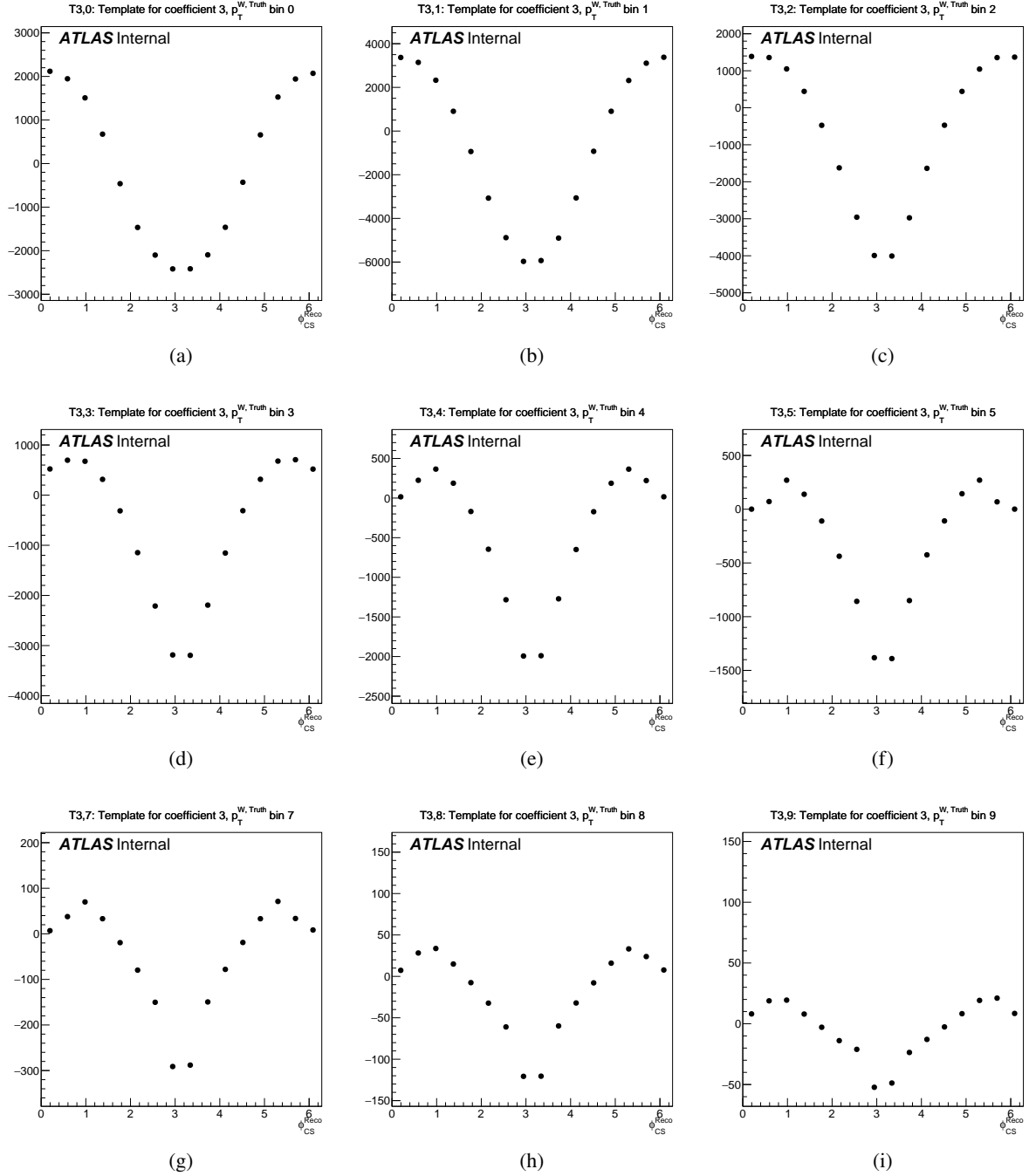


Figure 8: 1D Projections of folded polynomial templates onto ϕ_{CS}^{Reco} for T_{3j} for each $p_T^{W, Truth}$ bin for $W^- \rightarrow e^- \bar{\nu}$.

3.3 Minimization Technique

To extract the coefficients the likelihood is minimized. To do this we can express Equation 18 as

$$N_{\text{exp}}(\alpha, \theta) = \sum_i \left[\sum_j (T_{ij} \Pi_k \alpha_k) \right] S_i(\theta), \quad (25)$$

where T_{ij} are bin values of templates, α_k multiply templates, $S_i(\theta)$ are systematic variations on $\sum_j (T_{ij})$, and θ are NPs. For this analysis α are multipliers like A_{ij} and σ_j , or gamma parameters for MC statistics while θ are other NPs. The binned NLL is then

$$-\ln \mathcal{L} = \sum_n -\ln P(N_{\text{data}}|N_{\text{exp}}) + \sum_m P(N_{\text{MC}}|g_m N_{\text{MC}}) + \sum_1 G(0|\theta_1). \quad (26)$$

A layered cache is used to compute likelihood terms and derivatives in bottom-up approach compared to RooFit top-down approach as shown in Figure 9. Derivatives are implemented via chain rules for the NLL,

$$\frac{\delta(-\ln \mathcal{L})}{\delta\beta} = \sum_{n+m} \frac{\delta(-\ln P)}{\delta\beta} + \sum_1 \frac{\delta(-\ln G)}{\delta\beta}, \quad (27)$$

where the negative log Poisson is

$$\frac{\delta(-\ln P)}{\delta\beta} = \left(1 - \frac{N_{\text{obs}}}{N_{\text{exp}}}\right) \frac{\delta N_{\text{exp}}}{\delta\beta}. \quad (28)$$

Derivatives are computed within a layered cache similar to the 0th order values where it is optimized for derivatives we know are zero and utilize efficient sparse matrix in Eigen [22] for using the full Hessian $\nabla^2(-\ln \mathcal{L})$.

Full analytical Hessian information enables us to use a more efficient minimization algorithm based on Newton's method, where a more detailed description can be found in Appendix G which performs root finding on the gradient of the NLL to find the solution of $\nabla[-\ln \mathcal{L}(\vec{x})] = 0$. This is an iterative method where we chose to iterate until $|\ln \mathcal{L}(\vec{x}_{n+1}) - \ln \mathcal{L}(\vec{x}_n)| < \epsilon$, where ϵ is the chosen precision which for this analysis a value of $\epsilon = 10^{-4}$ is used.

A sanity check is also performed to make sure that $\nabla[-\ln \mathcal{L}(\vec{x})] = 0$ corresponds to a minima and not a maximum or saddle point. Checks are also done so that \mathbf{H} is positive definite, which is that is invertible and the covariance matrix, \mathbf{H}^{-1} has all positive diagonals.

Full information of the Hessian allows us to compute the full uncertainty decomposition quickly and accurately because the total uncertainty for a given parameter is the square root of the diagonal of the covariance matrix. For a more detailed description see Appendix G.

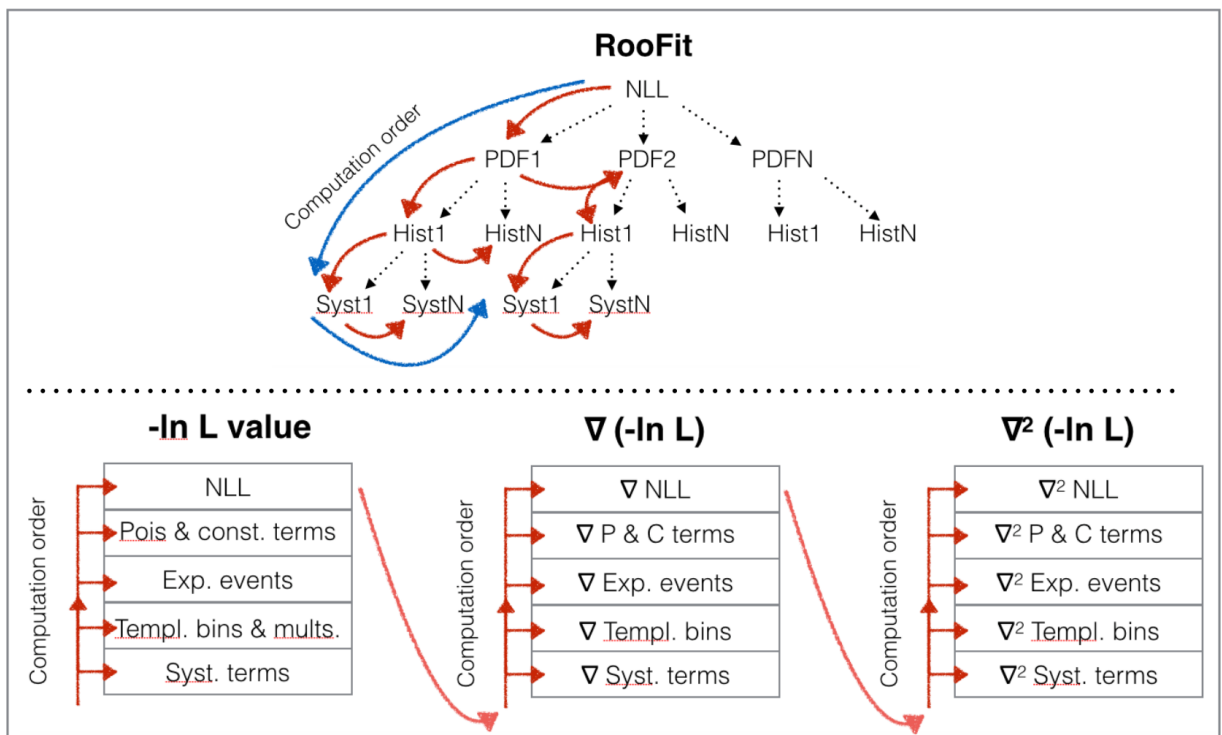


Figure 9: Comparison of computation order of RooFit which takes a top down approach to the bottom up approach taken for this analysis.

348 4 Analysis

349 This section describes the dataset consider in this analysis the event selection, the reconstruction of the
350 unmeasured longitudinal momentum of the neutrino.

351 4.1 Data Sets

352 This analysis uses ATLAS special runs at 13TeV taken in Fall 2017 and July 2018 with low pile-up
353 conditions ($\langle\mu\rangle \approx 2$) to optimise the measurement resolution of the hadronic system recoiling against the
354 bosons that gives a direct measurement of the W -boson transverse momentum. The following runs are
355 considered for analysis:

- 356 • $\sqrt{s} = 13\text{TeV}$ data taken in November 2017, ATLAS data period N, preliminary calibrated luminosity
357 144.931pb^{-1} with an uncertainty of $\pm 2.1\%$ (tag OffLumi-13TeV-010-lowmu),
358 GRL [data17_13TeV.periodN_DetStatus-v98-pro21-16_Unknown_PHYS.StandardGRL.All.Good.25ns.ignore.GLOBAL.LOWMU.xml](#):
359 Runs 341294, 341312, 341419, 341534, 341615, 341649
- 360 • $\sqrt{s} = 13\text{TeV}$ data taken in June 2018, ATLAS data period G4+J, preliminary calibrated luminosity
361 190.249pb^{-1} with an uncertainty of $\pm 1.5\%$ (tag OffLumi-13TeV-010-lowmu),
362 GRL [data18_13TeV.periodG4J_MERGED_PHYS.StandardGRL.All.Good.25ns.ignore.GLOBAL.LOWMU.xml](#):
363 Runs 354396, 355331, 355389, 355416, 355468

364 The total luminosity for the 13 TeV data amounts to 335.180pb^{-1} with an uncertainty of $\pm 1.5\%$. The 13
365 TeV data was taken with a fixed levelled luminosity at $\langle\mu\rangle = 2$ (with minor exceptions).

366 Several notable differences to the standard high μ data taking are present in addition to the lower pileup

- 367 • Lower topo-cluster thresholds are used in the reconstruction to optimise the response of topo-clusters
368 for the hadronic recoil; this has also implications for the electron response
- 369 • Single e and μ triggers with significantly lower thresholds and looser identification criteria are run
370 without prescale, most notably HLT_e15_1h1oose_nod0_L1EM12 and HLT_mu14

371 A more detailed description of the low- μ data condition can be found in [23].

372 4.2 MC Samples

373 Fully simulated and reconstructed ATLAS Monte Carlo samples (“MC”) from the MC16 production
374 campaigns are used to model the signal and background processes. All MC samples use settings specific
375 to the special data run conditions, i.e. specifically the pileup overlay, topo-cluster noise thresholds and
376 trigger menu are adapted. Given that the pileup distribution is already adjusted as to the dataset, no further
377 μ -reweighting is performed. The main event samples generated and simulated (“HITS”) for the main
378 ATLAS MC16d campaign are reused and processed further with dedicated settings in digitisation and
379 reconstruction. The simulation and reconstruction tags for these samples are s3126_r10244_r10210. These
380 samples correspond to the 2017 conditions except for the bunch structure of the beam, which was “8b4e”
381 in data and the more “standard” structure in the MC.

Process	Data set	Generator	$\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}}$ [nb] (th. unc.)	$N_{\text{evt}}^{\text{skim}}$ [10^6]	$N_{\text{evt}}^{\text{unskim}}$ [10^6]
$W^+ \rightarrow e^+ \nu$	361100	POWHEG+PYTHIA8	11.61 (5%)	40	40
$W^+ \rightarrow \mu^+ \nu$	361101	POWHEG+PYTHIA8	11.61 (5%)	40	40
$W^+ \rightarrow \tau^+ \nu$	361102	POWHEG+PYTHIA8	11.61 (5%)	0.28	5.0
$W^- \rightarrow e^- \bar{\nu}$	361103	POWHEG+PYTHIA8	8.630 (5%)	30	30
$W^+ \rightarrow e^+ \nu$	361104	POWHEG+PYTHIA8	8.630 (5%)	29	29
$W^- \rightarrow \tau^- \bar{\nu}$	361105	POWHEG+PYTHIA8	8.630 (5%)	0.24	4.0
$Z \rightarrow ee$	361106	POWHEG+PYTHIA8	1.910×1.025 (5%)	10	10
$Z \rightarrow \mu\mu$	361107	POWHEG+PYTHIA8	1.910×1.025 (5%)	10	10
$Z \rightarrow \tau\tau$	361108	POWHEG+PYTHIA8	1.910×1.025 (5%)	0.12	1.0
$ZZ(q\bar{q}\ell\ell)$	363356	SHERPA 2.2.1	0.01556×0.141 (10%)	0.0064	0.010
$WZ(q\bar{q}\ell\ell)$	363358	SHERPA 2.2.1	0.003433 (10%)	0.0063	0.010
$WW(q\bar{q}\ell\nu)$	363359	SHERPA 2.2.1	0.02472 (10%)	0.0093	0.020
$WW(\ell\nu q\bar{q})$	363360	SHERPA 2.2.1	0.02472 (10%)	0.0093	0.020
$WZ(\ell\nu q\bar{q})$	363489	SHERPA 2.2.1	0.01142 (10%)	0.0047	0.010
$ZZ(4\ell)$	364250	SHERPA 2.2.2	0.001252 (10%)	0.0057	0.010
$WZ(3\ell\nu)$	364253	SHERPA 2.2.2	0.004583 (10%)	0.0062	0.010
$WW(2\ell 2\nu)$	364254	SHERPA 2.2.2	0.01250 (10%)	0.0073	0.010
$WZ(\ell 3\nu)$	364255	SHERPA 2.2.2	0.003235 (10%)	0.0050	0.010
Wt	410013	POWHEG+PYTHIA8	0.03582 (10%)	0.0037	0.010
$W\bar{t}$	410014	POWHEG+PYTHIA8	0.03399 (10%)	0.0037	0.010
$t\bar{t}$ (nominal)	410470	POWHEG+PYTHIA8	0.8318×0.544 (7%)	1.2	2.0
t (t – chan. t)	410642	POWHEG+PYTHIA8	0.03699 (10%)	0.016	0.030
t (t – chan. \bar{t})	410643	POWHEG+PYTHIA8	0.02217 (10%)	0.011	0.020
t (s – chan. t)	410644	POWHEG+PYTHIA8	0.002027 (10%)	0.0050	0.010
t (s – chan. \bar{t})	410645	POWHEG+PYTHIA8	0.001268 (10%)	0.0052	0.010
$t\bar{t}$ (syst.)	410480	POWHEG+PYTHIA8	0.8318×0.438 (7%)	0.85	1.5
$t\bar{t}$ (syst.)	410482	POWHEG+PYTHIA8	0.8318×0.105 (7%)	0.40	0.50
$t\bar{t}$ (syst.)	410557	POWHEG+PYTHIA8	0.8318×0.438 (7%)	0.85	1.5
$t\bar{t}$ (syst.)	410558	POWHEG+PYTHIA8	0.8318×0.105 (7%)	0.40	0.50

Table 2: Monte Carlo samples at $\sqrt{s} = 13\text{TeV}$. Given is a short description of the process, the ATLAS MC data set number (DSID), the names and version numbers of the MC generator(s), the used value of the higher order cross section times any branching and filter efficiencies ($\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}}$) with the theoretical uncertainty in percent (“th. unc.”), and finally the number of events analysed after skimming at derivation production ($N_{\text{evt}}^{\text{skim}}$) as well as the number of events originally processed and simulated ($N_{\text{evt}}^{\text{unskim}}$). In the case of $Z \rightarrow \ell\ell$ samples, the given $\epsilon_{\text{filter}} > 1$ is related to the fact, that the cross sections were calculated for $66 < m_{\ell\ell} < 116\text{GeV}$, but the generated mass range is larger. The last section of $t\bar{t}$ samples refers to variation samples for systematics studies. The MC equivalent luminosity $N_{\text{evt}}^{\text{unskim}}/(\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}})$ is generally above 3fb^{-1} for signal and significant backgrounds, the exception are Powheg $W \rightarrow \tau\nu$ and $Z \rightarrow \tau\tau$ samples, that have about 0.45fb^{-1} only.

382 Table 2 summarizes the MC samples used in this analysis for signal templates as well as for EW+top
 383 background modeling. The cross-sections quoted in the table are used to normalize predicted event counts
 384 for the various samples and the respective uncertainties are listed as well.

385 The main signal event samples for W production are generated using the POWHEGEVENT generator
 386 [24–27] using CT10 [28] PDF, interfaced to PYTHIA8[29] using the AZNLO tune [30]. These
 387 POWHEG+PYTHIA8samples are interfaced to PHOTOS++ [31] to simulate the effect of final state QED
 388 radiation.

389 The effect of multiple interactions per bunch crossing (“pileup”) is modelled by overlaying simulated
 390 minimum bias events over the original hard-scattering event. The minimum bias events were generated
 391 with PYTHIA8using the NNPDF2.3LO set of PDFs [32] and the A3 tune [33], which improves on the
 392 previous A2 tune [34] by using some Run2 results at $\sqrt{s} = 13$ TeV and the Donnachie–Landshoff diffractive
 393 model that better describes the “visible” inelastic cross section compared to the default Schuler–Sjöstrand
 394 model.

395 For W processes, the samples are normalized to NNLO calculations performed using the DYTURBO
 396 program, an optimised version of DYNNLO [35, 36] using the MMHT2014nnlo PDF set [37]. The
 397 numerical values are taken from the corresponding ATLAS publications of the 2015 data at 13 TeV
 398 [38]. The uncertainties on those cross-sections arise from the choice of PDF set and PDF set internal
 399 uncertainties ($\sim 3 - 4\%$), from factorization and renormalisation scale dependence ($\sim 1\%$), and the strong
 400 coupling constant α_s ($\sim 1 - 2\%$). A conservative total uncertainty of 5% is taken as an uncertainty on any
 401 event count predictions normalized using these cross-sections, e.g. for use of subtraction of electroweak
 402 backgrounds.

403 Backgrounds from top-quark pair-production $t\bar{t}$ as well as single-top production (Wt , t-channel, s-channel)
 404 are generated with POWHEG+PYTHIA8. At $\sqrt{s} = 13$ TeV samples that can be used to derive systematics on
 405 the $t\bar{t}$ background were produced: the “rad-low” variation can be obtained by internal weights, the “rad-up”
 406 variation is obtained by dedicated samples. Various combinations of di-bosons $VV, V = W, Z$ are generated
 407 with Sherpa in all decay channels with at least one real lepton in the final state, for details see Table 2.

408 4.3 Event Selection and object definition

409 In this analysis, electrons or muons are considered as leptons. Electrons are reconstructed from energy
 410 clusters in the calorimeter associated to a track, while muons are reconstructed from a track that crosses
 411 both the inner detector and the muon chambers.

412 The object selection for this analysis follows the selection described in [39], however several studies were
 413 performed in order to maximise the sensitivity to the angular coefficients discussed in Appendix A.

414 $W \rightarrow \ell\nu$ candidates events are selected using the following criteria:

- 415 • Single electron (*HLT_e15_lhloose_nod0_L1EM12*) and muon (*HLT_mu14*) triggers were used for the
 416 electron and muon channels;
- 417 • Exactly one lepton in the event;
- 418 • Both electrons and muons are required to have $p_T > 25$ GeV;
- 419 • Electrons should have $|\eta| < 2.47$ and candidates in the crack region $1.37 < |\eta| < 1.52$ are excluded.
 420 Muons candidates are required to be within $|\eta| < 2.4$;

- 421 • The electron candidates must satisfy the *Tight LH* identification criteria. The muon candidates must
422 satisfy the *Medium* identification criteria;
- 423 • All leptons are required to be isolated. We define lepton as isolated if it satisfies both isolation
424 criteria: $ptvarcone20/p_T < 0.1$ and $topoetcone20/p_T < 0.05$;
- 425 • Leptons are required to originate from the primary vertex. The longitudinal impact parameter of
426 each lepton track is required to be less than 0.5 mm ($\Delta z_0 \times \sin \theta < 0.5$ mm); Furthermore, the
427 significance of the transverse impact parameter divided by its estimated uncertainty is required to
428 satisfy for electrons $|d_0|/\sigma(d_0) < 5$ and for muons $|d_0|/\sigma(d_0) < 3$.

429 Dedicated lepton calibrations and efficiency corrections obtained specifically for the low- μ conditions are
430 applied to the data and the MC and described in details in References [40, 41] for electrons and muons,
431 respectively.

432 The electron reconstruction scale factors are obtained by extrapolation of the standard high-pileup SFs to
433 the low- μ regime, and apply to both the 13 and 5 TeV datasets. The identification scale factors are measured
434 in-situ separately for the 13 and 5 TeV data. Isolation and trigger efficiencies and SF are measured in-situ
435 separately for the 13 and 5 TeV data sets. For the electron energy calibration, the global strategy follows
436 the methodology applied in high- μ standard calibration and described in Ref. [42]. Electron energy scale
437 and resolution corrections are measured in-situ using $Z \rightarrow ee$ events from the low pile-up dataset.

438 Muon reconstruction and *track-to-vertex association* requirements efficiencies measured insitu with
439 $Z \rightarrow \mu\mu$ events are found to be compatible with the corresponding high- μ measurements and are measured
440 to permille-level precision. The muon trigger and isolation efficiencies are measured insitu in the low- μ
441 data, for each dataset, to typically better than $< 1\%$ precision, limited by the size of the $Z \rightarrow \mu\mu$ data
442 samples. Finally, the muon calibration derived from the high- μ data has been cross-checked using both
443 $Z \rightarrow \mu\mu$ and $J/\psi \rightarrow \mu^+\mu^-$ events in the low- μ datasets and has been found to be adequate at the current
444 level of systematics. A dedicated correction for charge-dependent momentum biases, so-called sagitta
445 correction, has been derived using a combination of different methods using the 13 TeV low- μ dataset.

446 Additional efficiency corrections were calculated specifically for this analysis as discussed in Appendix
447 H for the *Tight LH* identification criteria applied to the electrons candidates and the additional isolation
448 requirement ($topoetcone20/p_T < 0.05$) for both electrons and muons.

449 4.4 Neutrino Reconstruction

450 In the context of W boson production a high precision computation of the neutrino kinematics is mandatory,
451 and this is done with the help of the *hadronic recoil*. In proton-proton collisions there is a non-zero
452 transverse momentum for vector boson production that can be thought of to originate from initial state
453 gluon and quark radiation:

$$\vec{p}_T(W/Z) = \vec{p}_T^{\text{lepton1}} + \vec{p}_T^{\text{lepton2}} = - \sum \vec{p}_T^{\text{ISRquark,gluons}} = -\vec{u}_T. \quad (29)$$

454 Here $p_T(W/Z)$ denotes the transverse momentum of the W or Z boson and p_T^{lepton} denote the transverse
455 momenta decay leptons (charged lepton or neutrino). The *hadronic recoil*, $\sum \vec{p}_T^{\text{ISRquark,gluons}}$, is the quantity
456 which accounts for the transverse momenta any partons recoiling from the colour-neutral boson, and is
457 denoted as \vec{u}_T . In the contest of low pileup precision W analysis the use of *particle flow objects* (PFOs) as
458 input constituents to the building of u_T , thus using the full available information on both topo-clusters

459 and tracks, represents a major improvement. The hadronic recoil in the transverse plane, \vec{u}_T , is computed
 460 as the vectorial sum of all charged and neutral PFOs. The modulus of this 2-vector is simply termed
 461 u_T . A critical quantity is ΣE_T that represents the total event activity and is related to the resolution of
 462 the u_T measurement. ΣE_T is defined as the scalar sum of the p_T of all PFOs. Naturally ΣE_T will be
 463 strongly dependent on the vector boson dynamics and grow with $p_T^V = u_T$. To disentangle these effects, an
 464 even more important quantity is defined by $\Sigma \vec{E}_T = \Sigma E_T - u_T$. $\Sigma \vec{E}_T$ may be thought of the event activity
 465 corrected for the “directed” recoil activity and thus represents the activity from underlying event, pileup,
 466 and emissions beyond first (hard) emission. ¹

467 The detailed description of the *hadronic recoil* definition and calibration is done in reference [43] and also
 468 summarised briefly here. The calibration is obtained as a function of $\Sigma \vec{E}_T$ and p_T^V of the boson, since the
 469 hadronic recoil is mainly sensitive to these. The correction is obtained in Z events and extrapolated to the
 470 W events.

471 The procedure consists of three steps :

- 472 • First, the $\Sigma \vec{E}_T$ distribution in the Monte-Carlo should be well modelled and match that of the data.
 473 More precisely, it is crucial to model correctly the correlation between $\Sigma \vec{E}_T$ and p_T^V , since we want
 474 to have a good description of the activity as a function of our measured physics observable. This is
 475 achieved thanks to a 2-dimensional reweighting, obtained in Z events. In the simulated W events,
 476 an additional reweighting of $\Sigma \vec{E}_T$ in bins of u_T is applied. A further 1-dimensional reweighting is
 477 obtained for each process (W^- , W^+ and Z) to recover the initial underlying p_T^{true} spectrum.
- 478 • Second, the direction of the recoil is corrected, taking the projections on x and y axes of the recoil in
 479 Z events, and correcting for any data to MC differences.
- 480 • Finally, response and resolution corrections are, once again, obtained in-situ in Z data events, where
 481 the parallel and perpendicular components can be extracted in the data as a function of $\Sigma \vec{E}_T$ and
 482 $p_T^{\ell\ell}$, and compared to the Monte-Carlo to extract corrective coefficients.

483 This Z boson based calibration is applied to W events ; uncertainties due to this extrapolation are included.
 484 A summary on these uncertainties, as well as their impact on the unfolded spectra, are discussed in
 485 section 7.

486 Once that the *hadronic recoil* is computed it is possible to determine indirectly the neutrino ν transverse
 487 momentum using the following expression:

$$E_T^{\text{miss}} := \vec{p}_T^\nu = -(\vec{u}_T + \vec{p}_T^\ell) \quad (30)$$

488 where E_T^{miss} is the missing transverse energy. The transverse momenta of additional muons with $p_T > 5$ GeV
 489 and satisfying the medium identification is added at the definition of E_T^{miss} while their calorimeter energy
 490 deposit is subtracted from the event.

491 Thanks to the clean environment guarantee by the low pileup dataset, in this analysis it was possible to
 492 maximise the sensitivity and enlarge the acceptance at high p_T^W without applying any requirements on
 493 m_T^W ² and E_T^{miss} .

¹ E.g. a $V + 1$ jet event has $\Sigma \vec{E}_T \sim 0$ on parton level for any jet p_T and thus a finite $\Sigma \vec{E}_T > 0$ is due to additional activity.

² defined as:

$$m_T = \sqrt{2p_T^\nu p_T^\ell (1 - \cos \Delta\phi^\nu)} \quad (31)$$

494 However the unmeasured longitudinal momentum of the neutrino is inferred by assuming that the
 495 dilepton pair of the Drell-Yan process has invariant mass equal to m_W . A massless charged lepton with
 496 four-momentum p^μ and a massless neutrino with four-momentum q^μ are considered in the laboratory
 497 frame:

$$498 \quad p^\mu = \begin{pmatrix} \vec{p} \\ |p| \end{pmatrix} \quad q^\mu = \begin{pmatrix} \vec{q} \\ |q| \end{pmatrix} \quad (32)$$

$$499 \quad \vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \quad \vec{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} \quad (33)$$

500 By imposing the m_W constrain:

$$501 \quad (p + q)^\mu (p + q)_\mu = m_W^2 \quad (34)$$

502 and using the relations:

$$503 \quad |p| = \sqrt{p_x^2 + p_y^2 + p_z^2} \quad |q| = \sqrt{q_x^2 + q_y^2 + q_z^2} \quad (35)$$

504 and factorizing the expression in powers of q_z , leads to:

$$505 \quad a q_z^2 + b q_z + c = 0 \quad (36)$$

506 with

$$507 \quad a = (p_x^2 + p_y^2) \quad (37)$$

$$508 \quad b = -p_z [2(p_x q_x + p_y q_y) + m_W^2] \quad (38)$$

$$509 \quad c = |p|^2 (q_x^2 + q_y^2) - [2(p_x q_x + p_y q_y) + \frac{m_W^2}{2}]^2 \quad (39)$$

510 Finally, it is possible to write the two solutions for longitudinal momentum of the neutrino as:

$$511 \quad q_z(1, 2) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (40)$$

512 The lepton momentum in the Collins-Soper framed can be expressed as [1]:

$$513 \quad p_x^{CS} = \frac{1}{2} \frac{m_W}{m_W^2 + p_{T,W}^2} (2p_x - p_{T,W}^2) \quad (41)$$

$$514 \quad p_y^{CS} = p_y \quad (42)$$

$$515 \quad p_z^{CS} = \pm \frac{m_W}{2} \sqrt{1 - \frac{(p_x^{CS})^2 + (p_y^{CS})^2}{m_W^2/4}} \quad (43)$$

and as in Eq. (30), the neutrino p_T^ν and ϕ^ν are estimated by the corresponding E_T^{miss} quantities.

516 The azimuthal and polar angles in the Collins-Soper frame are

$$517 \quad \phi_{CS} = \arctan p_y^{CS}/p_x^{CS} \quad (44)$$

$$518 \quad \theta_{CS} = \arctan \sqrt{(p_x^{CS})^2 + (p_y^{CS})^2}/p_z^{CS} \quad (45)$$

519 The transverse components p_x^{CS} and p_y^{CS} are uniquely determined from measurable laboratory frame
520 quantities. On the other hand, the longitudinal component p_z^{CS} is determined only up to a sign. The
521 twofold ambiguity in the reconstruction of the leptons momenta in the Collins-Soper frame translates in an
522 ambiguity on the sign of $\cos \theta_{CS}$, while ϕ_{CS} is determined. In general, the twofold ambiguity related to the
523 unmeasured longitudinal momentum of the neutrino results in two solutions for the rapidity of the W boson,
524 y_W , and two solutions for $\cos \theta_{CS}$ which differ by a sign, while $p_{T,W}$ and ϕ_{CS} are uniquely determined.

525 The above formulas assume that the z axis is collinear to the direction of one of the collider beams. However
526 in proton-proton collisions the sign of the z axis in the Collins-Soper frame is taken along the direction
527 of the longitudinal momentum of W , i.e, we have $\cos \theta_{CS} \rightarrow \cos \theta_{CS} \text{sign}(y_W)$ (see Eq. 5). With this
528 definition, there are events for which the two solutions have opposite sign for y_W , hence $\cos \theta_{CS}$ is uniquely
529 determined. These events, which are usually at central rapidities, allow to resolve the sign of $\cos \theta_{CS}$ in
530 a fraction of the dataset, and open the possibility to measure the forward-backward asymmetry (the A4
531 coefficient) in W -boson production. More details on this subject are given in Appendix B.

532 5 Multi-Jet Estimate

533 Two main categories of backgrounds affect single W boson final states : the electroweak (single and
 534 diboson) and top backgrounds, obtained from the appropriate MC samples, and the the so-called multi-jet
 535 (MJ) background which can't be reliably simulated and has to be derived from data. Depending on the
 536 lepton flavour, the MJ background has significant contributions from real leptons produced in semi-leptonic
 537 decays of heavy quarks, pion decays, or photon conversions.

538 The data sample discussed in this analysis has a pile-up of $\mu \sim 2$, about a factor of 15–20 smaller than the
 539 standard 13 TeV running conditions. Lower pile-up will reduce the MJ background yield for at least three
 540 reasons:

- 541 • a reduced pile-up contribution to the isolation distributions, improving the discriminating power of
 542 the isolation cuts;
- 543 • better E_T^{miss} and m_T resolution, with correspondingly improved discriminating power of the event
 544 selection cuts;
- 545 • reduced contribution from pile-up jets themselves to the contamination of the sample.

546 The fake lepton background (called in the following multijet background or simply MJ) is concentrated
 547 at lower values of p_T^ℓ , E_T^{miss} and m_T . By relaxing the kinematic cuts in our signal region definition we
 548 introduce significant amount of multijet background. The background estimation method relies on the
 549 same methodology described in the WpT analysis (see [44]), however there are number of W-Ai analysis
 550 specific details that are listed in the following.

551 5.1 General procedure

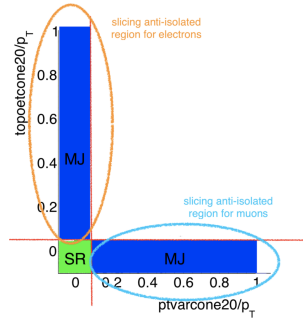


Figure 10: Sketch of the definition of the SR and CR1 for the multi jet estimate

552 The aim of the procedure is to estimate the multijet background yield in the signal region (SR). The multijet
 553 yields estimation relies on a fit on data using signal and MJ *templates* built from distributions as the p_T^ℓ ,
 554 E_T^{miss} and m_T . As the W-Ai analysis is measuring angles, additionally to p_T^ℓ , E_T^{miss} and m_T variables,
 555 the $|\Delta\phi(\ell, E_T^{\text{miss}})|$ variable was used as well in the fit. For the signal, EWK and Top backgrounds those
 556 distributions are obtained using simulation, and include calibration corrections. The corresponding MJ
 557 background *templates*, are obtained from the data in a separate control region where we applying the same
 558 kinematic selections but inverting the isolation cuts. The choice of lepton isolation definition to define the
 559 SR and the CR1 for electron and muon channel is sketch in Figure 10 and explained in Section 4.3.

560 The definition of the event categories used for the MJ extraction are defined following naming convention
 561 in Ref. [44]:

- 562 • fit region (FR): which correspond to the analysis signal region (SR), with one isolated lepton
 563 ($ptvarcone20/p_T < 0.1$ and $topoetcone20/p_T < 0.05$), but all distributions includes overflow
 564 bins;
- 565 • control region 1 (CR1): Multijet enriched CR where the lepton is required to be anti-isolated
 566 ($topoetcone20/p_T < 0.05$ and $ptvarcone20/p_T > 0.1$ for muons or $ptvarcone20/p_T < 0.1$ and
 567 $topoetcone20/p_T > 0.05$ for electrons) all distributions includes overflow bins;

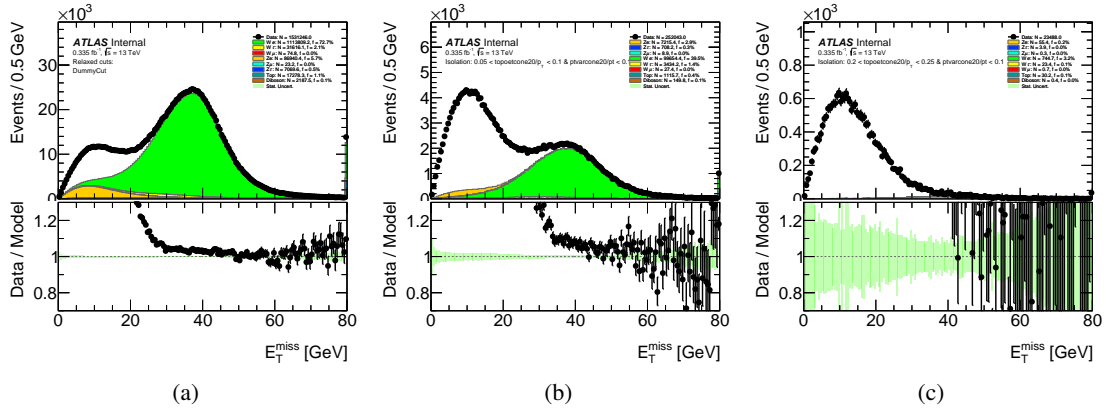


Figure 11: The E_T^{miss} distribution for $W^+ \rightarrow e^+ \nu$ candidates in the fit region (a) and in the CR1 for first anti-isolated slice $0.05 < topoetcone20/p_T < 0.1$ (b) and anti-isolated slice $0.2 < topoetcone20/p_T < 0.25$ (c). The difference between data and simulation is attributed to the missing MJ contribution.

568 As example, the E_T^{miss} distribution for data in the FR is compared only with the signal and the electroweak
 569 and top backgrounds expectation form MC is illustrated in Figure 11(a). The difference between data and
 570 the MC simulation in the fit region is expected to be due to multijet background contribution. CR1 by
 571 construction is dominated by non isolated object which characterised multijet production, and serves to
 572 determine the multijet template distribution used in the FR. Appendix E.1 describe in details the correction
 573 applied on the definition of the recoil reconstruction in the CR1 as also described in Ref. [44]. As it is
 574 explained in the Appendix E.2 in this analysis the CR1 are built scanning only one isolation requirement at
 575 the time.³

576 The MJ template used in the FR is extracted from CR1 while subtracting from the data the contribution
 577 of signal, electroweak and top processes as simulated by MC. Figure 11(b) shown E_T^{miss} distribution for
 578 data and the MC in the CR1 where the lepton is required to to have $0.05 < topoetcone20/p_T < 0.1$
 579 and Figure 11(c) shown E_T^{miss} distribution for data and the MC in the CR1 defined with a lepton to have
 580 $0.2 < topoetcone20/p_T < 0.25$. In Figure 11(b) the fraction of the electroweak and top related process
 581 contribution is 44.6%, and the fraction is drastically reduced in Figure 11(c), however in this slice also the
 582 data statistic is much lower.

583 To study the dependence of the multijet templates on the anti-isolation criterion used to define CR1, the CR1
 584 is split into successive intervals (slice) of the isolation variable, instead of a single, integrated anti-isolated

³ Only track-based isolation is relaxed and sliced for μ^\pm channels and for e^\pm channels only calo-based isolation is relaxed and sliced (as displayed in Figure 10).

585 region. The isolation slices used to define $CR1(i)$ to extract the MJ templates are defined as the following
 586 intervals of lepton isolation :

- 587 • For $W \rightarrow \mu\nu$ $ptvarcone20/pT$ in $[0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5]$ and $topoetcone20/pT <$
 588 0.05
- 589 • For $W \rightarrow e\nu$ $topoetcone20/pT$ in $[0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45]$ and $ptvarcone20/pT <$
 590 0.1

591 Once the $CR1(i)$ slices are defined, the extraction of the number of multijet background events in the FR
 592 proceeds as follows, for a given isolation slice i :

- 593 • The multijet background template in each anti-isolation slices (also called bin), $N_{MJ}^{CR1(i)}$, is derived
 594 as follows:

$$N_{MJ}^{CR1(i)} = N_{data}^{CR1(i)} - N_{EW}^{CR1(i)} \quad (46)$$

595 where $N_{MJ}^{CR1(i)}$ is obtained from each one of the kinematic distributions used in the fit, p_T^ℓ , E_T^{miss} ,
 596 m_T or $|\Delta\phi(\ell, E_T^{miss})|$.

- 597 • The fraction fit is performed for p_T^ℓ , E_T^{miss} , m_T or $|\Delta\phi(\ell, E_T^{miss})|$ distributions in the FR for
 598 each multijet template obtained in each anti-isolation slices. Figure 12 shows as an example the
 599 E_T^{miss} distribution in the FR, after fitting the multijet templates, $N_{MJ}^{CR1(i)}$, derived from different
 600 anti-isolation slices in $CR1(i)$. It's written as:

$$N_{data}^{FR} = \alpha \cdot N_{MC}^{FR} + T(i) \cdot N_{MJ}^{CR1(i)}. \quad (47)$$

601 The fitting parameter $T(i)$ gives the estimated number of MJ events in SR: $N_{MJ}^{SR} \approx T(i) \cdot N_{MJ}^{CR1(i)}$.
 602 An overall normalisation factor for the signal and EW+top contribution (N_{MC}^{FR}), α , in this analysis is
 603 fixed to 1 during the fit.

604 In Eqs. 47 the multijet distributions in the fitting region are approximated by their anti-isolated counterparts
 605 from $CR1(i)$. The residual dependence of the multijet background estimate on the anti-isolation criteria
 606 choose to built the template, introduces a systematic effect in the procedure. To reduce this dependence the
 607 MJ yields extracted in the FR using the different anti-isolation slices are used to extrapolate the final MJ
 608 yields in the signal region at very low isolation values. The final MJ background estimation is therefore
 609 extrapolated toward the SR for both the total yield and the template shape. Section 5.2 describe the final
 610 MJ yield extraction, and Section 5.3 the MJ template shape extrapolation.

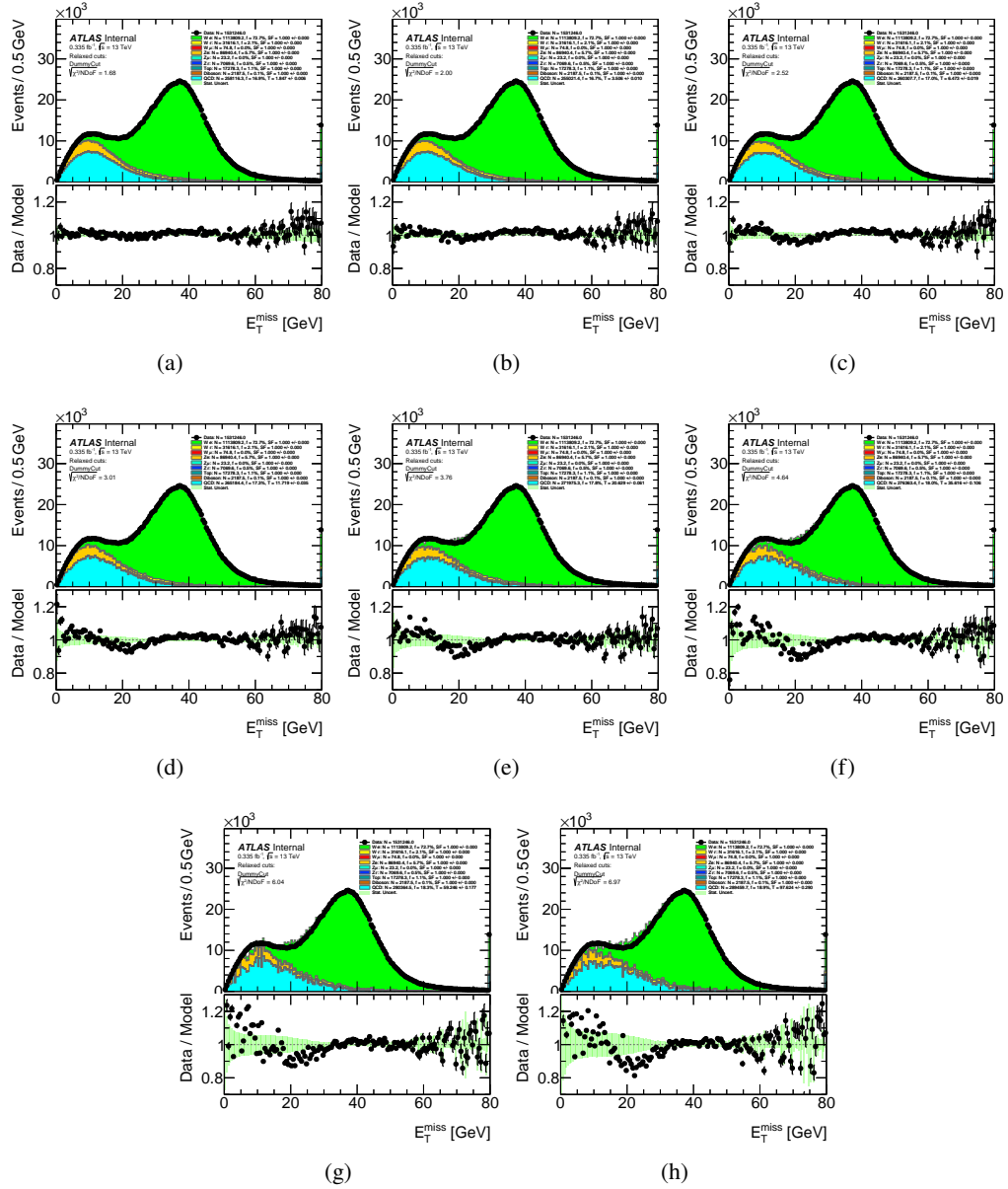


Figure 12: The multijet template distributions used in the fraction fit for $W^+ \rightarrow e^+ \nu$ channel. The MJ distributions are derived in CR1 and fitted to the data in FR and indicated by light-blue color. 8 control region slices with different $topocone20/p_T$ cuts are constrained from (a) $0.05 < topocone20/p_T < 0.1$ to (h) $0.4 < topocone20/p_T < 0.45$.

611 5.2 MJ yield normalization

612 The total MJ yield in the analysis SR is evaluated by extrapolating the number of MJ events extracted from
 613 the fraction fit using different MJ template build from different anti-isolated bin towards the isolated regions.
 614 In the FR the MJ yield is extracted performing the fraction fit using Eq. 47 for each anti-isolation template
 615 $N_{MJ}^{CR1(i)}$ in different discriminating distributions. The MJ yields extracted using different anti-isolation
 616 slice for all the discriminating variable (p_T , m_T , E_T^{miss} and $|\Delta\phi(\ell, E_T^{miss})|$) are shown in Figure 13. The

617 error on each measured MJ yields is multiplied by $\sqrt{\chi^2/NDof}$ resulting from the fraction fit to account
 618 for possible mismodelling in the template of the considered variables.

619 A linear fit is used to extrapolate the MJ yield extracted using different anti-isolation slices to the signal
 620 region (SR) separately for each discriminative variables p_T , m_T , E_T^{miss} and $|\Delta\phi(\ell, E_T^{miss})|$. The average
 621 isolation for the final extrapolation of the MJ background is taken as $X = 0.025$ from $b\bar{b}$ and $c\bar{c}$ samples
 622 studies [44]. The final central value of the MJ yield in SR is considered to be the average of the linear
 623 extrapolation at $X = 0.025$ of the four observables estimate. Figure 13 presents linear fit used for the
 624 the yield extrapolation to SR, for each channel, where p_T , m_T , E_T^{miss} and $|\Delta\phi(\ell, E_T^{miss})|$ ideally should
 625 converge at $X = 0.025$.

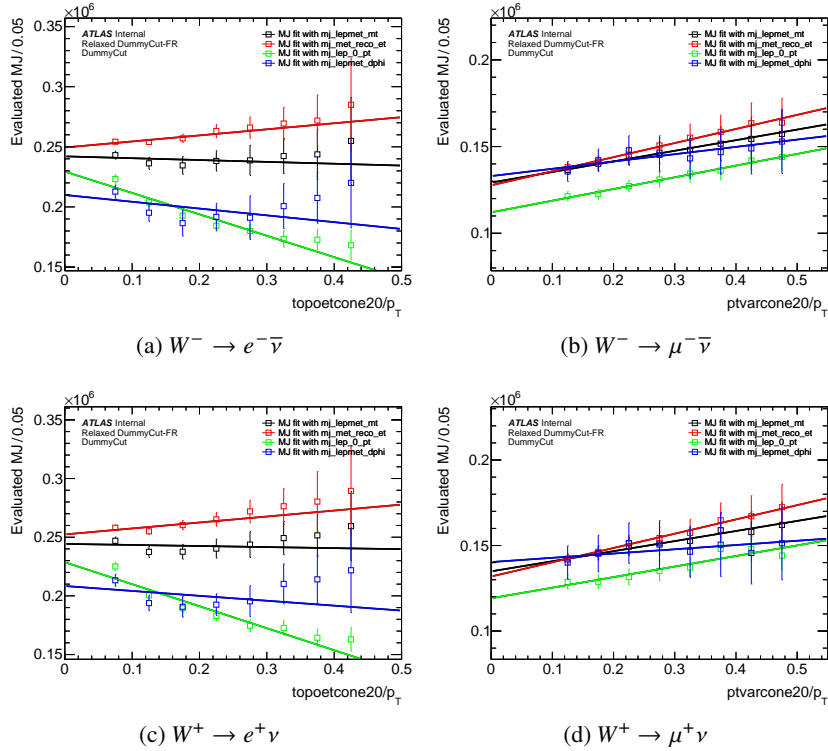


Figure 13: The estimated total number of the multijet background in SR scanned as a function of the isolation slice. Isolation scans in the $W^- \rightarrow e^- \bar{\nu}$, $W^- \rightarrow \mu^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$ and $W^+ \rightarrow \mu^+ \nu$ with recoil correction. Each point is plotted as the central of each isolation slice and correspond to the result of Eq. 47. Every set of points corresponds to the one of four multijet discriminative variables m_T , E_T^{miss} , p_T , and $|\Delta\phi(\ell, E_T^{miss})|$.

626 The results for the final yield estimation as well as the uncertainty breakdown in the $W^- \rightarrow e^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$,
 627 $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels is showed in Tab. 3. All the numbers and plots related to this
 628 estimation method are presented in the Appendix E.3.

629 The numbers of the MJ background estimated in the electron and positron channels are similar as expected
 630 due to the fake rates of a jet to fake an electron and to fake a positron should be the same. Good compatibility
 631 is observed between the two charges. Accounting for the systematics uncertainty the MJ yields in muon
 632 and anti-muon channels shows no significant discrepancy.

633 A closure test has been performed on this method using $b\bar{b} + c\bar{c}$ MC samples as a source of multijet
 634 templates in the CR1 passing all kinematic and anti-isolation selection. Results are in consistency with

635 data-driven method. Details on this are in the Appendix E.4.

636 5.3 Correction of the multijet background shapes in the signal region

637 The MJ templates from anti-isolated control regions themselves cannot correctly represent the MJ shape in
 638 the isolated regions. To solve this issue, a shape correction is applied to all MJ templates in CR1 according
 639 to the prescription below. The MJ shape is defined to be:

$$H_{MJ}^{[0.A,0.B]}[X] = H_{data}^{[0.A,0.B]}[X] - H_{EW+Top}^{[0.A,0.B]}[X] \quad (48)$$

640 where $H_{MJ}^{[0.A,0.B]}[X]$ is the normalized distribution of each kinematic variable X in CR1 after subtraction
 641 of the EW and top processes, and satisfying $A < isoCone/p_T < B$, where $isoCone$ is $ptvarcone20$ for
 642 $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ and $topoetcone20$ for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$. The shape correction
 643 representing the dependence on isolation variable (as example when $isoCone$ is $ptvarcone20$) is calculated
 644 via:

$$\Delta^{CR1}[X] = \frac{1}{4} \left\{ \frac{H_{MJ}^{0.1,0.15} - H_{MJ}^{0.3,0.35}}{4} + \frac{H_{MJ}^{0.15,0.2} - H_{MJ}^{0.35,0.4}}{4} + \frac{H_{MJ}^{0.2,0.25} - H_{MJ}^{0.4,0.45}}{4} + \frac{H_{MJ}^{0.25,0.3} - H_{MJ}^{0.45,0.5}}{4} \right\} \quad (49)$$

645 which helps estimate the MJ shape in SR:

$$H_{MJ}^{SR}[X] = H_{MJ}^{0.1,0.15}[X] + 2 \cdot \Delta^{CR1}[X] \quad (50)$$

646 The correction term $\Delta[X]$ is purely built upon the MJ shape in 8 anti-isolated slices of CR1. With the
 647 above correction, the shapes of MJ templates each CR1(i) are extrapolated to an uniquely determined shape
 648 in SR as shown in Figure 14 to 17, which means that the fraction fit is effectively carried out only once for
 649 a given kinematic distribution.

650 Same strategy, but with different binning is used in case when $isoCone$ is $topoetcone20$ for $W^- \rightarrow e^- \bar{\nu}$
 651 and $W^+ \rightarrow e^+ \nu$ channels.

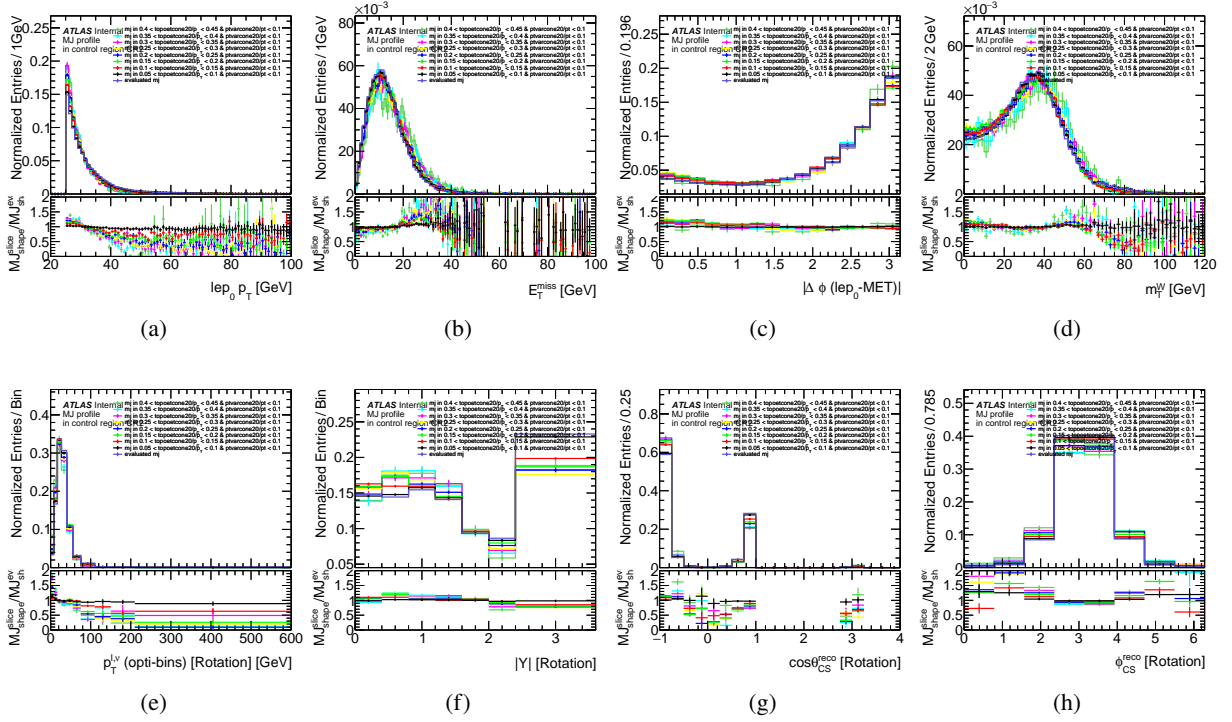


Figure 14: Normalized MJ shapes in anti-isolated slices of CR1 for $W^- \rightarrow e^- \bar{\nu}$ channel, along with the shape corrected to SR.

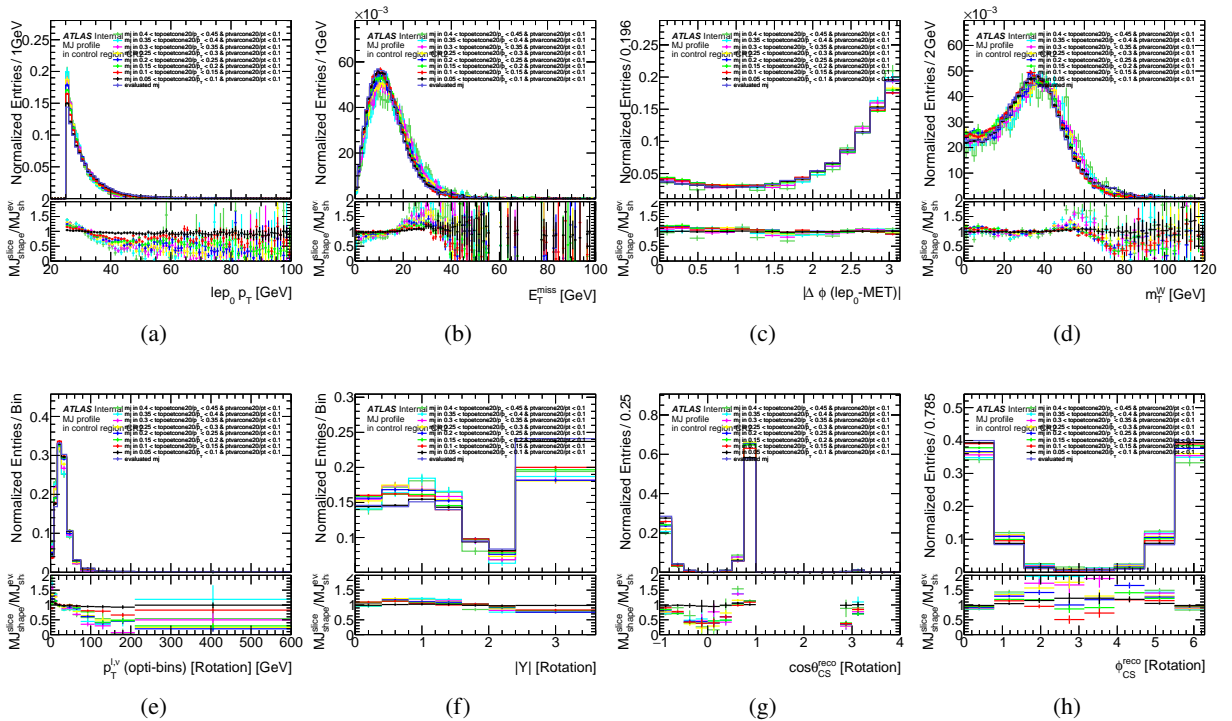


Figure 15: Normalized MJ shapes in anti-isolated slices of CR1 for $W^+ \rightarrow e^+ \nu$ channel, along with the shape corrected to SR.

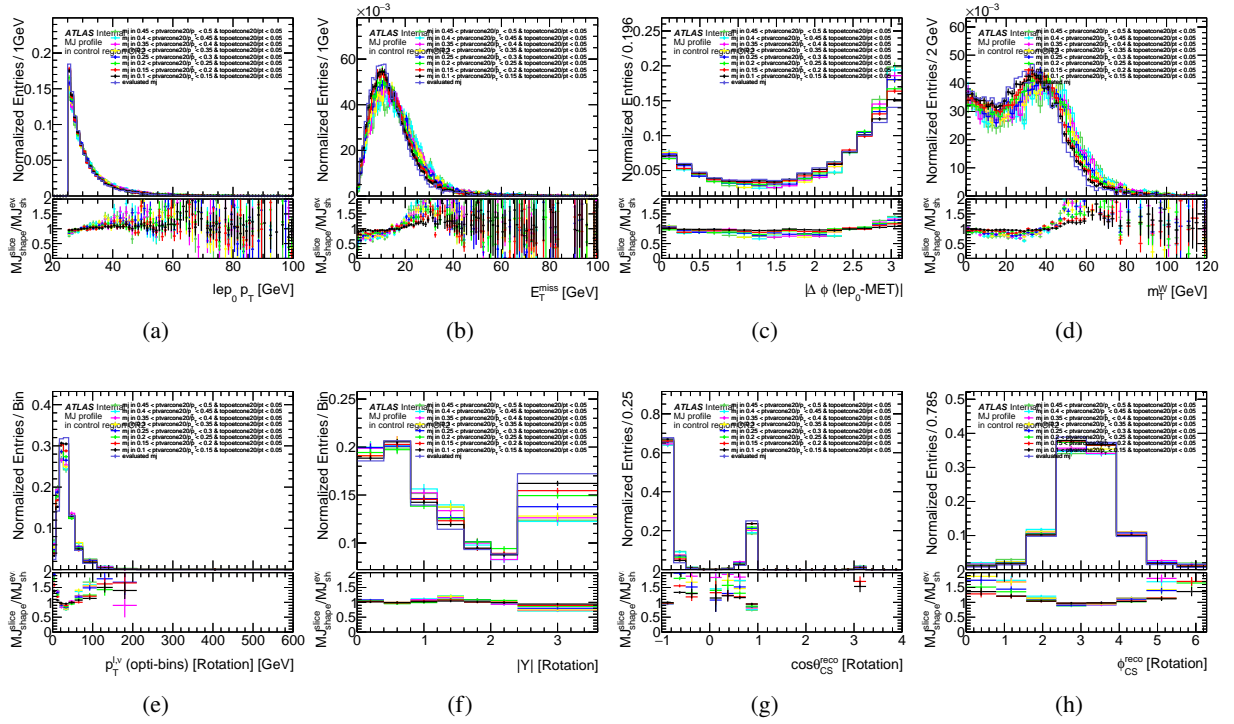


Figure 16: Normalized MJ shapes in anti-isolated slices of CR1 for $W^- \rightarrow \mu^- \bar{\nu}$ channel, along with the shape corrected to SR.

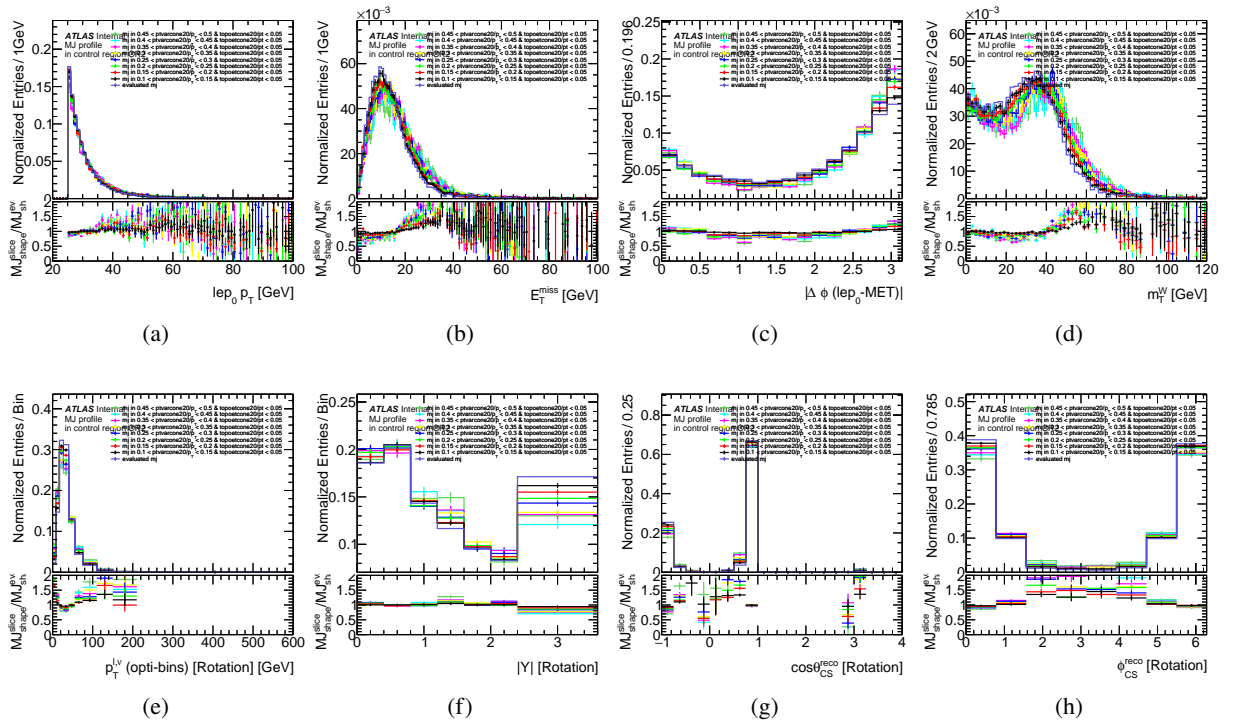


Figure 17: Normalized MJ shapes in anti-isolated slices of CR1 for $W^+ \rightarrow \mu^+ \nu$ channel, along with the shape corrected to SR.

652 5.4 MJ acceptance correction

653 Figure 18 shows the final 2D multijet templates as function of $\cos \theta_{CS}$ and ϕ_{CS} distributions normalised
 654 to the total multijet yield for the inclusive signal region.

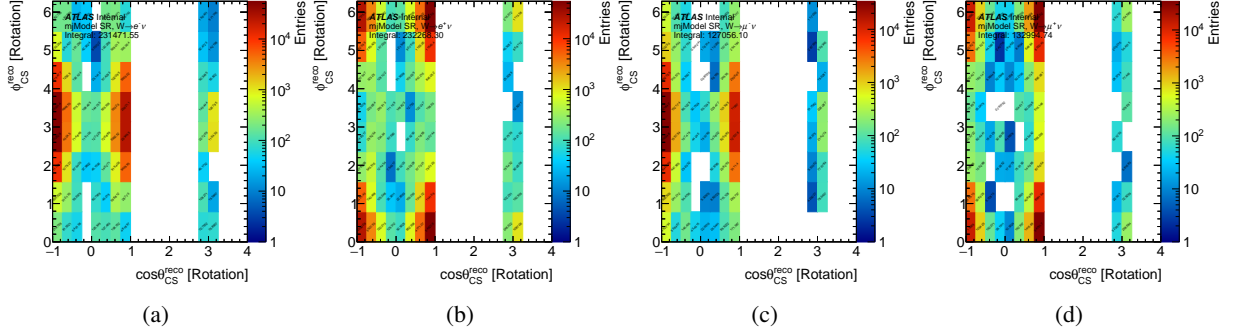


Figure 18: The multijet templates of the $\cos \theta_{CS}$ vs. ϕ_{CS} distributions for $W^- \rightarrow e^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$, $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels for the inclusive signal region.

655 In this analysis MJ templates need to be provided as function of $\cos \theta_{CS}$ and ϕ_{CS} for each $p_T^{\ell, \nu}$ and $|Y|$
 656 measurement bin. As a reminder the extraction of the angular coefficient is done in $p_T^{\ell, \nu}$ and $|Y|$ bins as
 657 defined in the following:

$$658 \quad p_T^{\ell, \nu} : [0, 8, 17, 27, 40, 55, 75, 110, 150, 210, 600]; \quad (51)$$

$$659 \quad |Y| : [0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 3.6] \quad (52)$$

660 Running directly the data-driven method to provide multijet shape and yield estimation for each $p_T^{\ell, \nu}$ or $|Y|$
 661 bin individually would not work due to significant statistics drop in the CR1 regions used to extract MJ
 662 templates. However using a single 2D multijet template ($\cos \theta_{CS}^{reco}$ vs. ϕ_{CS}^{reco}) derived from the inclusive
 663 SR for all $p_T^{\ell, \nu}$ and $|Y|$ bins wont work because there are strong dependence of the $\cos \theta_{CS}$ multijet shape
 664 as function $|Y|$ and ϕ_{CS} as function of $p_T^{\ell, \nu}$ as it is shown in Figure 19. Therefore the 2D $\cos \theta_{CS}$ vs ϕ_{CS}
 665 MJ distribution extracted in the inclusive signal region (Figure 18 shows this distributions for $W^- \rightarrow e^- \bar{\nu}$,
 666 $W^+ \rightarrow e^+ \nu$, $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels) could be extrapolated in each $p_T^{\ell, \nu}$ and $|Y|$ bins using
 667 so called *acceptance correction functions* to account the kinematic changes in the different $p_T^{\ell, \nu}$ and $|Y|$
 668 bins : $[A_j = h_{MC}^{bin_j} / h_{MC}^{Inc.}]$

669 To built those *acceptance correction* special background MC samples that are expected to have kinematics
 670 similar to MJ kinematics are used. The $b\bar{b}+c\bar{c}$ MC samples for $W^\pm \rightarrow \mu^\pm \nu$ channels and JF17 MC
 671 samples for $W^\pm \rightarrow e^\pm \nu$ channels are used to this scope (more details in Appendix E.5.1).

672 To validate this procedure the full MJ extraction was also extracted directly from data in 3 coarse $p_T^{\ell, \nu}$ and
 673 $|Y|$ regions define as :

$$674 \quad p_T^{\ell, \nu} : [0, 17, 55, 600]; \quad (53)$$

$$675 \quad |Y| : [0, 0.8, 2.0, 3.6] \quad (54)$$

676 In this coarse regions the full data-driven multijet estimation described in Section 5.1 was repeated to
 677 estimate the MJ yield and shape. The 2D MJ template extracted form data in coarse $p_T^{\ell, \nu}$ and $|Y|$ regions

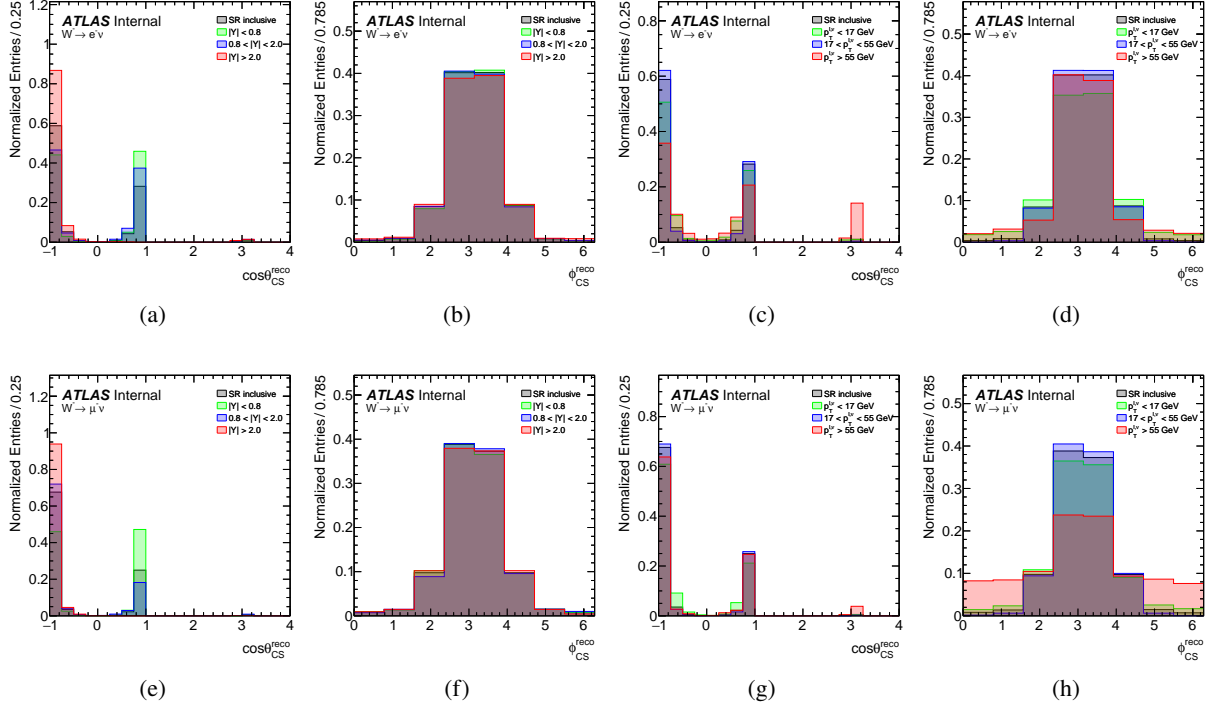


Figure 19: Projections for data-driven multijet templates derived for the inclusive signal region compared with 3 different $p_T^{\ell,\nu}$ or $|Y|$ bins in for $W^- \rightarrow e^- \bar{\nu}$ (top) and $W^- \rightarrow \mu^- \bar{\nu}$ (bottom) channels.

678 are compared with the 2D MJ template extracted from the data inclusive region and corrected with the
679 corresponding *acceptance correction* [$B_j = h_{MC}^{bin_k} / h_{MC}^{Inc}$, where the index k run on the coarse region define in
680 Formula 53]. Results of the closure test are shown on Figure 20 for $W^- \rightarrow e^- \bar{\nu}$ and $W^- \rightarrow \mu^- \bar{\nu}$ channels.
681 As results form this closure test it was verified that in particular for the electron channel extrapolating the
682 data driven inclusive MJ 2D template might results in suboptimal results.

683 Therefore the extraction of the final 2D multijet background template for a given $p_T^{\ell,\nu}$ or $|Y|$ bin j (define
684 in Eq. 51) proceeds as follows:

- 685 • Derive the shape of multijet background events as function of $\cos \theta_{CS}^{reco}$ and ϕ_{CS}^{reco} distribution from
686 data in 3 coarse bin, k , define in Eq. 53 ($h_{MJ,data}^{bin_k}$). The correction on the multijet background
687 shapes described in the Section 5.3 is applied.
- 688 • With the dedicated MC samples a multijet template is built for the coarse $|Y|$ or $p_T^{\ell,\nu}$ bins k ($h_{MC}^{bin_k}$)
689 and for the fine j -bin ($h_{MC}^{bin_j}$).
- 690 • *Acceptance correction* for each $p_T^{\ell,\nu}$ or $|Y|$ measurement bin are computed in MC as the difference
691 in multijet templates between coarse bin k to a given j bin shape : $A_j = h_{MC}^{bin_j} / h_{MC}^{bin_k}$.
- 692 • Apply acceptance correction to the data-driven template from the coarse bin k to extrapolate to get
693 template shape for a given j bin:

$$h_{MJ}^{bin_j} = h_{MJ,data}^{bin_k} \times A_j \quad (55)$$

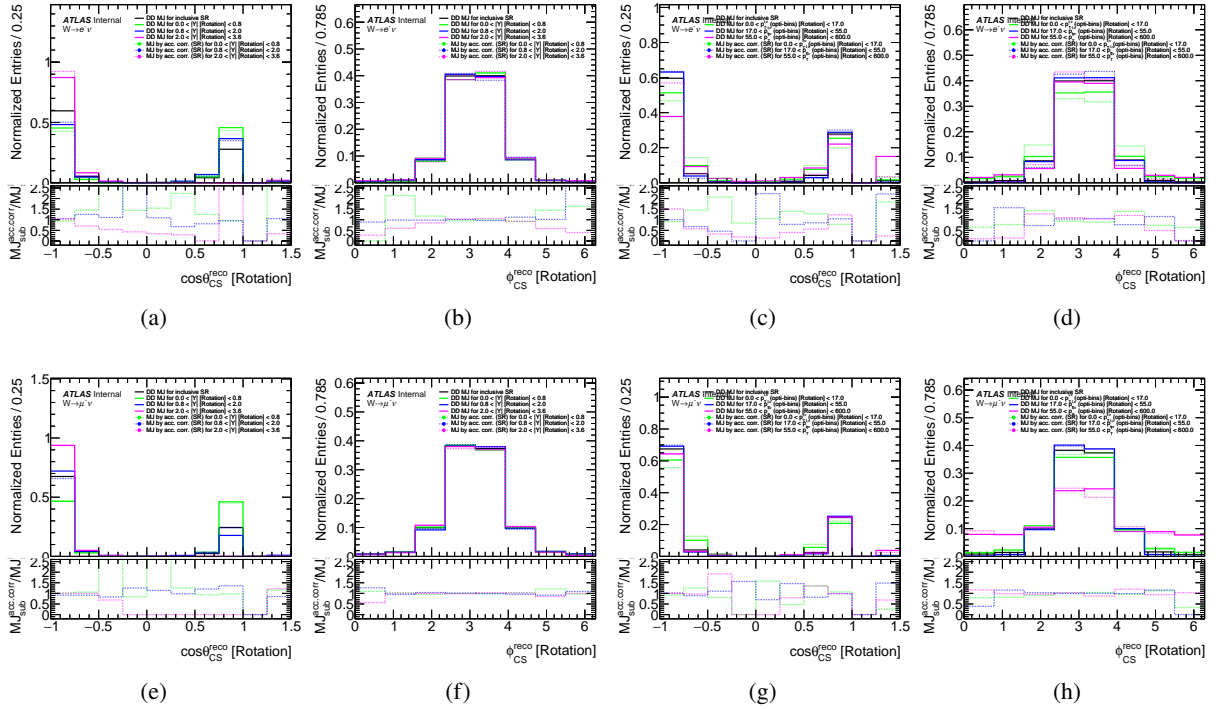


Figure 20: Projections for data-driven multijet templates (solid lines) and templates provided by acceptance correction method (dashed lines) derived for 3 individual subregions for $W^- \rightarrow e^- \bar{\nu}$ (top) and $W^- \rightarrow \mu^- \bar{\nu}$ (bottom) channels. Daniil can you specify in the here in the caption what is in the ratio plot of the figure ?

694 This procedure is shown for selected number of $|Y|$ and $p_T^{\ell, \nu}$ bins on Figures 21-22 for the 1D pro-
695 jection in $\cos \theta_{CS}^{reco}$ and ϕ_{CS}^{reco} distributions for $W^- \rightarrow e^- \bar{\nu}$ channel. Figures 23-24 shows same
696 distributions for $W^- \rightarrow \mu^- \bar{\nu}$ channel. The 1D distributions are shown for simplicity: the accept-
697 ance corrections for the 2D distributions follows the same steps. Daniil can you please add also
698 here just for illustration a plot of the 1D projection of the acceptance correction themself ? eg the
699 the azure curve in slide 8 [https://indico.cern.ch/event/1218457/contributions/5125277/
700 attachments/2540781/4373906/20221103_Ponomarenko_WAi_MJ.pdf](https://indico.cern.ch/event/1218457/contributions/5125277/attachments/2540781/4373906/20221103_Ponomarenko_WAi_MJ.pdf) Tow sources of uncertainty
701 are consider for this acceptance correction. One source of systematic is define as the relative difference
702 between $h_{MC}^{bin_k}$ and $h_{MJ,data}^{bin_k}$ and is taken correlated within the coarse bin k is , the other one is the statistical
703 uncertainty of the MC samples template derived for j bins, $h_{MC}^{bin_j}$, which is included into analysis as fully
704 uncorrelated uncertainty on acceptance correction functions.

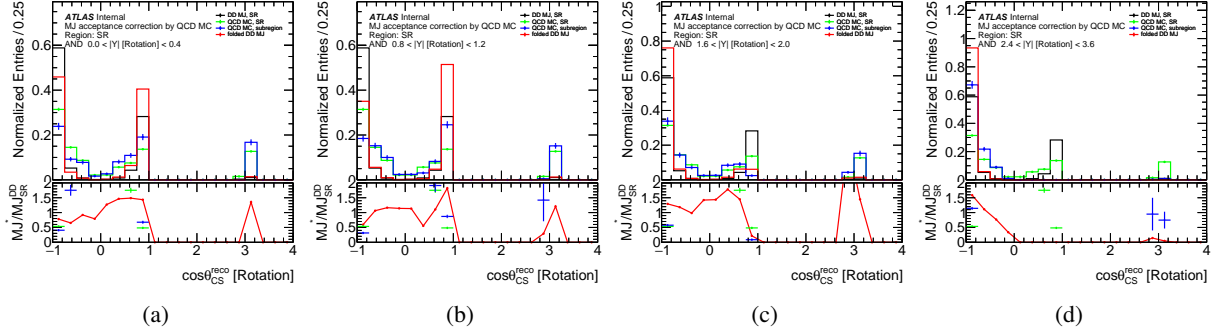


Figure 21: Multijet background template acceptance correction functions for some of $|Y|$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel. Daniil not clear to me if this is done form the inclusive SR or form the coarse bins, can you specify ?

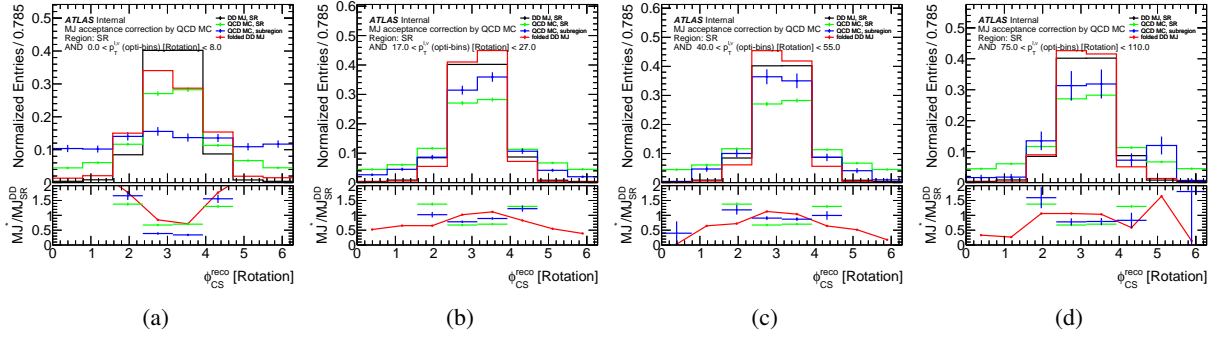


Figure 22: Multijet background template acceptance correction functions for some of $p_T^{\ell, \nu}$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

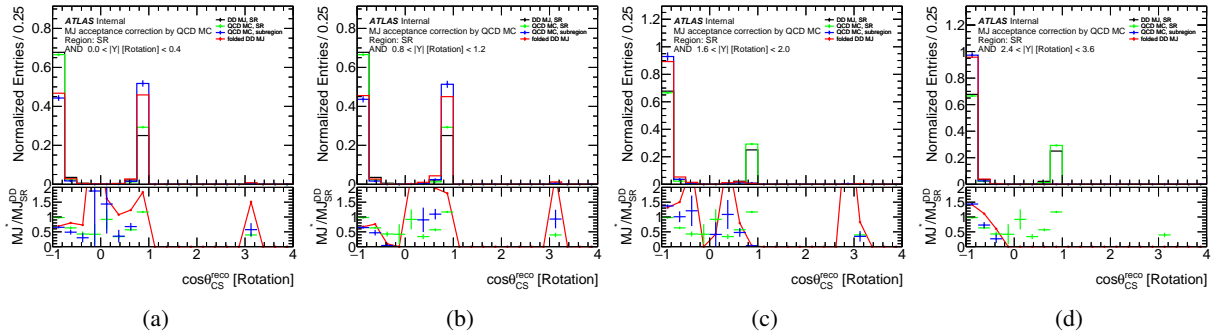


Figure 23: Multijet background template acceptance correction functions for some of $|Y|$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

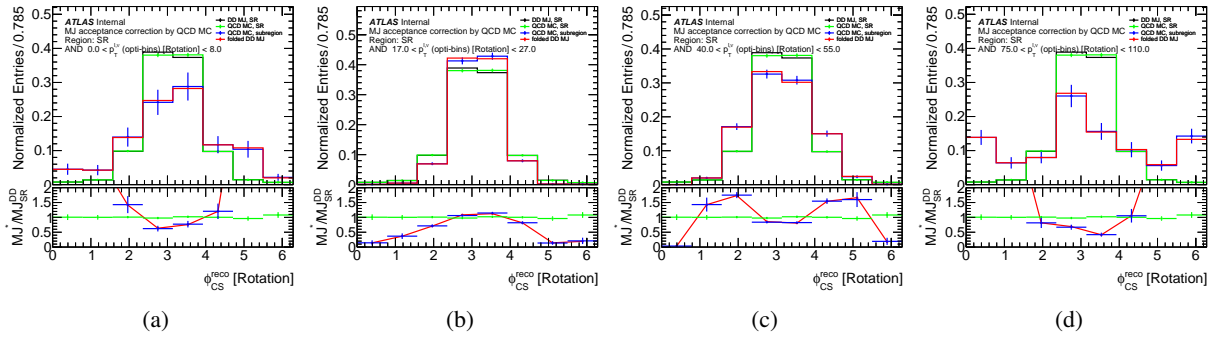


Figure 24: Multijet background template acceptance correction functions for some of $p_T^{\ell, \nu}$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

705 5.5 Sources of uncertainty in the MJ background estimation

706 The uncertainty of MJ background estimation arises from 3 major sources: total yield, the shape and
 707 statistics. These three components have different correlation patterns: the uncertainty of the total yield as
 708 well as the shape uncertainty is fully correlated across the measurement bin, while statistical uncertainty is
 709 uncorrelated due to its statistical nature.

710 The uncertainty breakdown for the final MJ estimation in the inclusive signal region for the $W^- \rightarrow e^- \bar{\nu}$,
 711 $W^+ \rightarrow e^+ \nu$, $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels is showed in Tab. 3. The complete brake down of the
 712 different sources of uncertainty investigated in this analysis are discussed below.

Channel	$W^- \rightarrow e^- \bar{\nu}$	$W^+ \rightarrow e^+ \nu$	$W^- \rightarrow \mu^- \bar{\nu}$	$W^+ \rightarrow \mu^+ \nu$
Total Number of MJ bkg	231472	232268	127056	132995
Luminosity and cross section	± 7199 (3.11%)	± 9098 (3.92%)	± 547 (0.43%)	± 698 (0.52%)
Intersection point	± 22928 (9.91%)	± 24910 (10.72%)	± 13313 (10.48%)	± 12276 (9.23%)
Extrapolation target	± 1246 (0.54%)	± 1172 (0.50%)	± 1575 (1.24%)	± 1433 (1.08%)
Choice of hist	± 7643 (3.30%)	± 8303 (3.57%)	± 4438 (3.49%)	± 4092 (3.08%)
Isolation correction	± 453 (0.20%)	± 686 (0.30%)	± 466 (0.37%)	± 381 (0.29%)
Correlated Uncertainty	± 24033 (10.38%)	± 26521 (11.42%)	± 13324 (10.49%)	± 12296 (9.25%)
Data Stat.	± 490 (0.21%)	± 463 (0.20%)	± 795 (0.63%)	± 809 (0.61%)
MC Stat.	± 262 (0.11%)	± 229 (0.10%)	± 990 (0.78%)	± 880 (0.66%)
Shape Correction	± 2961 (1.28%)	± 2769 (1.19%)	± 927 (0.73%)	± 915 (0.69%)
Uncorrelated Uncertainty	± 3012 (1.30%)	± 2816 (1.21%)	± 1572 (1.24%)	± 1505 (1.13%)

Table 3: MJ background yield and the impact on the uncertainty on the yield due to shape uncertainty breakdown for inclusive signal region in the $W^- \rightarrow e^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$, $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels.

713 The following sources of uncertainty are evaluated for the MJ background yield extraction described in
 714 Section 5.2:

- 715 • Uncertainty due to the luminosity and the cross section of the simulated samples. In several
 716 anti-isolated slice in CR1 the contribution from the MC backgrounds is largest. This source of
 717 uncertainty is included in the uncertainty of MJ background.
- 718 • Intersection point: Uncertainty in the yield caused by the spread of the linear extrapolation as a
 719 function of isolation $k^i X + b^i$ using the different discriminating variable i (p_T , m_T , E_T^{miss} and
 720 $|\Delta\phi(\ell, E_T^{miss})|$). This uncertainty mainly affects the total number of the MJ background, which is
 721 estimated with the maximum and minimum $k^i X + b^i$ values. Here X is the isolation value for the
 722 central value, which is 0.025. These 4 lines used in the uncertainty sources which could lead to an
 723 bad intersection point. This is one of the reason that the statistical uncertainty is ignored from the
 724 uncertainty of the extrapolation to avoid double counting.
- 725 • Uncertainty from the choice of the extrapolation target, estimated by changing the isolation target
 726 from 0.025 to 0.
- 727 • Choice of discriminating variable: The uncertainty due to the choice of the distributions used to
 728 extract the MJ in the FR eg p_T , m_T , E_T^{miss} and $|\Delta\phi(\ell, E_T^{miss})|$, is estimated by removing from the
 729 linear extrapolation one of the results from the MJ yield extraction using a discriminative variable.
 730 The maximum difference is quoted as the uncertainty.

731 • Isolation correction: The uncertainty due to the isolation correction is estimated by changing the
732 isolation correction by 10%. This uncertainty is negligible comparing to the statistical uncertainty
733 of the first slice of CR1. So only the contribution to the total multijet number is considered.

734 The following sources of uncertainty are estimated for the shape of the MJ background:

735 • Bin-by-bin uncorrelated data and MC statistics: the statistical uncertainty from the limited data and
736 MC samples is taken from the results of shape extrapolation according to Eq. 50.

737 • Uncertainty due to the shape correction (described in details in 5.3), which mainly affects the shape
738 of the MJ background. Shape uncertainty is defined to be the difference between the extrapolated
739 shape in SR and the MJ shape of the first slice in CR1 (Eq. 49).

740 As the final output of MJ estimation three sources of uncertainty are propagated to the measurement
741 (Table 5) as combination of uncertainties listed above. The total uncertainty related to the MJ yield
742 extraction and the uncertainty related to the shape of the MJ background are taken as fully correlated
743 across the measurement bin. The systematic uncertainty from the *acceptance correction* which raise
744 from the $b\bar{b} + c\bar{c}$ or *JF17* MC statistics in the given p_T^ℓ or $|Y|$ bin is taken as fully uncorrelated across
745 the measurement bin and a correlated source across coarse bin k defined in Eq. [eq:coreseBin] is also
746 accounted to take care of possible data/MC differences .

747 6 Control Plots

748 In Table 4 $W^\pm \rightarrow \ell^\pm \nu$ signal selection event yields for the 13 TeV dataset are shown. The yields for the
 749 signal and for the electroweak and top backgrounds are obtained from MC samples. The estimate of the
 750 multijet background passing due selection of leptons produced in semi-leptonic decays of heavy quarks,
 751 in-flight decays, or misidentification of photon conversions or hadrons, is done performed with data-driven
 752 techniques described in Section 5.

Channel	$W \rightarrow e^- \nu$	$W \rightarrow e^+ \nu$	$W \rightarrow \mu^- \nu$	$W \rightarrow \mu^+ \nu$
Wenu	863254.04 ± 267.09 (stat)	1113809.20 ± 304.28 (stat)	3.17 ± 0.52 (stat)	4.51 ± 0.63 (stat)
Wmumu	66.76 ± 2.56 (stat)	74.80 ± 2.88 (stat)	964361.45 ± 284.37 (stat)	1269812.83 ± 323.72 (stat)
Wtaunu	26349.67 ± 146.26 (stat)	31616.08 ± 166.55 (stat)	25398.98 ± 143.14 (stat)	31343.83 ± 165.80 (stat)
Zee	82869.92 ± 66.43 (stat)	86940.36 ± 67.86 (stat)	0.68 ± 0.22 (stat)	0.36 ± 0.19 (stat)
Zmumu	26.61 ± 1.59 (stat)	23.16 ± 1.30 (stat)	69322.58 ± 59.46 (stat)	74223.55 ± 61.39 (stat)
Ztautau	6934.17 ± 77.68 (stat)	7069.61 ± 78.17 (stat)	7610.59 ± 87.26 (stat)	7686.91 ± 87.63 (stat)
Top	16581.90 ± 55.33 (stat)	17278.30 ± 57.85 (stat)	16299.96 ± 54.65 (stat)	17028.52 ± 58.24 (stat)
Diboson	2170.16 ± 48.59 (stat)	2187.45 ± 51.63 (stat)	2202.69 ± 49.58 (stat)	2321.34 ± 52.10 (stat)
MultiJet	231471.55	232268.30	127056.10	132994.74
Data	1259751.00 ± 1122.39 (stat)	1531246.00 ± 1237.44 (stat)	1206859.00 ± 1098.57 (stat)	1533100.00 ± 1238.18 (stat)

Table 4: Number of data yield and the expected contribution from the MC process and the MJ. Errors quoted are just statistical evaluated as simple \sqrt{N} for MC statistic and data statistic, for systematic error related to MJ estimate refer to Section 5.5

753 Control plots for the $W^- \rightarrow e^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$ (Figures 25-26) and $W^- \rightarrow \mu^- \bar{\nu}$, $W^+ \rightarrow \mu^+ \nu$ (Figures 27-28)
 754 channels at 13 TeV are provided here after applying the event selection and all the corrections described in
 755 Section 4.3. Only MJ systematics uncertainty is shown in the ratio plots. Above 100 GeV the data/MC
 756 agreement in the $p_T^{\ell, \nu}$ distributions degradate due to missing truth p_T^W reweighting correction above this
 757 range. This discrepancy doesn't affects W-Ai measurement as analysis is performed in the $p_T^{\ell, \nu}$ bins.
 758 However p_T^W reweighting above 100 GeV would be addressed in the next NTuple iterations.

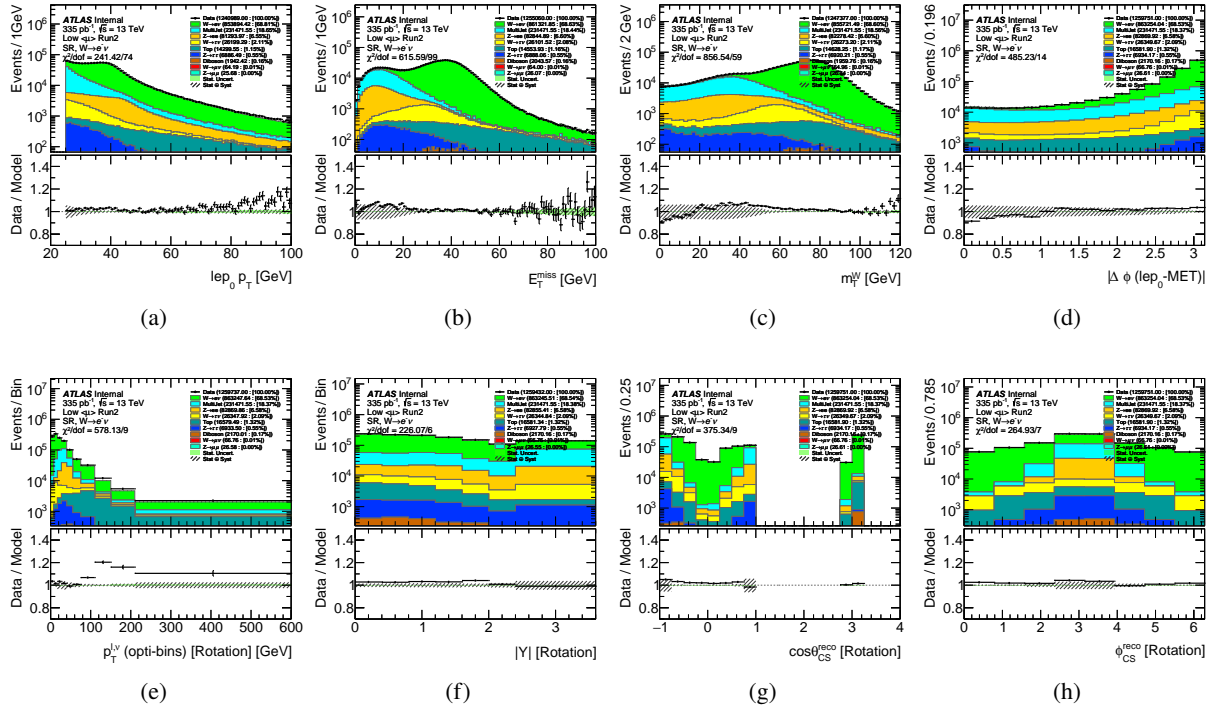


Figure 25: Control plots for inclusive signal region for $W^- \rightarrow e^- \bar{\nu}$ channel. Only overall multijet systematics uncertainty is shown.

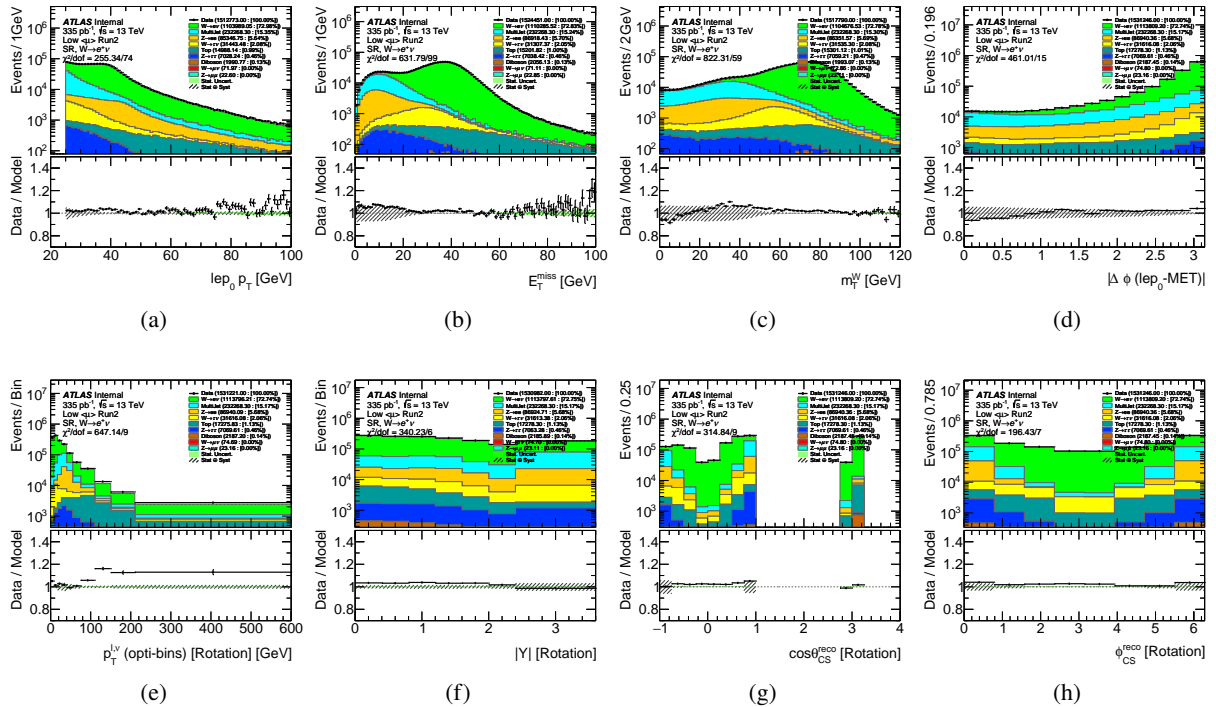


Figure 26: Control plots for inclusive signal region for $W^+ \rightarrow e^+ \nu$ channel. Only overall multijet systematics uncertainty is shown.

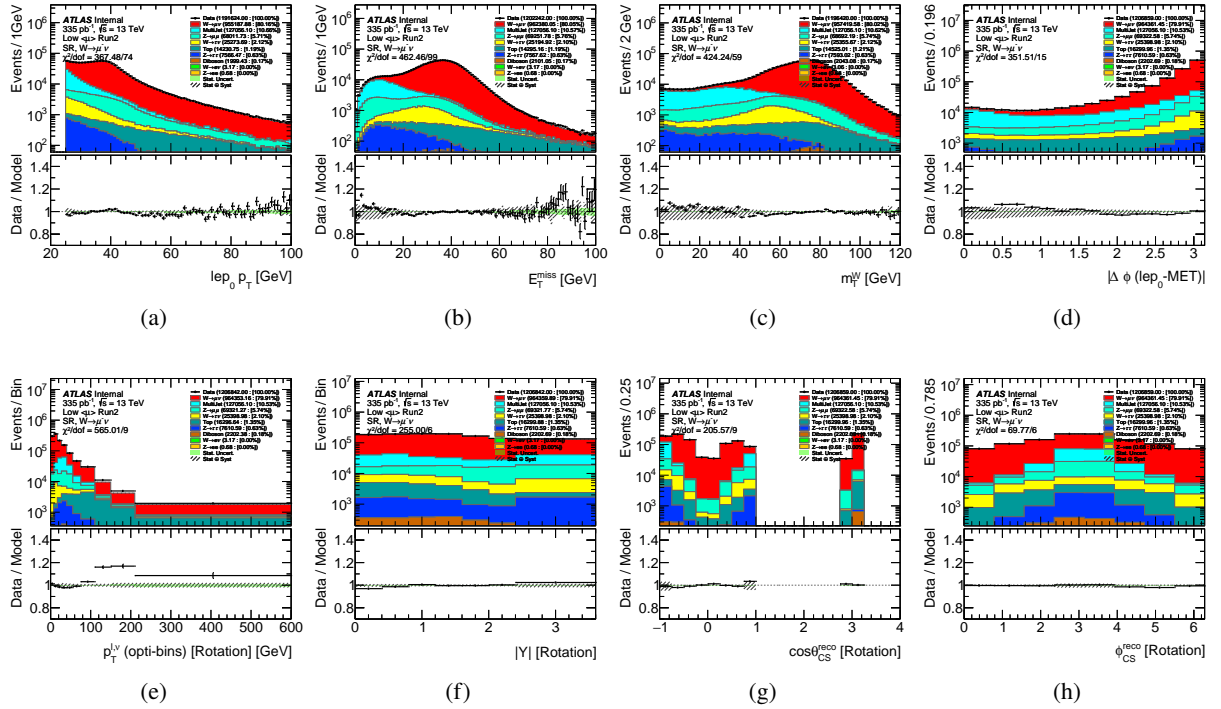


Figure 27: Control plots for inclusive signal region for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Only overall multijet systematics uncertainty is shown.

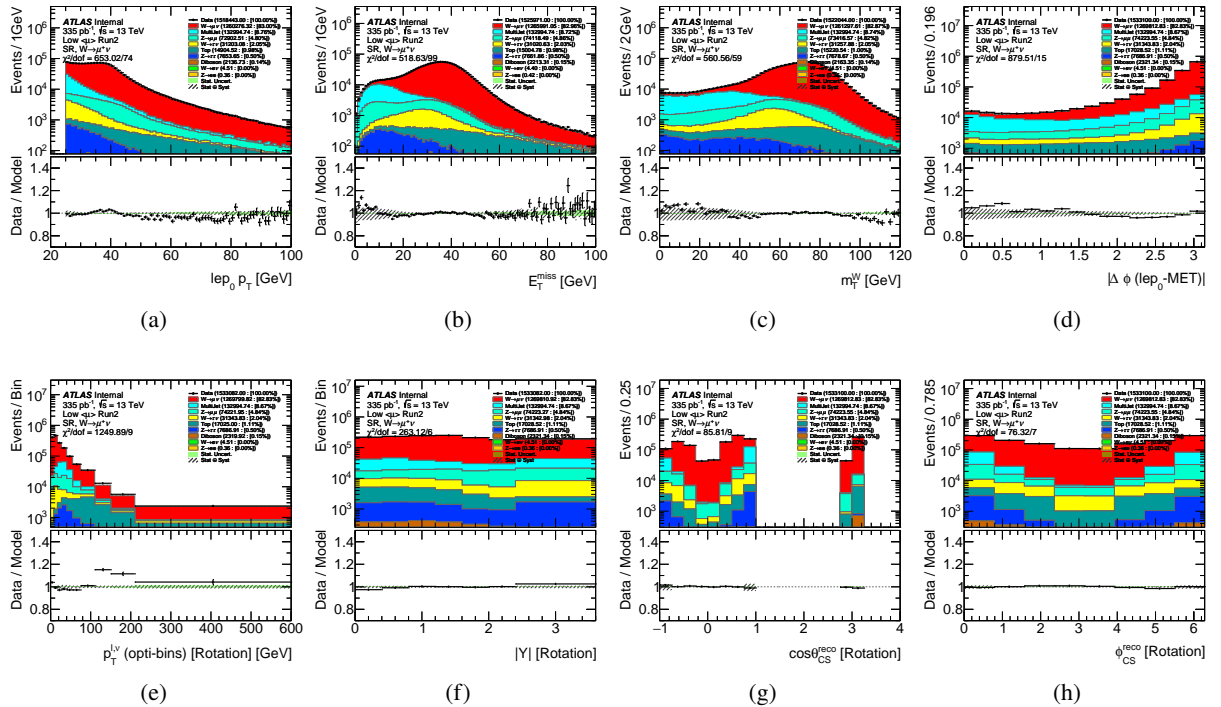


Figure 28: Control plots for inclusive signal region for $W^+ \rightarrow \mu^+ \nu$ channel. Only overall multijet systematics uncertainty is shown.

759 **7 Uncertainties**

760 The dominant source of uncertainty in this measurement is the statistical uncertainties in the data. These
 761 uncertainties are described in Section 7.1. The systematic uncertainties due to impact of experimental and
 762 theoretical uncertainties are described in Section 7.2 and 7.3.

763 **7.1 Statistical Uncertainties**

764 Uncertainties due to the finite data statistics can be divided into several categories as shown in Figure 29 for
 765 an example of A_0 in $p_T^{W, Truth}$ bin 0. Uncertainties due to the parameter of interest are labeled as data-stat
 766 uncertainties where the available statistics of the data has the largest impact but resolution in the angular
 767 binning also impacts this value. The columns correspond to each coefficient while the rows correspond to
 768 the $p_T^{W, Truth}$ bin. For example, the first column is for parameters in the same $p_T^{W, Truth}$ as the parameter
 769 of interest and for rows 2-8 correspond to the other coefficients these are labeled as "shape" parameters
 770 which are non-migrational. While the first row corresponds to parameters for the same coefficient but in
 771 different $p_T^{W, Truth}$ bins and are self-migrational parameters. The combination of different $p_T^{W, Truth}$ and
 772 different coefficient number are labeled as "cross-migration". Lastly, this separation is also done for
 773 the normalization factors which are labeled "norm" and "norm-migration". The separated migration
 774 uncertainties for each coefficient can be found in Appendix D. There also is a non-migrational uncertainty
 775 which comes from the uncorrelated statistical uncertainty in the likelihood fit.

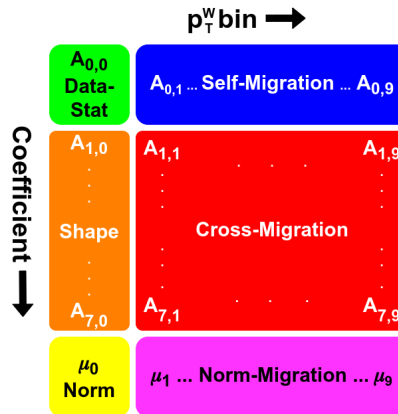


Figure 29: Categorization of parameters for the data-statistical uncertainty on the measured coefficients for an example of A_0 in $p_T^{W, Truth}$ bin 0.

776 **7.2 Experimental Uncertainties**

777 The main experimental uncertainties affecting this measurement are expected to arise from hadronic recoil,
 778 resolution, and calibration. Lepton related systematic are expected to be subdominant. Multi-jet and
 779 EW+top background estimates contribute as well to the relevant experimental uncertainties which need to
 780 be evaluate. These experimental uncertainties only impact the numerator of the templates discussed in
 781 Section 3.2.

782 The systematic uncertainties connected to the hadronic recoil calibration (discussed in Section 4.4) are
783 separated into 6 categories with a combined 62 nuisance parameters corresponding to the resolution,
784 response, and the ΣE_T . Appendix C detailed some dedicated studies on the recoil systematics on the fit
785 results.

786 *The lepton systematic related to the leptons calibration and the leptons efficiency correction (discussed*
787 *in Section 4.3) are expected to be subdominant for the but still need to be implemented. For the electron*
788 *channels the SF efficiency SF are separated for the identification, isolation, reconstruction, and trigger.*
789 *For the muon channels these are separated for the isolation, reconstruction, sagitta bias, trigger, and*
790 *track-to-vertex-association.*

791 The uncertainties for EW+top background estimates are taken as a flat uncertainty on their cross-sections.
792 For single boson samples this is 5%, diboson samples 10%, and top backgrounds range from 7-10%;
793 specifics for each sample can be found in Table 2 in Section 4.1. These variations are grouped into 3
794 categories for a total of 3 nuisance parameters based on the background type into boson, diboson, and
795 top.

796 The uncertainties for the multi-jet background are discussed in Section 5 and Appendix E and are separated
797 into 3 categories with a combined $(3 \times 10) = 30$ nuisance parameters corresponding to the acceptance,
798 shape, and yield.

799 **7.3 Theoretical Uncertainties**

800 The only theoretical uncertainties considered are due to PDFs which have an impact on both the numerator
801 and denominator of the templates. These are described by 26 NPs and are subdominant for $A_0 - A_7$ and
802 approximately $\sim 3\%$ for the differential cross-section. Examples of the total expected PDF uncertainty can
803 be found in Figures 38 and 47. Other theoretical uncertainties such as parton shower, generator modelling,
804 and QED/electroweak corrections are not considered as they were found to be negligible for the Z angular
805 coefficient measurement at $\sqrt{s} = 8$ TeV which had more precision than this measurement.

806 **7.4 Expected Uncertainties**

807 **7.4.1 p_T^W Differential**

Uncertainty	Description
Response_Sys	Systematic uncertainty on the response correction
Response_ExtSys	Systematic uncertainty on the response correction due to the extrapolation from Z to W
Response_Stat0_1-15	Statistical uncertainty on the response correction
Response_Stat1_1-15	Statistical uncertainty on the response correction
Resolution_ExtSys	Systematic uncertainty on the resolution correction due to the extrapolation from Z to W
Resolution_Stat0_1-14	Statistical uncertainty on the resolution correction
Resolution_Stat1_1-14	Statistical uncertainty on the resolution correction
SET_Sys	Systematic uncertainty on the ΣE_T
XSec_Boson	Flat 5% variation on the cross-section for all single boson samples
XSec_Diboson	Flat 10% variation on the cross-section for all diboson samples
XSec_Top	Flat variation between 7-10% on the cross-section for all top associated samples
MJ_Acc	Uncertainty on acceptance correction functions
MJ_Shape	Uncertainty on MJ template shape
MJ_Yield	Uncertainty on yield from intersection point
MC Stat	Poisson variation for the limited MC statistics used in template generation
PDF_1-26	Systematic uncertainty on the parton distribution function

Table 5: Description of the systematics nuisance parameters used in the analysis.

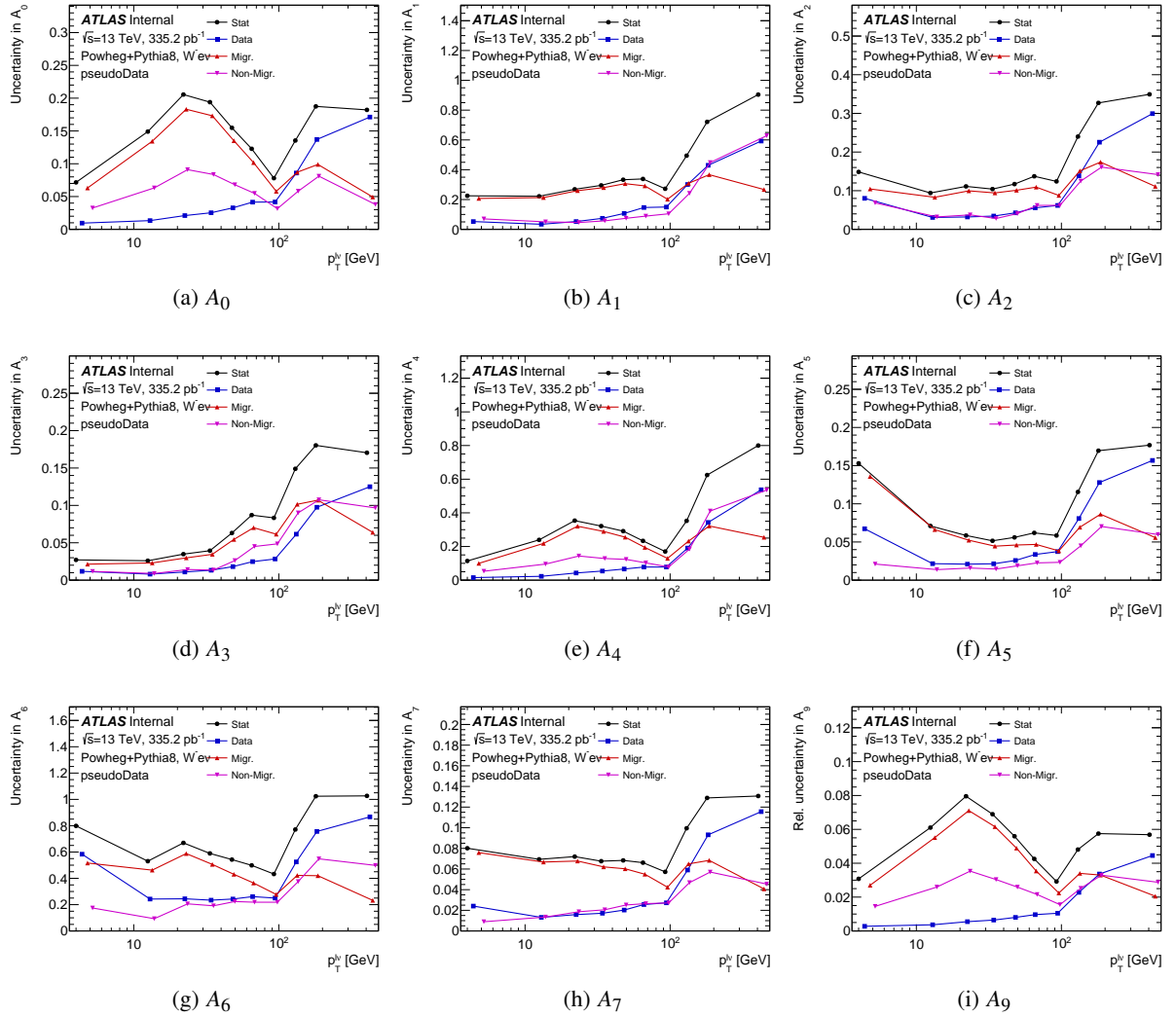


Figure 30: The uncertainty comparison for the statistical uncertainties in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 104, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

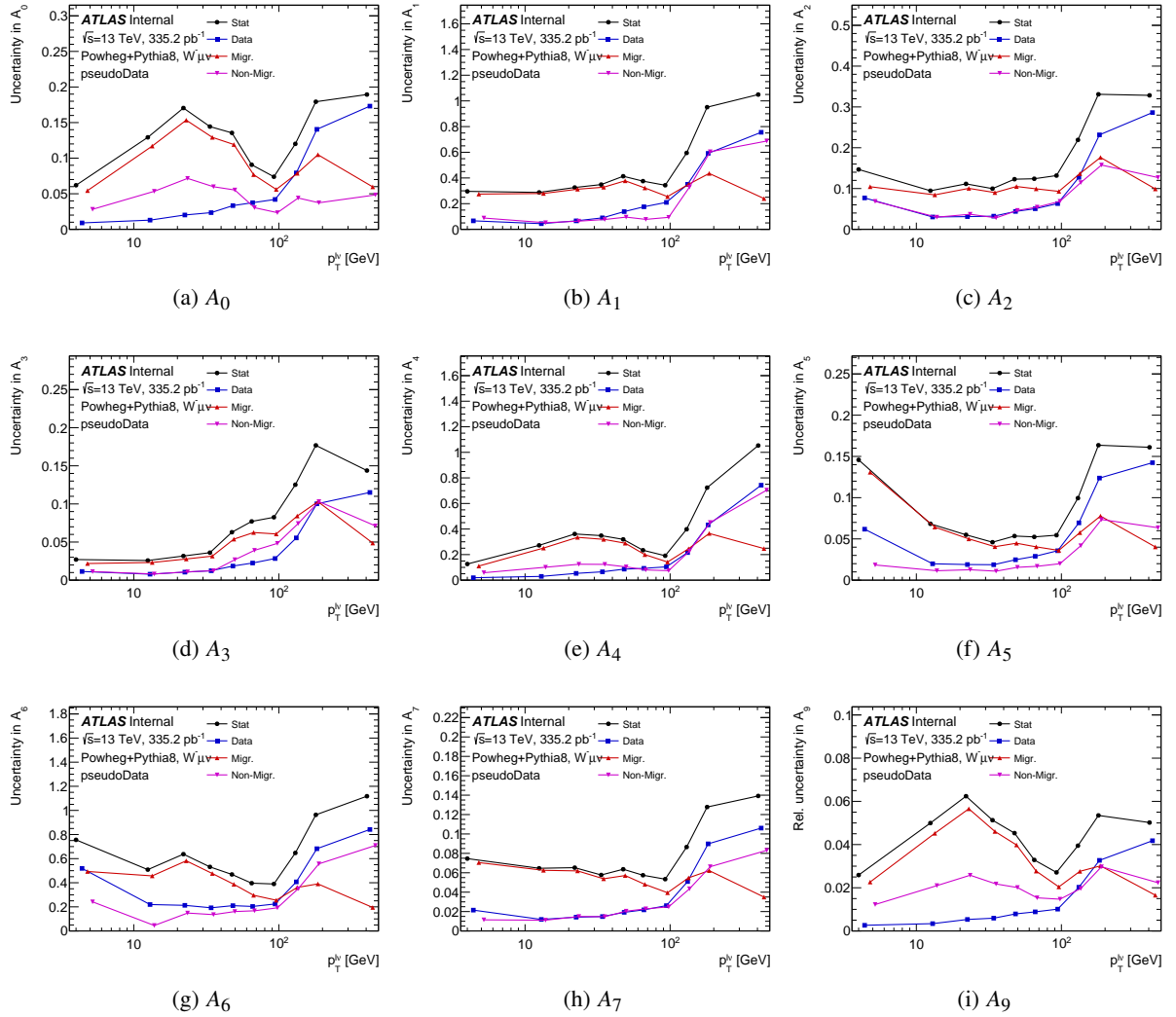


Figure 31: The uncertainty comparison for the statistical uncertainties in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 105, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

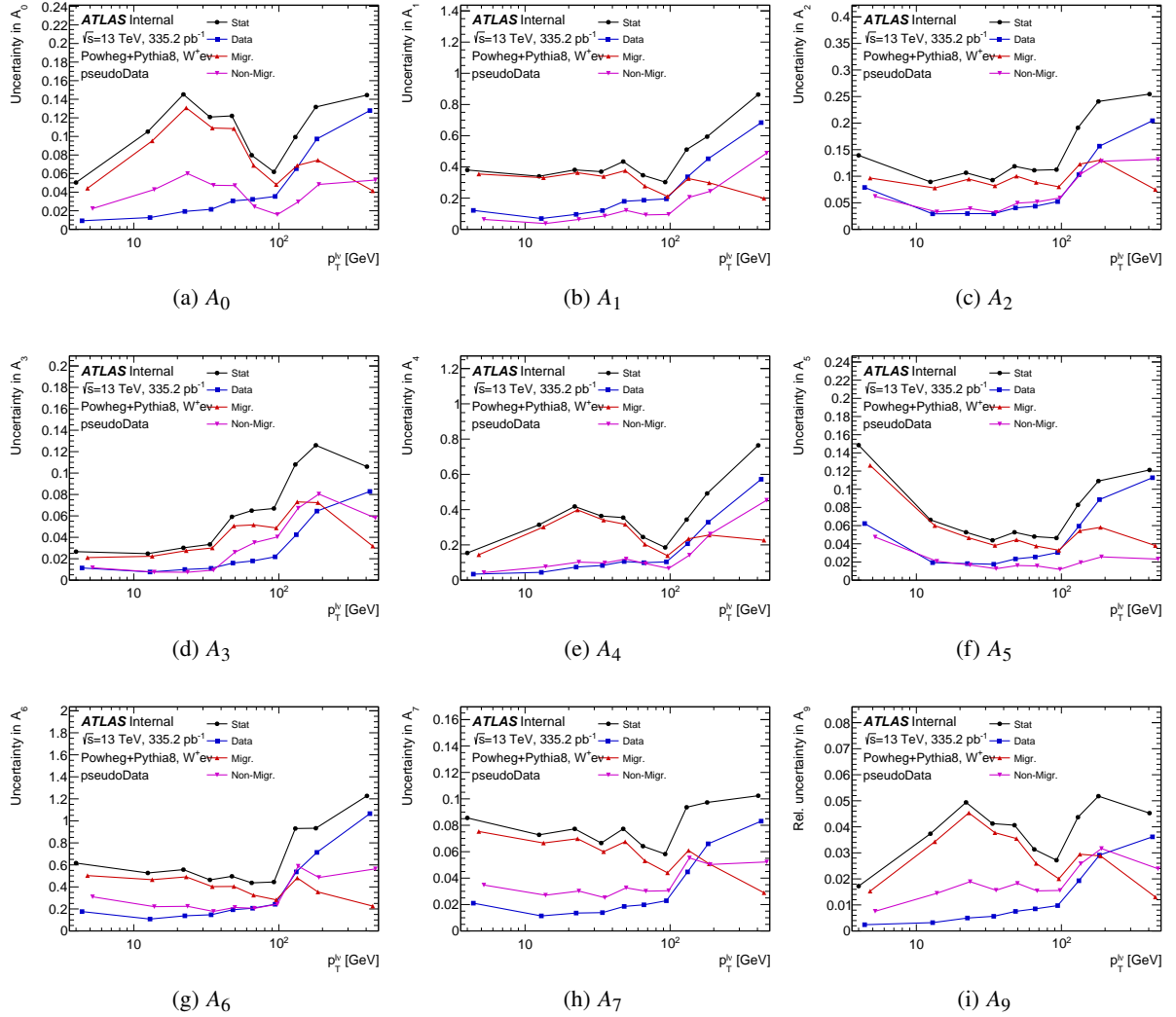


Figure 32: The uncertainty comparison for the statistical uncertainties in the $W^+ \rightarrow e^+ \nu$ channel for measurement in p_T^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 106, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

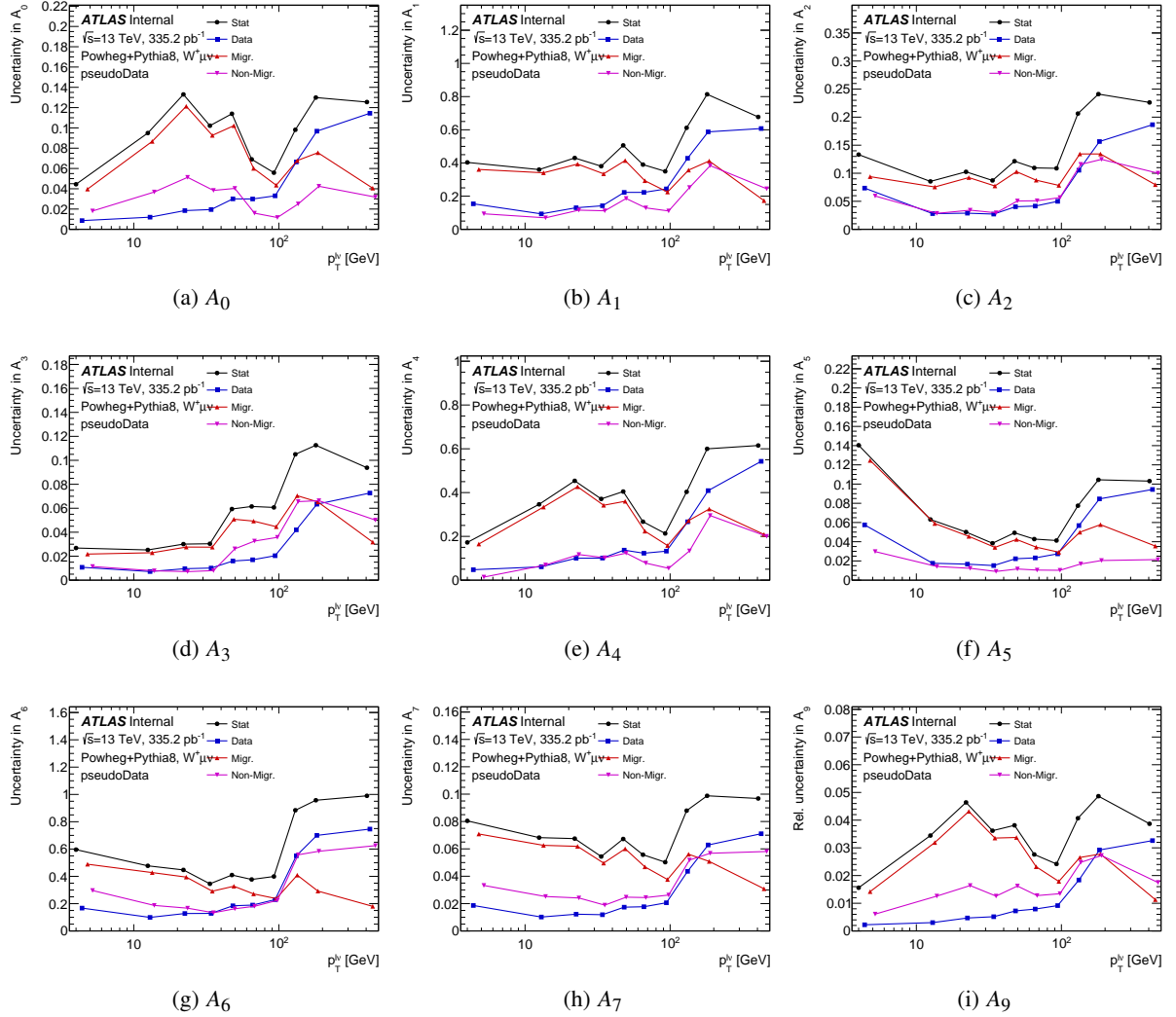


Figure 33: The uncertainty comparison for the statistical uncertainties in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in p_T^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 107, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

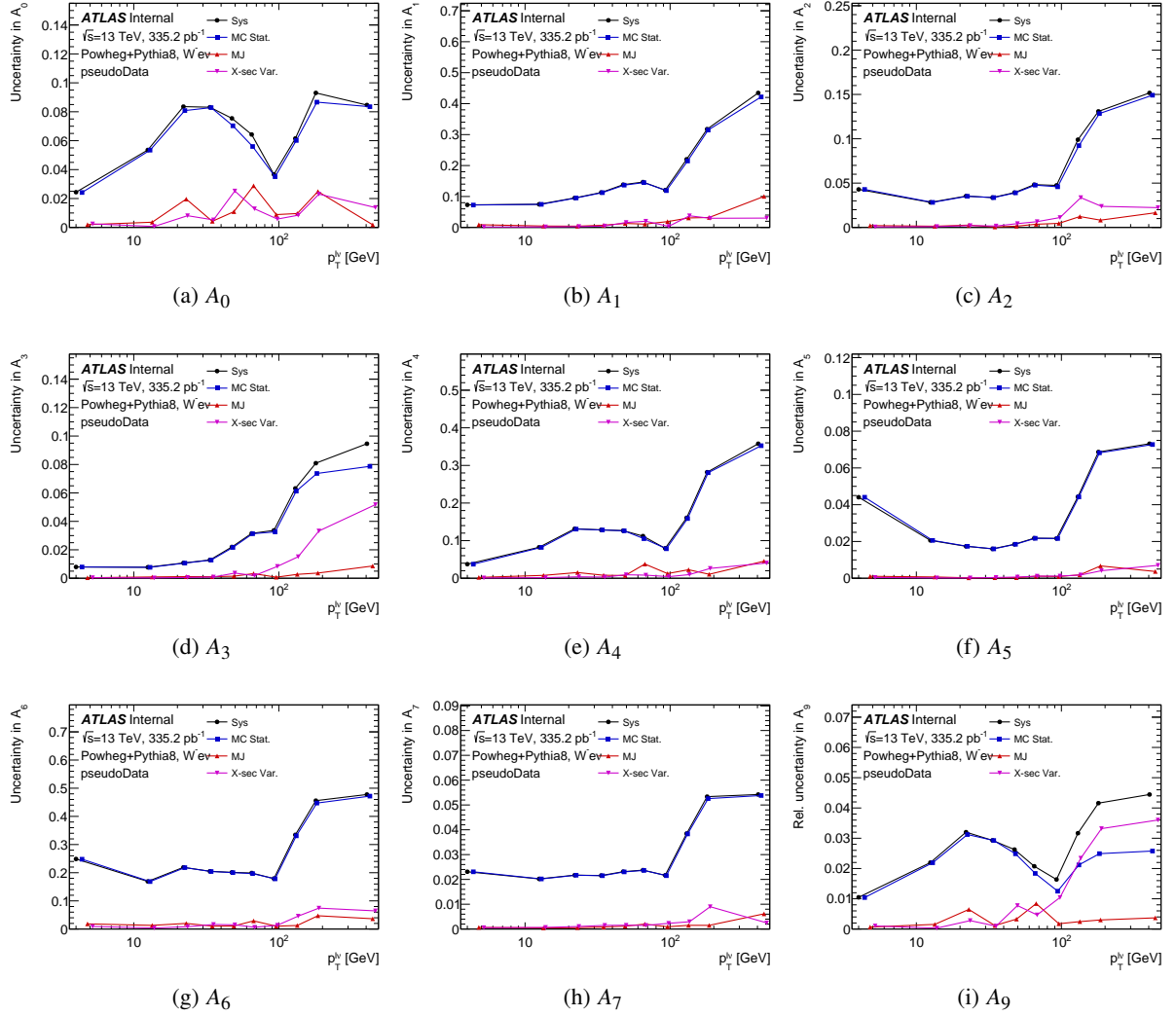


Figure 34: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

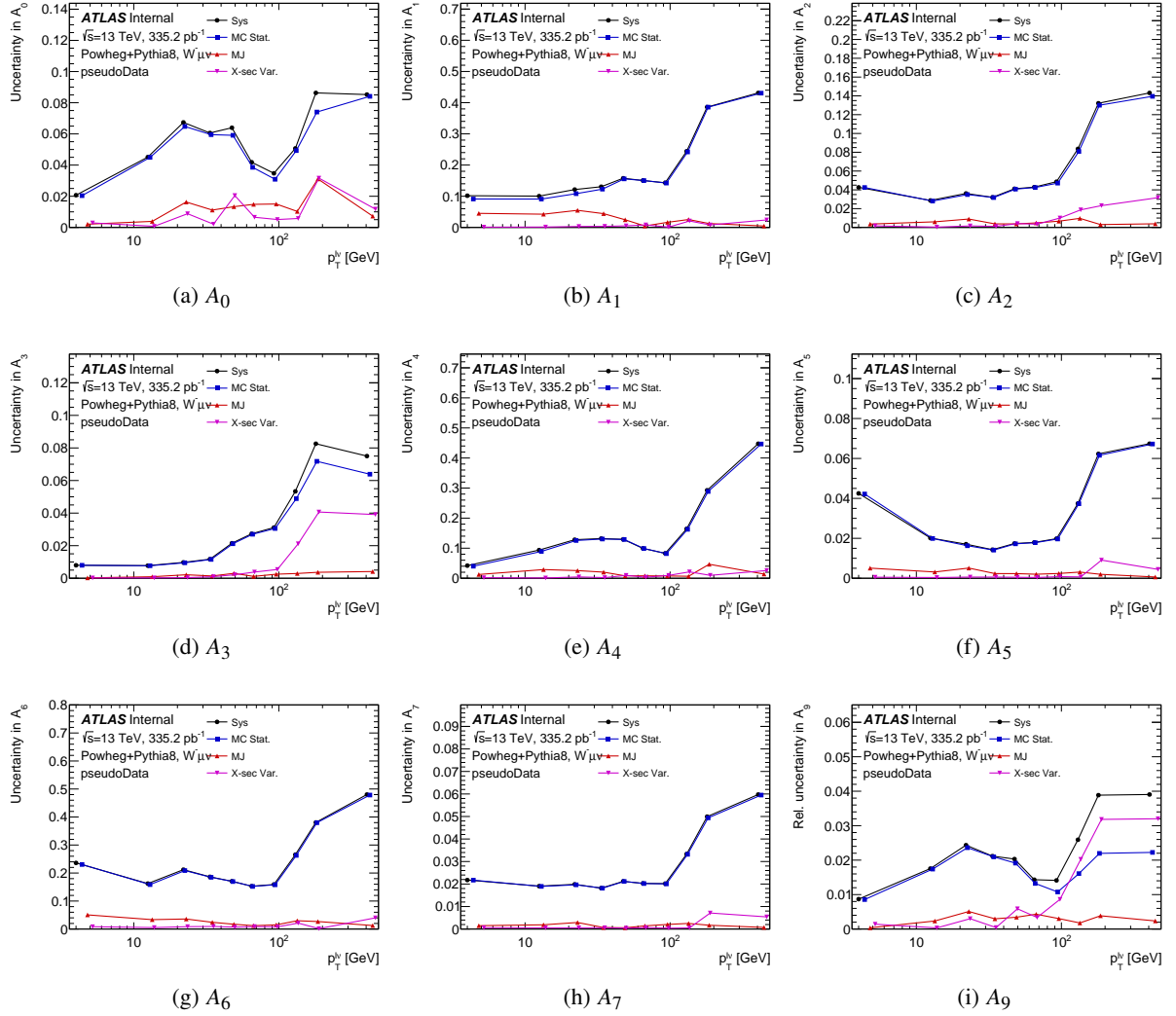


Figure 35: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

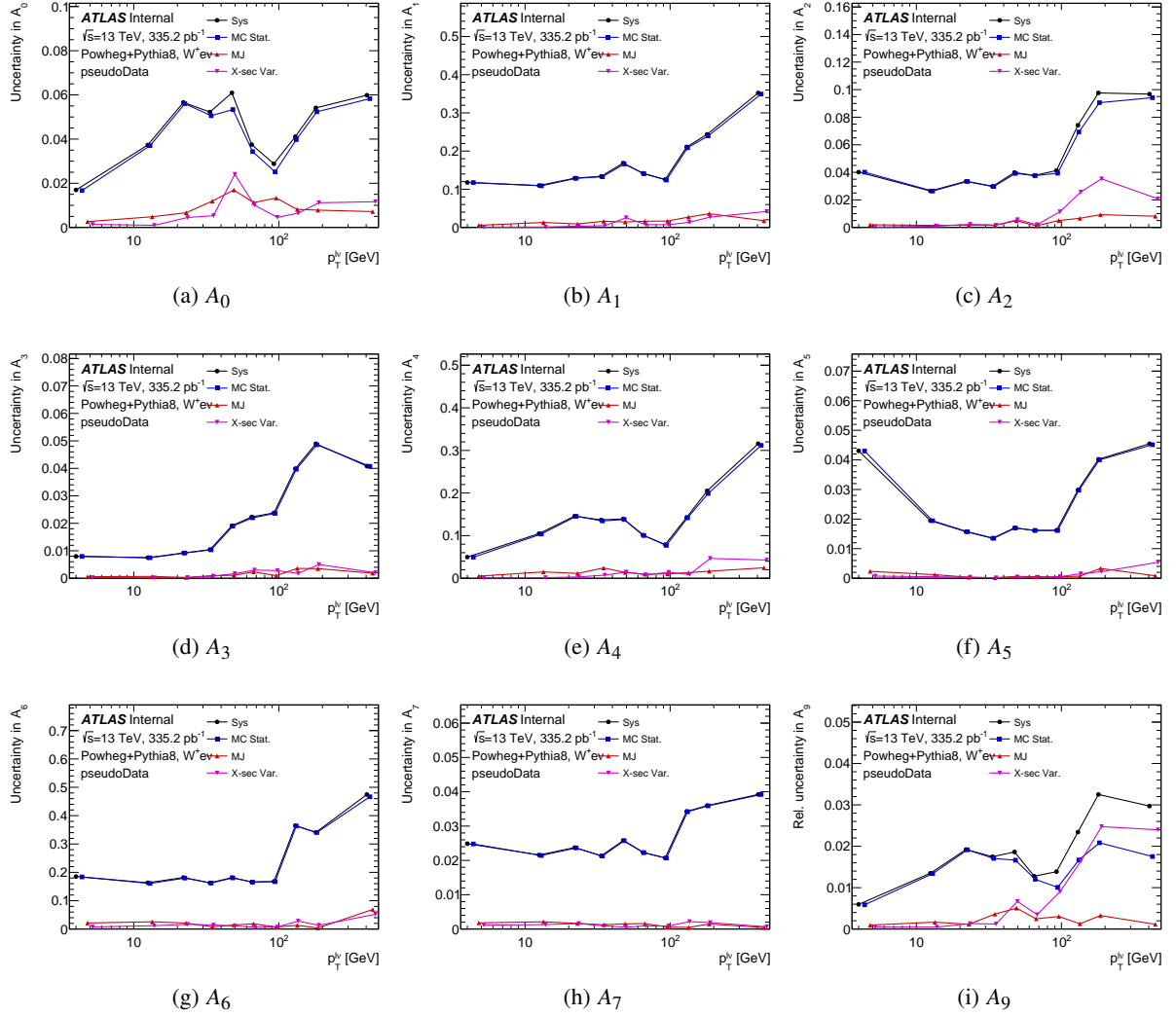


Figure 36: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^+ \rightarrow e^+\nu$ channel for measurement in p_T^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

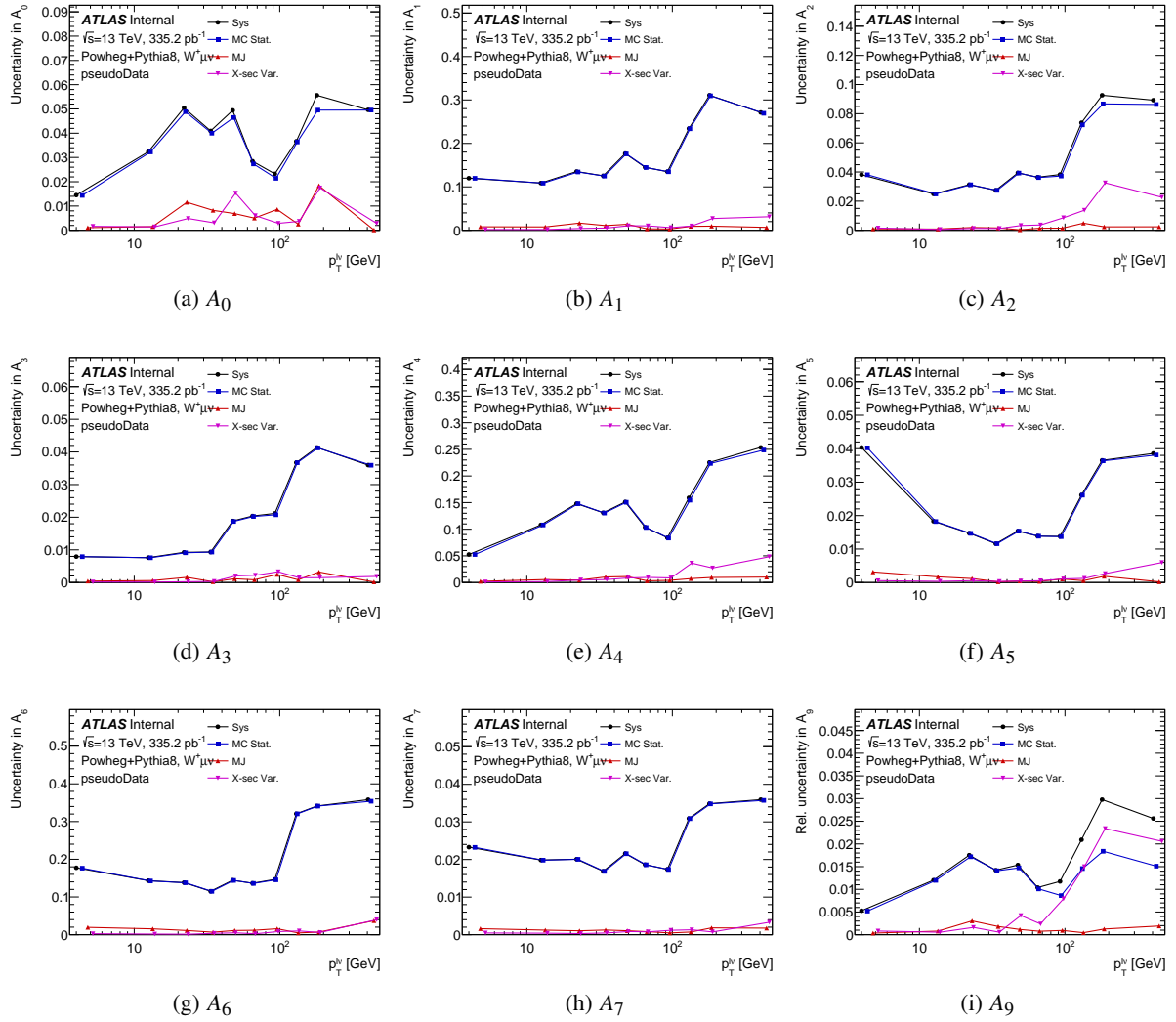


Figure 37: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in p_T^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

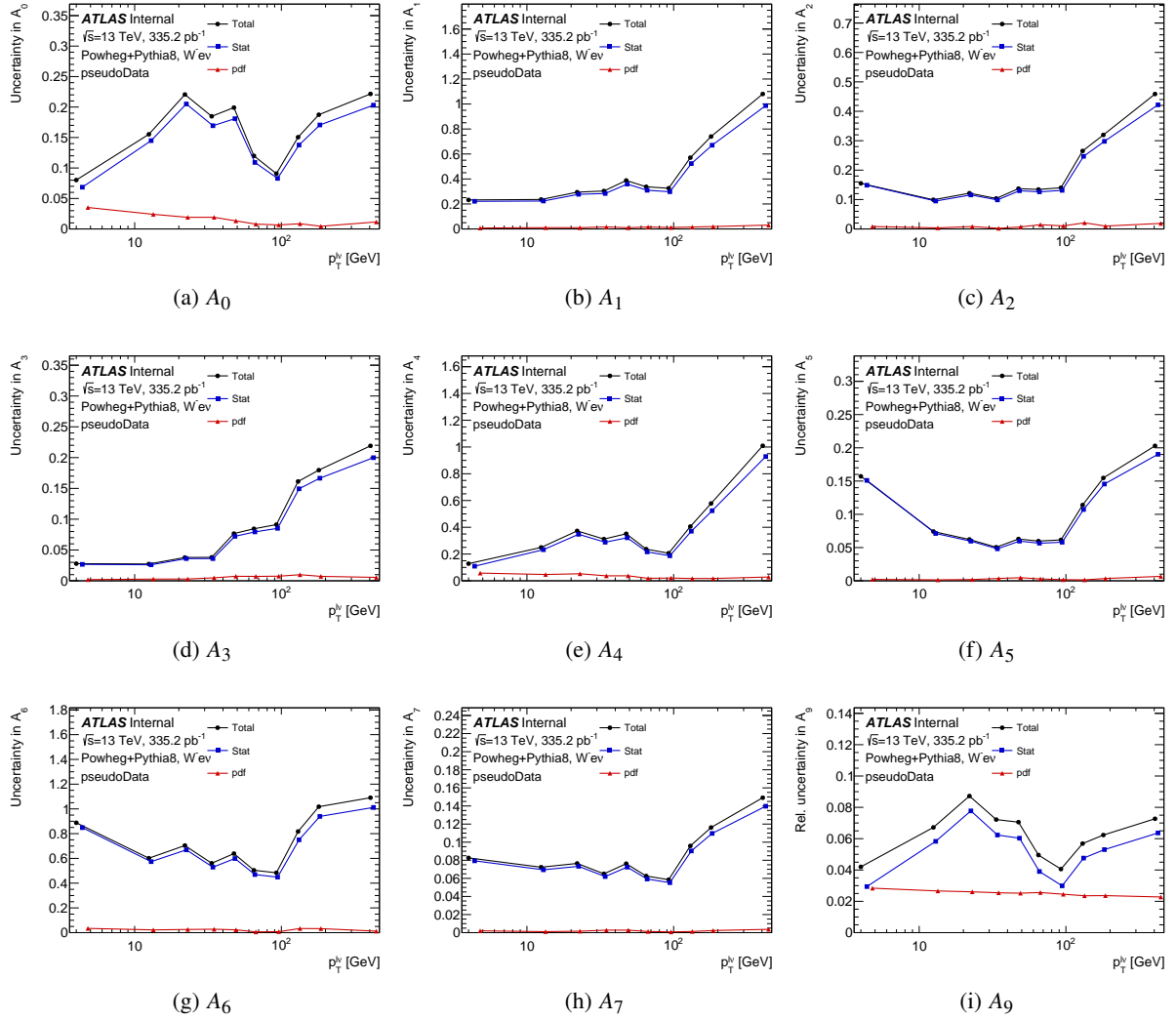


Figure 38: Comparison of the PDF uncertainty in red to the statistical uncertainty in blue in the $W^- \rightarrow e^- \bar{\nu}$ channeling for measurement in p_T^W .

808 7.4.2 y^W Differential

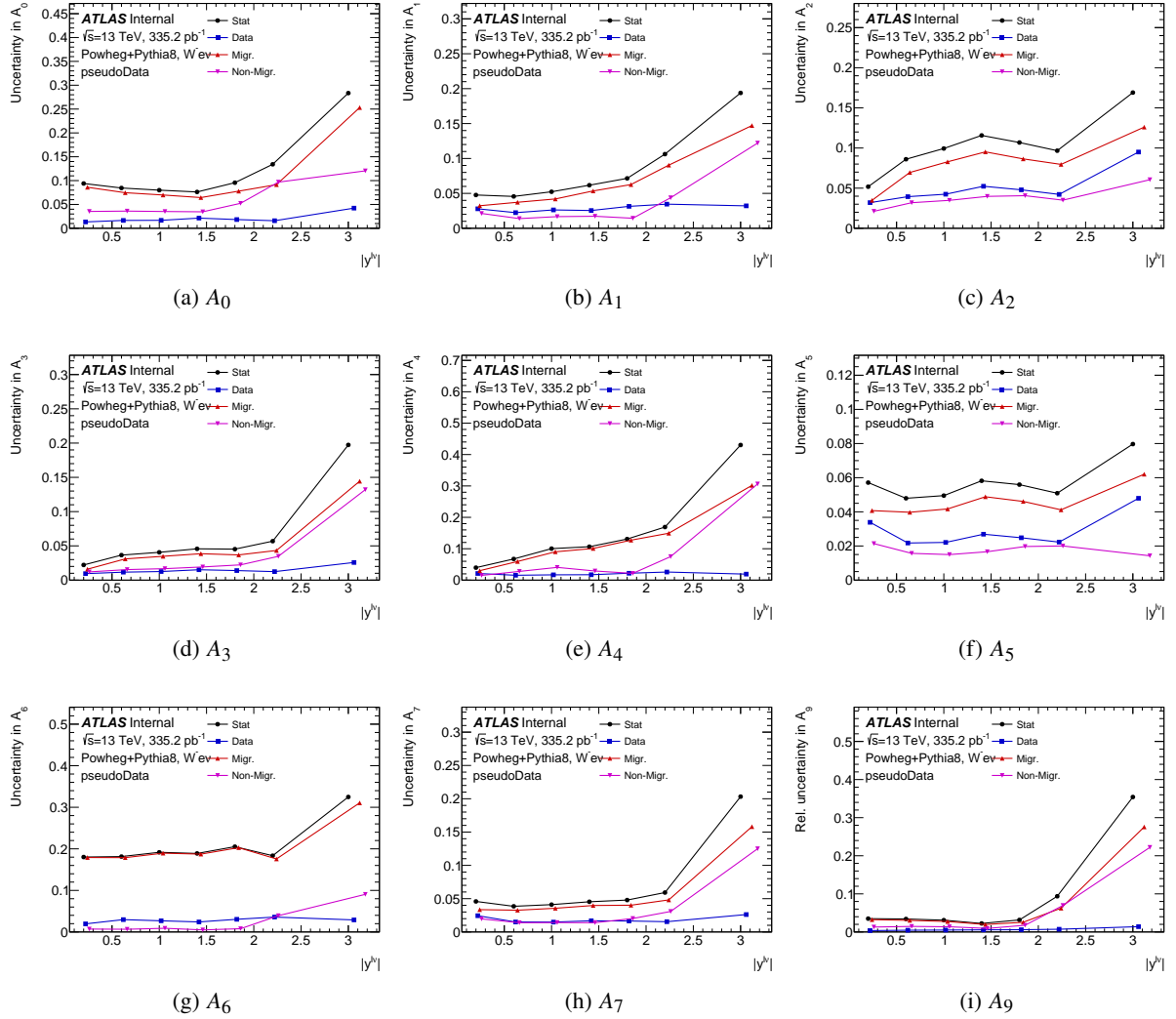


Figure 39: The uncertainty comparison for the statistical uncertainties in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in y^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 108, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

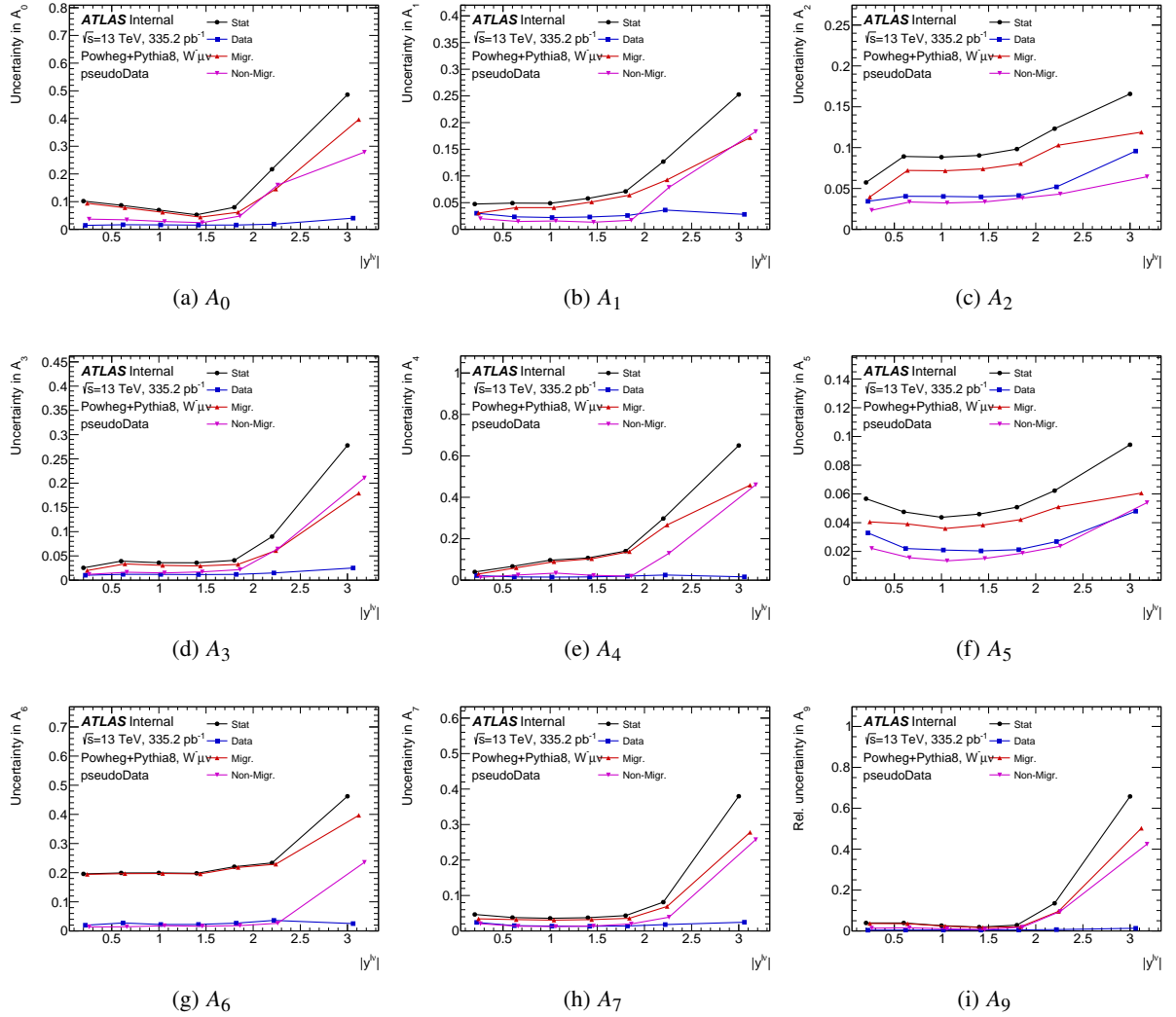


Figure 40: The uncertainty comparison for the statistical uncertainties in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in y^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 109, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

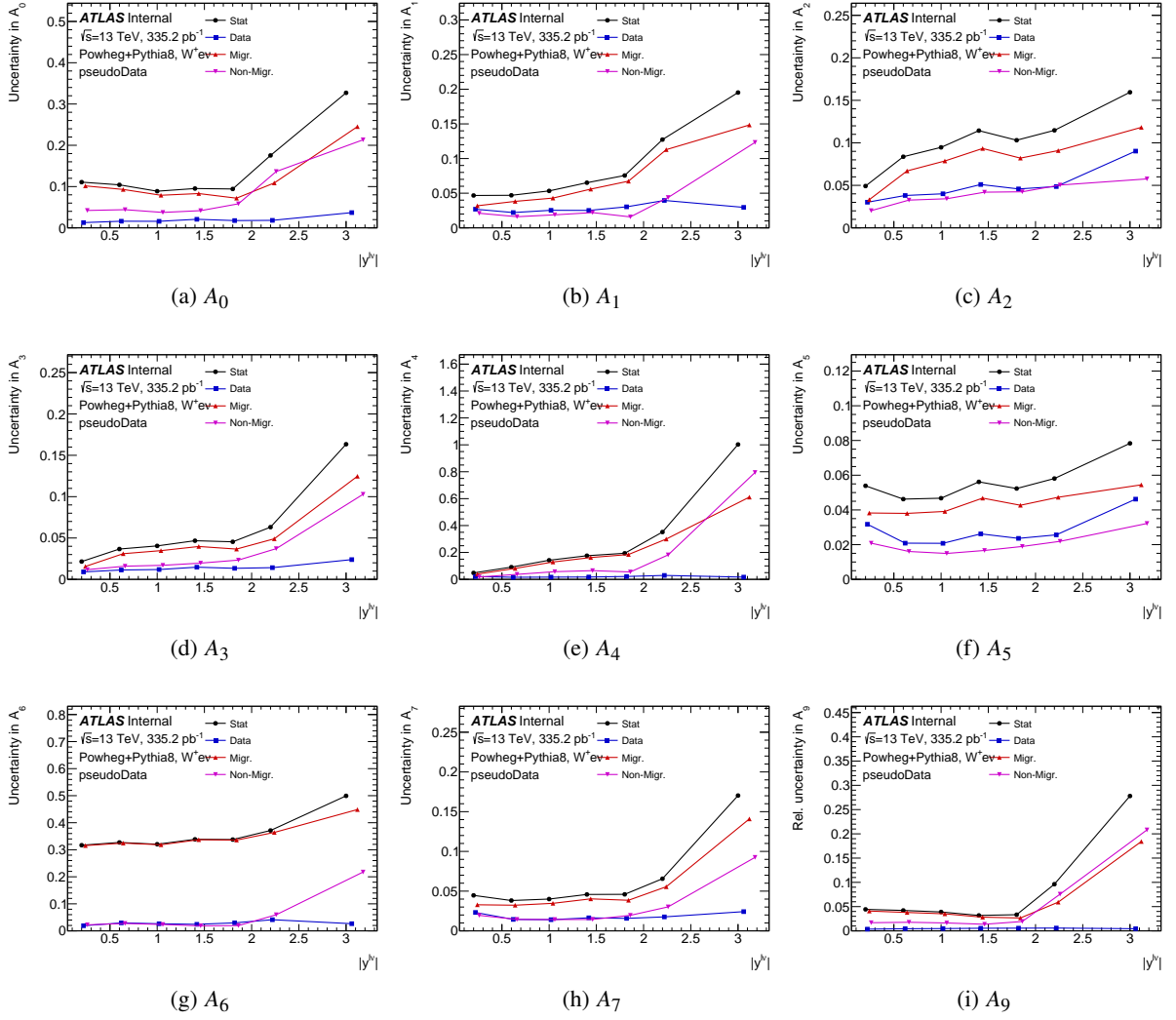


Figure 41: The uncertainty comparison for the statistical uncertainties in the $W^+ \rightarrow e^+ \nu$ channel for measurement in y^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 110, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

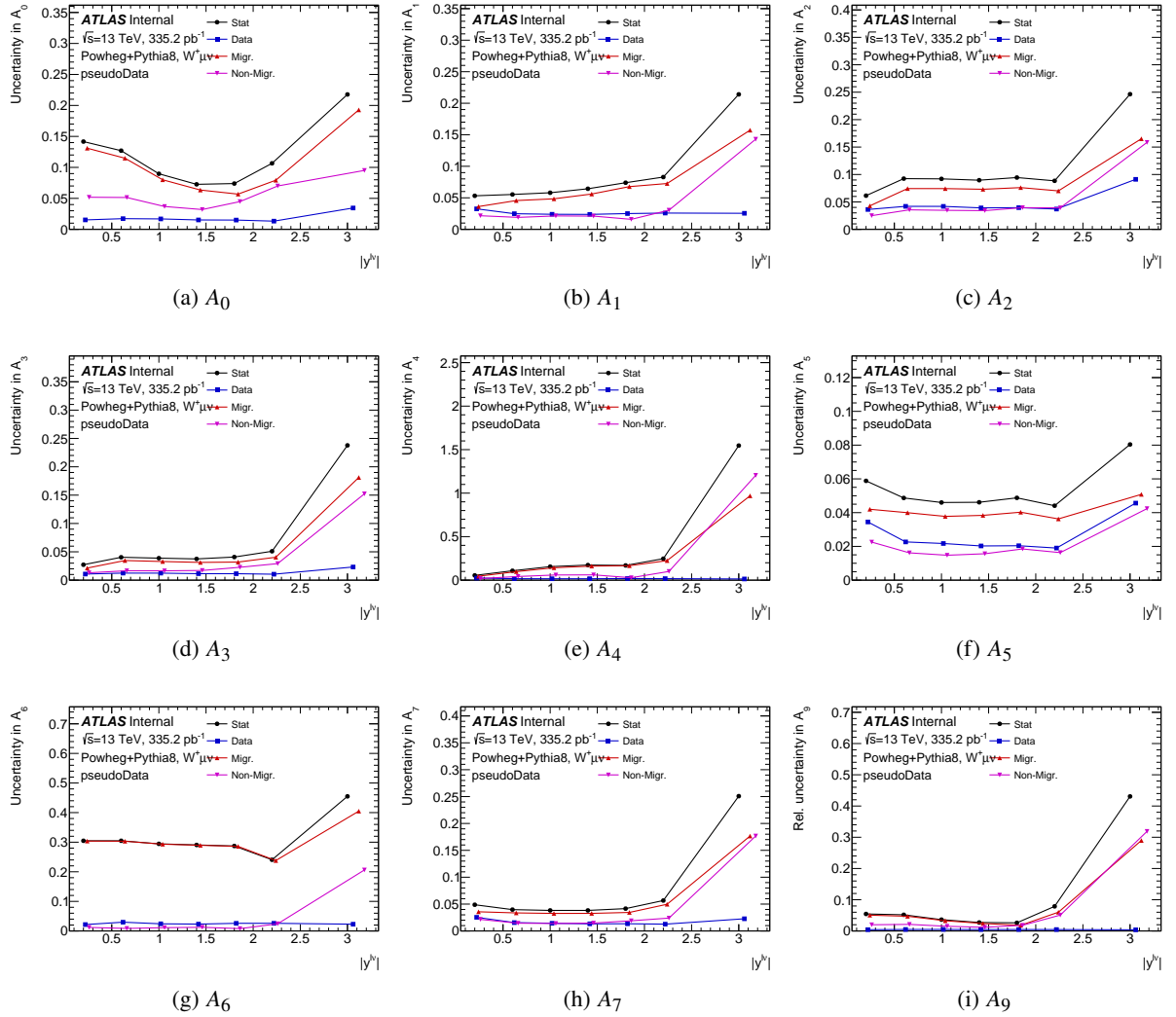


Figure 42: The uncertainty comparison for the statistical uncertainties in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in y^W . Blue is the uncertainty due to the limited data statistics in each bin, red is the total migration uncertainty which is decomposed further in Figure 111, and pink are the uncorrelated statistical uncertainty from the likelihood fit. Black is the combination of all of these components to give the total statistical uncertainty.

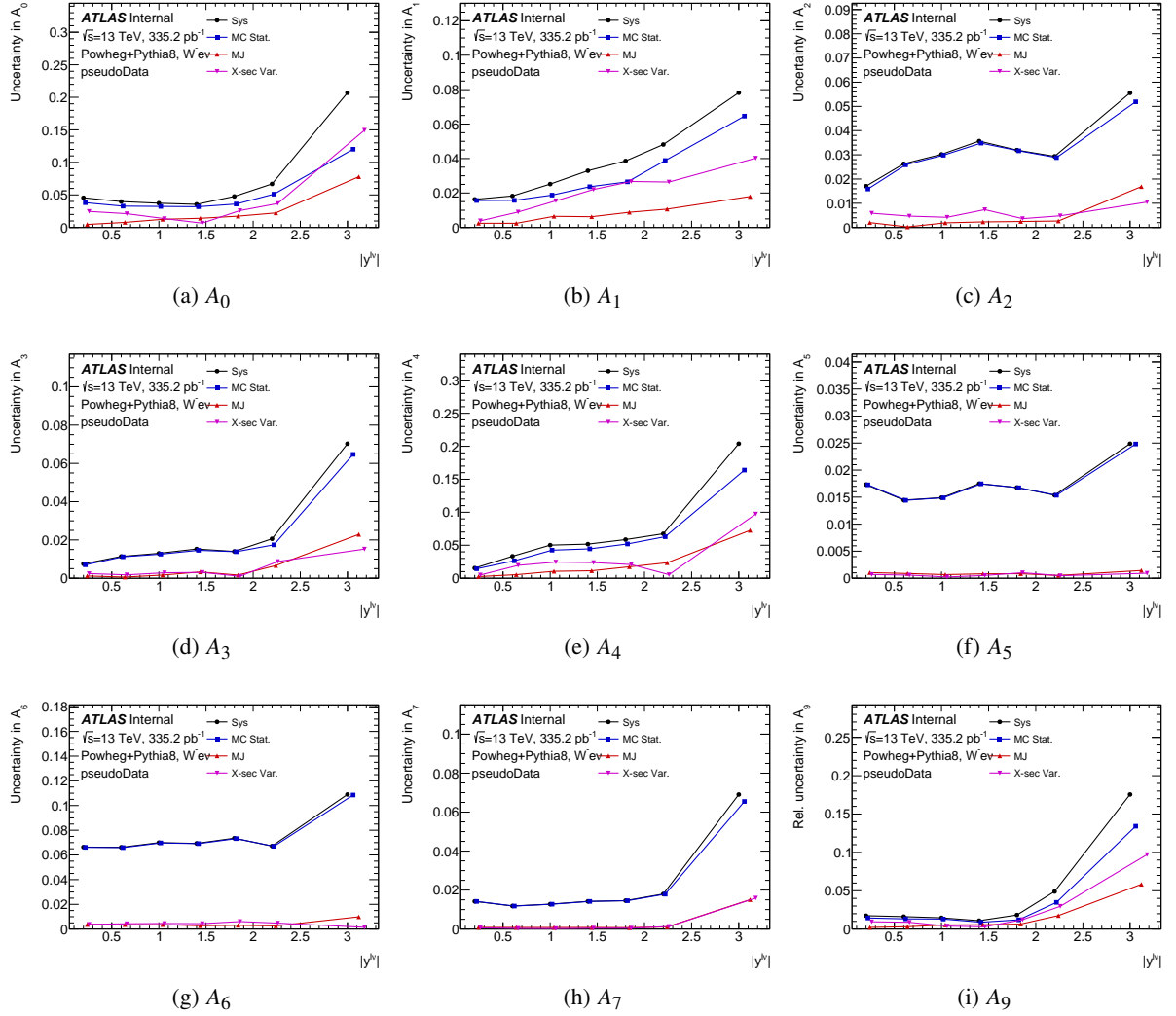


Figure 43: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in y^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

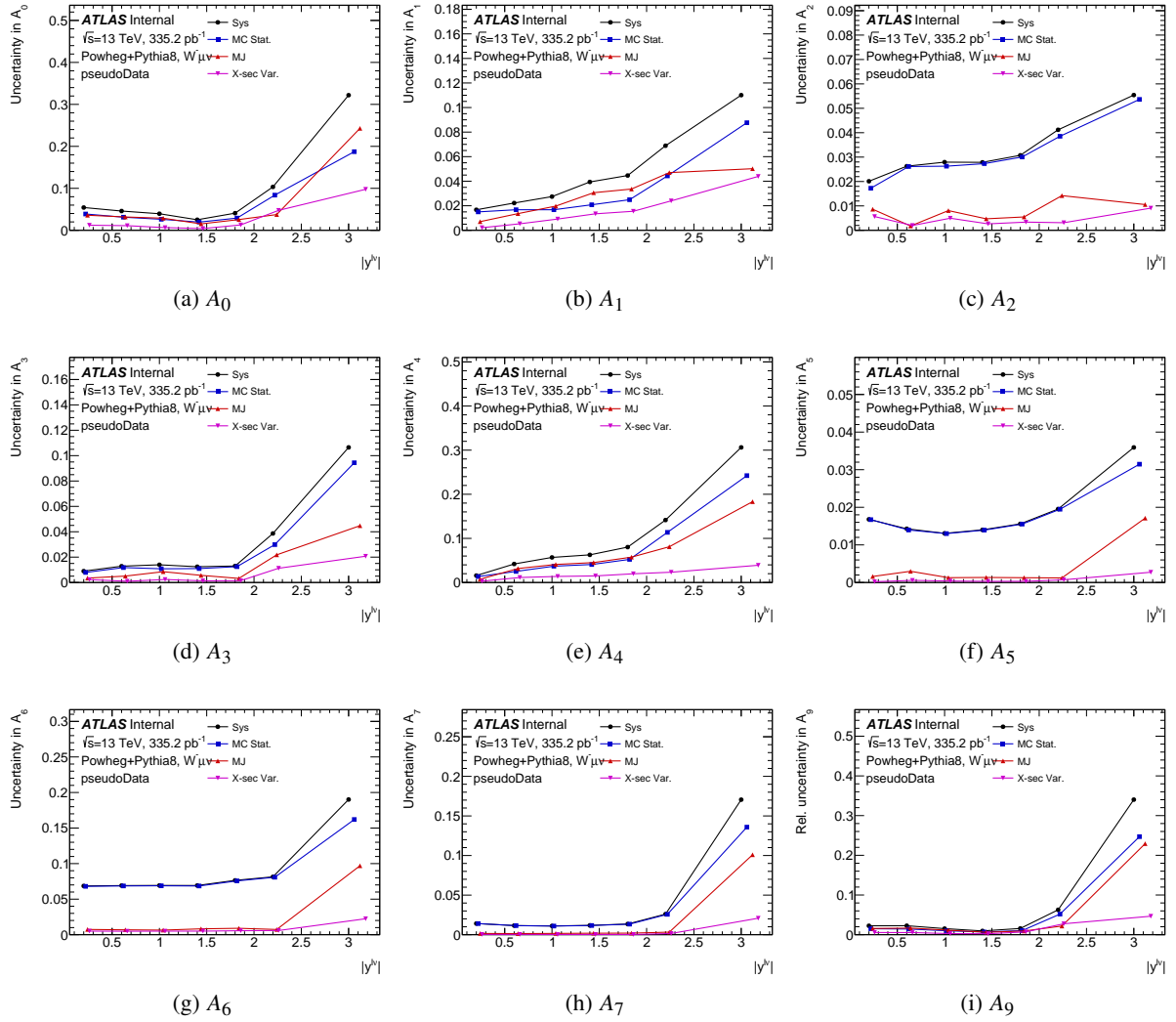


Figure 44: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in y^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

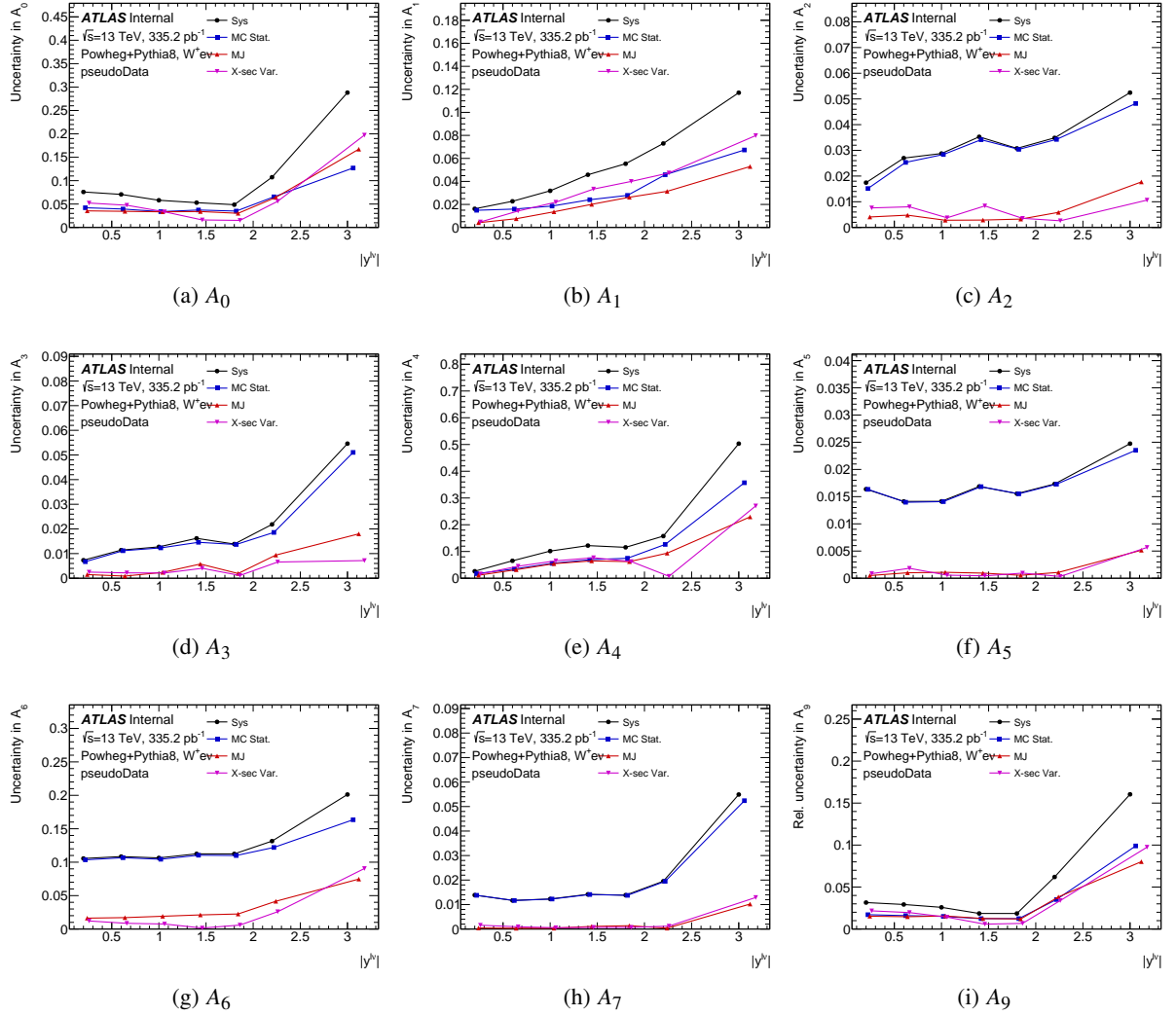


Figure 45: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^+ \rightarrow e^+ \nu$ channel for measurement in y^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

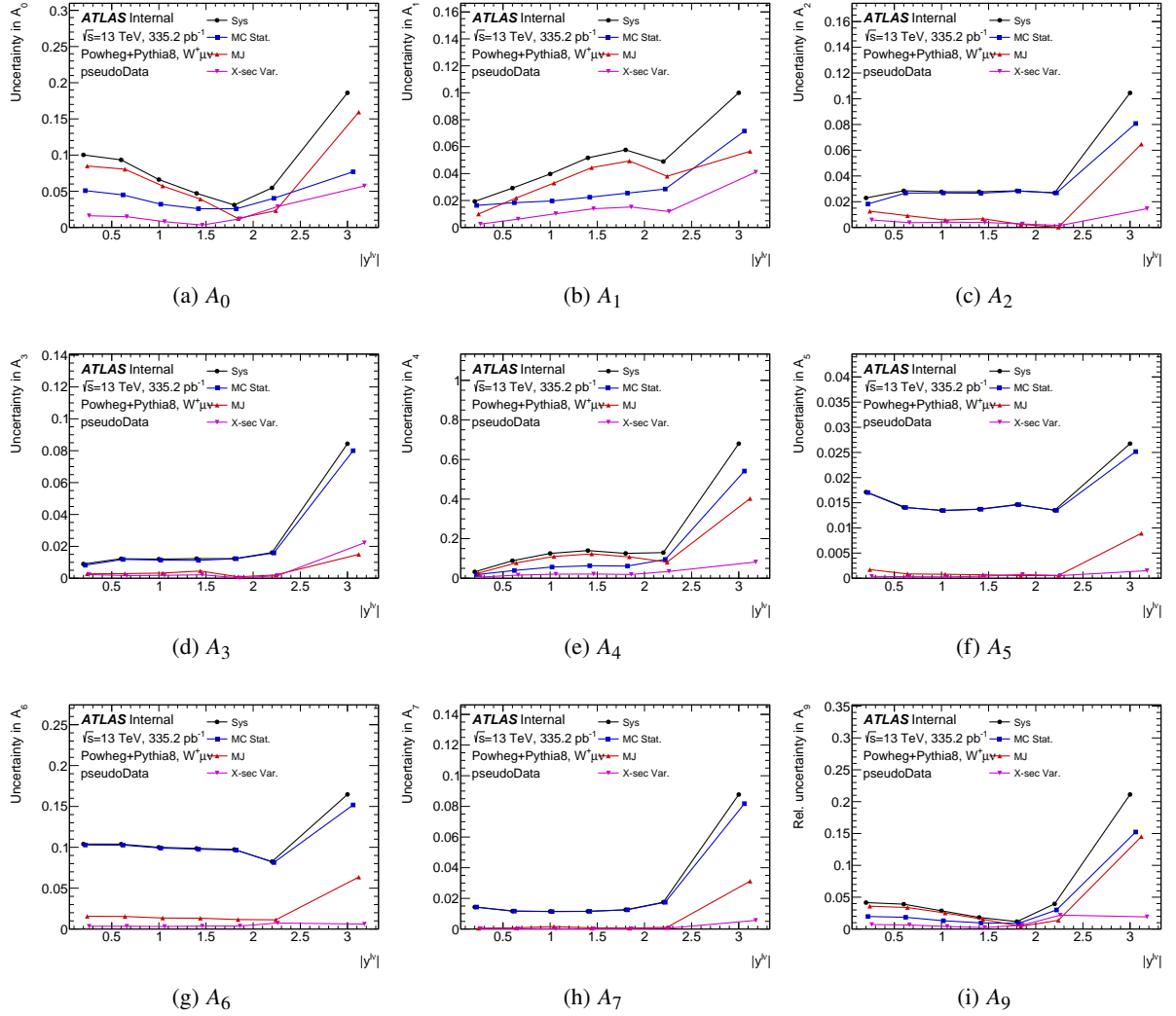


Figure 46: The uncertainty comparison for the background related systematic uncertainties and the dominant MC stat. systematic in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in y^W . Blue is the uncertainty related to the limited MC statistics used to generate the templates, MJ is the uncertainty related to the MJ background estimate, and pink is the uncertainty from varying the background cross section. Black is the combination of these three systematic uncertainties.

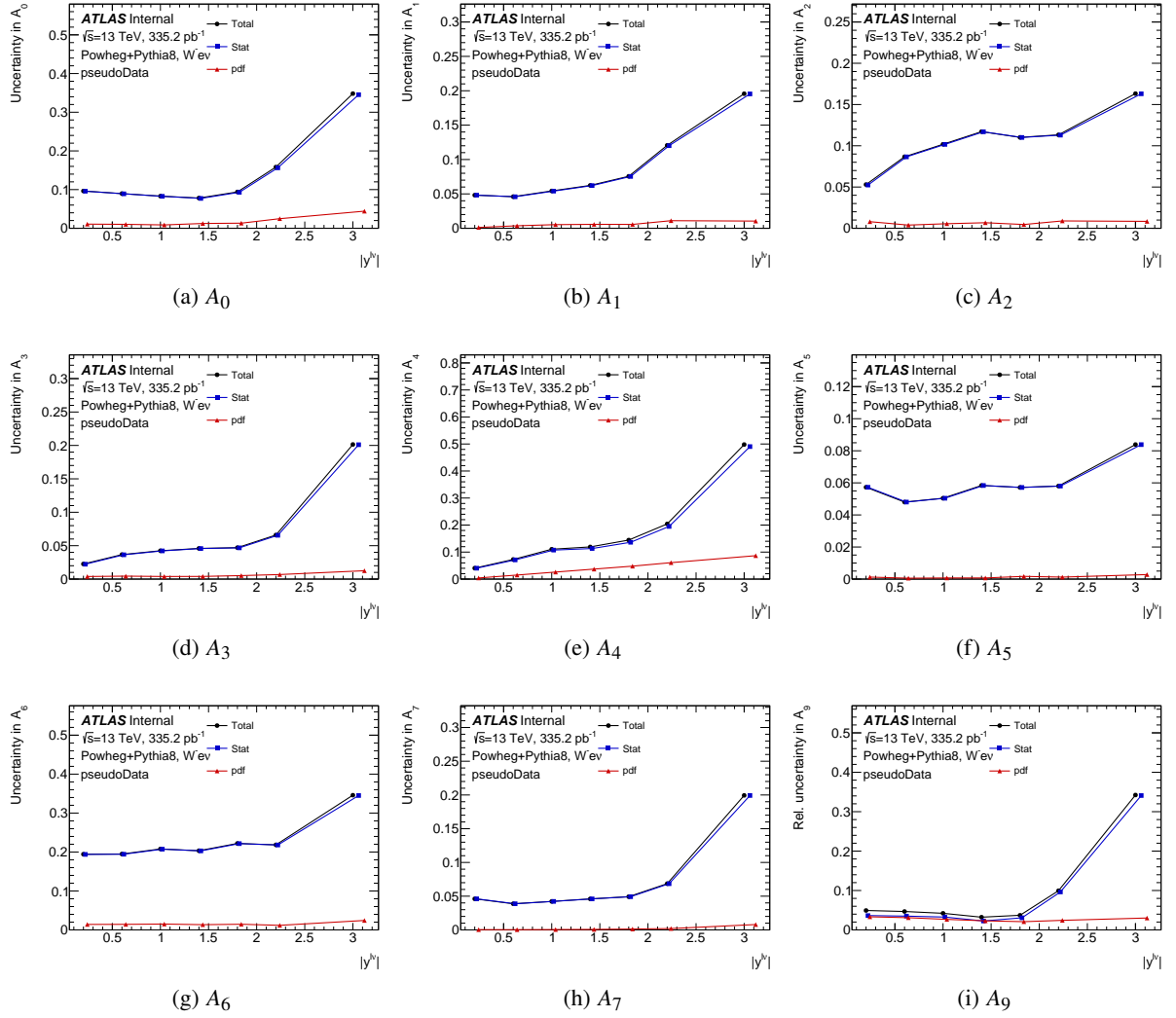


Figure 47: Comparison of the PDF uncertainty in red to the statistical uncertainty in blue in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in y^W .

809 **7.5 Summary of Uncertainties**

810 A summary of the expected statistical and systematic uncertainties associated to the measurement of the A_i
811 coefficients as a function of p_T^W is presented in Section 7.5.1 and as function of y^W is presented in Section
812 7.5.2 for each category of events ($W^- \rightarrow e^- \bar{\nu}$, $W^+ \rightarrow e^+ \nu$, $W^- \rightarrow \mu^- \bar{\nu}$, and $W^+ \rightarrow \mu^+ \nu$). The dominant
813 uncertainty on the measurement of the A_i coefficients is the statistical uncertainty for all bins for both
814 p_T^W and y^W differential measurements. The next dominant uncertainty is due to the signal MC statistical
815 uncertainty, which is $\sim 1/3$ of the data-statistical uncertainty due since the signal MC was produced with
816 approximated ten times more events than the data sets used.

817 **7.5.1 p_T^W Differential**

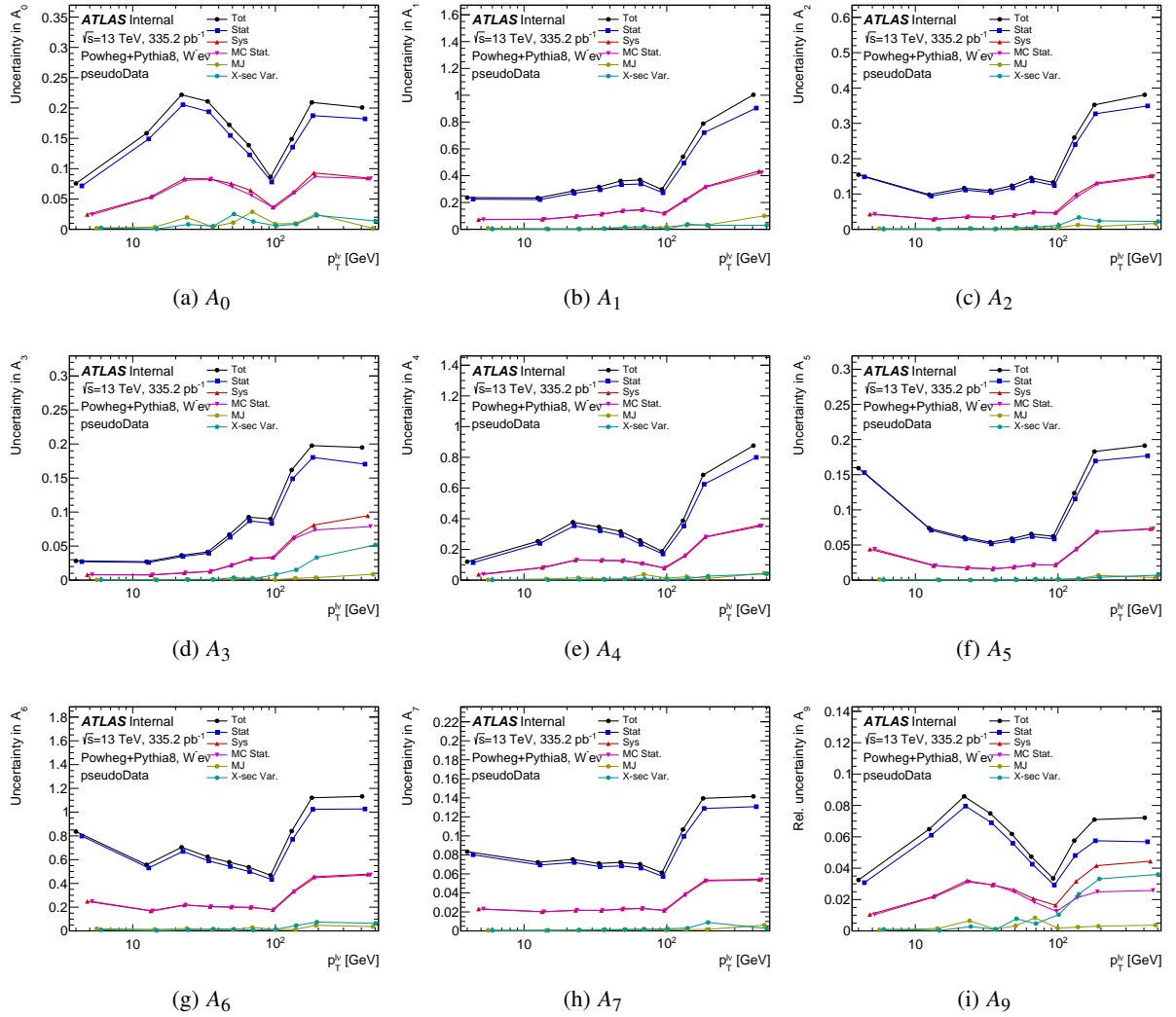


Figure 48: The uncertainty comparison for all contributions in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

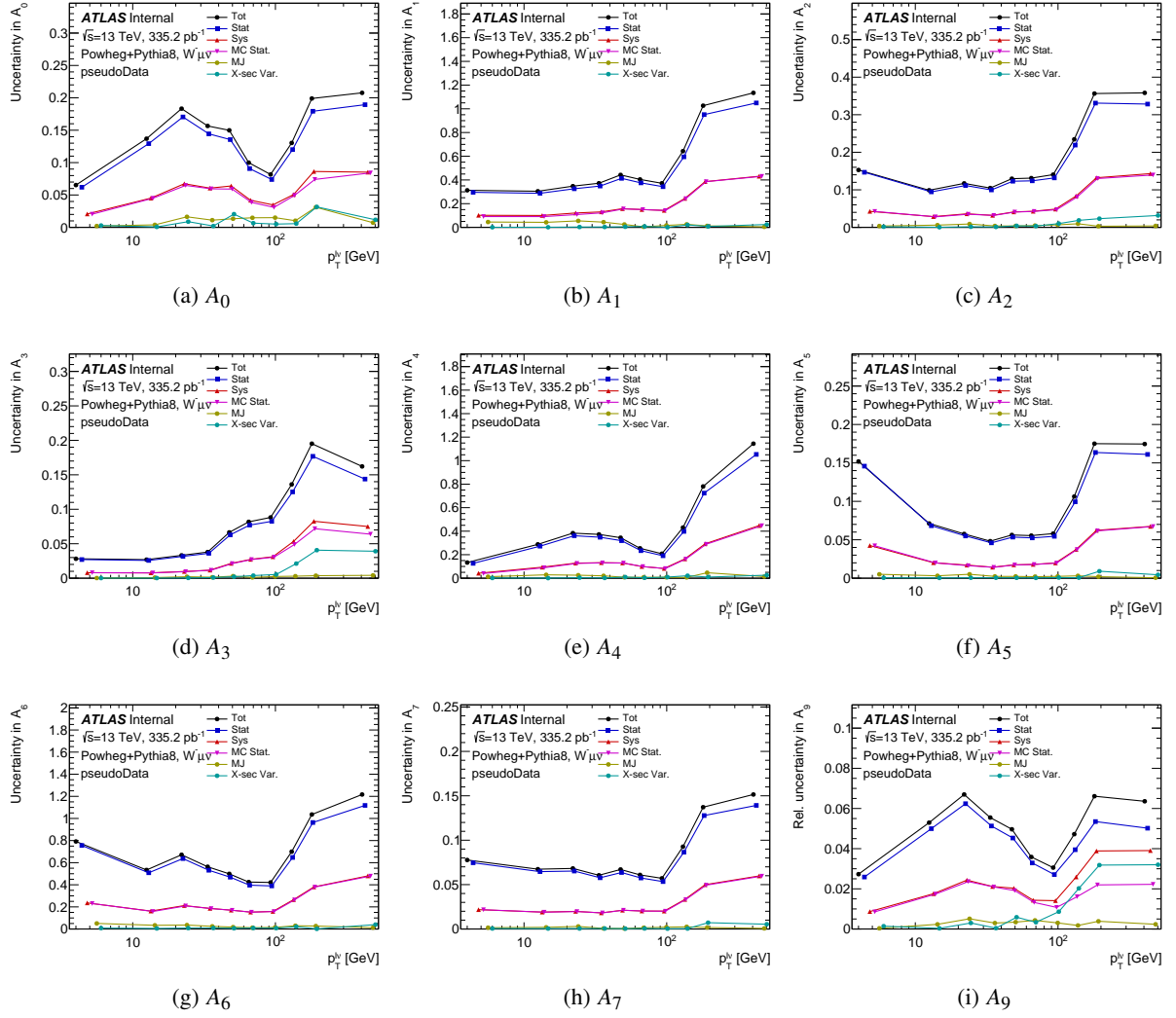


Figure 49: The uncertainty comparison for all contributions in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

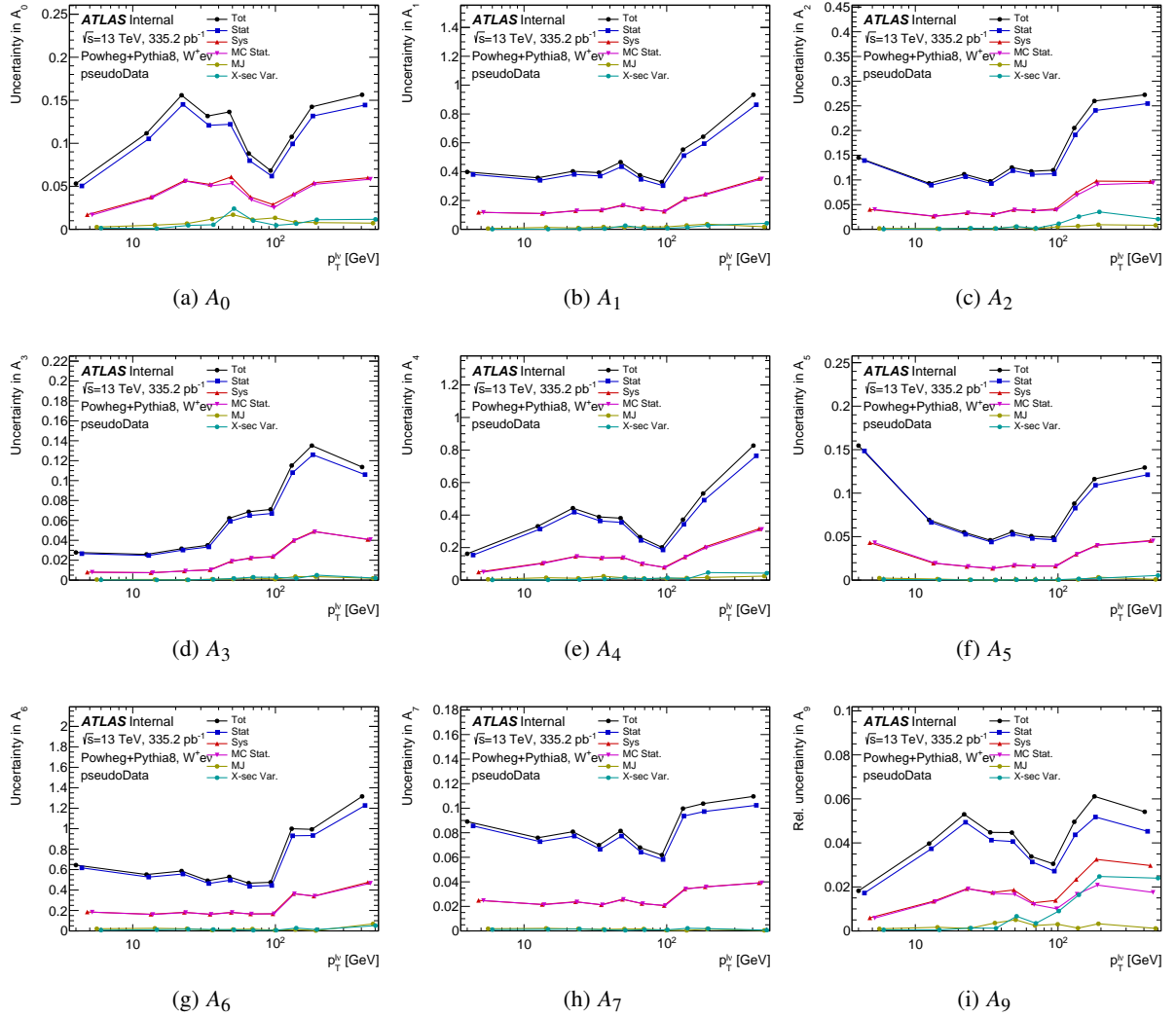


Figure 50: The uncertainty comparison for all contributions in the $W^+ \rightarrow e^+\nu$ channel for measurement in p_T^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

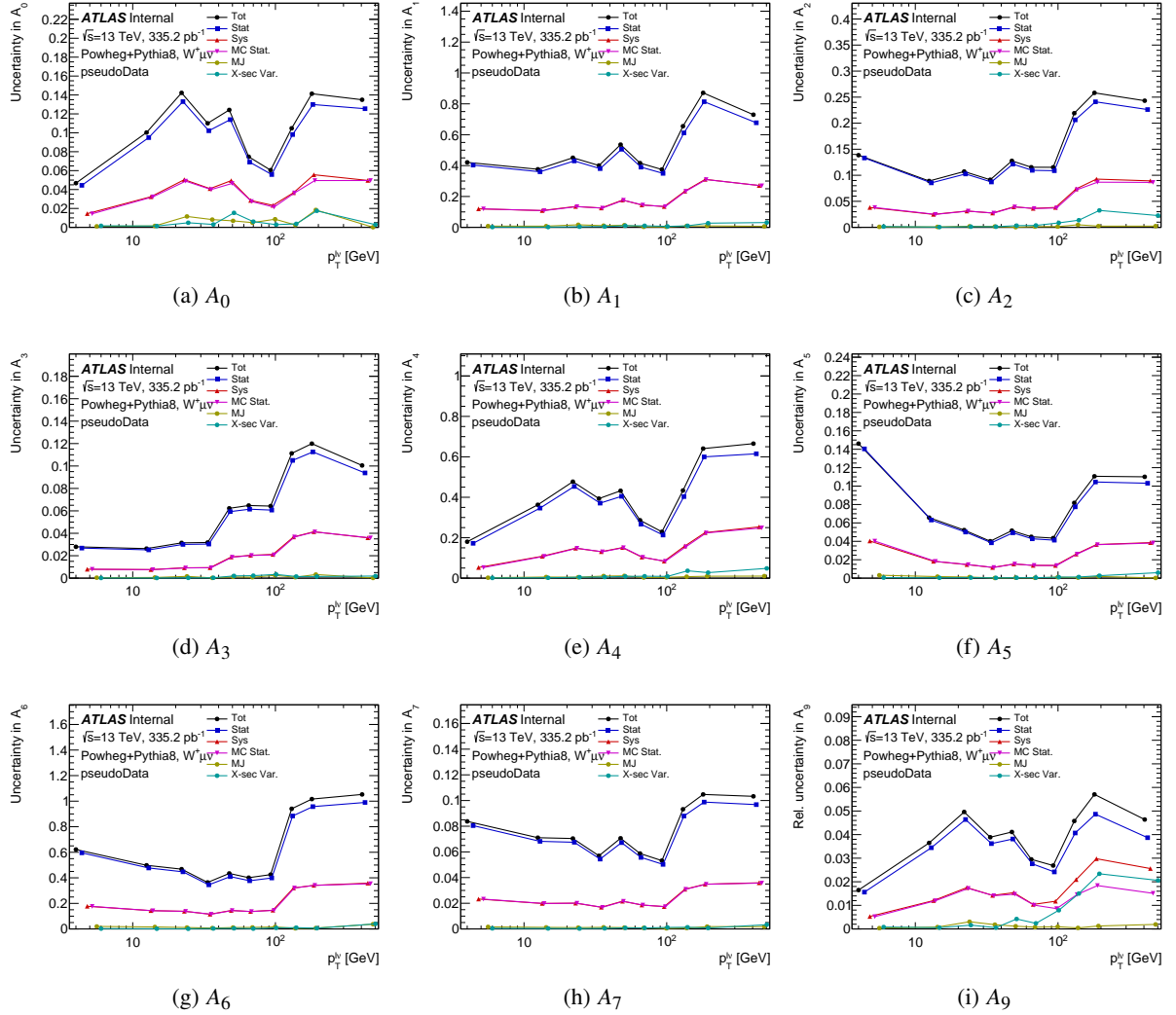


Figure 51: The uncertainty comparison for all contributions in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in p_T^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

818 7.5.2 y^W Differential

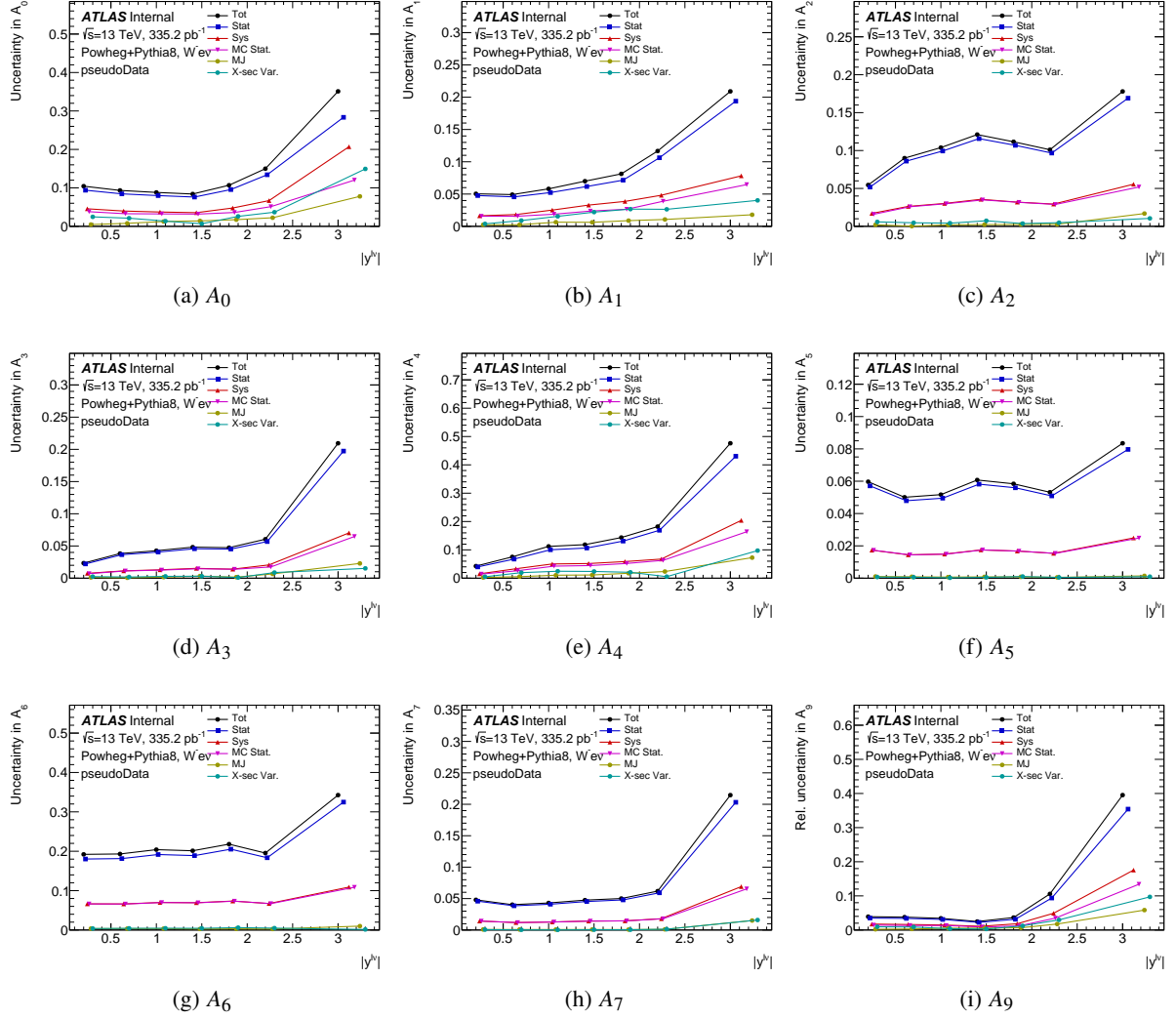


Figure 52: The uncertainty comparison for all contributions in the $W^- \rightarrow e^- \bar{\nu}_W$ channel for measurement in y^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

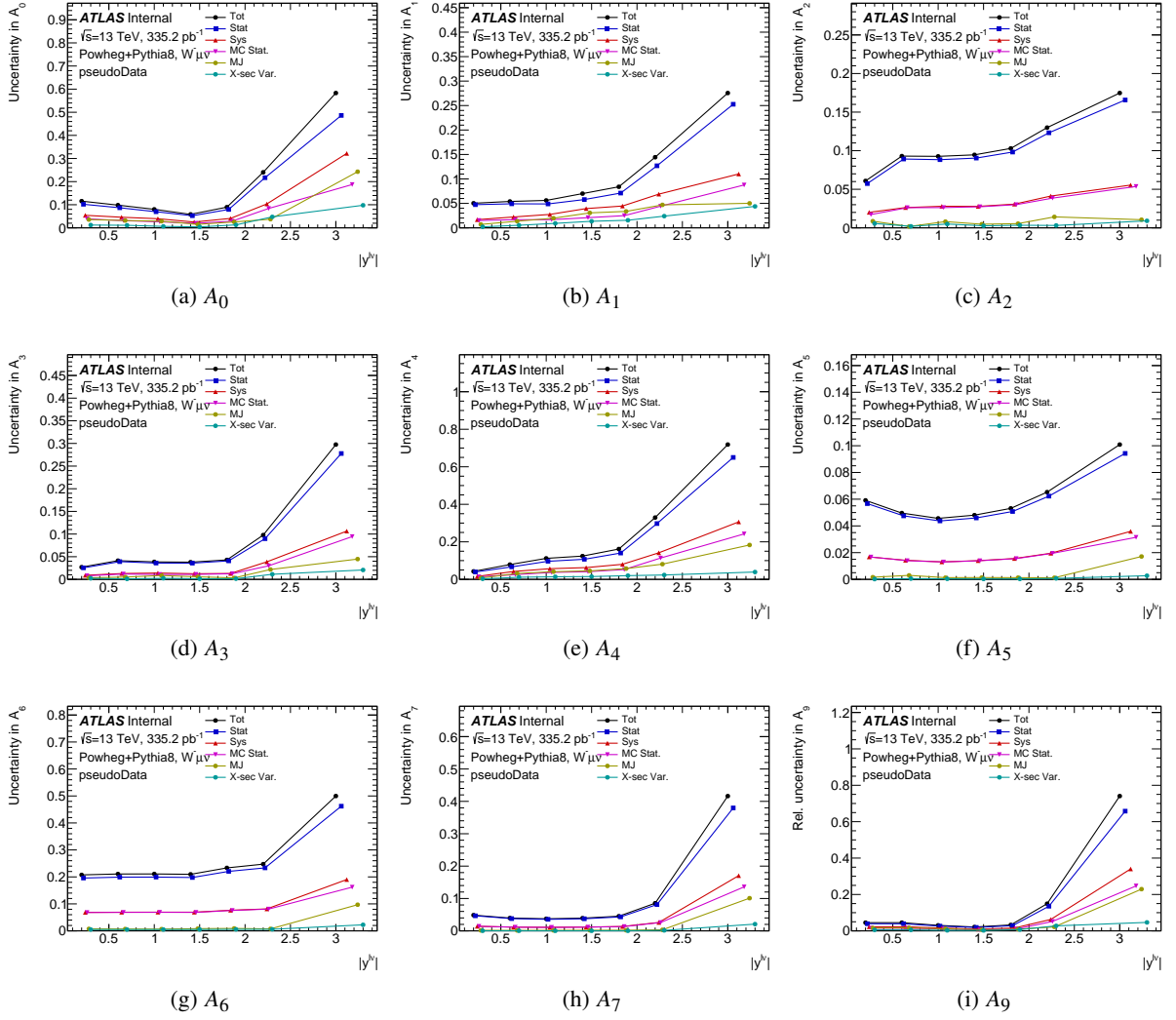


Figure 53: The uncertainty comparison for all contributions in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in y^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

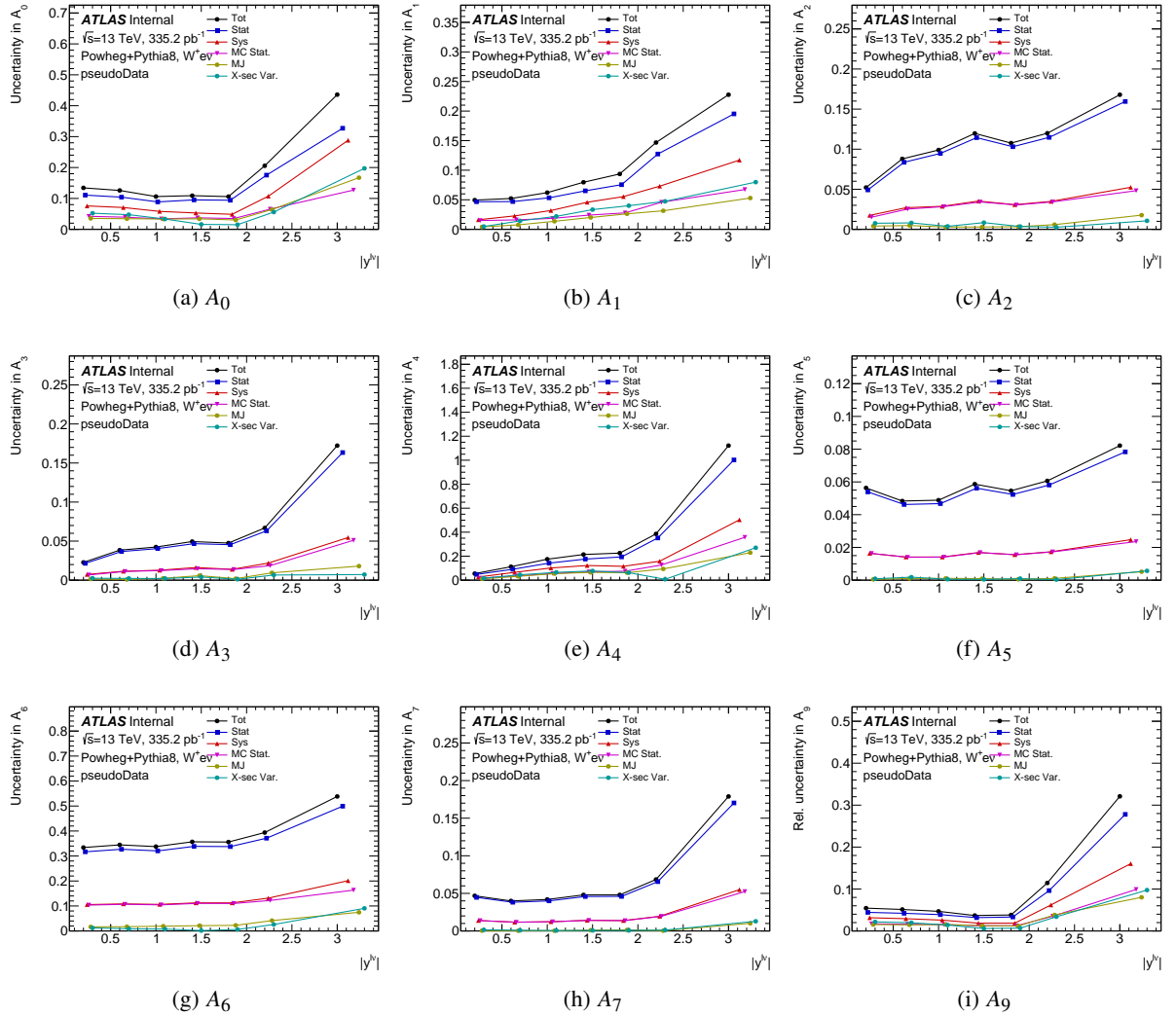


Figure 54: The uncertainty comparison for all contributions in the $W^+ \rightarrow e^+ \nu$ channel for measurement in y^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

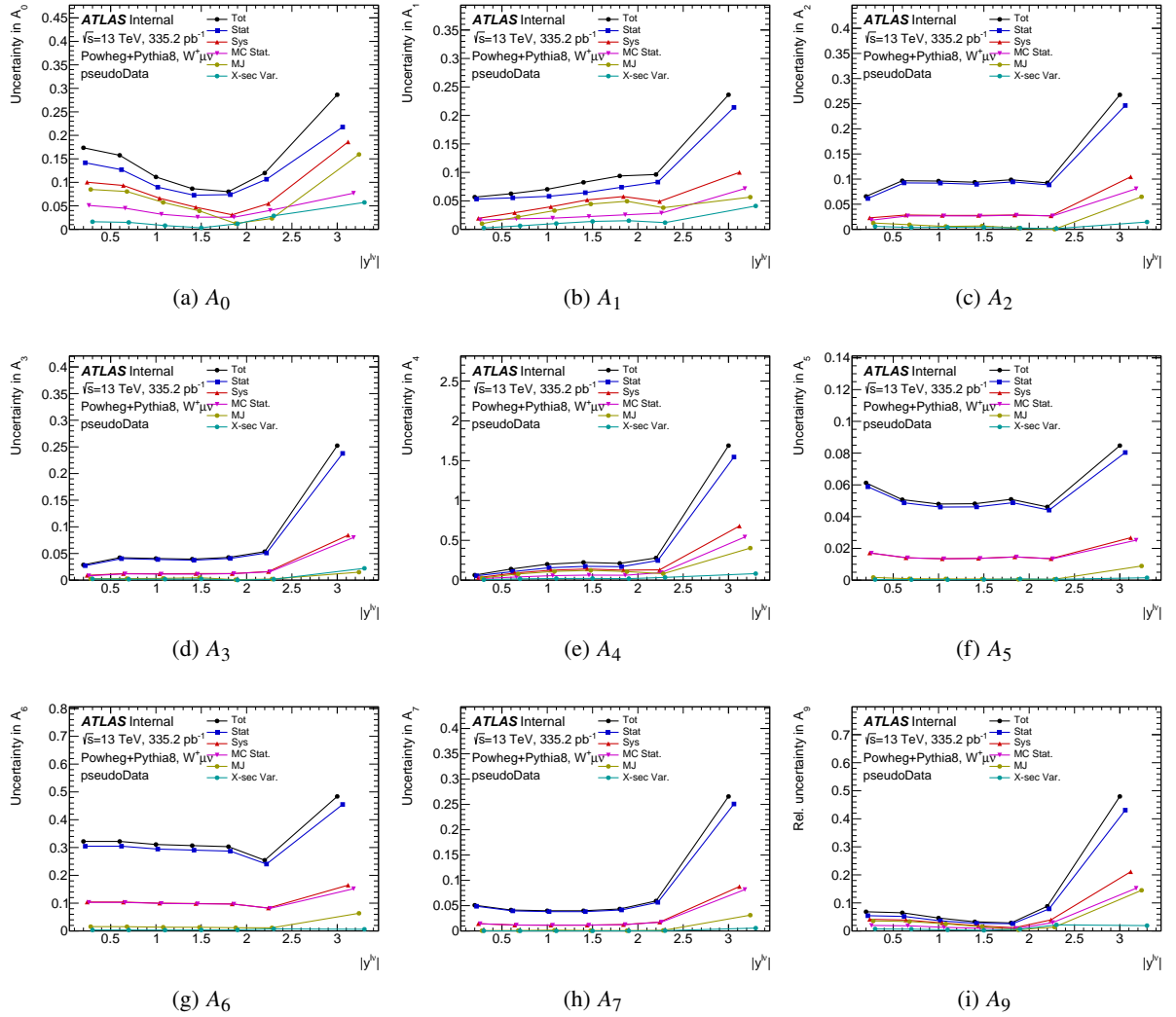


Figure 55: The uncertainty comparison for all contributions in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in y^W . Blue is the total statistical uncertainty, red is the total systematic uncertainty, and black is the combination of the statistical and systematic uncertainties. The components of the systematic uncertainty are shown in pink, green, and teal to compare them to the statistical uncertainty directly.

819 **8 Results**

820 Results will be here once the coefficients are extracted using data. In the meantime results using pseudo
 821 data are currently used as placeholders.

822 **8.1 Expected Results**

823 Here are the expected extracted results for the coefficients that are expected to be measured using pseudo data
 824 which is generated by randomly sampling the expected number events according to a Poisson distribution.

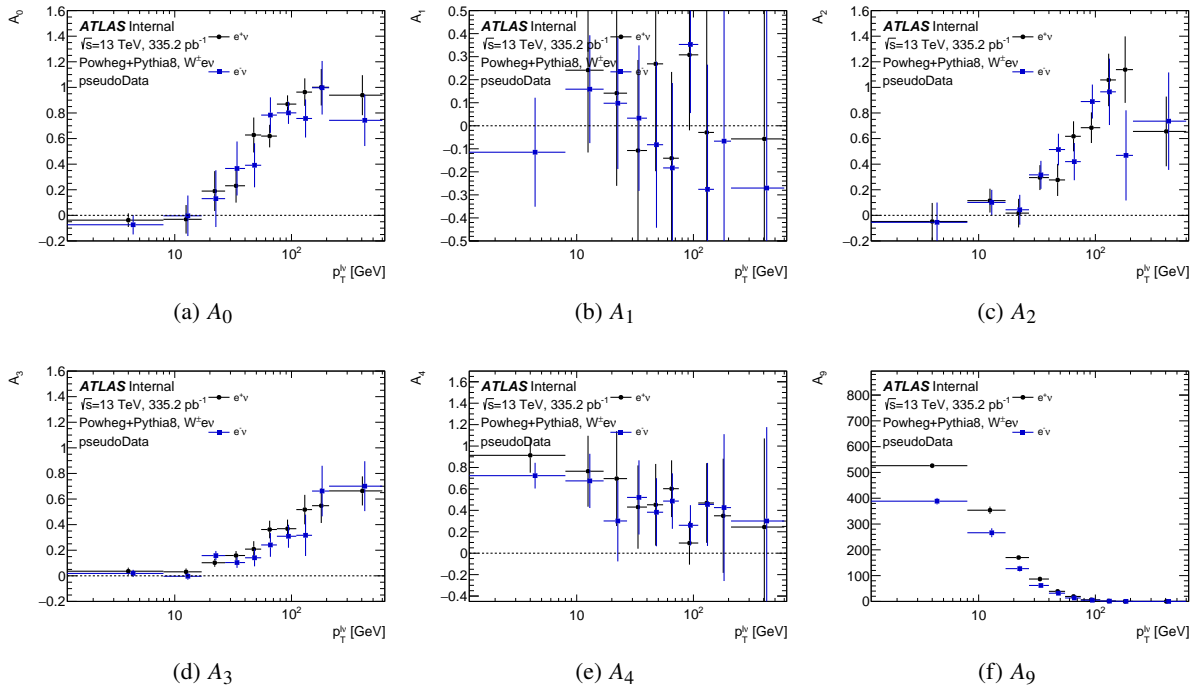


Figure 56: The coefficients extracted using pseudo data for channels $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ as a function of p_T^W . The errors represent the total uncertainty expected on the measurement.

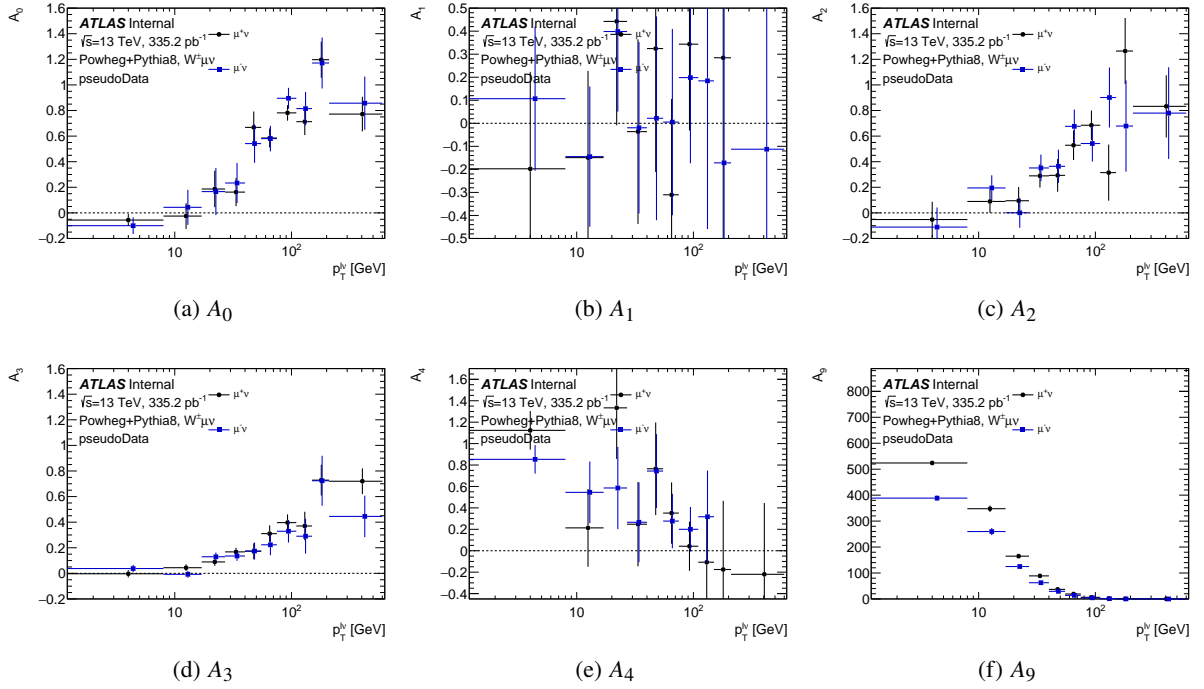


Figure 57: The coefficients extracted using pseudo data for channels $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ as a function of p_T^W . The errors represent the total uncertainty expected on the measurement.

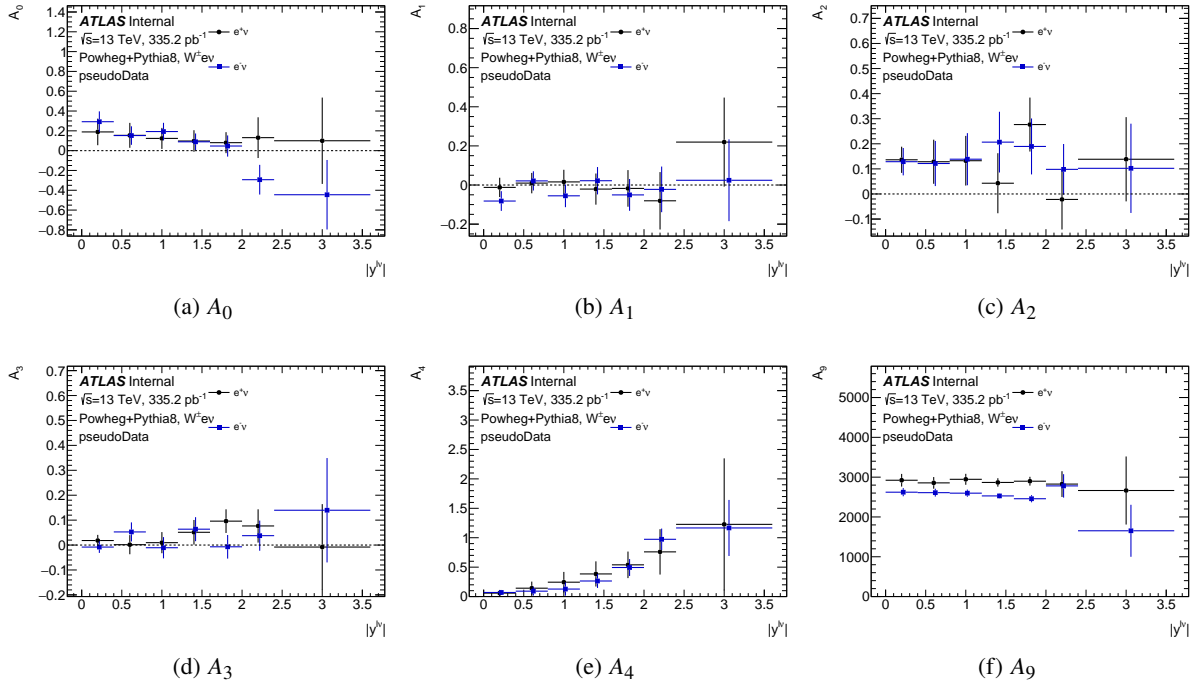


Figure 58: The coefficients extracted using pseudo data for channels $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ as a function of y^W . The errors represent the total uncertainty expected on the measurement.

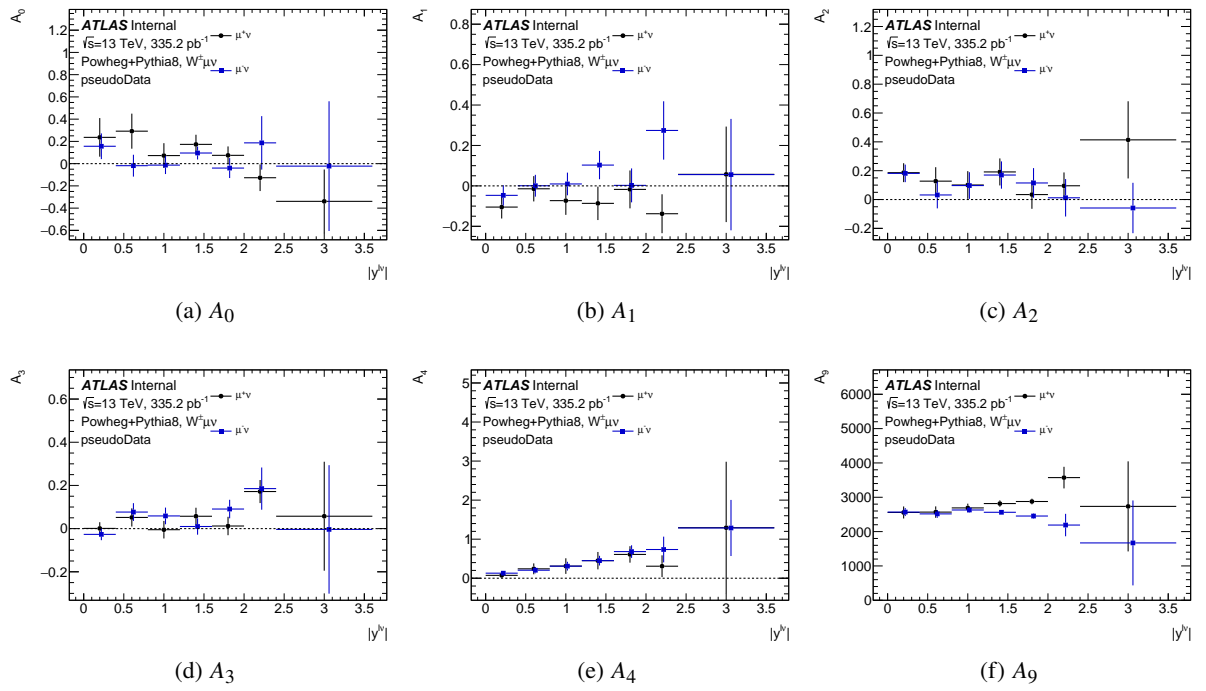


Figure 59: The coefficients extracted using pseudo data for channels $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ as a function of y^W . The errors represent the total uncertainty expected on the measurement.

825 **8.2 Channel Compatibility**

826 Before extracting the measurements themselves for each channel, it is important to assess the compatibility
 827 between different channels in such a complex fitting procedure in terms of the reconstructed observables.
 828 The likelihood in Equation 19 was reparameterized in terms of new free parameters $\Delta A_{ij} = A_{ij}^{\mu\nu} - A_{ij}^{e\nu}$ and
 829 keeping the original $A_{ij}^{\mu\nu}$. The likelihood is maximized over this new set of parameters and uncertainties
 830 on ΔA_{ij} are extracted as described in Section 3.3. The expectation is that all the differences would be
 831 compatible with zero within the total uncertainty. This is performed in the p_T^W differential analysis in
 832 Section 8.2.1 for the differences $e^+\nu - \mu^+\nu$ and $e^-\nu - \mu^-\nu$ and in the y^W differential analysis in Section
 833 8.2.2.

834 The observed compatibility between the channels is within the uncertainty of these measurements in most
 835 cases. It should be noted that the size of the statistical uncertainty on the differences between the channels
 836 is approximately twice the size of the expected statistical uncertainty on the combination between the
 837 two channels (one loses a factor $\sqrt{2}$ with respect to the individual channel statistical uncertainty when
 838 evaluating the difference between the two channels and one gains this same factor when combining them).
 839 The correlated systematic uncertainties are less important in these comparisons since these cancel, while
 840 the experimental ones are uncorrelated and add in quadrature between the two channels. The compatibility
 841 doesn't include the MC stat systematic but isn't needed as the channels are compatible just statistically.

842 To demonstrate the compatibility the goodness of fit is calculated which is summarized with the associated
 843 p-values in Tables 6.

Channel	χ^2/ndf	p-value
$e^-\nu - \mu^-\nu, p_T^W$ differential	1526.91/1430	0.0371
$e^+\nu - \mu^+\nu, p_T^W$ differential	1443.61/1429	0.3879
$e^-\nu - \mu^-\nu, y^W$ differential	925.51/954	0.7401
$e^+\nu - \mu^+\nu, y^W$ differential	931.64/952	0.6753

Table 6: Summary of goodness of fit and p-values for compatibility between channels.

844 8.2.1 p_T^W Differential $e^+\nu$ vs $\mu^+\nu$

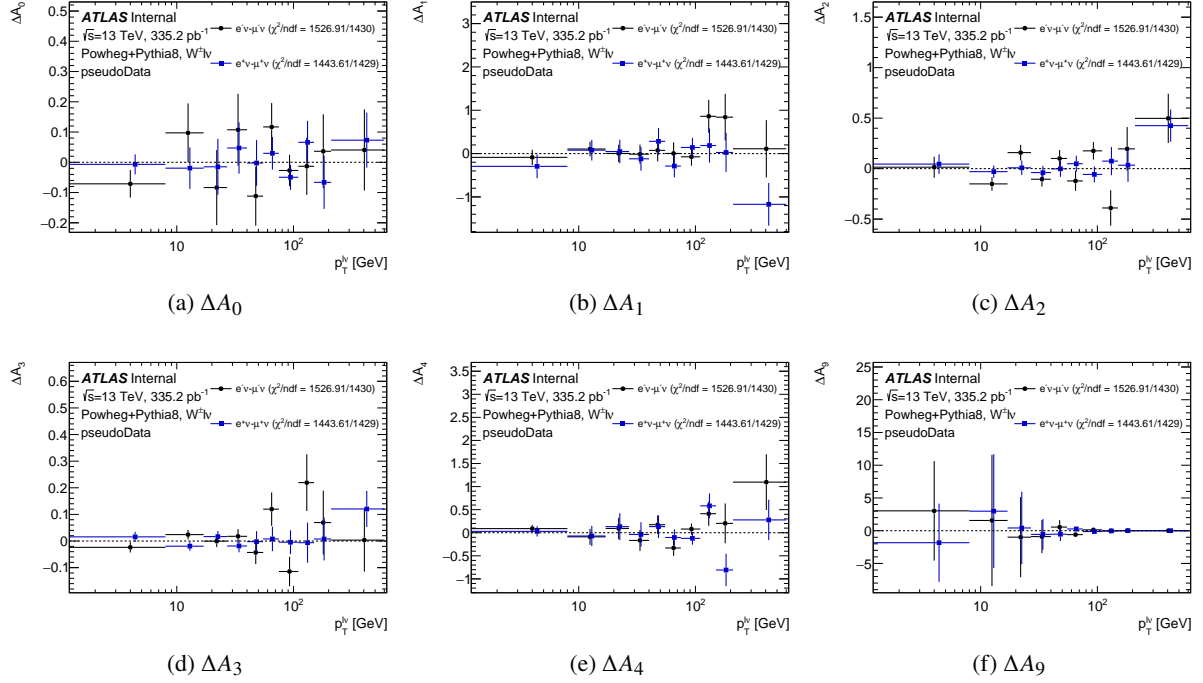


Figure 60: Differences as a function of p_T^W between the $W^- \rightarrow e^-\bar{\nu}$ and $W^- \rightarrow \mu^-\bar{\nu}$ channels in black and $W^+ \rightarrow e^+\nu$ and $W^+ \rightarrow \mu^+\nu$ channels in blue for the measured coefficients. The points represent the measured difference using pseudo data in each p_T^W bin. The goodness of fit, χ^2/ndf , for all coefficients is also listed.

845 **8.2.2 y^W Differential $e^+\nu$ vs $\mu^+\nu$**

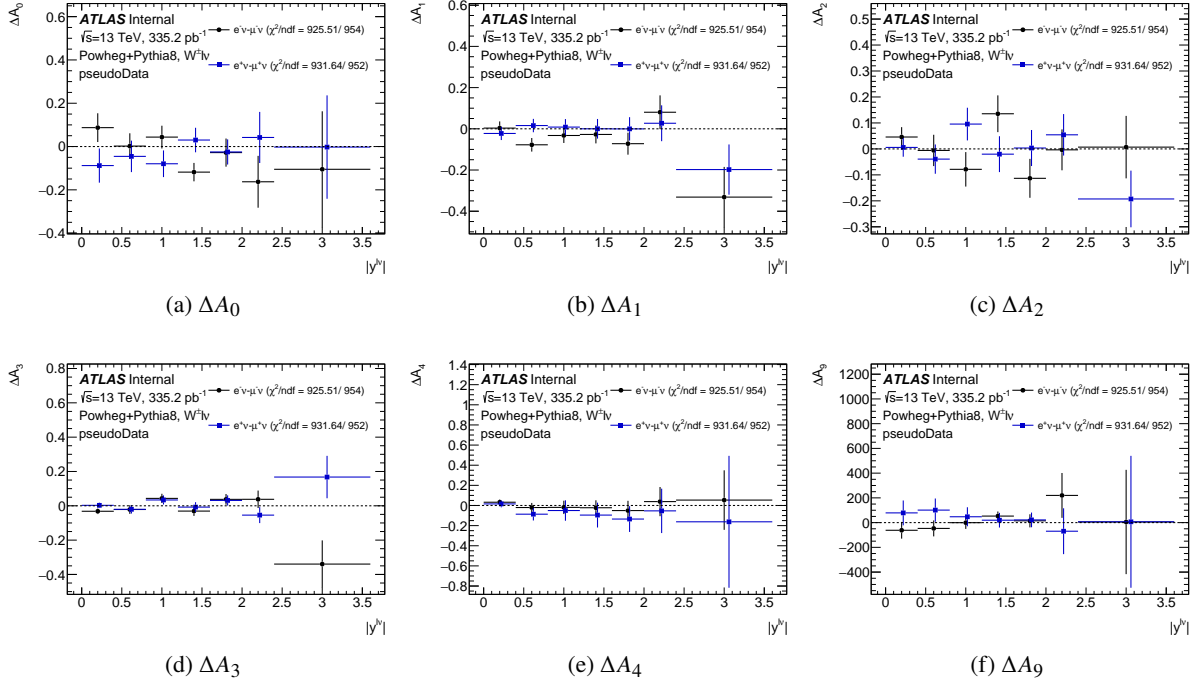


Figure 61: Differences as a function of y^W between the $W^- \rightarrow e^-\bar{\nu}$ and $W^- \rightarrow \mu^-\bar{\nu}$ channels in black and $W^+ \rightarrow e^+\nu$ and $W^+ \rightarrow \mu^+\nu$ channels in blue for the measured coefficients. The points represent the measured difference using pseudo data in each y^W bin. The goodness of fit, χ^2/ndf , for all coefficients is also listed.

8.3 Channel Combination

With compatible channels we are able to combine them to gain a $\sim \sqrt{2}$ statistical sensitivity gain than either individual channel. The individual likelihoods are combined through a simple likelihood multiplication, keeping in mind that constraints on correlated systematics only keep one constraint term. Each likelihood can be decomposed into two types of terms: that which contains the observed data, denoted $\mathcal{L}_{\text{data}}$, and the auxiliary terms that constrain the nuisance parameters, θ , denoted $\mathcal{A}_i(\theta_i)$. The combined likelihood can be written as:

$$\mathcal{L}_{\text{cb}} = \left\{ \prod_i^{\text{channels}} \mathcal{L}_i(A, \theta | N_i) \right\} \left\{ \prod_i^M \mathcal{A}_i(\theta_i) \right\} \quad (56)$$

The combination results for p_T^W differential can be found in red in Figures 62 and 63 for $e^- \nu + \mu^- \nu$ and $e^+ \nu + \mu^+ \nu$ respectively as well as the results for the individual channels. Similarly the results for y^W differential can be found in Figures 64 and 65. The overall agreement between the combined and individual measurements can be more qualitatively grasped when presented in this way as opposed to the ΔA_{ij} measurement.

8.3.1 p_T^W Differential

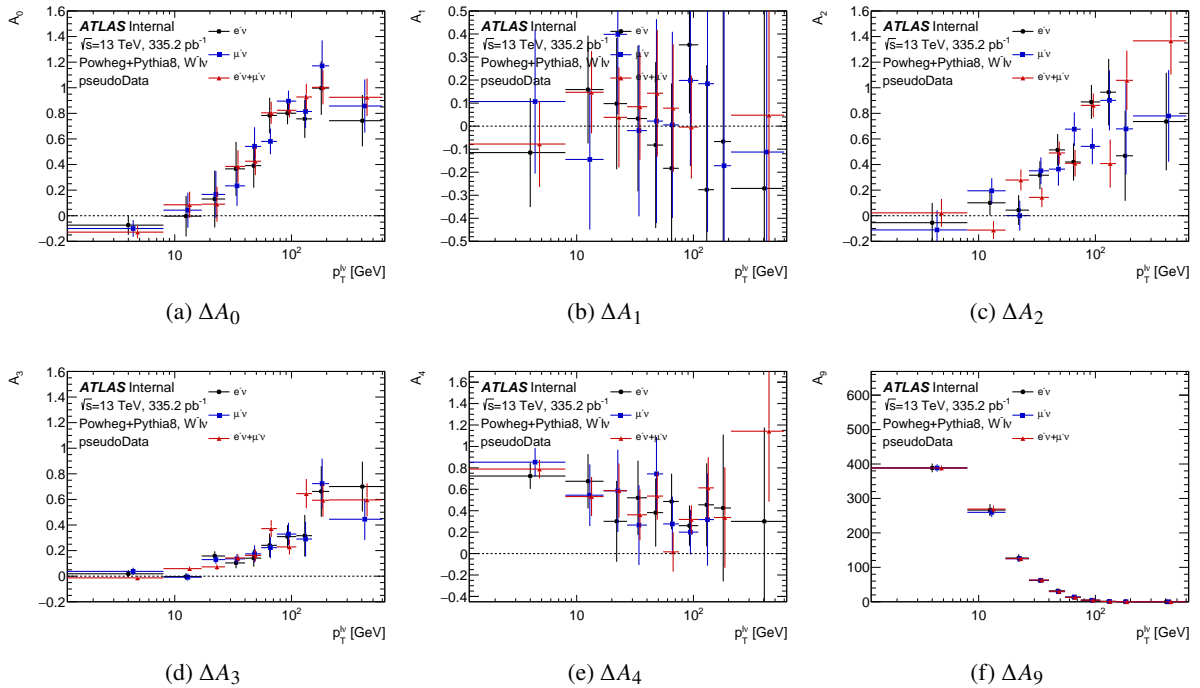


Figure 62: Combined measurement of coefficients for W^- as a function of y^W differential in rapidity. The measured coefficients from the individual channels are overlaid in black and blue. The error bars represent the total uncertainty on the combined measurements.

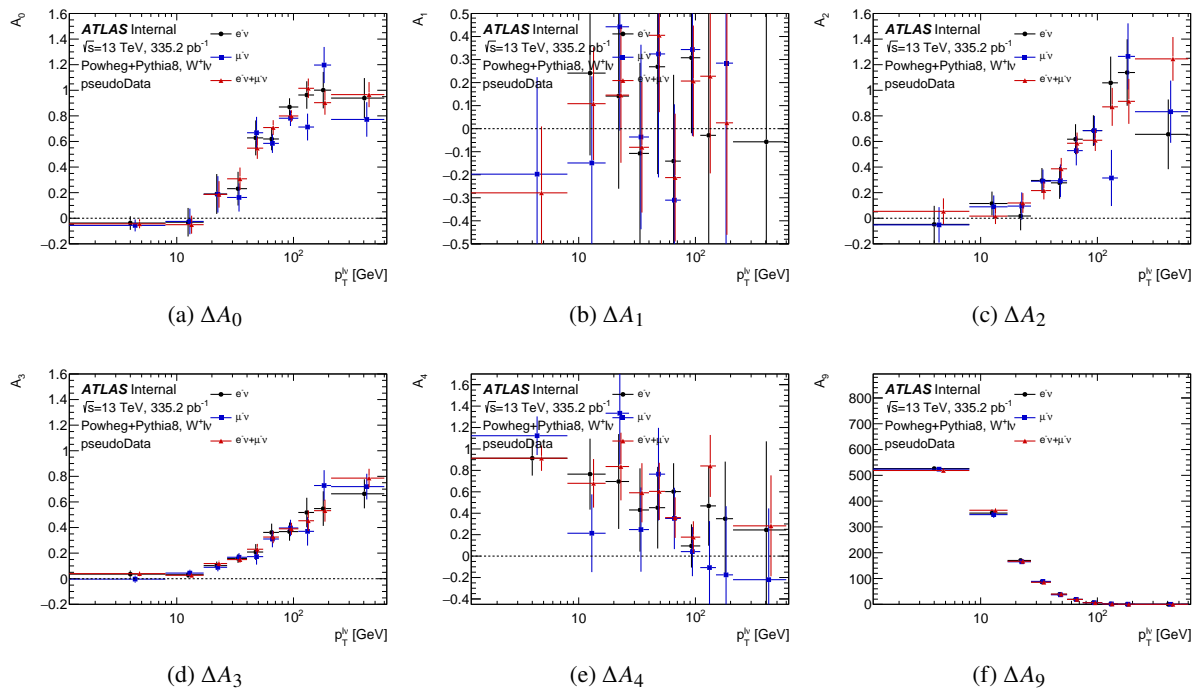


Figure 63: Combined measurement of coefficients for W^+ as a function of y^W differential in rapidity. The measured coefficients from the individual channels are overlaid in black and blue. The error bars represent the total uncertainty on the combined measurements.

859 8.3.2 y^W Differential

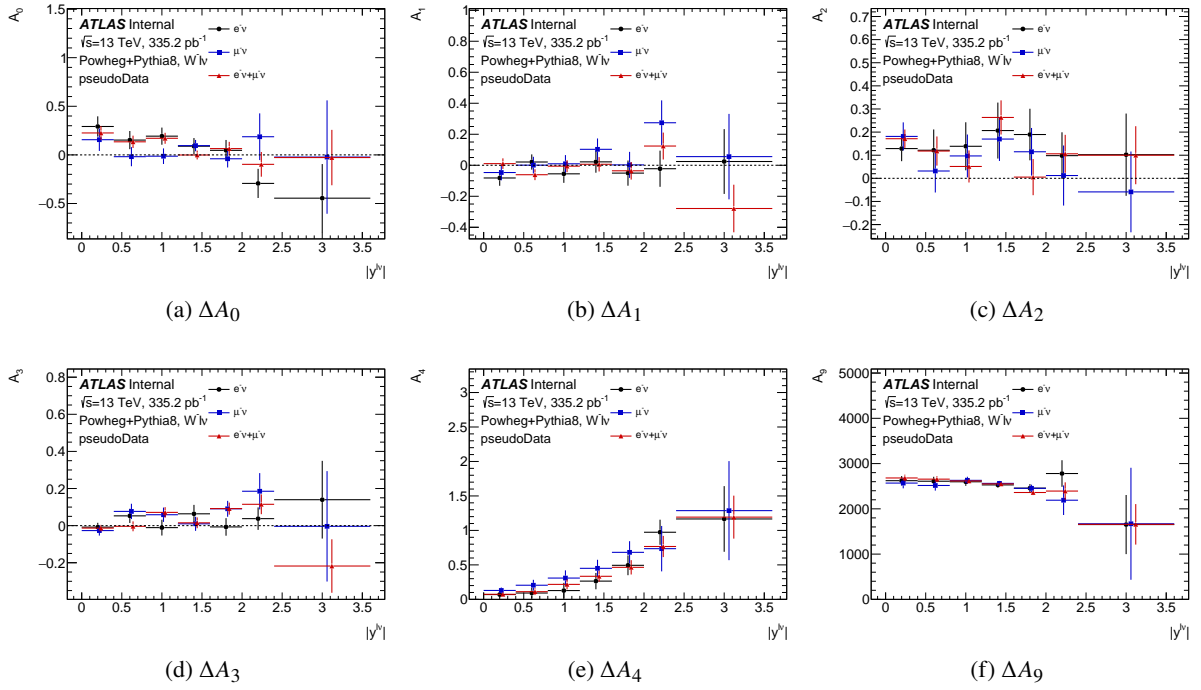


Figure 64: Combined measurement of coefficients for W^- as a function of p_T^W differential in transverse momentum. The measured coefficients from the individual channels are overlaid in black and blue. The error bars represent the total uncertainty on the combined measurements.

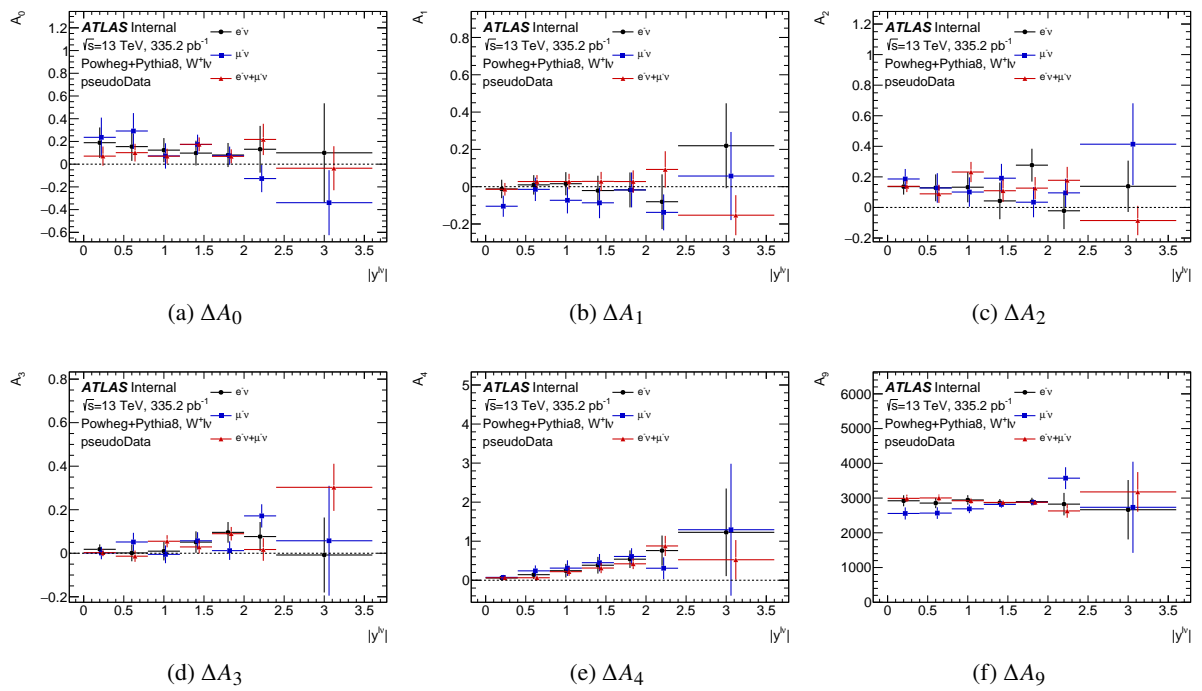
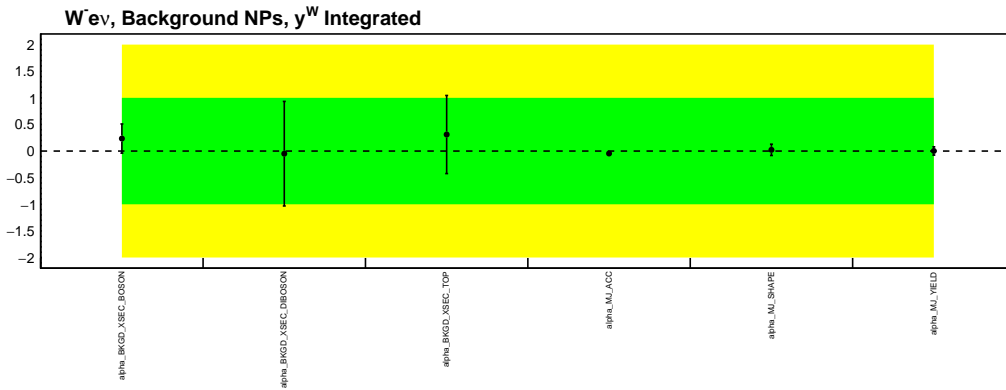


Figure 65: Combined measurement of coefficients for W^- as a function of p_T^W differential in transverse momentum. The measured coefficients from the individual channels are overlaid in black and blue. The error bars represent the total uncertainty on the combined measurements.

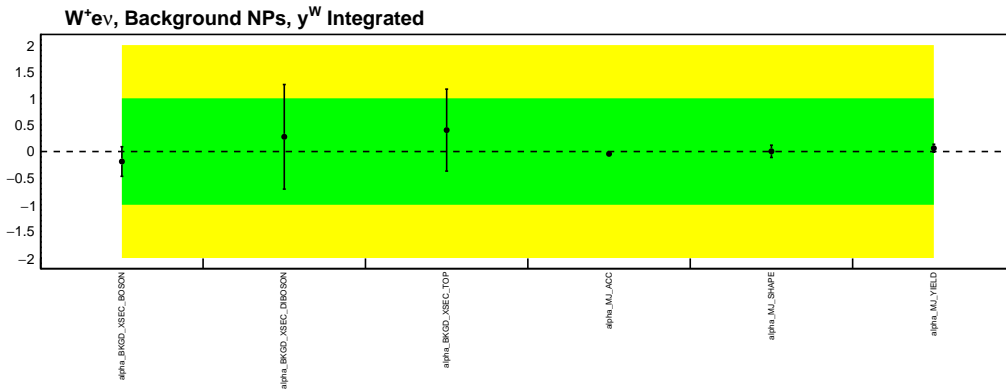
860 **8.4 Nuisance Parameter Pulls**

861 The best fit values of each nuisance parameter along with their post-fit constraints are presented here.
 862 Most parameters should have a fit value close to zero with a constraint close to unity. If a systematic
 863 variation does not mimic the PDF of the polynomials in the model, the floating coefficients won't absorb
 864 the variation and the result will be that the likelihood value changes in the variation to compensate. That is,
 865 the data will become incompatible with the PDF under the variation and the constraint on the nuisance
 866 parameter will be less than unity. In most cases this only happens by a small amount, but systematics
 867 that are over-conservative to begin with or which could even be measured using angular distributions
 868 can become significantly constrained. The pulls for each channel are shown for in the p_T^W differential
 869 measurement in Figures 66 - 69 and for the y^W differential measurement in Figures 70 - 73.

870 **8.4.1 p_T^W Differential**

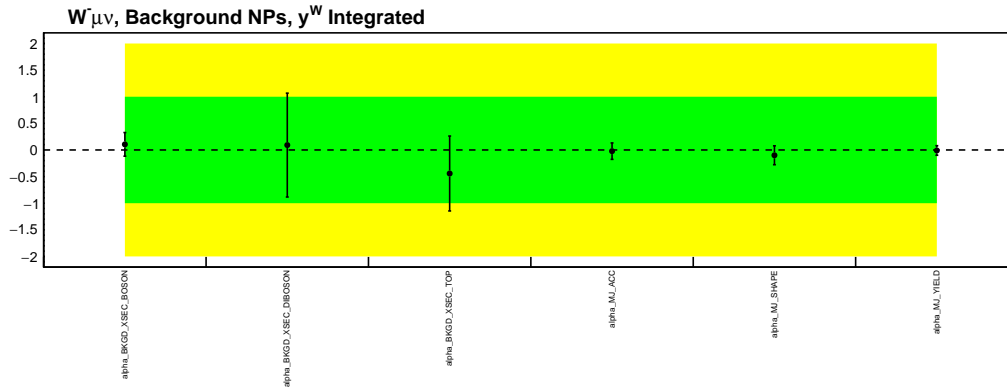


(a) $W^- \rightarrow e^- \bar{\nu}$ background NPs

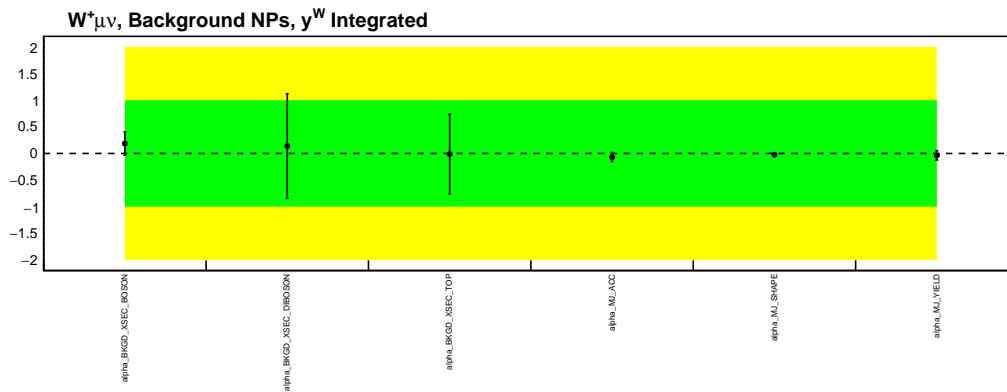


(b) $W^+ \rightarrow e^+ \nu$ background NPs

Figure 66: Best fit and post-fit constraint of the background nuisance parameters in the p_T^W differential configuration for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

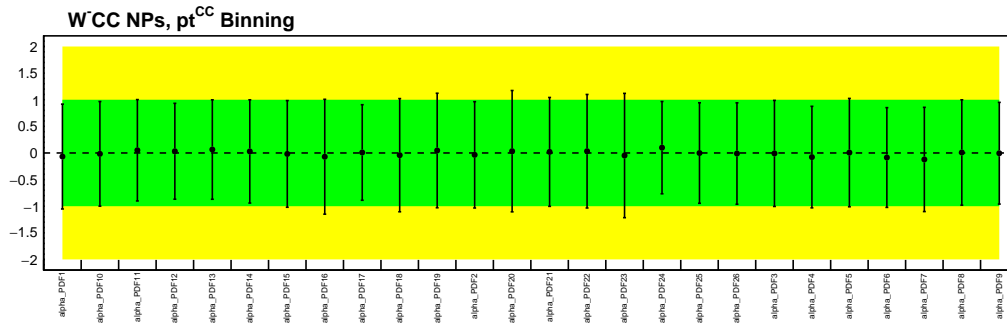


(a) $W^- \rightarrow \mu^- \bar{\nu}$ background NPs

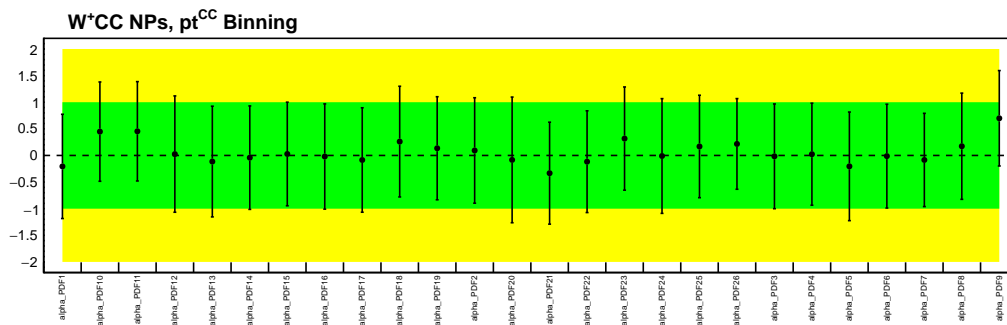


(b) $W^+ \rightarrow \mu^+ \nu$ background NPs

Figure 67: Best fit and post-fit constraint of the background nuisance parameters in the p_T^W differential configuration for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

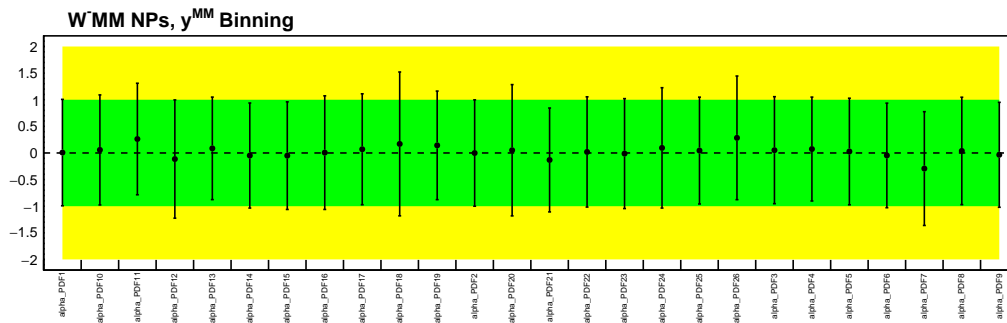


(a) $W^- \rightarrow e^- \bar{\nu}$ pdf NPs

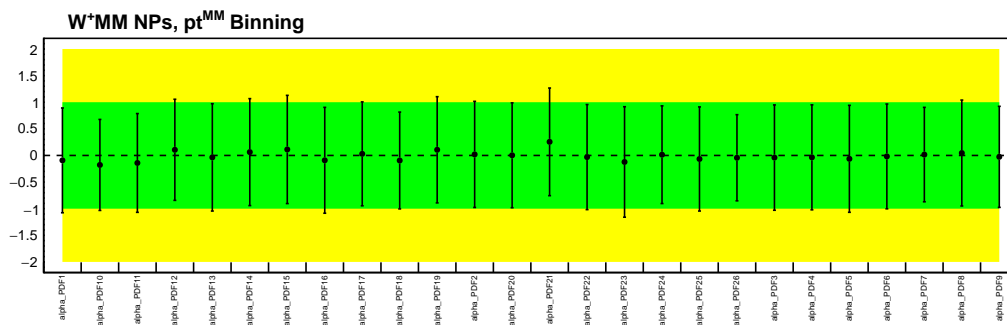


(b) $W^+ \rightarrow e^+ \nu$ pdf NPs

Figure 68: Best fit and post-fit constraint of the pdf nuisance parameters in the p_T^W differential configuration for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.



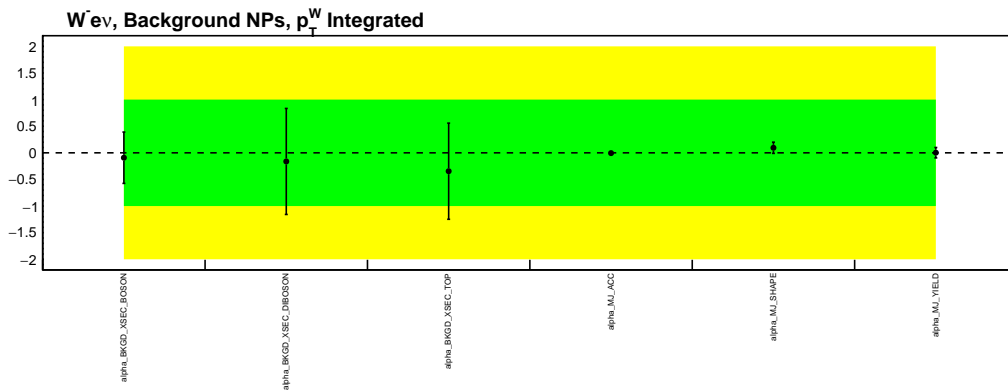
(a) $W^- \rightarrow \mu^- \bar{\nu}$ pdf NPs



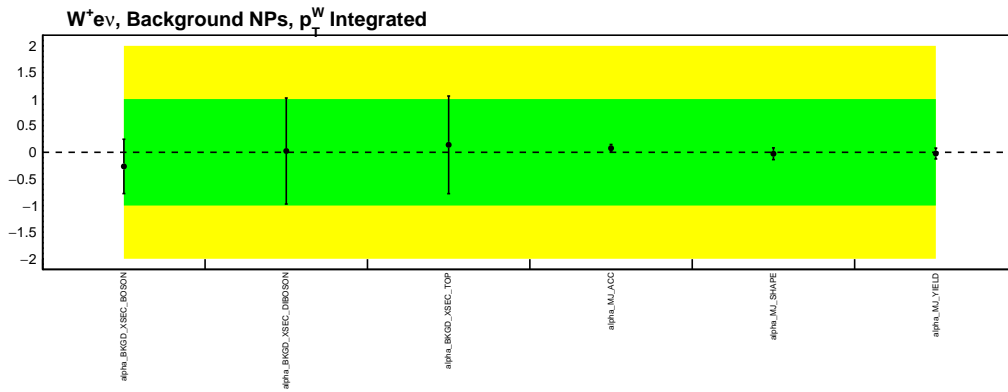
(b) $W^+ \rightarrow \mu^+ \nu$ pdf NPs

Figure 69: Best fit and post-fit constraint of the pdf nuisance parameters in the p_T^W differential configuration for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

871 8.4.2 y^W Differential

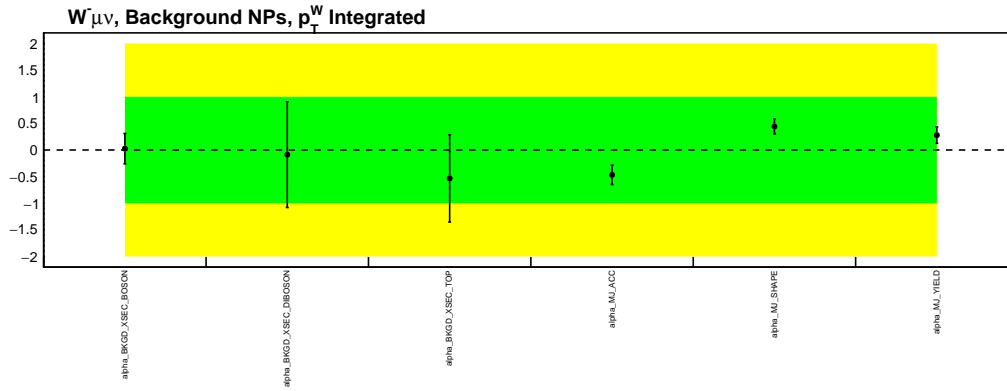


(a) $W^- \rightarrow e^- \bar{\nu}$ background NPs

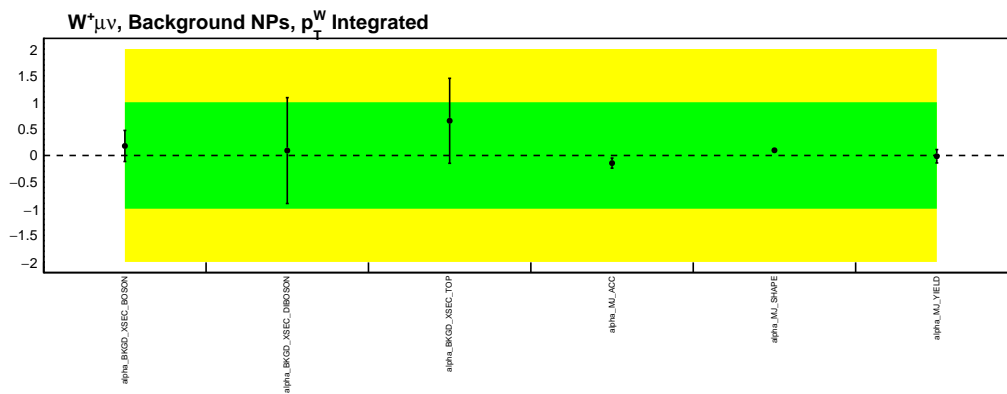


(b) $W^+ \rightarrow e^+ \nu$ background NPs

Figure 70: Best fit and post-fit constraint of the background nuisance parameters in the y^W differential configuration for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

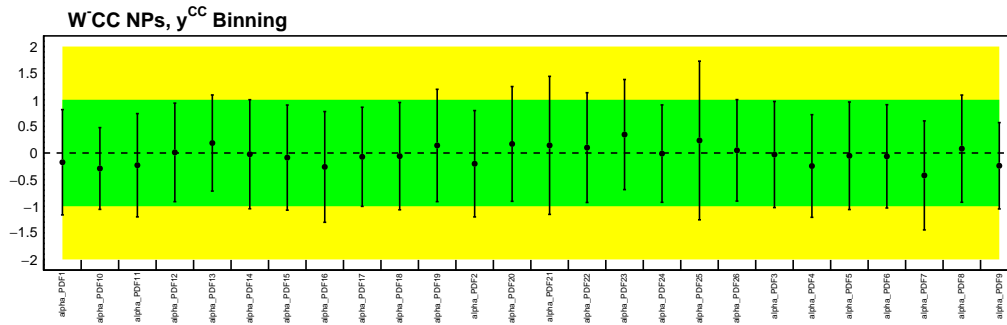


(a) $W^- \rightarrow \mu^- \bar{\nu}$ background NPs

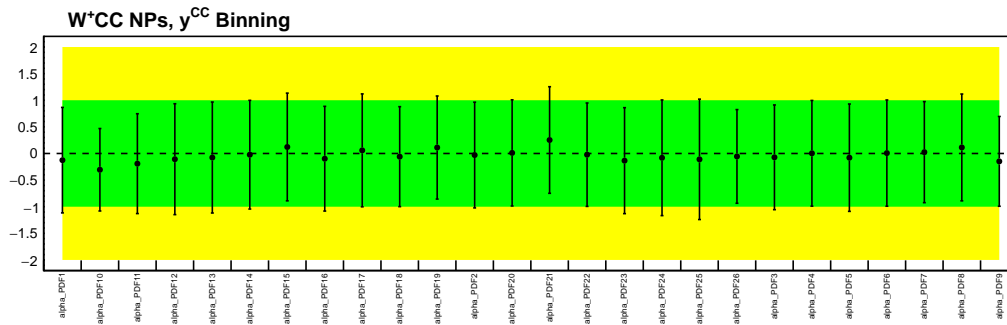


(b) $W^+ \rightarrow \mu^+ \nu$ background NPs

Figure 71: Best fit and post-fit constraint of the background nuisance parameters in the y^W differential configuration for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

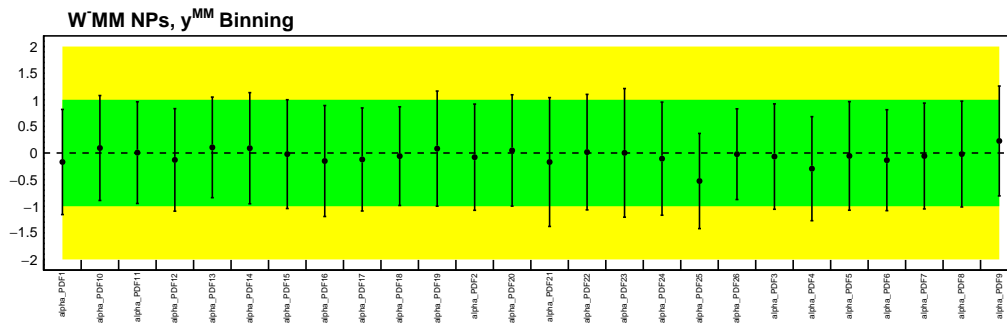


(a) $W^- \rightarrow e^- \bar{\nu}$ pdf NPs

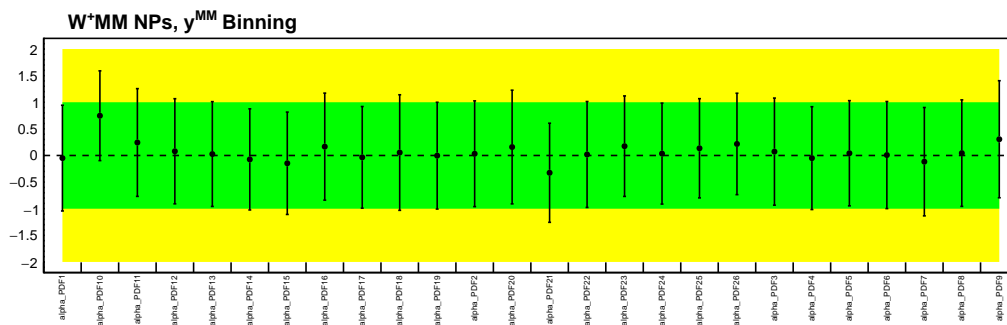


(b) $W^+ \rightarrow e^+ \nu$ pdf NPs

Figure 72: Best fit and post-fit constraint of the pdf nuisance parameters in the y^W differential configuration for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.



(a) $W^- \rightarrow \mu^- \bar{\nu}$ pdf NPs



(b) $W^+ \rightarrow \mu^+ \nu$ pdf NPs

Figure 73: Best fit and post-fit constraint of the pdf nuisance parameters in the y^W differential configuration for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels. The green and yellow bands represent $\pm 1\sigma$ and $\pm 2\sigma$ from zero.

872 9 Comparison with Theoretical Predictions

873 *This chapter will contain the comparison of results to theoretical predictions from Section 2 when those are*
874 *compiled.*

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¹⁰⁰² **List of contributions**

Ludovica Aperio Bella	Analysis design, initial sensitivity studies, Ntuple generation, hadronic recoil studies
Aaron Armbruster	Overall fitting methodology
Alexander Bachiu	Initial sensitivity studies, identification and isolation scale factors, coefficient extraction, systematic uncertainties
Stefano Camarda	Analysis design, neutrino reconstruction, initial sensitivity studies, theoretical predictions
¹⁰⁰³ Nicolo De Groot	Advising
Ruth Magdalena Jacobs	Initial Multi-jet studies, neutrino reconstruction studies
Daniil Ponomarenko	Multi-jet studies and generation. Implementation of the multijet analysis (fraction fits, anti-isolation scan; extrapolation to signal region and different W-Ai measurement bins; closure tests; systematics).
Manuella Vincter	Advising
¹⁰⁰⁴ Grigorii Tolkachev	PDF systematic uncertainties

1005 Appendices

1006 A Sensitivity studies

1007 Given the limited luminosity recorded by ATLAS with low pile-up condition during the Run-2 data-taking
1008 (335.180pb^{-1} at 13 TeV), in order to increase the overall analysis sensitivity, different sensitivity studies were
1009 performed to maximise that statistical significance of the measurement. In the following the optimisation
1010 of the choice is kinematic cuts, identification and isolation criterion selection on the lepton are shown
1011 together of detailed studies on how to handle events where there is no solution for the neutrino longitudinal
1012 momentum.

1013 A.1 Kinematic cuts choice

1014 One method to gain statistical sensitivity is by relaxing the cuts on kinematics to get a larger acceptance
1015 of events. However, these cuts are in place to reduce the amount of background which will need to be
1016 addressed in another way. This is discussed in the Section A.2 where tighter constraints are applied on
1017 the identification and isolation. This study was to see whether there is a meaningful gain with relaxed
1018 sensitivity and where in the distributions the gain is. It used the nominal p_z^ν solution method as discussed in
1019 Section A.3 but was repeated using the rotation p_z^ν methodology with no relative difference in results. The
1020 study did not include background and was done to see the impact on just the signal statistical uncertainty.

1021 There are three main kinematic cuts that were varied p_T^ℓ , E_T^{miss} , and m_T which is their order in Figures 74
1022 and 75. The most dominant for this analysis is the MJ and the kinematic cuts are one way of reducing it as
1023 it mostly is at low E_T^{miss} and m_T . As seen in Figure 74, reducing both of these cuts increases the statistical
1024 sensitivity by a large amount especially at high $p_T^{\ell\nu}$ where the analysis is limited. The MJ is predominantly
1025 at low $p_T^{\ell\nu}$ so it does not interfere with this statistical gain. The last thing to note was there was not a
1026 significant enough gain by adding events $20 \leq p_T^\ell \leq 25\text{GeV}$ to warrant the gain in MJ.

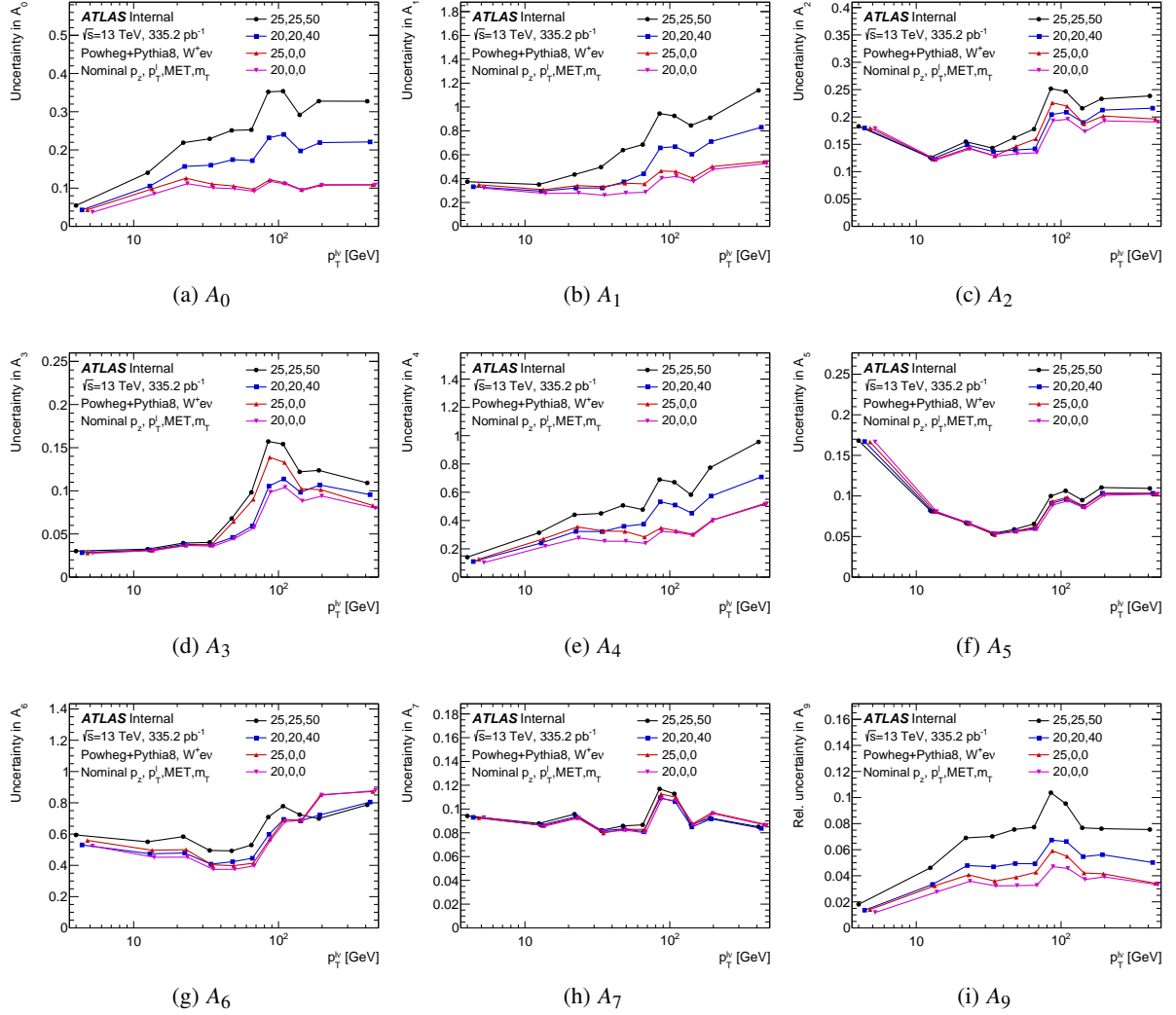


Figure 74: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for different selection criteria in $p_T^{\ell\nu}$ binning for $W^+ \rightarrow e^+\nu$. The legend lists the various selection criteria in GeV for p_T^ℓ , E_T^{miss} , and m_T respectively; for example in black is when $p_T^\ell > 25\text{GeV}$, $E_T^{\text{miss}} > 25\text{GeV}$, and $m_T \lesssim 50\text{ GeV}$. This study was done using the nominal p_2^ν choice from the study in Section A.3.

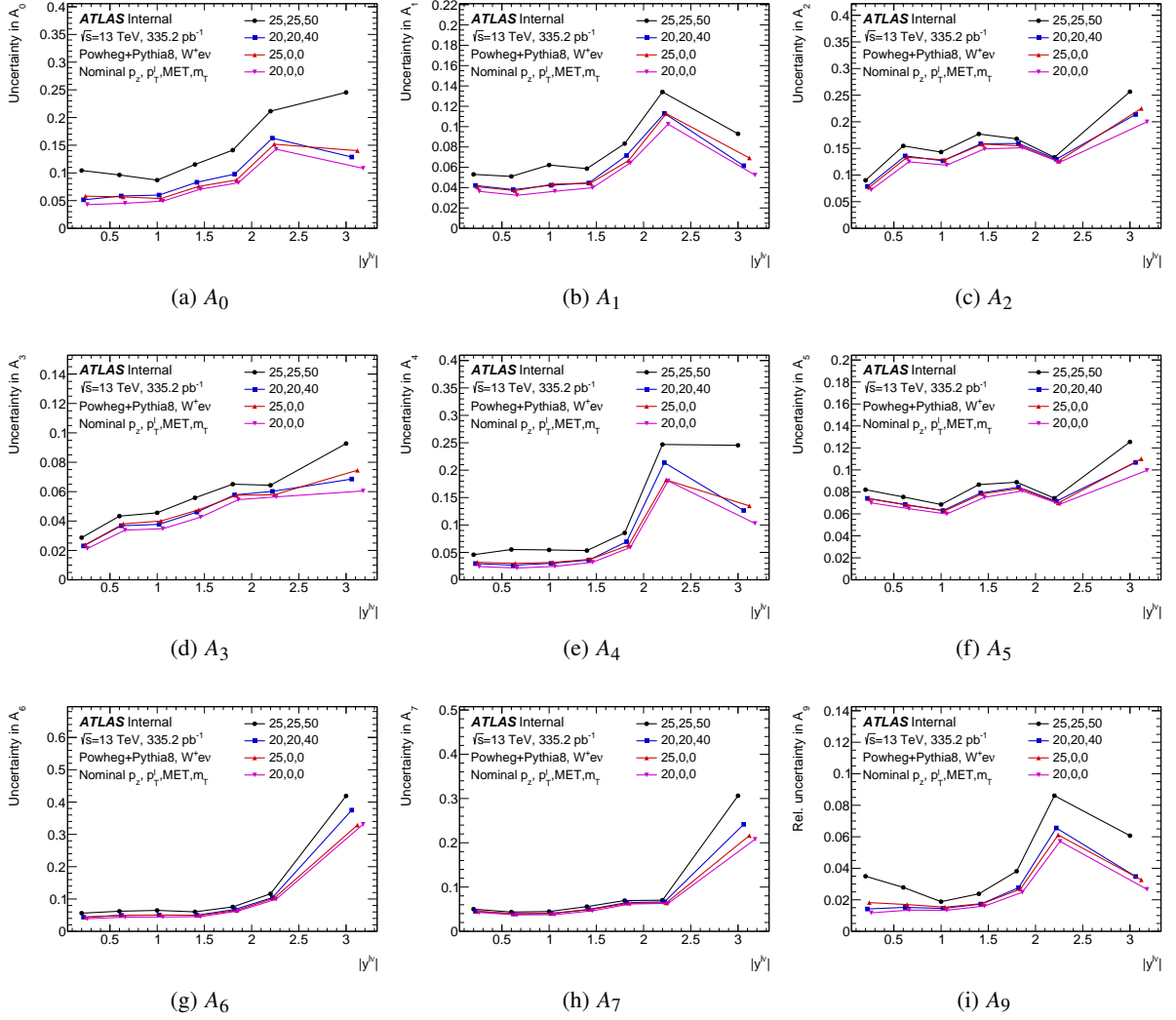


Figure 75: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for different selection criteria in $y^{\ell\nu}$ binning for $W^+ \rightarrow e^+\nu$. The legend lists the various selection criteria in GeV for p_T^ℓ , E_T^{miss} , and m_T respectively; for example in black is when $p_T^\ell > 25\text{GeV}$, $E_T^{\text{miss}} > 25\text{GeV}$, and $m_T \geq 50\text{ GeV}$. This study was done using the nominal p_z^ν choice from the study in Section A.3.

1027 **A.2 Lepton selection optimisation**

1028 Having relaxed kinematic cuts allows more MJ background but can be reduced by having stricter
 1029 identification and/or isolation requirements. The electron channel has more MJ so it needs both a tighter
 1030 identification requirement while both electron and muon channels use the isolation requirement. Figures
 1031 76 - 79 show that these requirements do not remove too much statistical sensitivity with the inclusion of the
 1032 identification and isolation cuts. There is net statistical significance gain with the relaxed kinematic cuts
 1033 discussed in Section A.1 and these additional identification and isolation requirements.

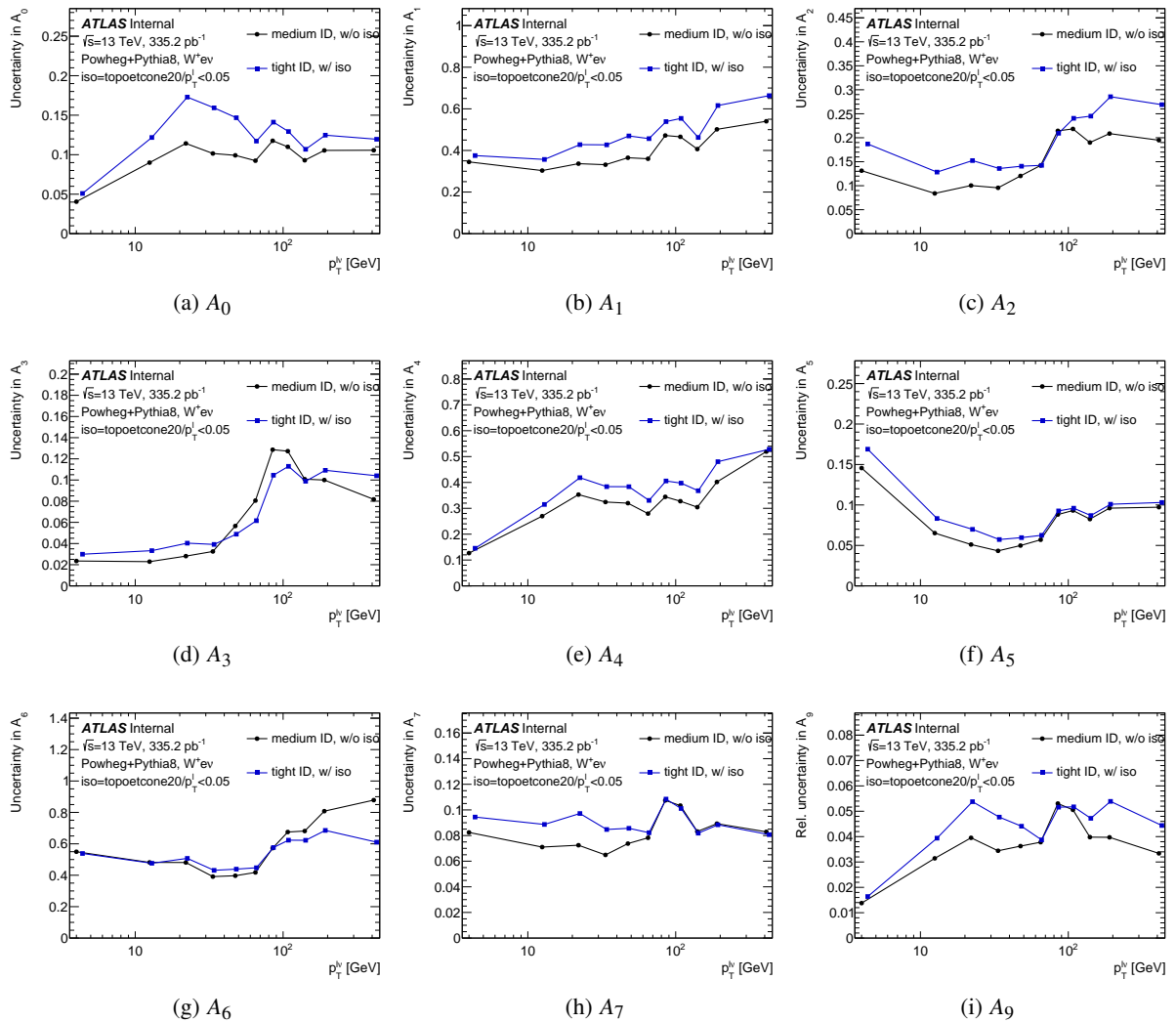


Figure 76: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the inclusion of adding isolation and a tight ID selection in $p_T^{\ell\nu}$ binning for $W^+ \rightarrow e^+ \nu$.

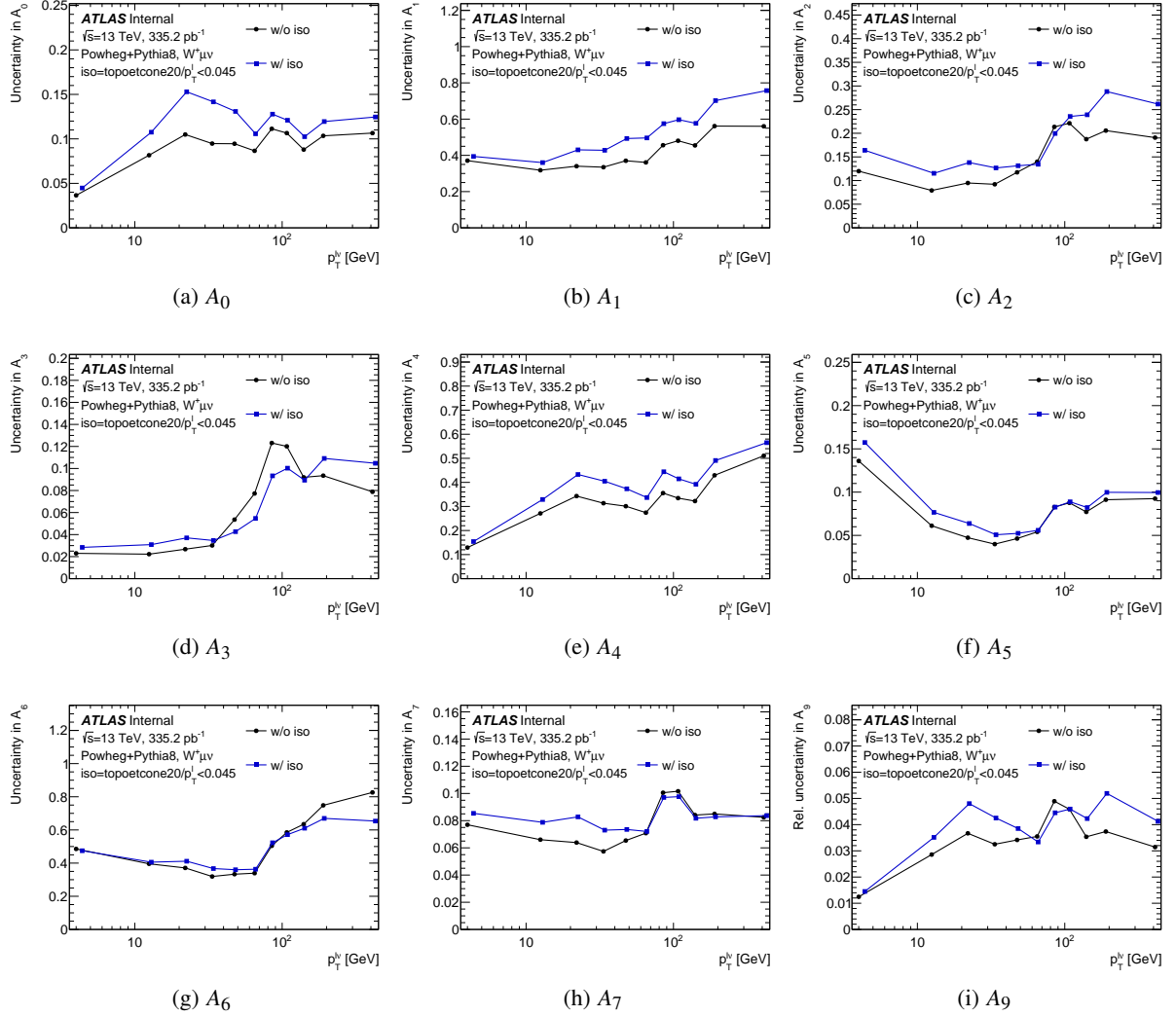


Figure 77: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the inclusion of adding isolation selection in $p_T^{\ell\nu}$ binning for $W^+ \rightarrow \mu^+\nu$.

1034 A.3 Neutrino p_z^ν no solution

1035 The mass constraint results in an imaginary solution when $\Delta < 0$, which occurs when $m_T > m^W$. When this
 1036 happens, events are not being able to be fully reconstructed, however, the ϕ_{CS} value can still be calculated
 1037 as normal for these events. To use these events the templates were extended to have an additional 8×8
 1038 phase space in order to not loose ϕ_{CS} shape information and not interfere with the real solutions. However,
 1039 what to use for the $\cos \theta_{CS}$ value had to be decided, four ideas were tested:

1040 Nominal

1041 The nominal choice is to not assign any $\cos \theta_{CS}$ information about these events by storing them within the
 1042 same bin and just use the ϕ_{CS} information. This is done by setting $\cos \theta_{CS} = 0$ and not solving for p_z^ν when

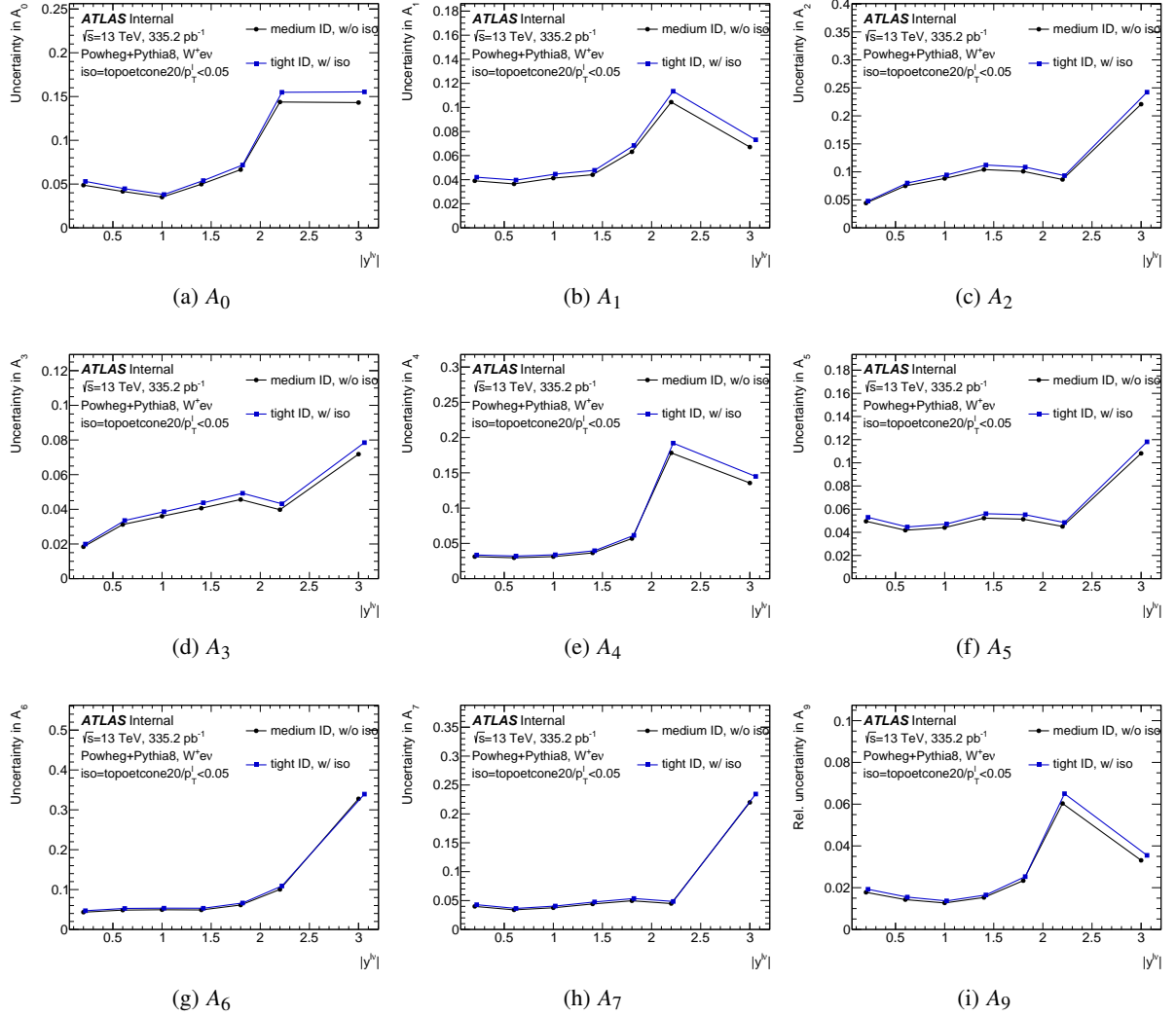


Figure 78: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the inclusion of adding isolation and a tight ID selection in $y^{\ell\nu}$ binning for $W^+ \rightarrow e^+\nu$.

1043 $\Delta < 0$. A drawback to this approach is that no attempt on the sign of $\cos\theta_{CS}$ is made so the W is always
1044 assumed to be traveling perpendicular to the negatively charged lepton (or neutrino in W^+ case) in the CS
1045 frame. The rest of the methods used were done in order to try and gain some of this $\cos\theta_{CS}$ information
1046 back.

1047 VH Style

1048 The second method tried, labeled VH style, took inspiration from a VH measurement just removes the
1049 imaginary part of the solution to $p_z^\nu = -b/2a$ [45]. This method allowed some solutions to have a negatively
1050 signed $\cos\theta_{CS}$ value but overall only increased the sensitivity to A_1 compared to the other methods.

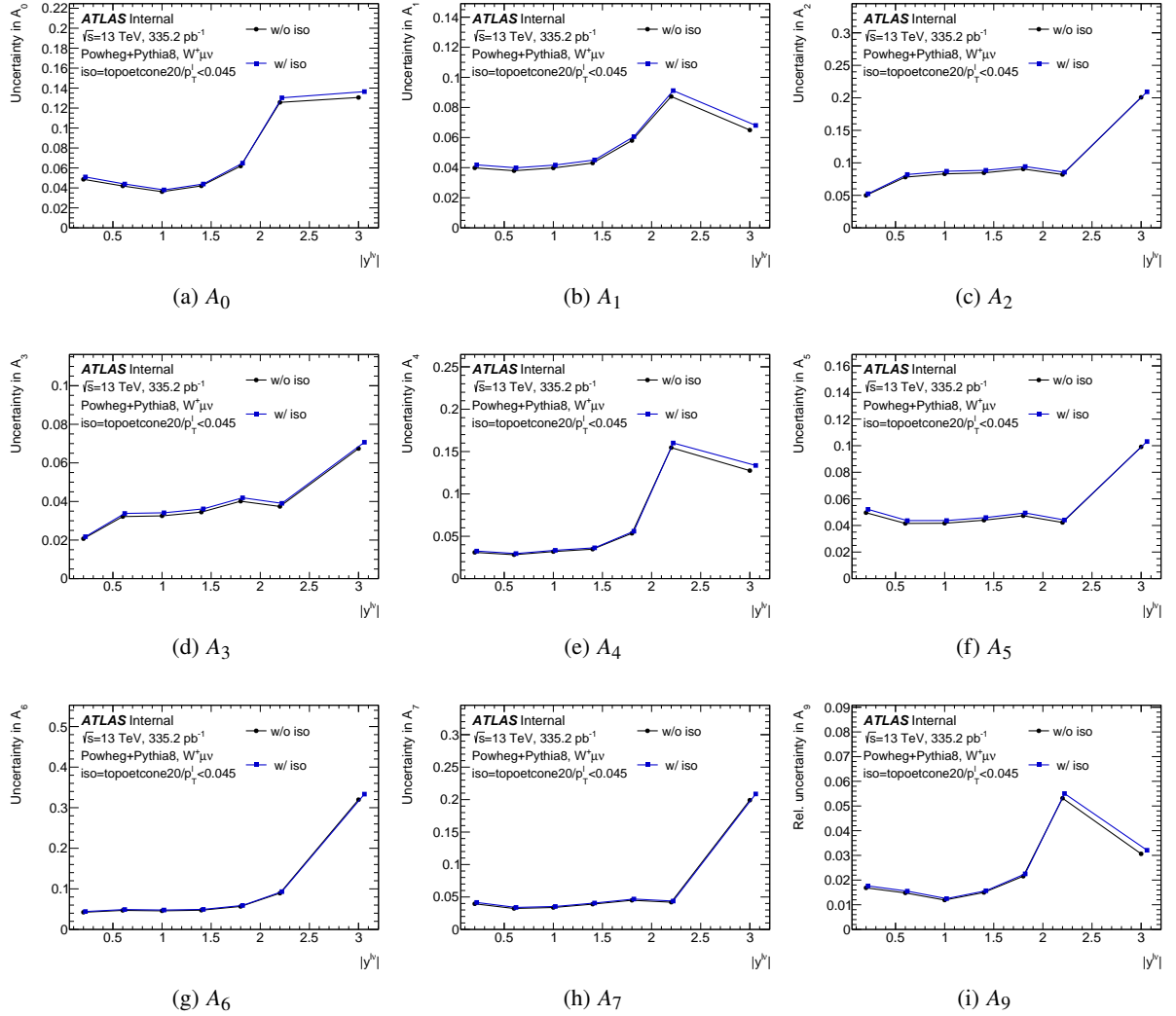


Figure 79: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the inclusion of adding isolation selection in $y^{\ell\nu}$ binning for $W^+ \rightarrow \mu^+\nu$.

1051 **Rotation p_z^ν**

1052 The rotation p_z^ν method gets its name from rotating the E_T^{miss} by the minimum amount needed for a solution.
 1053 This rotation changes the values of the neutrino p_T and ϕ resulting in a different ϕ_{CS} value for these events.
 1054 It was determined to give the best overall increase in sensitivity as there is a noticeable difference for low
 1055 $p_T^{\ell\nu}$ bins for $A_0, A_2, A_3, A_5,$ and A_7 as well as a slight increase in sensitivity over the nominal.

1056 **Shifted m_T**

1057 The last method tested utilizes why we do not get a solution, which is when $m_T > m^W$. For the events that
 1058 do not get an initial solution using the PDG m^W the solution is recalculated using $m_T + 100\text{MeV}$ for the
 1059 mass constraint. The 100MeV shift ensures that $\Delta \neq 0$ for all events like the VH style and this applied shift

1060 is well within the resolution of m_T . One downside to this method is that even using this method there are
 1061 still events with $\Delta < 0$ depending on the rest of the parameters while calculating it.

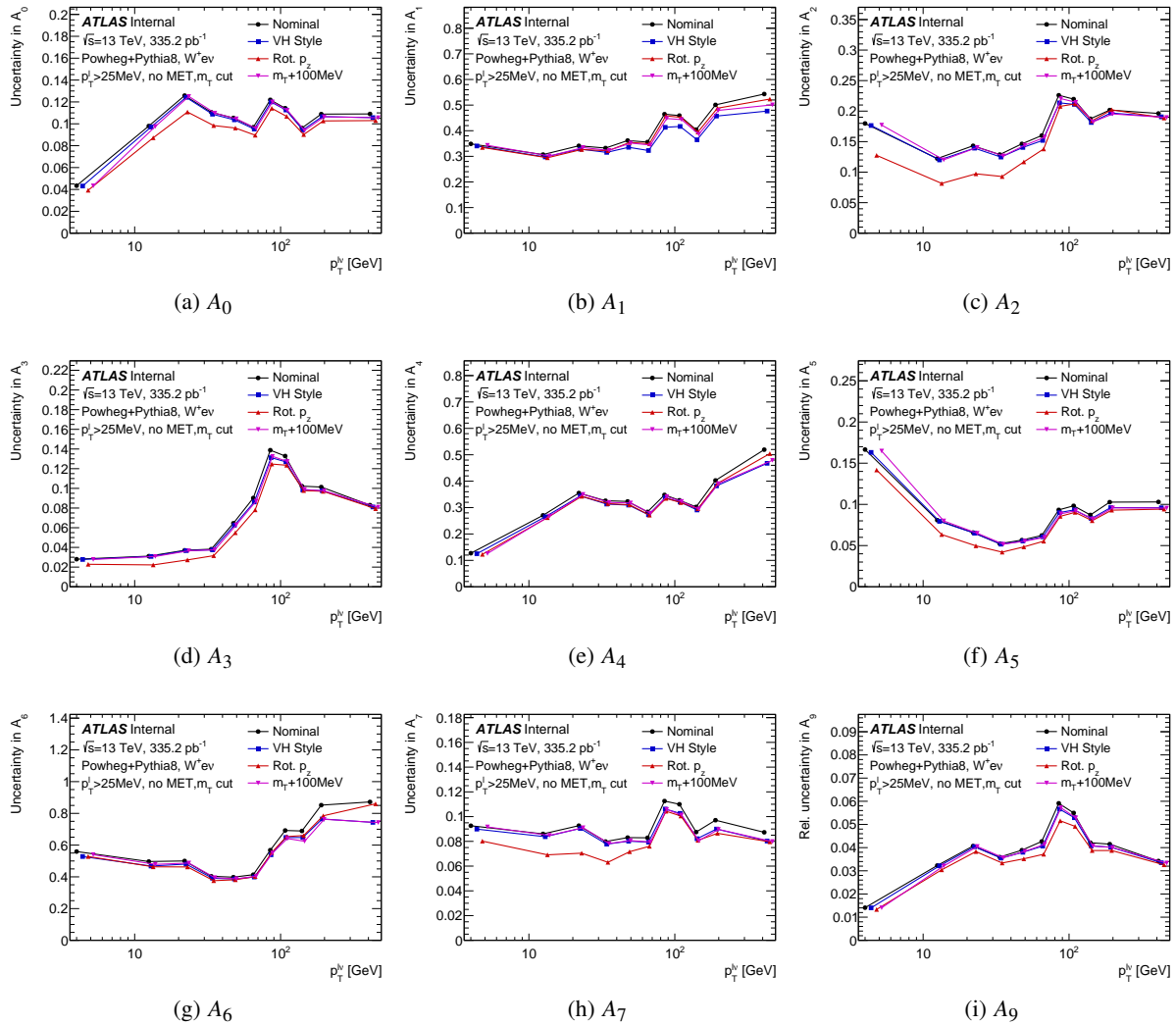


Figure 80: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the different neutrino solutions in $p_T^{\ell\nu}$ binning for $W^+ \rightarrow e^+\nu$.

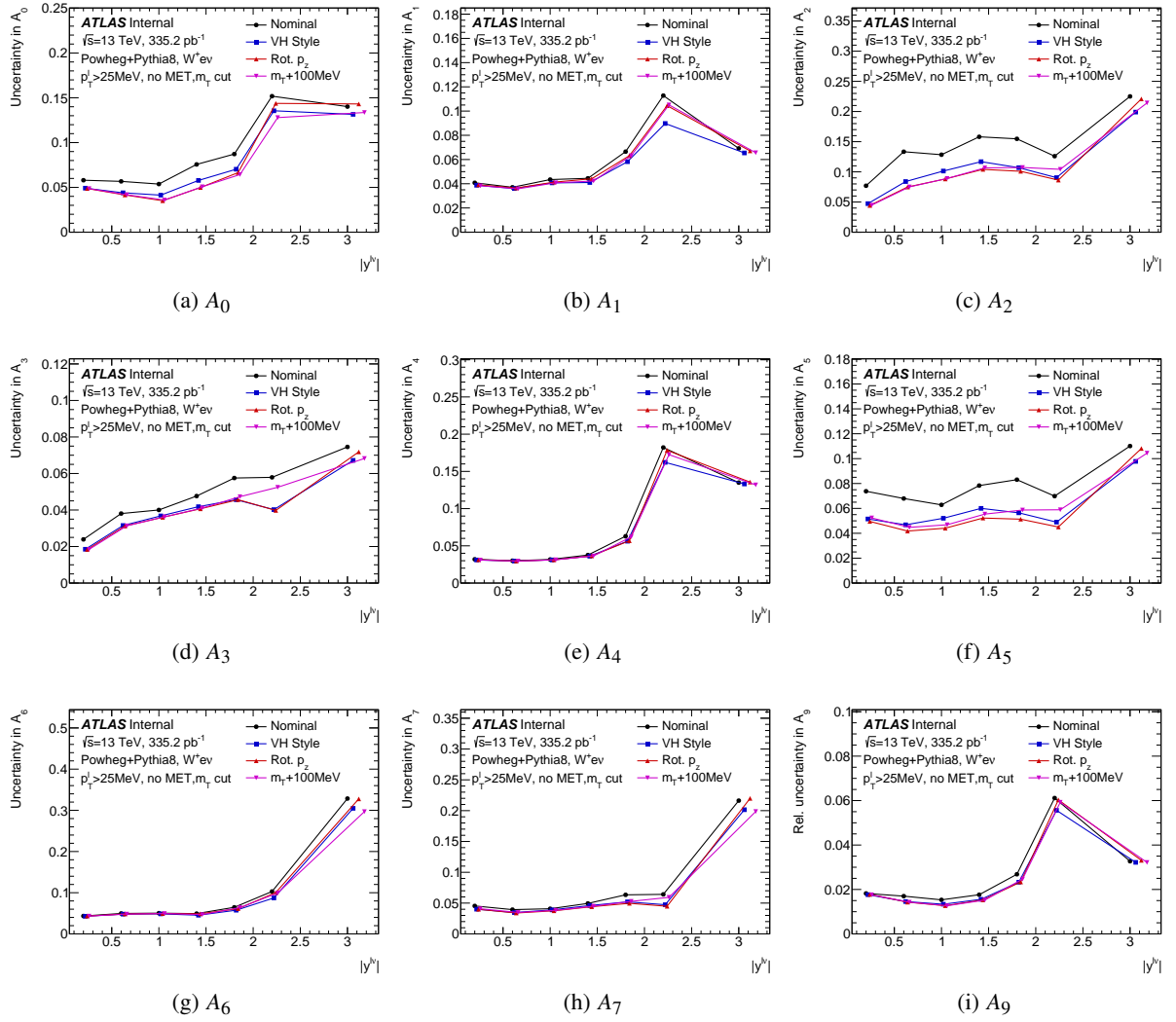


Figure 81: Comparison of the total statistical uncertainty for the coefficients $A_0 - A_7$ and relative statistical uncertainty for the total cross-section, A_9 for the different neutrino solutions in $y^{\ell\nu}$ binning.

1062 A.4 Fixing $A_5 - A_7$

1063 The coefficients $A_5 - A_7$ are expected to have small values since they don't start contributing until $O(\alpha_s^2)$.
 1064 This means this analysis won't be sensitive to them but they need to be managed in the fit. We tested three
 1065 different ways to see what the outcome on the other coefficients we are sensitive to:

- 1066 • **Fit $A_5 - A_7$** - Extract the coefficients and allow them to float in the fit where no assumptions are
 1067 made about them. This is considered to be the default option.
- 1068 • **Fixed $A_5 - A_7$ reference** - Treat the coefficients as constants in the fit where they are set to and
 1069 assumed as their reference values.
- 1070 • **Fixed $A_5 - A_7$ zero** - Treat the coefficients as constants in the fit but set them to 0 and assume they
 1071 are irrelevant.

1072 Fit $A_5 - A_7$ is the default and the other two methods were compared to it as seen in Figure 82 and 83 which
 1073 take the ratio of the uncertainty on the coefficient divided by the uncertainty of the nominal. We can see
 1074 the fixing $A_5 - A_7$ to their reference value reduces the uncertainty on the order of 1%. In contrast we see
 1075 that fixing $A_5 - A_7$ to zero cause a reduction of uncertainty in some bins $\lesssim 5\%$ but a gain in other bins, this
 1076 seesaw shape is seen in all coefficients except the cross-section A_9 . This region is known to be the least
 1077 stable part of the fit because there is a transition of shape in the signal templates as well as this region is
 1078 where the background makes a more meaningful contribution.

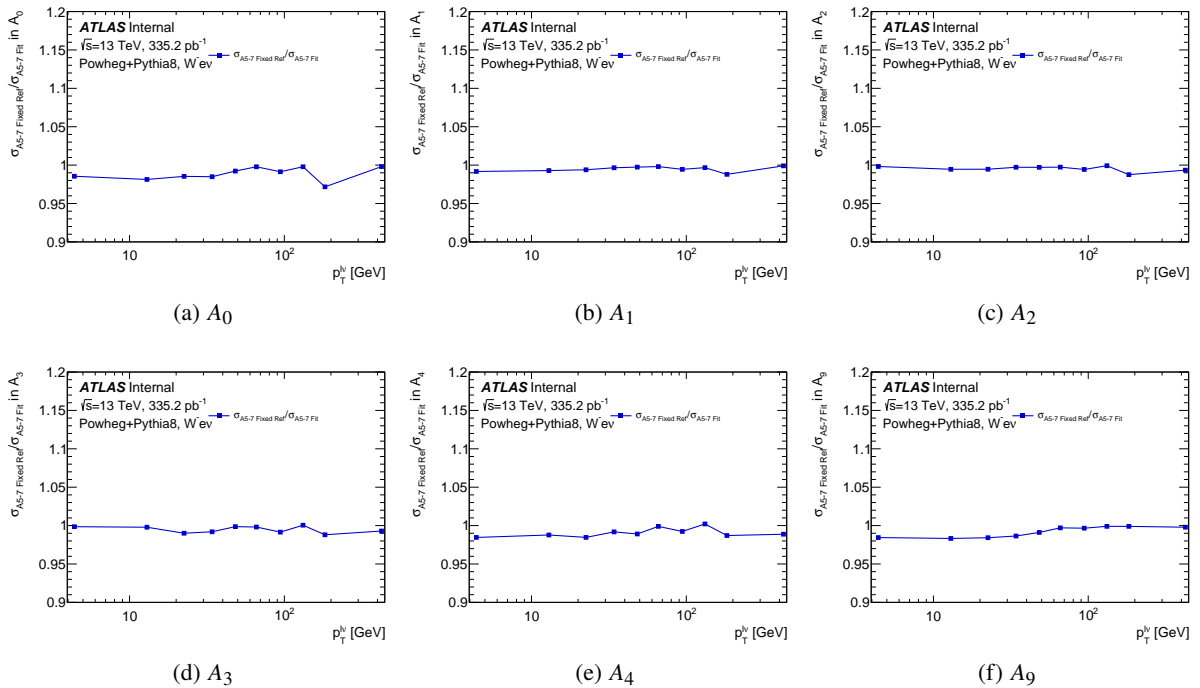


Figure 82: Ratio of the uncertainty on each coefficient if $A_5 - A_7$ is fixed to their reference value as a constant in the fit over the uncertainty of letting $A_5 - A_7$ float in the fit. There is a consistent reduction in uncertainty on the order of 1% when fixing $A_5 - A_7$ to their reference values.

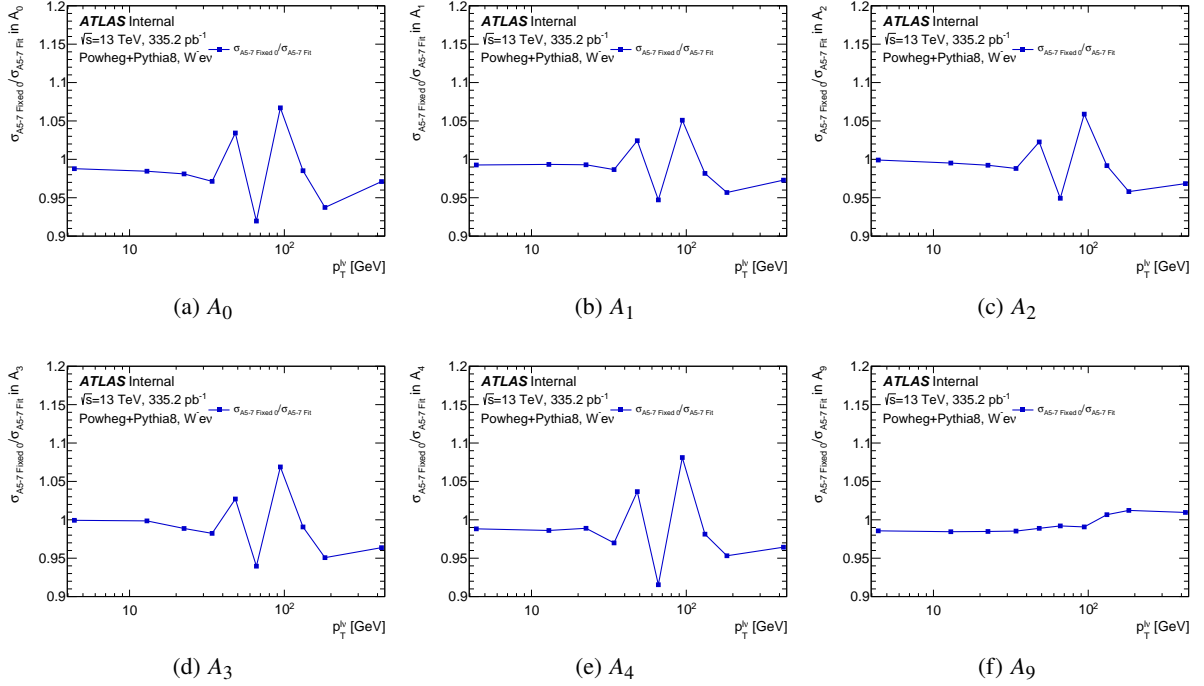


Figure 83: Ratio of the uncertainty on each coefficient if $A_5 - A_7$ is fixed to zero as a constant in the fit over the uncertainty of letting $A_5 - A_7$ float in the fit. There is a common seesaw shape which suggests an instability in the fit.

1079 Comparing the values of the three methods as shown in Figure 84 we can see that fixing the coefficients to
 1080 their reference values doesn't affect the value of the other coefficients in the fit while fixing them to zero
 1081 does. From this study we concluded to stay with our default value. Fixing them to zero causes instability
 1082 in the fit seen by the seesaw action of uncertainty and the change in values while fixing them at their
 1083 reference values has very marginal gain but introduces an assumption where a systematic uncertainty might
 1084 be needed to account for. The other thing to note is that χ^2/ndf only changed on the order of 10^{-3} so these
 1085 other methods didn't suggest a better fit.

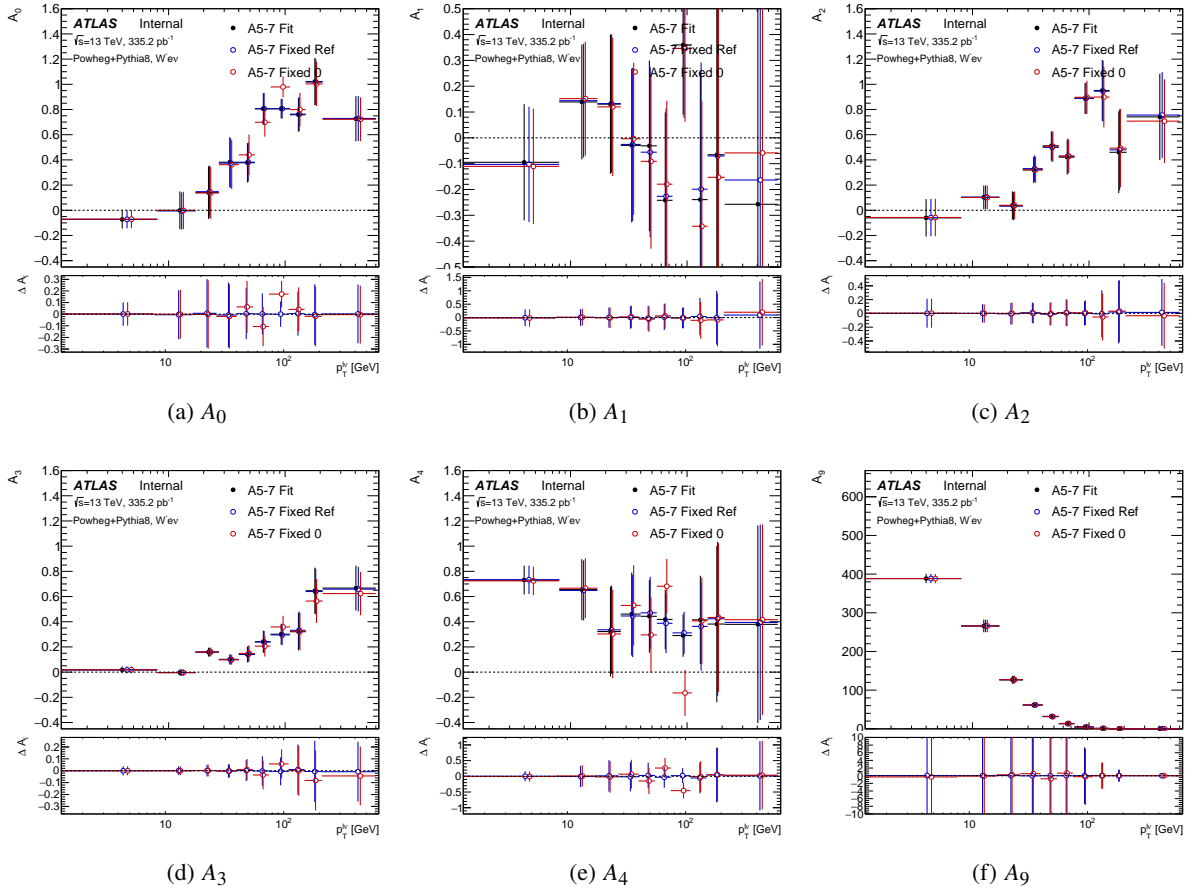


Figure 84: Comparison of the values for each treatment of $A_5 - A_7$. Fixing the coefficients to their reference values doesn't change the values for $A_0 - A_4$ and A_9 while fixing $A_5 - A_7$ to zero does cause a change to the values.

1086 B Neutrino p_z^ν Sign Ambiguity

1087 As discussed in Section 4.4, taking the mass constraint in order get p_z^ν results in a quadratic equation with
 1088 a two-fold ambiguity. When $m_T < m^W$ there are two real solutions which results in a sign ambiguity
 1089 in $\cos \theta_{CS}$. This appendix covers how choosing a solution at random we are able to statistically resolve
 1090 the right distributions and have sensitivity to measure A_4 which depends on having the correct sign of
 1091 $\cos \theta_{CS}$.

1092 To understand this we look at Equation 5 which has two parts that contribute to the sign ambiguity. The
 1093 first is $p_z^{\ell\nu}/|p_z^{\ell\nu}|$ which can be rewritten as the sign of $y^{\ell\nu}$ and the second is $\Delta P_\ell = (P_1^+ P_2^- - P_1^- P_2^+)$. The
 1094 main source of solving the ambiguity in $\cos \theta_{CS}$, which happens 25% of the time, is when the two solutions
 1095 have opposite signs in $y^{\ell\nu}$ which also causes a sign flip in ΔP_ℓ and both solutions of $\cos \theta_{CS}$ are equal.
 1096 This happens predominately for central boson rapidity as seen in Figure 85 in red.

1097 This sign flip can be understood by looking at the $W^- \rightarrow e^- \bar{\nu}$ case. In this case P_1 is fixed because we
 1098 measure the lepton so ΔP_ℓ is simplified to

$$\Delta P_\ell = P_1^+ (E_\nu - p_z^\nu) - P_1^- (E_\nu + p_z^\nu). \quad (57)$$

1099 If ΔP_ℓ flips sign then

$$\begin{aligned} E_\nu - p_z^\nu > E_\nu + p_z^\nu &\Rightarrow p_z^\nu < 0 && \text{for solution 1,} \\ E_\nu - p_z^\nu < E_\nu + p_z^\nu &\Rightarrow p_z^\nu > 0 && \text{for solution 2.} \end{aligned} \quad (58)$$

1100 The sign flip of $y^{\ell\nu}$ can be understand by looking at the definition of y

$$y = \frac{1}{2} \ln \left(\frac{E + p_z c}{E - p_z c} \right), \quad (59)$$

1101 where if $y^{\ell\nu}$ flips sign then

$$\begin{aligned} E^{\ell\nu} + p_z^{\ell\nu} c > E^{\ell\nu} - p_z^{\ell\nu} c &&& \text{for solution 1,} \\ E^{\ell\nu} + p_z^{\ell\nu} c < E^{\ell\nu} - p_z^{\ell\nu} c &&& \text{for solution 2.} \end{aligned} \quad (60)$$

1102 The sign flip happens mostly for central $y^{\ell\nu}$ due to fact the p_z^ν sign flip is more likely to occur when $p_z^\ell + p_z^\nu$
 1103 is close to zero. This can be seen by using $p_z^{\ell\nu} = p_z^\ell + p_z^\nu$ in Equation 60 which can be rearranged into

$$\begin{aligned} p_z^\ell + p_z^\nu > 0 &&& \text{for solution 1,} \\ p_z^\ell + p_z^\nu < 0 &&& \text{for solution 2.} \end{aligned} \quad (61)$$

1104 The other time that we can resolve this sign ambiguity is when the one of the solutions violates the Bjorken
 1105 condition of $x < 1$ and is unphysical. This occurs when either $|p_z^\nu| > \sqrt{s}/2$ or $|p_z^\ell + p_z^\nu| > \sqrt{s}/2$ which is
 1106 when the solution causes the longitudinal momenta of the neutrino or the W boson to exceed the beam
 1107 energy. These events aren't frequent as seen in blue in Figure 85.

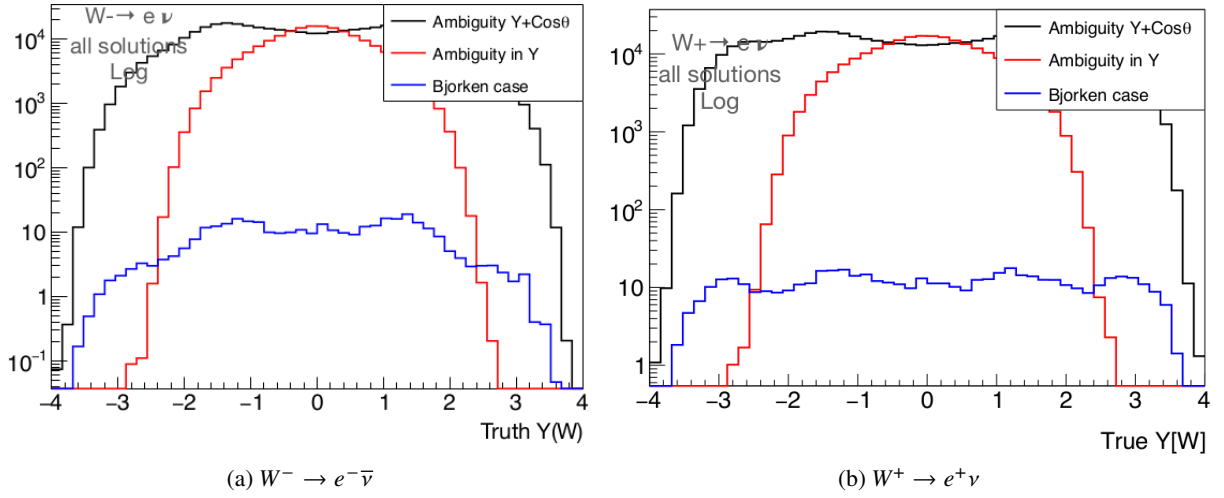


Figure 85: The distribution of events based on the ability to resolve the sign ambiguity. In black are events where there is both a sign ambiguity in $\cos \theta_{CS}$ and in $y^{\ell\nu}$, in red are events where the ambiguity in $\cos \theta_{CS}$ is solved by sign flipping cancelling but there is still an ambiguity in the sign of $y^{\ell\nu}$, and in blue are events where one solution violates Bjorken $x < 1$ and only one solution is valid.

1108 To further understand the sensitivity it is important to look at what happens when we chose both the right
 1109 and wrong p_z^{ν} solution, where we define the right p_z^{ν} as when the solution is closest to the truth p_z^{ν} . Both
 1110 right and wrong solutions happen an equal amount as shown in Figure 86 where the wrong solution in blue
 1111 is covering the right solution in red.

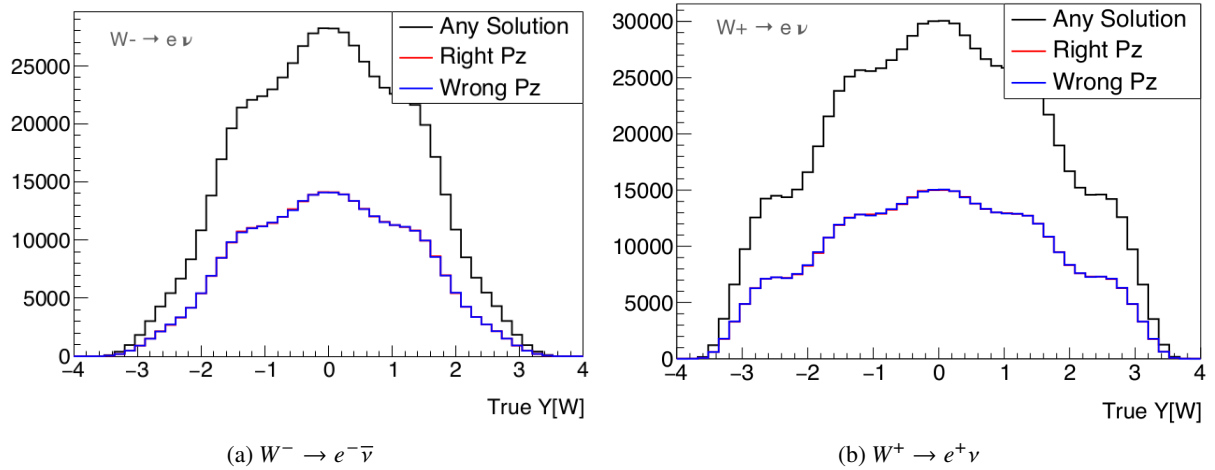


Figure 86: The distribution of the right (red) and wrong (blue) choice of p_z^{ν} which are equally distributed in $y^{W, Truth}$.

1112 There are four possible combinations of choosing the correct solution of p_z^{ν} and getting the correct sign of
 1113 $\cos \theta_{CS}$. The first is when we chose the correct p_z^{ν} and get the correct $\cos \theta_{CS}$ as shown in Figure 87 in red.
 1114 These events are important for having sensitivity at large rapidity while the more central region we would
 1115 get the correct sign of $\cos \theta_{CS}$ regardless. Second are events where we chose the correct p_z^{ν} but due the
 1116 limited resolution the sign of $\cos \theta_{CS}$ flipped as shown in Figure 87 in blue. The events in where we do

1117 better than the random choice and gain the most sensitivity to A_4 in when we chose the wrong p_z^{ν} but get
 1118 the right sign of $\cos \theta_{CS}$ because the sign of $y^{\ell\nu}$ also flipped. These events are shown in green in Figure 87
 1119 and are seen to be more central. Lastly, are events where we have the wrong p_z^{ν} and wrong sign of $\cos \theta_{CS}$
 1120 which dilutes our sensitivity mostly at high $y^{\ell\nu}$ as shown in pink Figure 87.

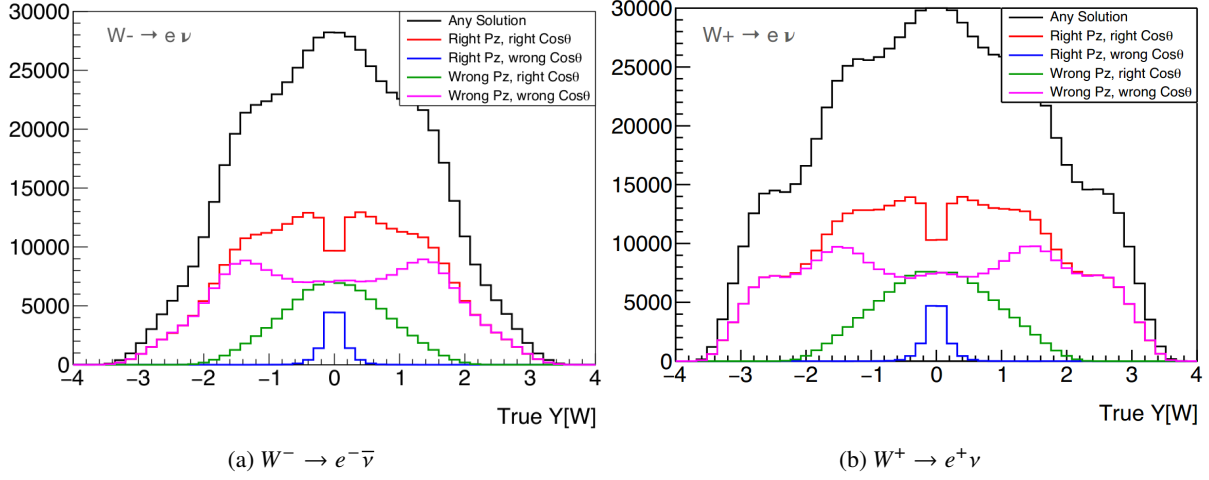


Figure 87: Distribution of events based for the four combinations of the choice of p_z^{ν} solution and the correct sign of $\cos \theta_{CS}$.

1121 To show the sensitivity gained, A_4 was extracted dependent on the ambiguity level based on the three
 1122 categories in Figure 85. Using only events that have both an ambiguity in the sign of $\cos \theta_{CS}$ and $y^{\ell\nu}$, where
 1123 there is only a 50% chance to get the correct sign, we can see there isn't any sensitivity to A_4 as shown in
 1124 Figures 88 & 89 in black. By combining these events with the events that only have a sign ambiguity in
 1125 $y^{\ell\nu}$, where we get the right sign every time, we gain a large sensitivity as shown in blue in Figures 88 & 89.
 1126 Lastly, for events that violate Bjorken $x < 1$ we gain a very small sensitivity increase as seen in red in
 1127 Figures 88 & 89. In Figure 89 we also see that most of our sensitivity gain is in central $y^{\ell\nu}$ as expected.

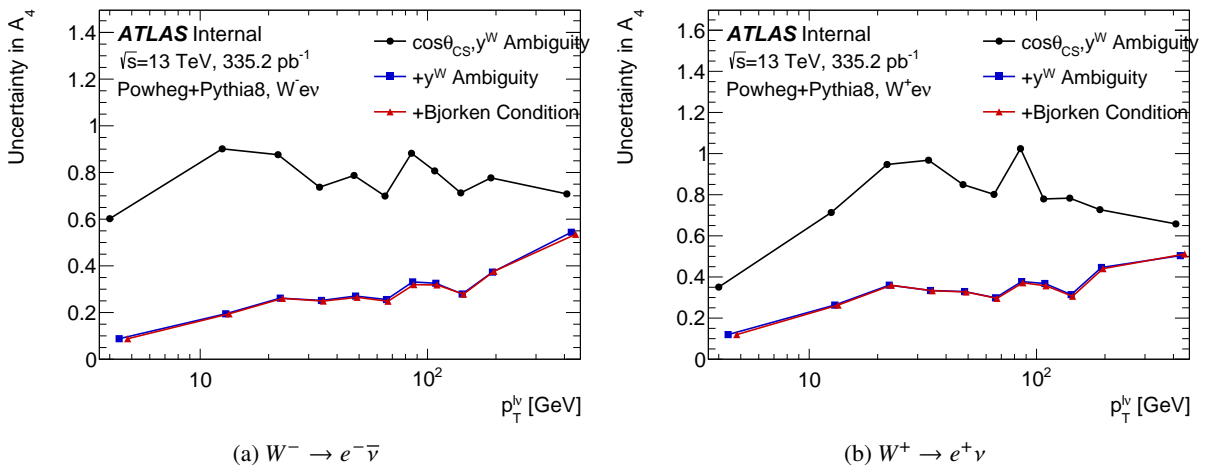
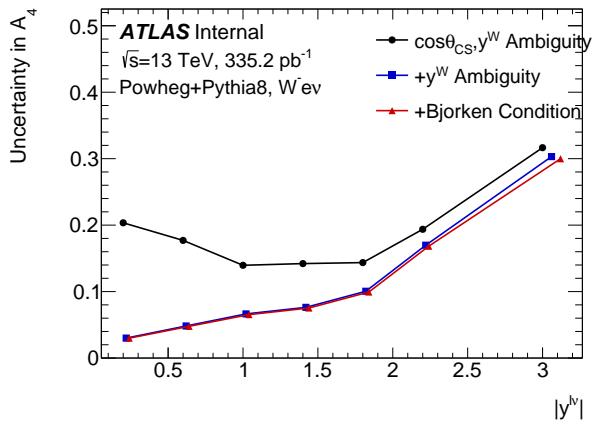
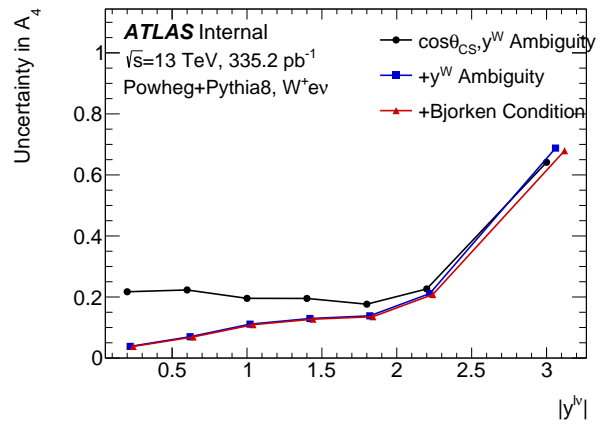


Figure 88: Comparison of the total statistical uncertainty for A_4 for the inclusion of events dependent on the ambiguity in the sign of $\cos \theta_{CS}$ and y^W in $p_T^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$.



(a) $W^- \rightarrow e^- \bar{\nu}$



(b) $W^+ \rightarrow e^+ \nu$

Figure 89: Comparison of the total statistical uncertainty for A_4 for the inclusion of events dependent on the ambiguity in the sign of $\cos\theta_{CS}$ and y^W in $y^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$.

1128 C Hadronic Recoil Systematics

1129 The hadronic recoil is expected to be the most dominant experimental systematic uncertainty for this
 1130 measurement. There are 62 nuisance parameters associated to the statistical uncertainty on the resolution
 1131 correction, statistical uncertainty of the response correction, systematic uncertainty on the response
 1132 correction, systematic uncertainty on the ΣE_T , systematic uncertainty on the resolution correction due to the
 1133 extrapolation from Z to W , and systematic uncertainty on the response correction due to the extrapolation
 1134 from Z to W . More information on these corrections can be found in the support note [46].

1135 C.1 Kinematic Distribution Impact

1136 To understand the impact of the NPs variations on the hadronic recoil the angular distributions for
 1137 $W^- \rightarrow e^- \bar{\nu}$ were plotted, Figures 90-93 show the variation for the four largest variations. The largest
 variation in the angular distributions is in Figure 91 where $\cos \theta_{CS}$ varies at a 0.8% level.

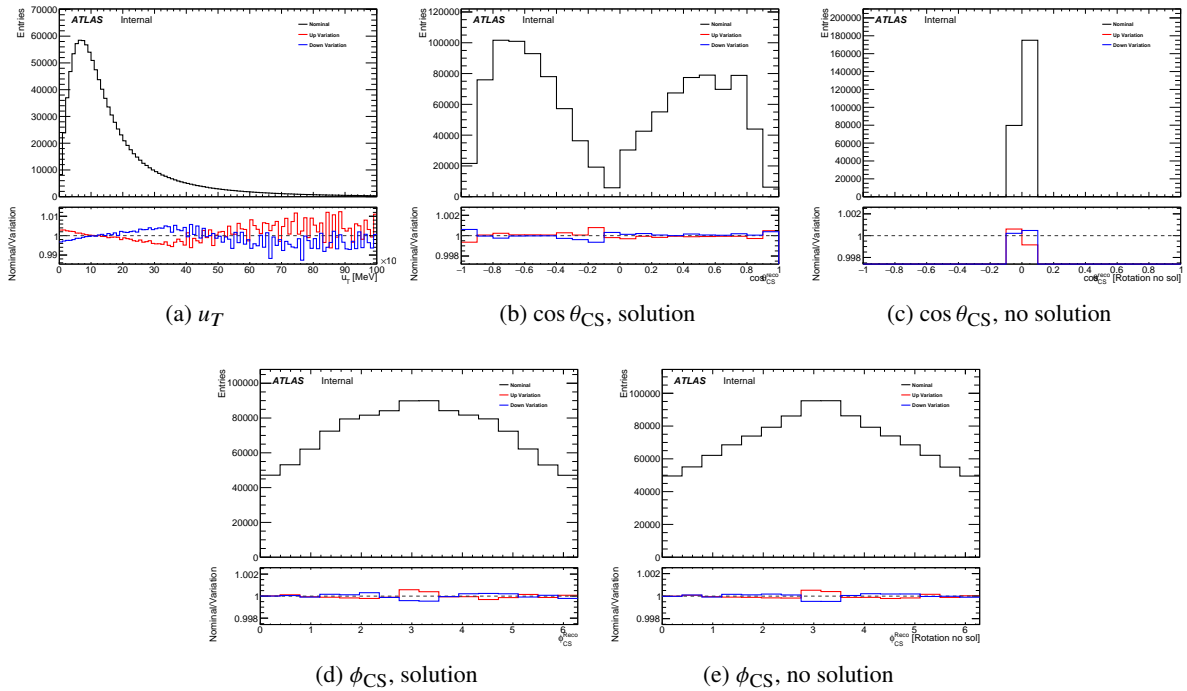


Figure 90: Systematic variation of resolution extrapolation systematic on the hadronic recoil and the impact on the angular distributions.

1138

1139 C.2 Template Variation

1140 The variations in Section C.1 show the impact of some of the nuisance parameters on the integrated angular
 1141 distribution level but this section will cover the impact the systematic variations have on the templates
 1142 for $W^- \rightarrow e^- \bar{\nu}$. Examples of the impact on three NPs is shown for three $p_T^{\ell\nu}$ bins in Figures 94 - 96 and

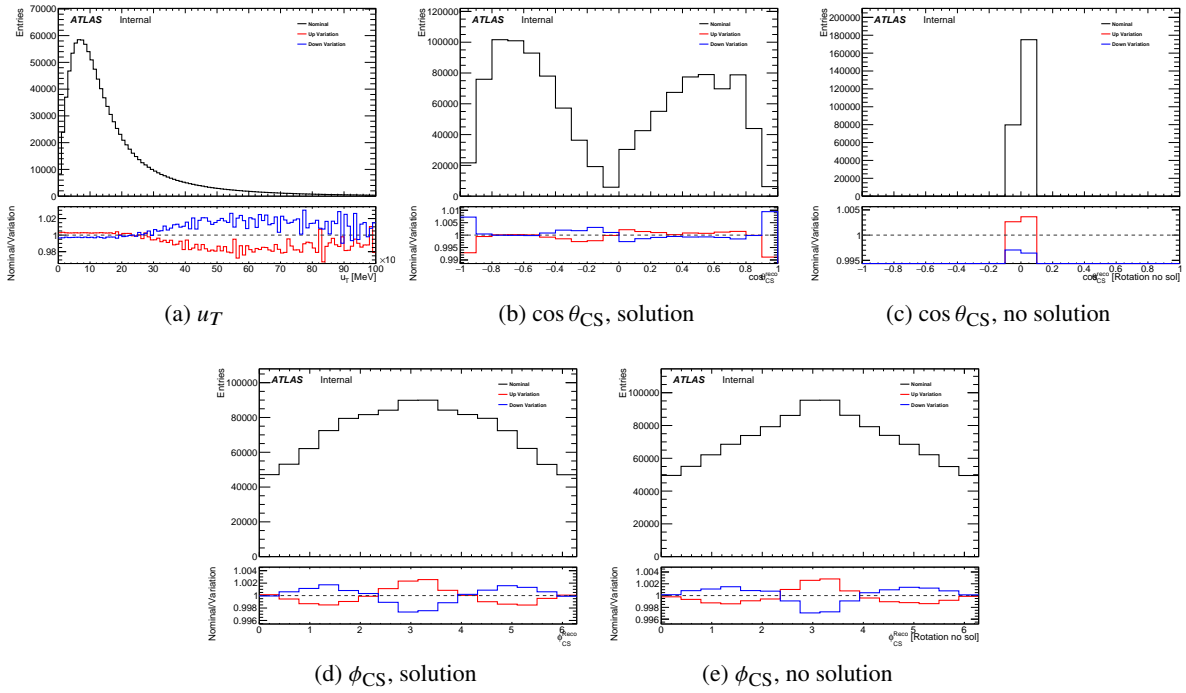


Figure 91: Systematic variation of response extrapolation systematic on the hadronic recoil and the impact on the angular distributions.

1143 three $y^{\ell\nu}$ bins in Figures 97 - 99. They are plotted to show the percent difference between nominal and the
 1144 variation.

1145 C.3 Coefficient Impact

1146 All NPs are propagated through the A_i framework in order to estimate their expected uncertainty on the
 1147 fit. They are combined into three categories, Resolution, Response, and ΣE_T (labeled as SET) as shown
 1148 in Figures 100 and 101. For the ΣE_T NP, the effects cancel for the coefficients $A_0 - A_7$, while the for
 1149 the differential cross-section, A_9 , they do not as the systematic is modeling the non-closure of the ΣE_T
 1150 modeling procedure discussed in [46]. The total impact of the hadronic recoil systematics is not expected
 1151 to be large as shown in Figures 102 and 103 where it is labeled as Sys and does not contribute to the total
 1152 uncertainty on the same level as the statistical uncertainty.

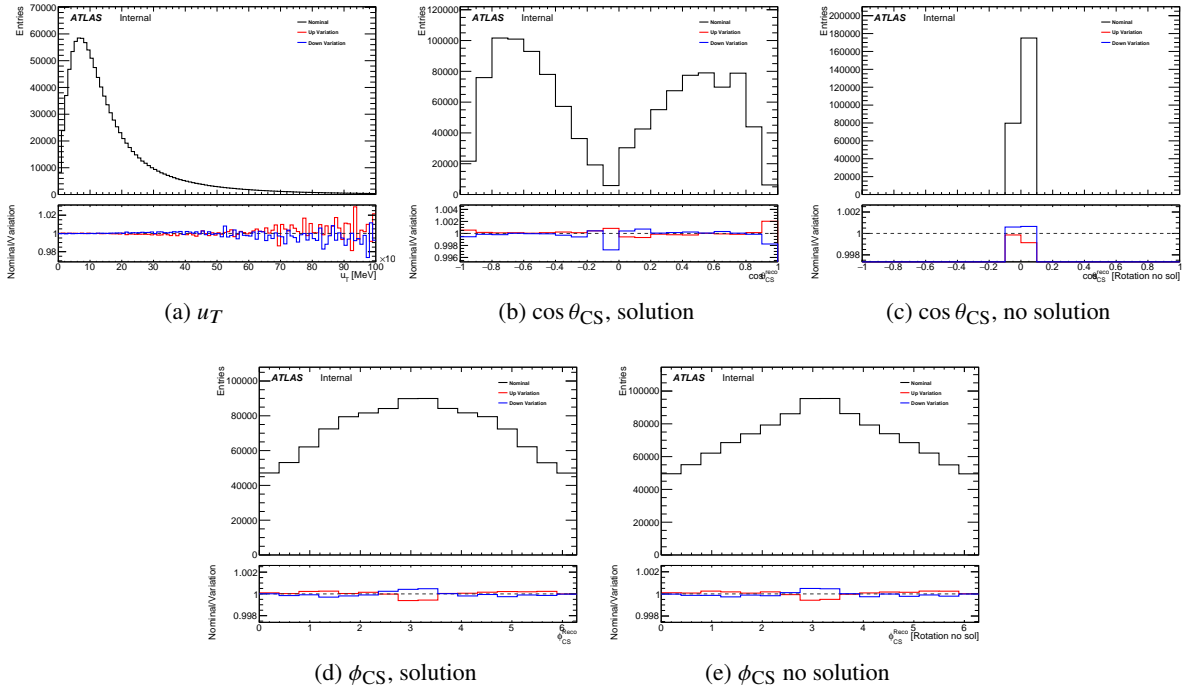


Figure 92: Systematic variation of response systematic on the hadronic recoil and the impact on the angular distributions.

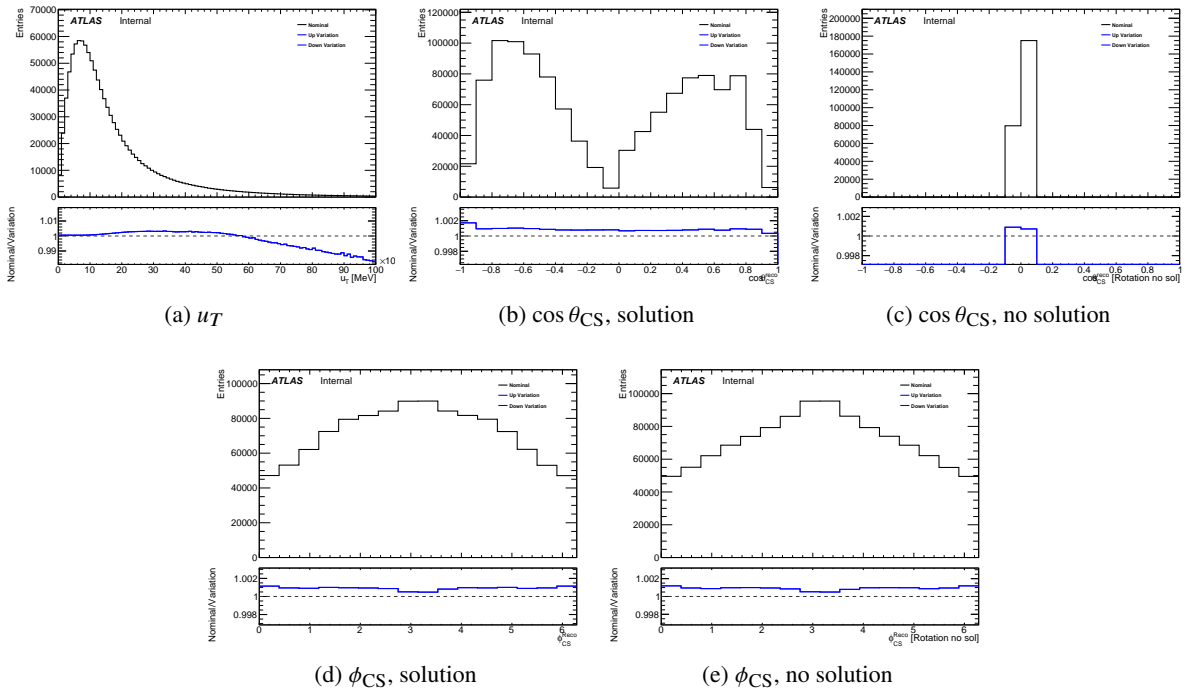


Figure 93: Systematic variation of ΣE_T systematic on the hadronic recoil and the impact on the angular distributions.

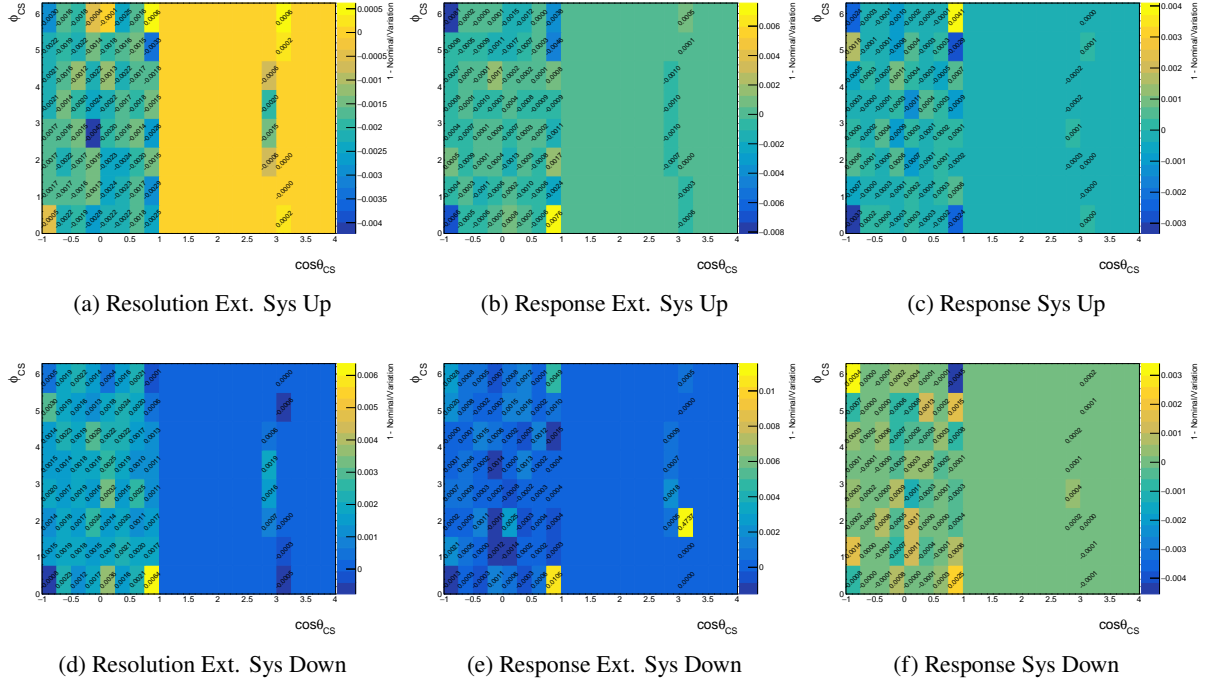


Figure 94: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $0 < p_T^{\ell\nu} < 8\text{GeV}$.

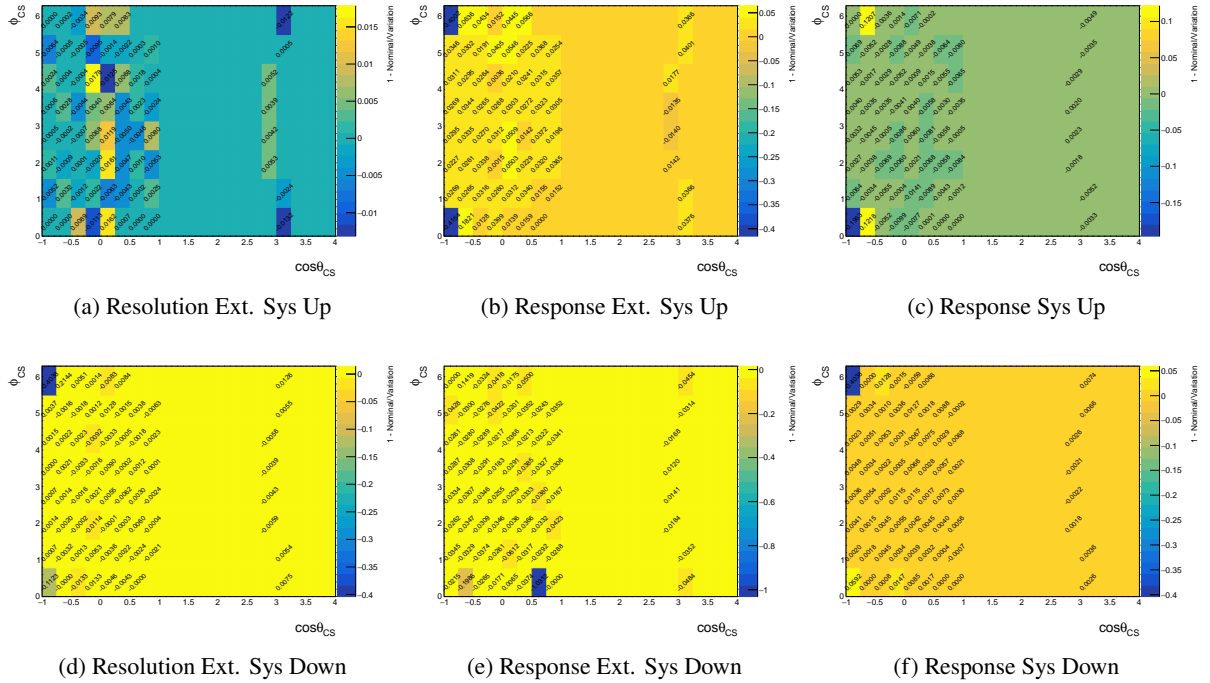


Figure 95: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $50 < p_T^{\ell\nu} < 55\text{GeV}$.

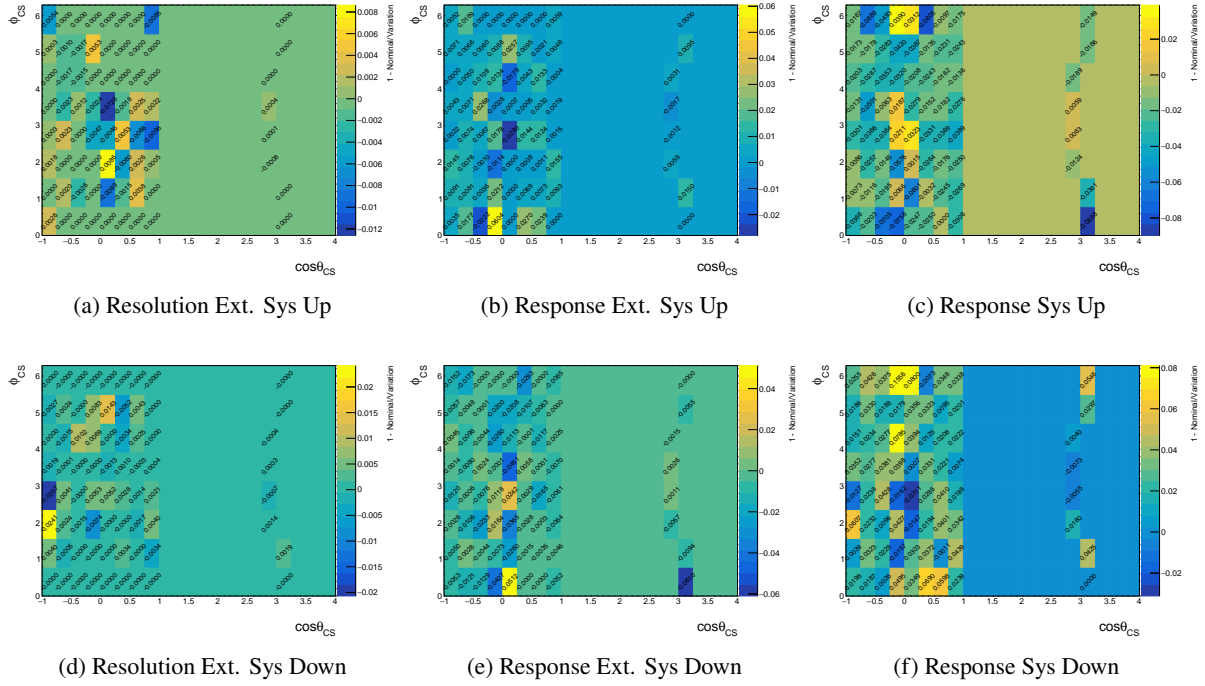


Figure 96: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $120 < p_T^{\ell\nu} < 160\text{GeV}$.

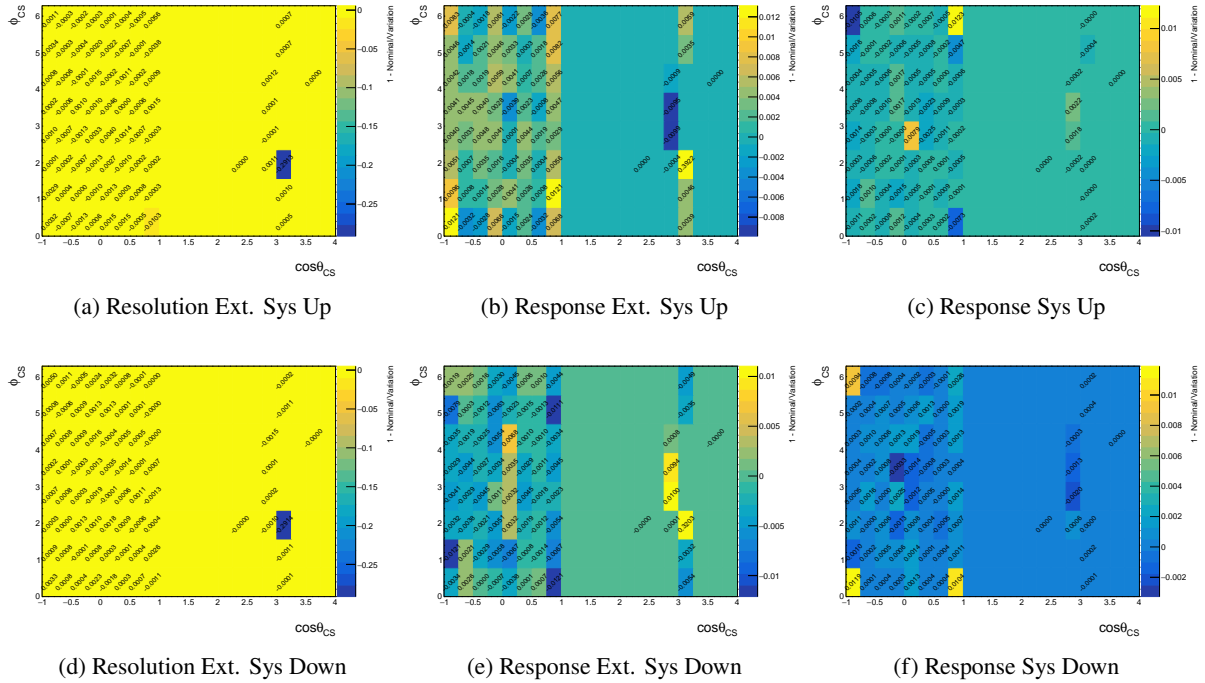


Figure 97: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $0 < y^{\ell\nu} < 0.4$.

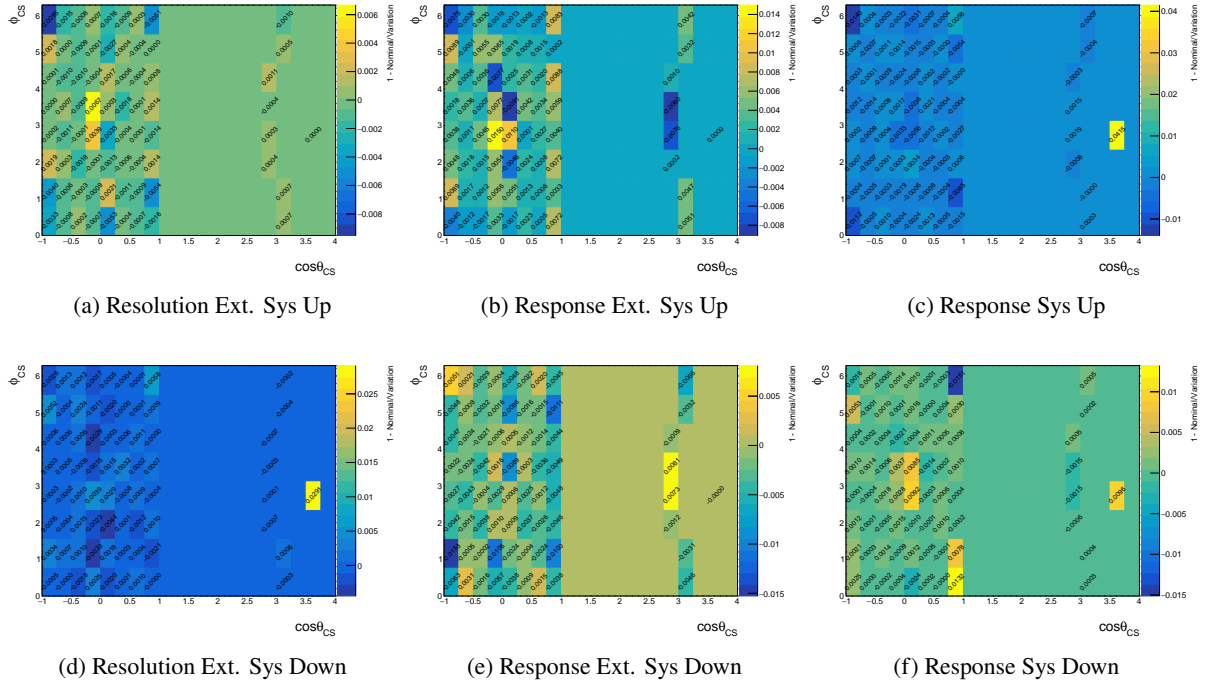


Figure 98: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $1.2 < y^{\ell\nu} < 1.6$.

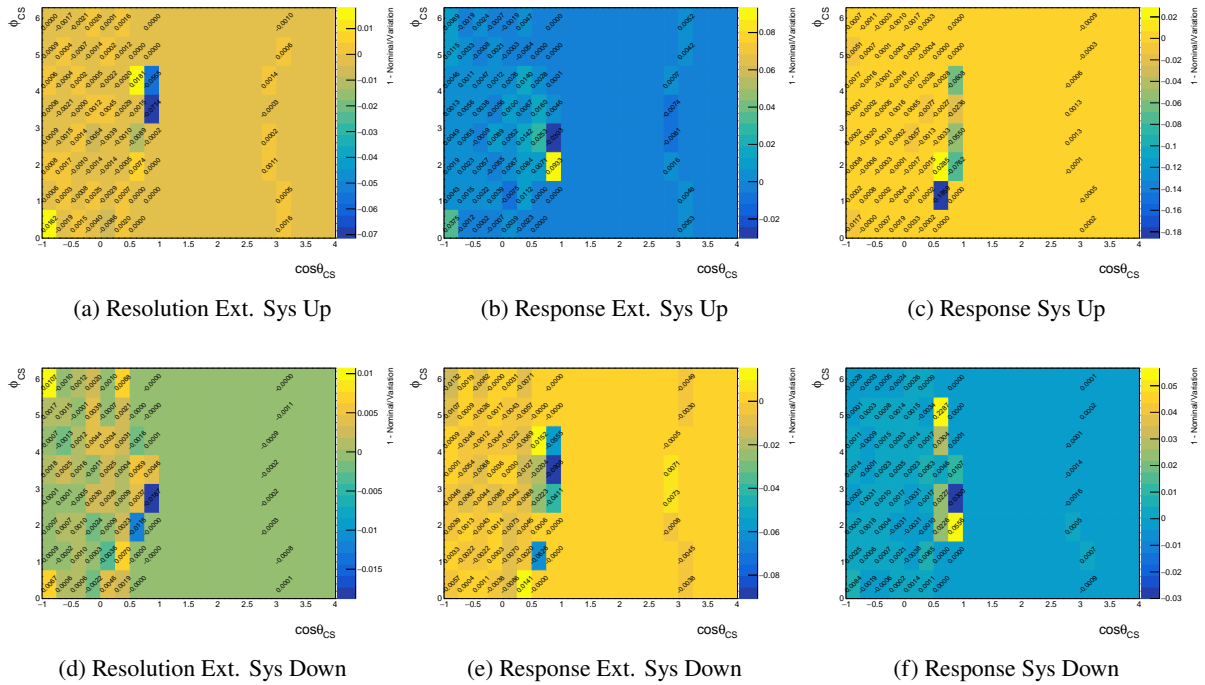


Figure 99: Up and down variation in the templates used to extract the coefficients for $W^- \rightarrow e^- \bar{\nu}$ for three different hadronic recoil NPs for the range $2.0 < y^{\ell\nu} < 2.4$.

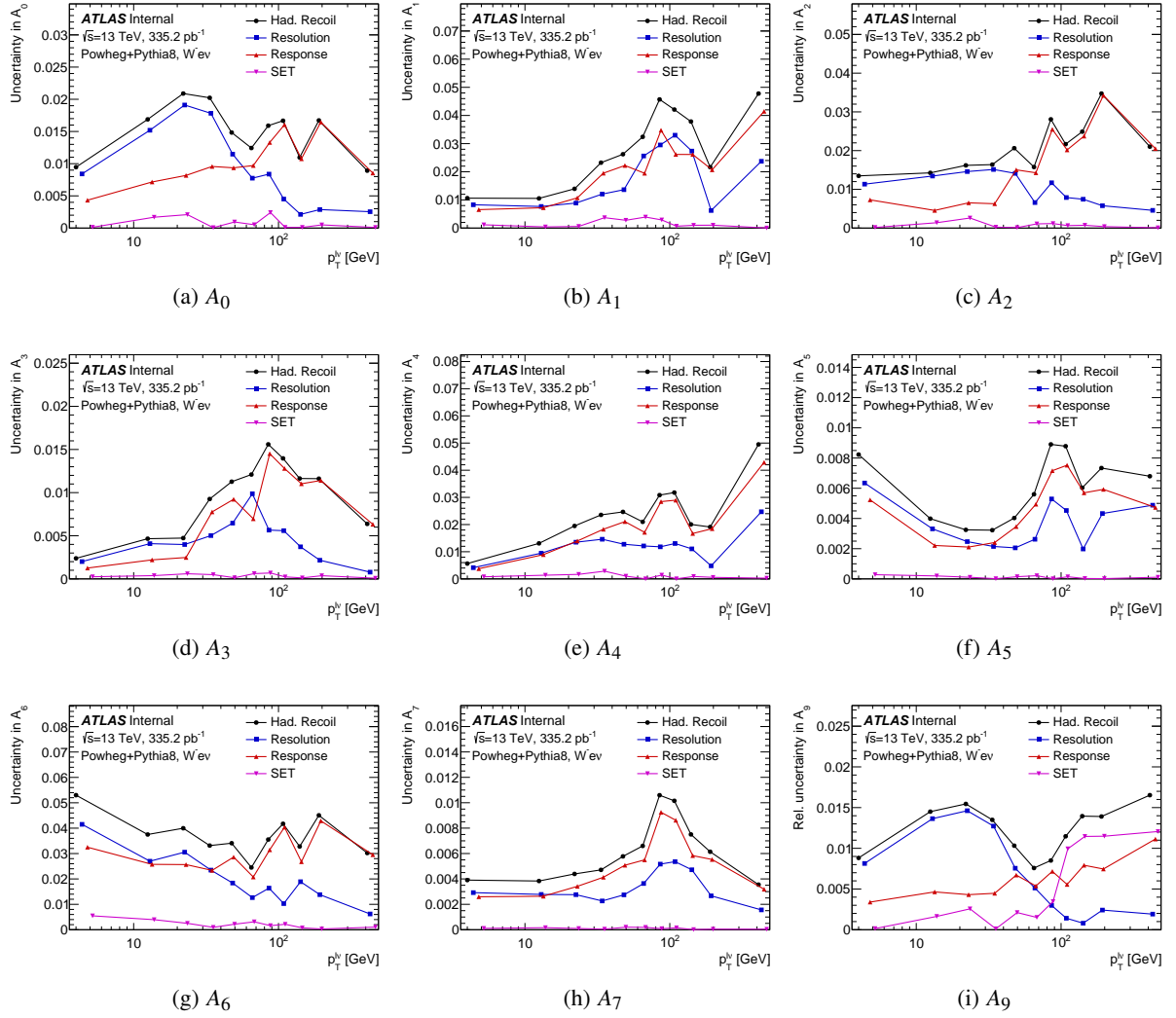


Figure 100: Hadronic recoil systematic uncertainty breakdown for the coefficients $A_0 - A_7$ in $p_T^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$.

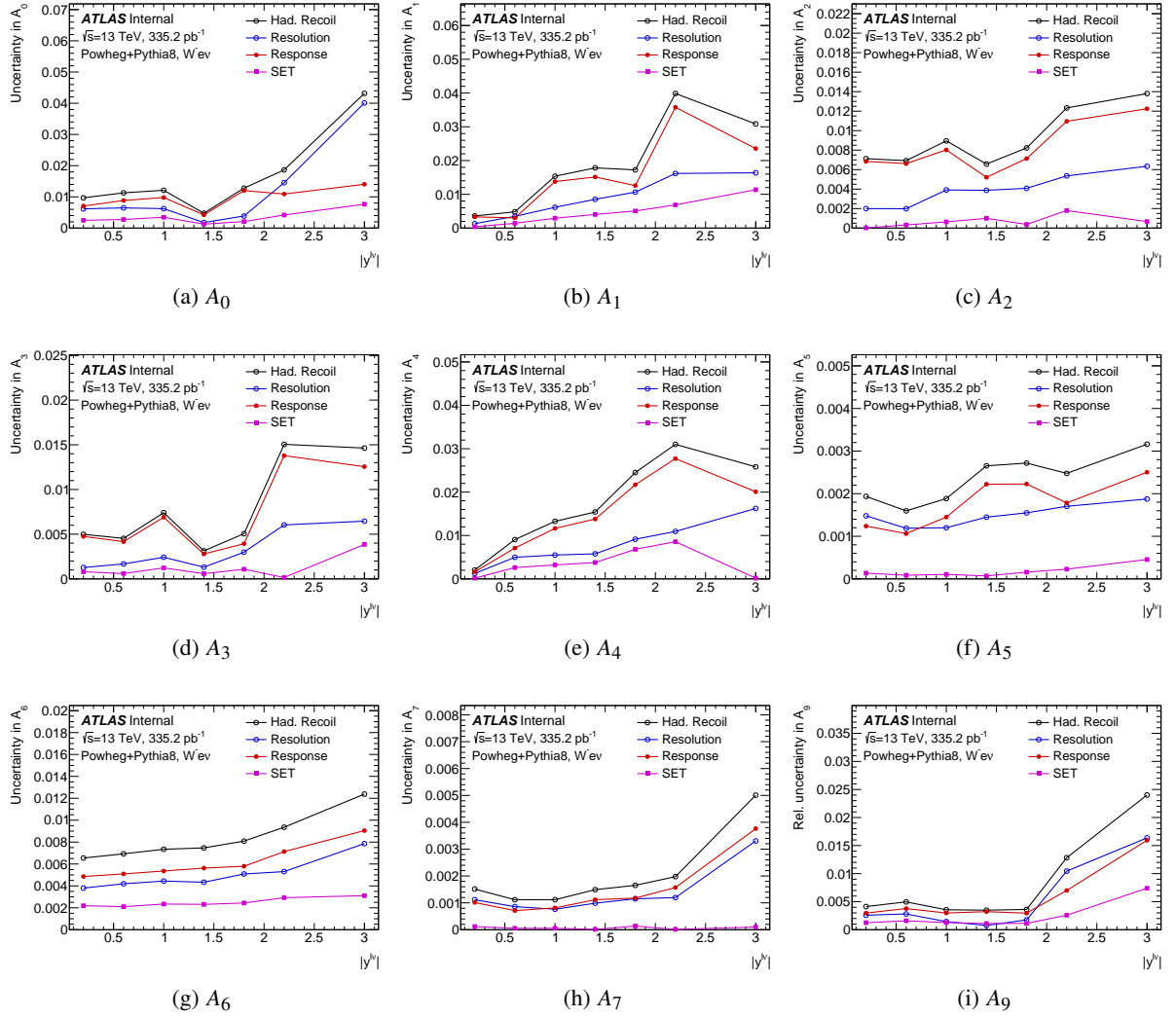


Figure 101: Hadronic recoil systematic uncertainty breakdown for the coefficients $A_0 - A_7$ in $y^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$.

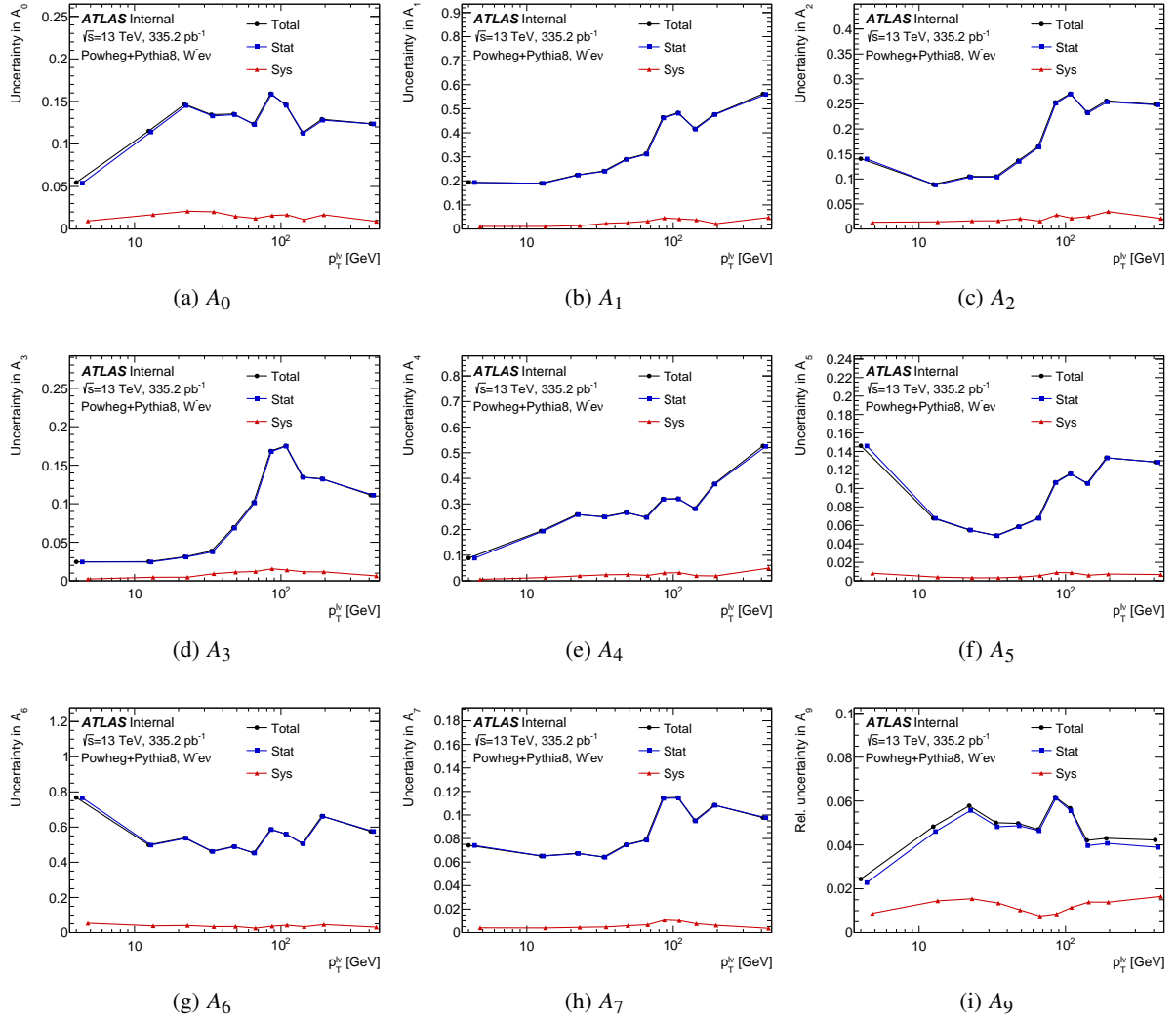


Figure 102: Total uncertainty breakdown for the two components statistical and hadronic recoil systematic for the coefficients $A_0 - A_7$ in $p_T^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$.

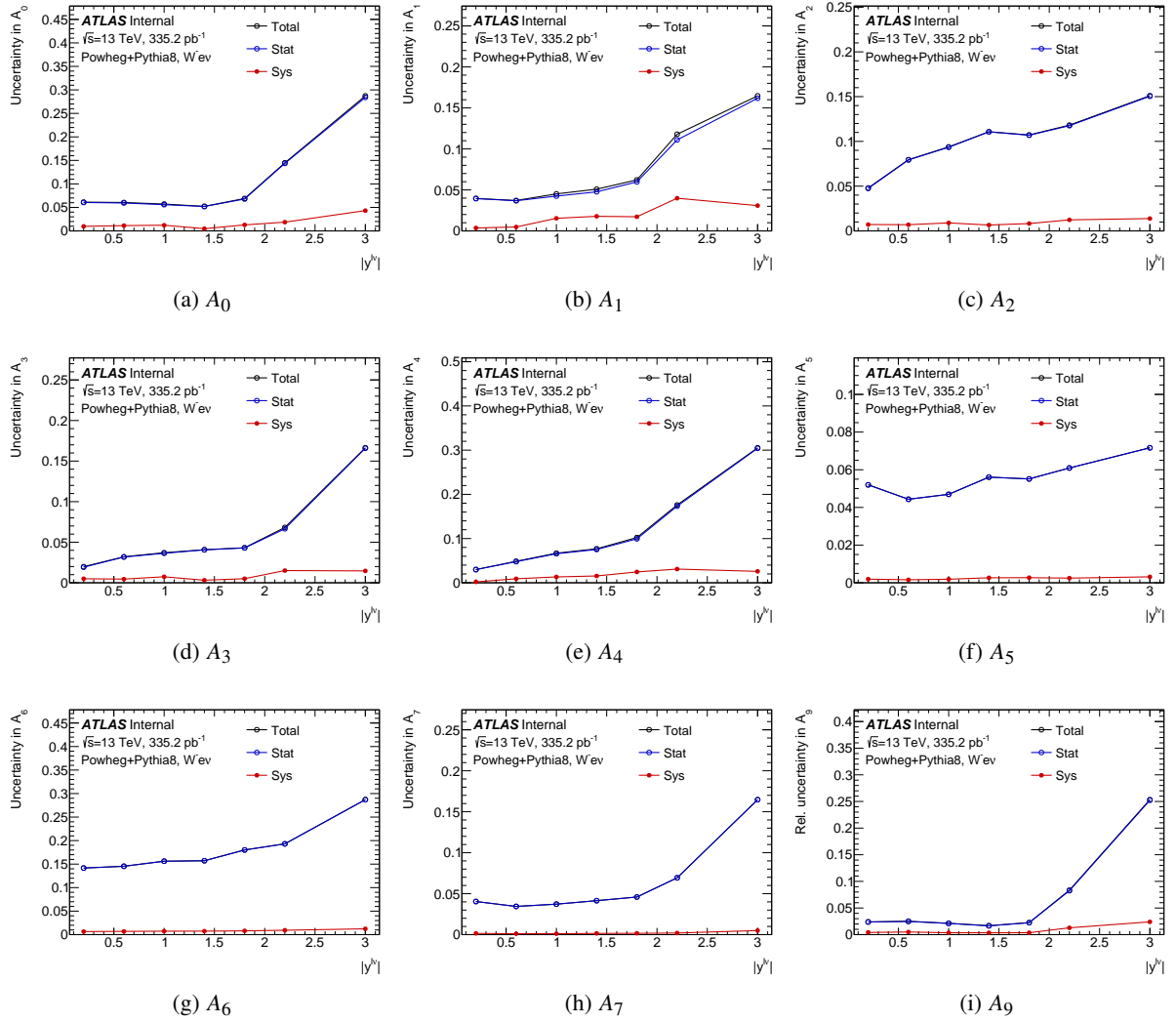


Figure 103: Total uncertainty breakdown for the two components statistical and hadronic recoil systematic for the coefficients $A_0 - A_7$ in $y^{\ell\nu}$ binning for $W^- \rightarrow e^- \bar{\nu}$.

1153 **D Migration Uncertainties**

1154 This appendix covers the breakdown of each migration uncertainty category for each coefficient as described
1155 in Section 7.1. There are three categories of migration uncertainty: self which is the migration between
1156 $p_T^{W, \text{Truth}}$ or $y^{W, \text{Truth}}$ bins for the same coefficient, shape which is the migration between coefficients in
1157 the same $p_T^{W, \text{Truth}}$ bin, and norm which is the self migration for the normalization. Note that the self
1158 and norm migrations affect each coefficient but the differential cross-section, A_9 , only is affected by the
1159 norm migration. The coefficients are ratios and are affected by their own measurement as well as the
1160 normalization, while A_9 is just a measurement of the normalization.

1161 **D.1 p_T^W Differential**

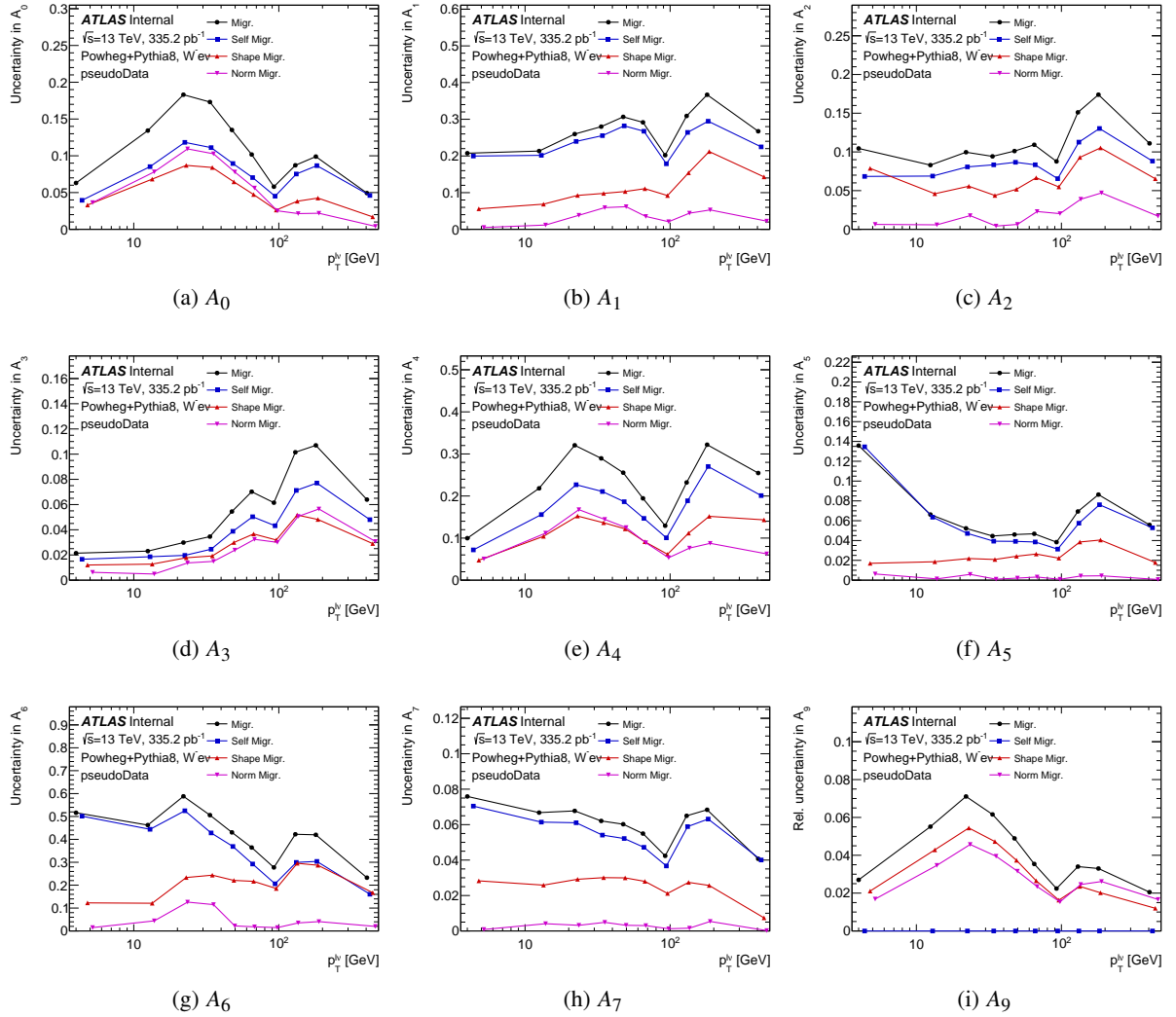


Figure 104: The uncertainty comparison for migration related uncertainties in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the self migration between $p_T^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

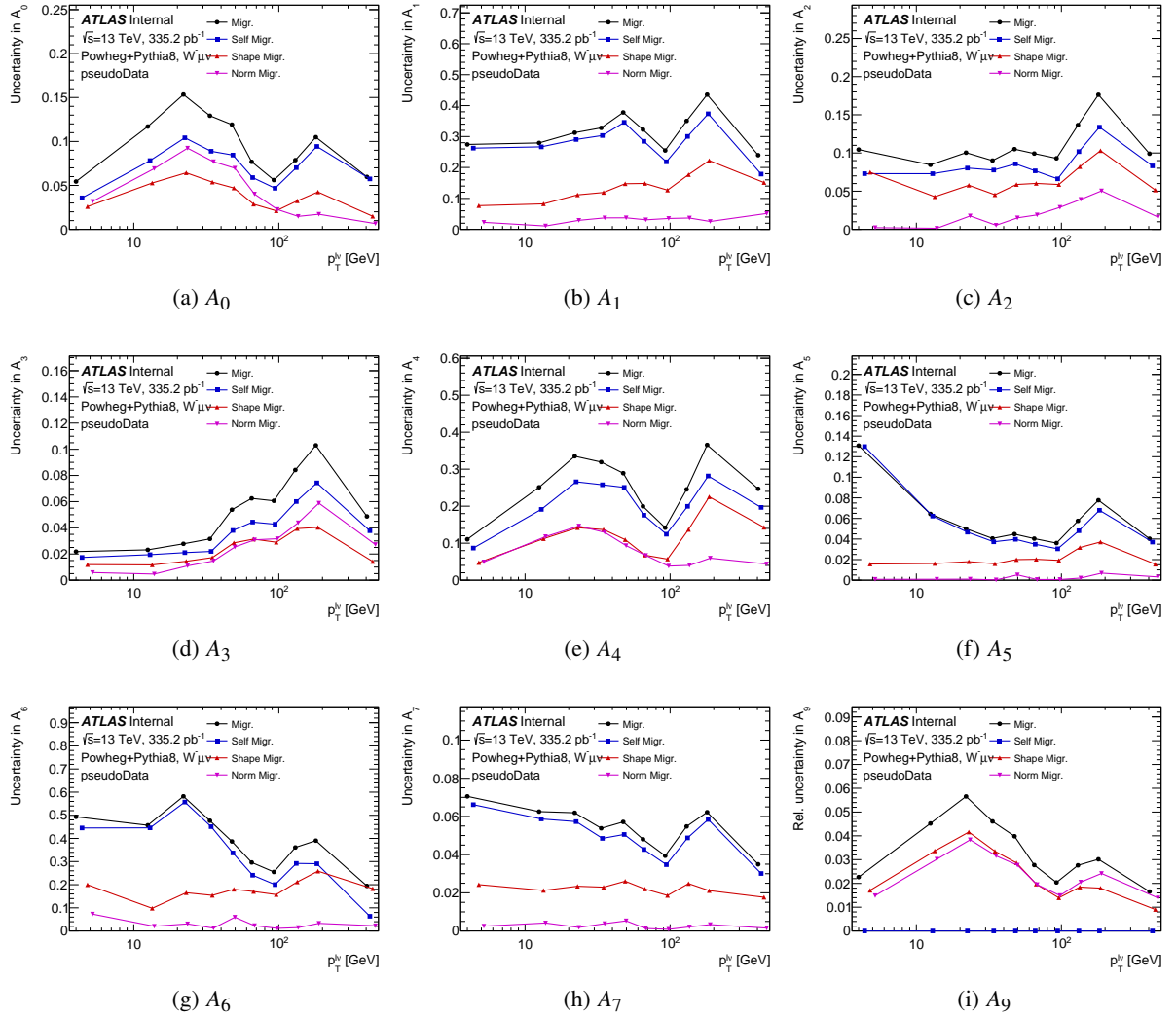


Figure 105: The uncertainty comparison for migration related uncertainties in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in p_T^W . Blue is the self migration between $p_T^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

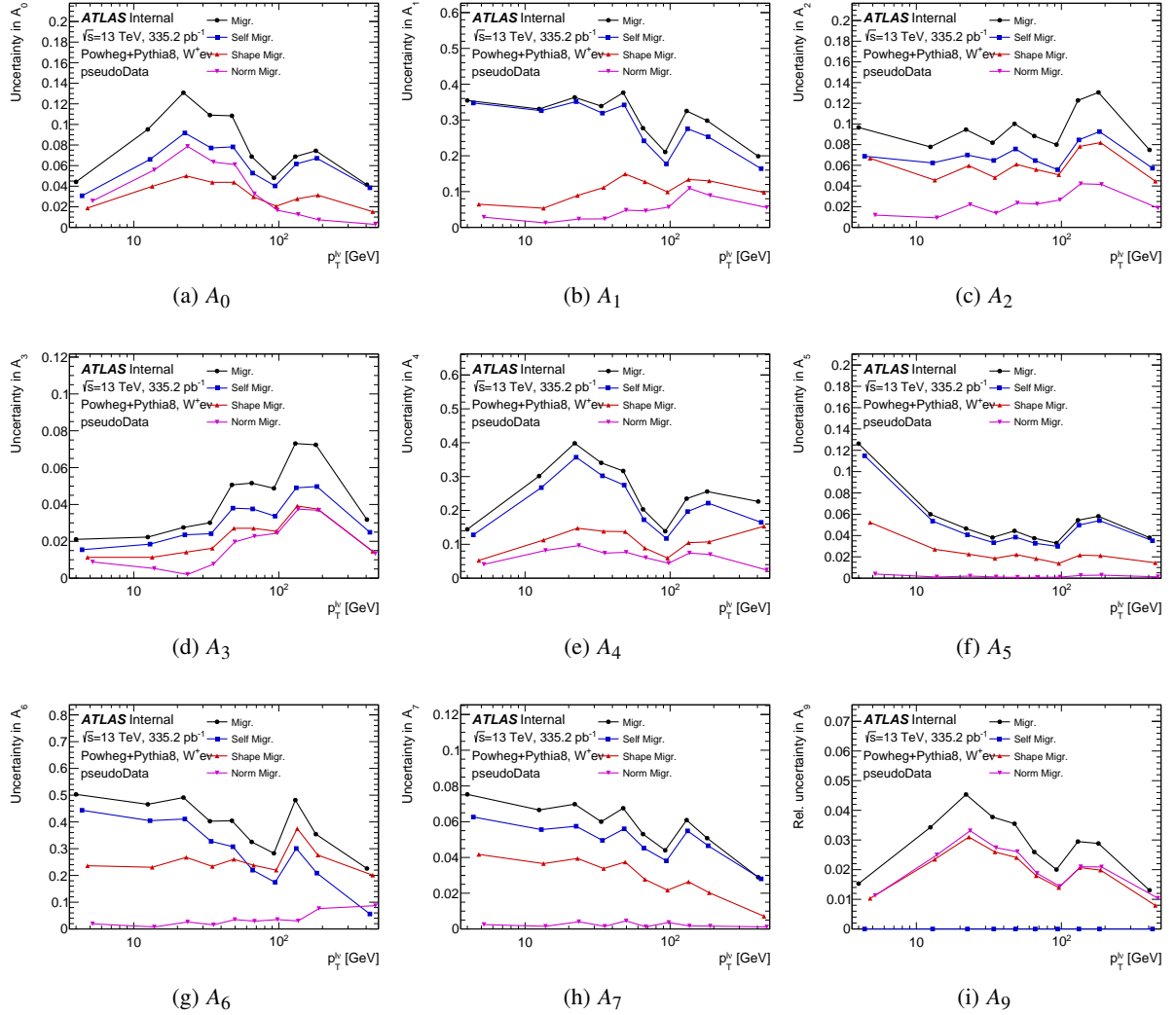


Figure 106: The uncertainty comparison for migration related uncertainties in the $W^+ \rightarrow e^+ \nu$ channel for measurement in p_T^W . Blue is the self migration between $p_T^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

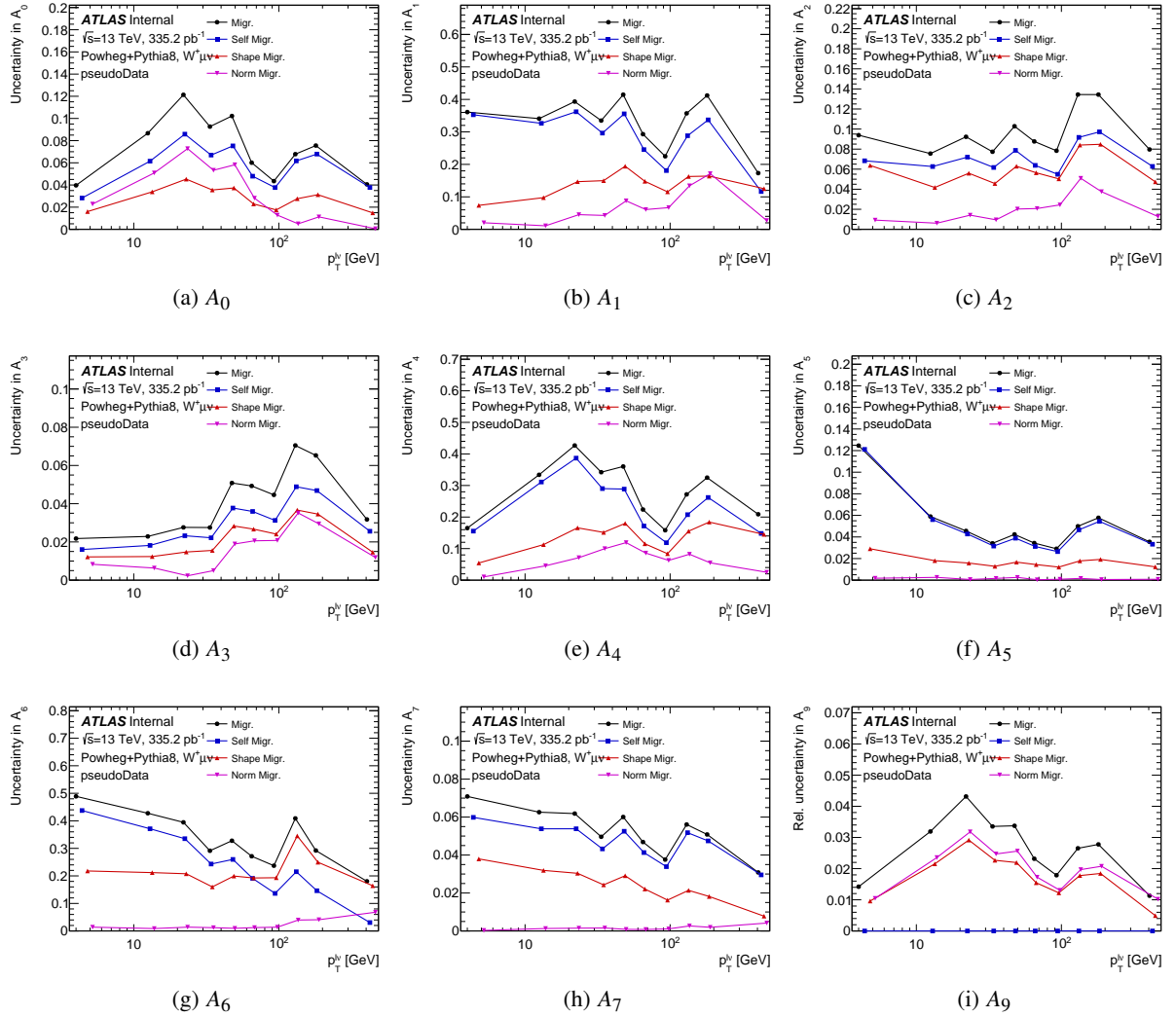


Figure 107: The uncertainty comparison for migration related uncertainties in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in p_T^W . Blue is the self migration between $p_T^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

1162 **D.2 y^W Differential**

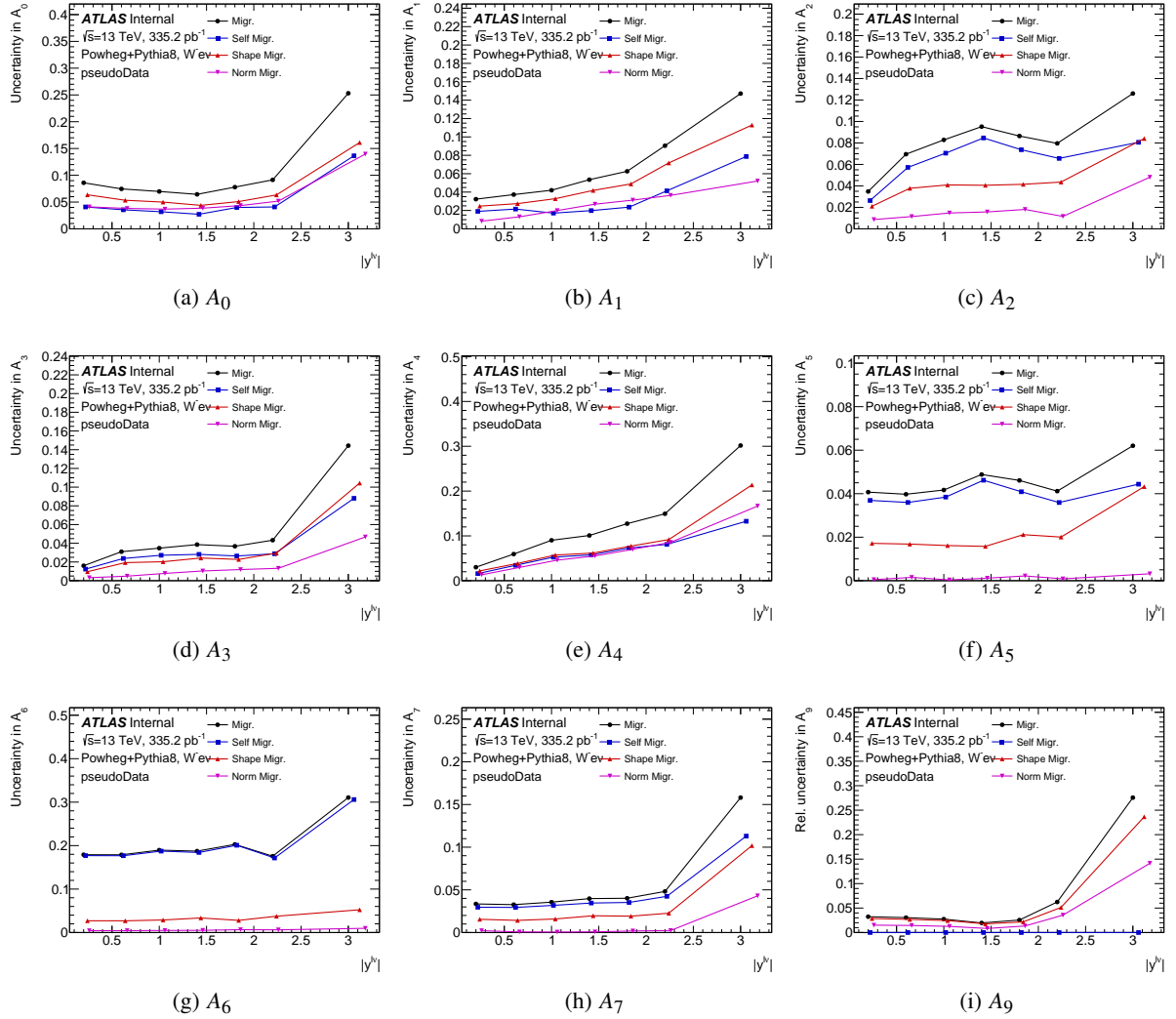


Figure 108: The uncertainty comparison for migration related uncertainties in the $W^- \rightarrow e^- \bar{\nu}$ channel for measurement in y^W . Blue is the self migration between $y^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

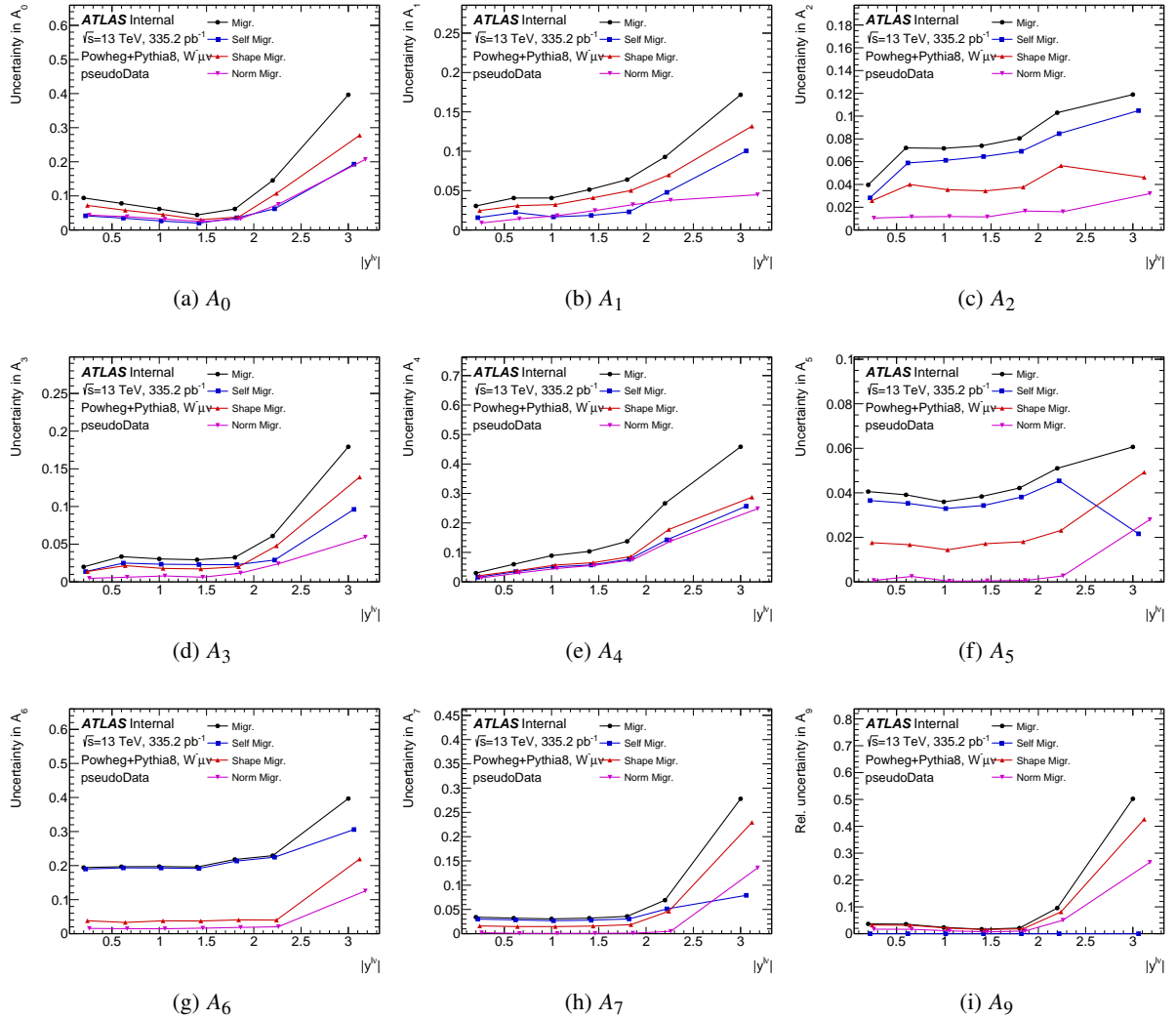


Figure 109: The uncertainty comparison for migration related uncertainties in the $W^- \rightarrow \mu^- \bar{\nu}$ channel for measurement in y^W . Blue is the self migration between $y^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

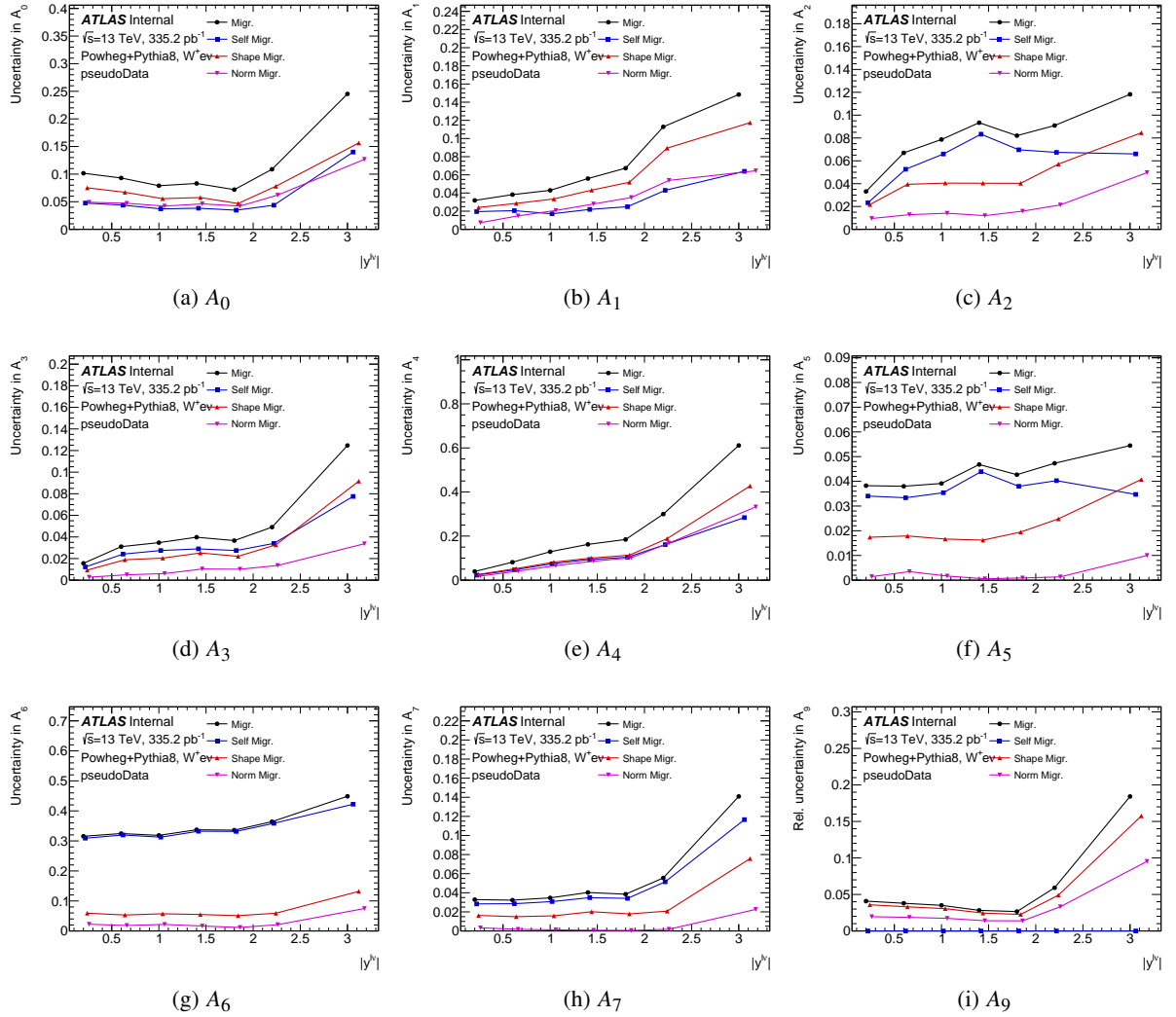


Figure 110: The uncertainty comparison for migration related uncertainties in the $W^+ \rightarrow e^+ \nu$ channel for measurement in y^W . Blue is the self migration between $y^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

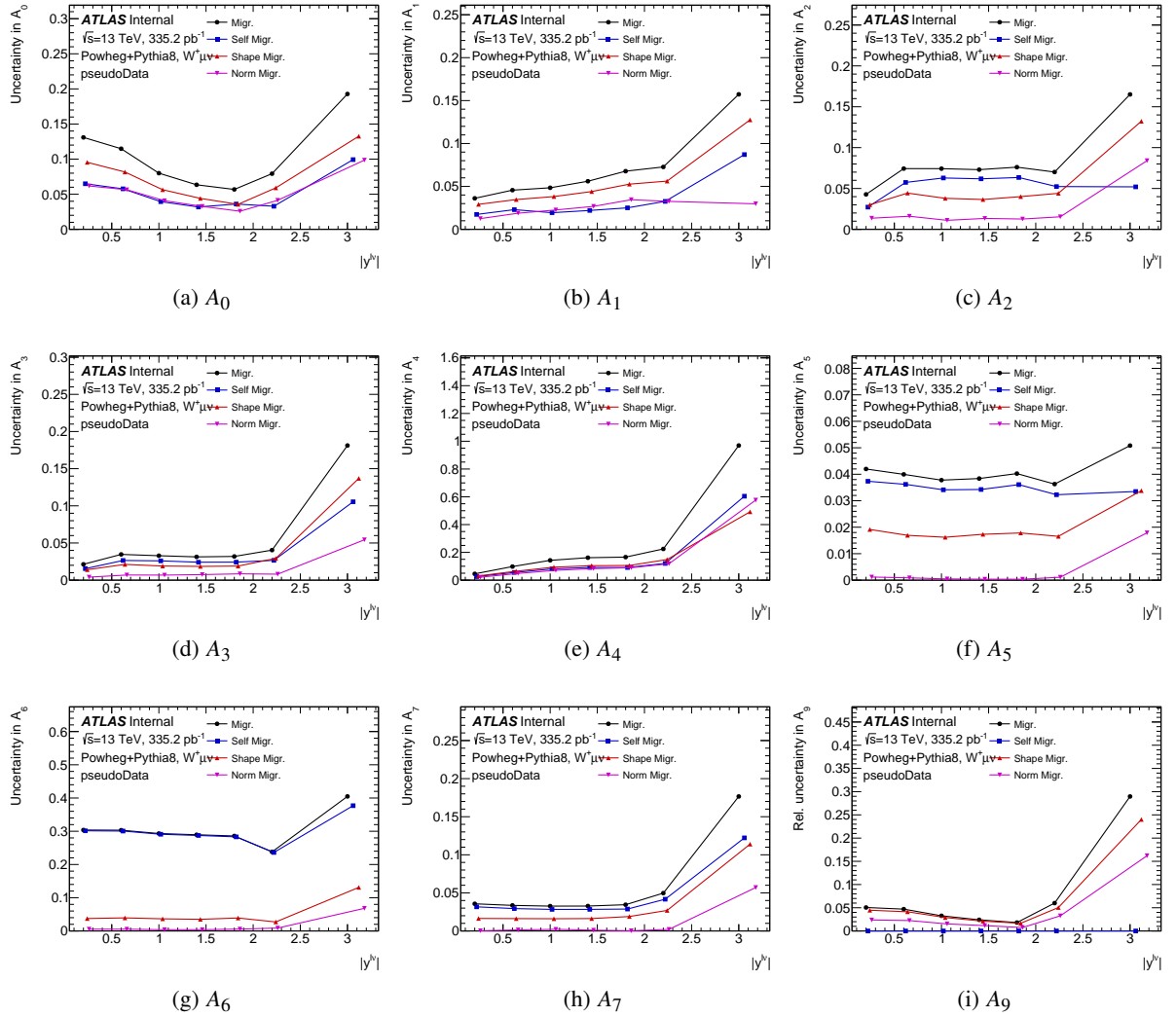


Figure 111: The uncertainty comparison for migration related uncertainties in the $W^+ \rightarrow \mu^+ \nu$ channel for measurement in y^W . Blue is the self migration between $y^{W, \text{Truth}}$ bins for the same coefficient, red is the shape migration for migration between coefficients, and pink is the self migration for the normalization. Black is the combination of the all of these migration uncertainty components.

1163 E Multi Jet Figures

1164 Place your MJ studies here.

1165 E.1 Recoil isolation correction

1166 The recoil reconstruction, described in detail in Ref. [43], uses all PFOs in the event, excluding a cone of
 1167 $\Delta R = 0.2$ around the selected leptons. To compensate for the underlying event and pile-up energy activity
 1168 removed by this procedure, another same-size cone of activity in the event is selected, centred at the same
 1169 η , but away in azimuth from any lepton and from hard activity; a vector, of magnitude given by the energy
 1170 observed in this cone and oriented along the removed lepton, is added to the recoil. This procedure is
 1171 successful in the signal region, where leptons are isolated and typically surrounded by soft activity.

1172
 1173 However, for events in anti-isolated regions the selected lepton is on average close to a jet, and the cone
 1174 replacement procedure fails. For example, for background events with a selected lepton of $p_T^\ell > 30$ GeV
 1175 and $ptcone20/p_T > 0.1$, the total track energy around the lepton is at least $0.1 \times p_T^\ell \sim 3$ GeV. This energy
 1176 is removed in the recoil reconstruction, and replaced by an inappropriately soft contribution. This effect
 1177 biases the recoil, hence E_T^{miss} , m_T and $|\Delta\phi(\ell, E_T^{miss})|$, and worsens for increasing isolation values.

1178
 1179 The projection of the recoil on the lepton momentum direction, u_{\parallel}^ℓ , is most affected, as this is the
 1180 direction affected by the cone replacement procedure. This projection is of particular importance for the
 1181 reconstruction of E_T^{miss} and the W transverse mass.

1182
 1183 This effect is corrected for in a dedicated hadronic recoil reconstruction, used for the present back-
 1184 ground analysis. For a given lepton, *isocone20* (*ptcone20* or *topoetcone20*), defined as the sum of the
 1185 transverse momenta of tracks (calorimeter deposit) in a cone of $\Delta R = 0.2$ around it, is correlated to the
 1186 removed energy around the lepton. This suggests the following correction:

$$1187 \vec{u}^{corr} = \vec{u}^{baseline} + \vec{u}^{iso}, \text{ where} \quad (62)$$

$$1188 \vec{u}^{iso} \equiv k_{iso} \times isocone20 \cdot \vec{u}_\ell \quad (63)$$

1189 where $\vec{u}^{baseline}$ is the standard recoil reconstruction, \vec{u}^{iso} represents the present correction, and \vec{u}_ℓ is the
 1190 unit vector aligned with the lepton direction and k_{iso} is a parameter accounting for a possible difference
 1191 in scale between the *isocone20* energy and the energy removed around the lepton. As the correlation
 1192 coefficient between *isocone20* and the energy of removed hard activities, k_{iso} has to be measured with
 1193 data. The effect of this correction is by construction negligible when approaching signal region, hence not
 1194 affecting the signal templates, used in the fraction fits, in a significant way.

1195 To determine k_{iso} it is assumed that the average value of u_{\parallel}^ℓ should be independent of the anti-isolation
 1196 value. So $\langle u_{\parallel}^{corr} \rangle$ should be a constant after this correction. Denoting b_{iso} this constant:

$$1197 \begin{aligned} \langle u_{\parallel}^{corr} \rangle &= \langle u_{\parallel}^{baseline} \rangle + \langle u_{\parallel}^{iso} \rangle \equiv b_{iso} \\ \langle u_{\parallel}^{corr} \rangle &= \langle u_{\parallel}^{baseline} \rangle + k_{iso} \langle isocone20 \rangle \equiv b_{iso} \\ \langle u_{\parallel}^{baseline} \rangle &= -k_{iso} \langle isocone20 \rangle + b_{iso} \end{aligned} \quad (64)$$

k_{iso} parameter	$W^- \rightarrow e^- \bar{\nu}$	$W^+ \rightarrow e^+ \nu$	$W^- \rightarrow \mu^- \bar{\nu}$	$W^+ \rightarrow \mu^+ \nu$
<i>ptcone20</i>	-	-	1.51 ± 0.02	1.50 ± 0.02
<i>topoetcone20</i>	1.38 ± 0.02	1.41 ± 0.02	-	-

Table 7: The fitted values of k_{iso} , the uncertainty are the statistical uncertainty from the fitting.

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While this assumption may not hold to high accuracy, as the different isolation slices have different properties in terms of jet structure, potentially affecting u_{\parallel}^{ℓ} , it is justified to compensate for the mis-match in the energy replacement procedure for anti-isolated leptons. So the correction factor k_{iso} can be fitted from the linear relationship between $\langle u_{\parallel}^{\ell} \rangle$ and $isocone20$. The effect of this correction is illustrated in Figure 112 and 113, which shows a large dependence of $\langle u_{\parallel}^{\ell} \rangle$ on the value of $isocone20$ before correction and that this dependence is significantly reduced after correction. The k_{iso} value fitted for a given channel is the opposite of the slope obtained from the linear fit between [5, 25] GeV, the results of which are listed in Table 7.

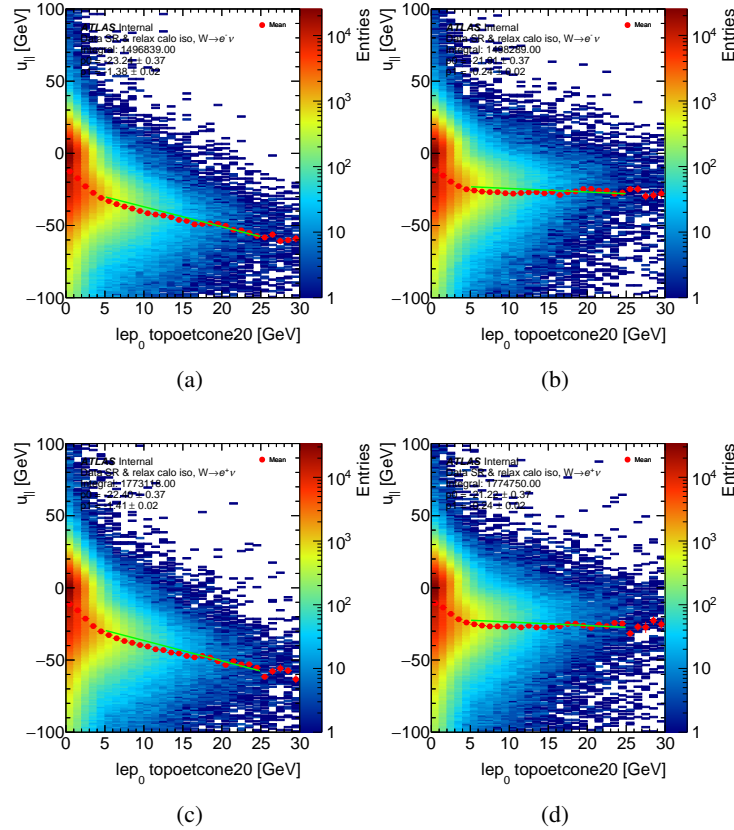


Figure 112: $\langle u_{\parallel}^{\ell} \rangle$ as a function of *topoetcone20*, before (left) and after (right) correction in FR + CR1 for $W^- \rightarrow e^- \bar{\nu}$ (a, b) and $W^+ \rightarrow e^+ \nu$ channel (c, d).

1207

Example fraction fits before and after this correction are compared in Figure 114 and 115 for $W^- \rightarrow e^- \bar{\nu}$

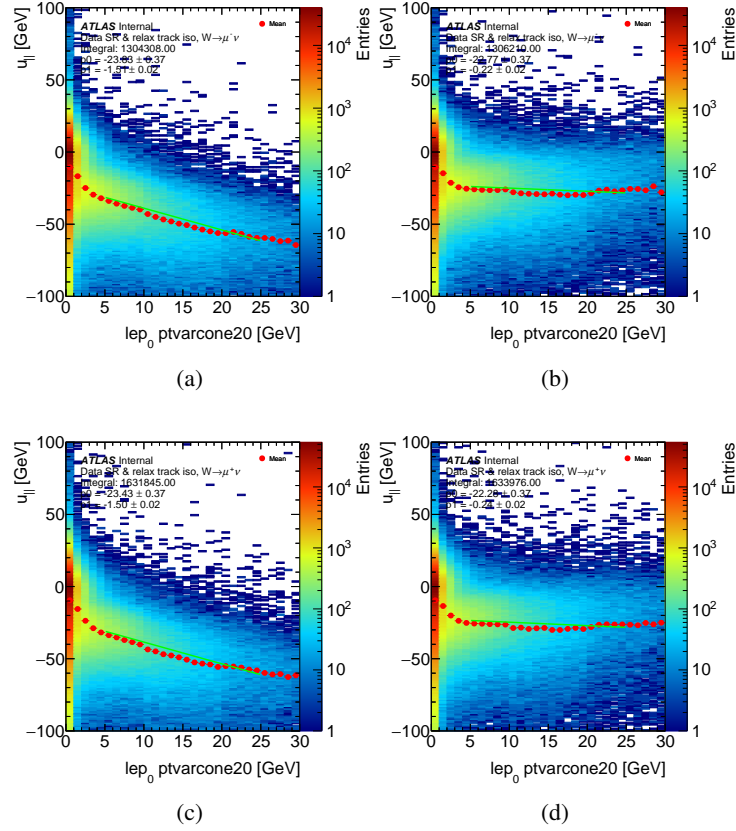


Figure 113: $\langle u_{\parallel}^{\ell} \rangle$ as a function of $ptvarcone20$, before (left) and after (right) correction in FR + CR1 for $W^{-} \rightarrow \mu^{-}\bar{\nu}$ (a, b) and $W^{+} \rightarrow \mu^{+}\nu$ channel (c, d).

1208 with the first and the third anti-isolation slices. The m_T , E_T^{miss} and $|\Delta\phi(\ell, E_T^{miss})|$ fits are visibly improved,
 1209 as these variables directly benefit from the improved recoil definition; this is not the case for p_T^{ℓ} , which
 1210 shows satisfactory agreement before the correction. The mis-modelling observed at high values for all
 1211 distributions is driven by a mismodelling of the p_T^W distribution in the simulation.

1212 Figures 116 and 117 show the effect of this correction on the $ptvarcone20/p_T$ (for μ^{\pm}) and $topoetcone20/p_T$
 1213 (for e^{\pm}) scan in the electron and muon channels. The dependence of the fitted yield on the isolation range
 1214 is affected.

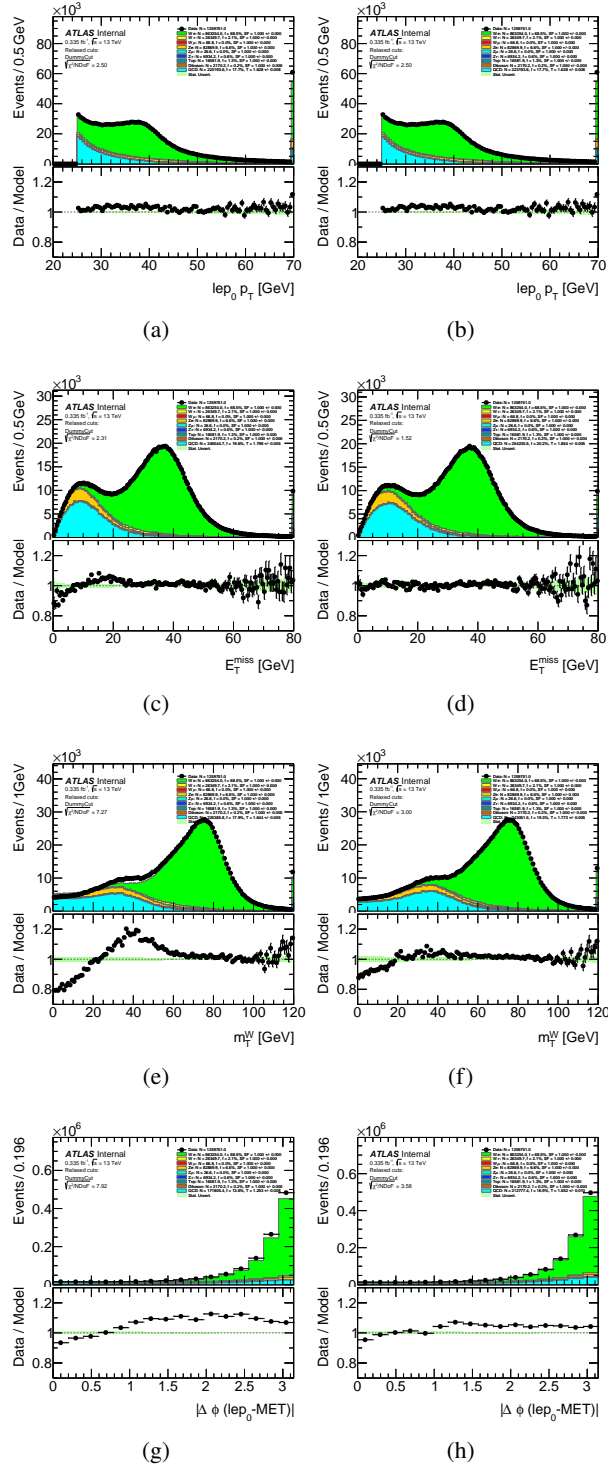


Figure 114: Multi-jet background template fits performed in distributions of $p_T^{e^-}$ (top), E_T^{miss} (middle) and m_T (bottom) with the MJ template obtained from isolation slice $0.05 < \text{topoetcone20}/p_T < 0.1$. The fits are presented in $W^- \rightarrow e^- \bar{\nu}$ channel before (left) and after (right) recoil correction.

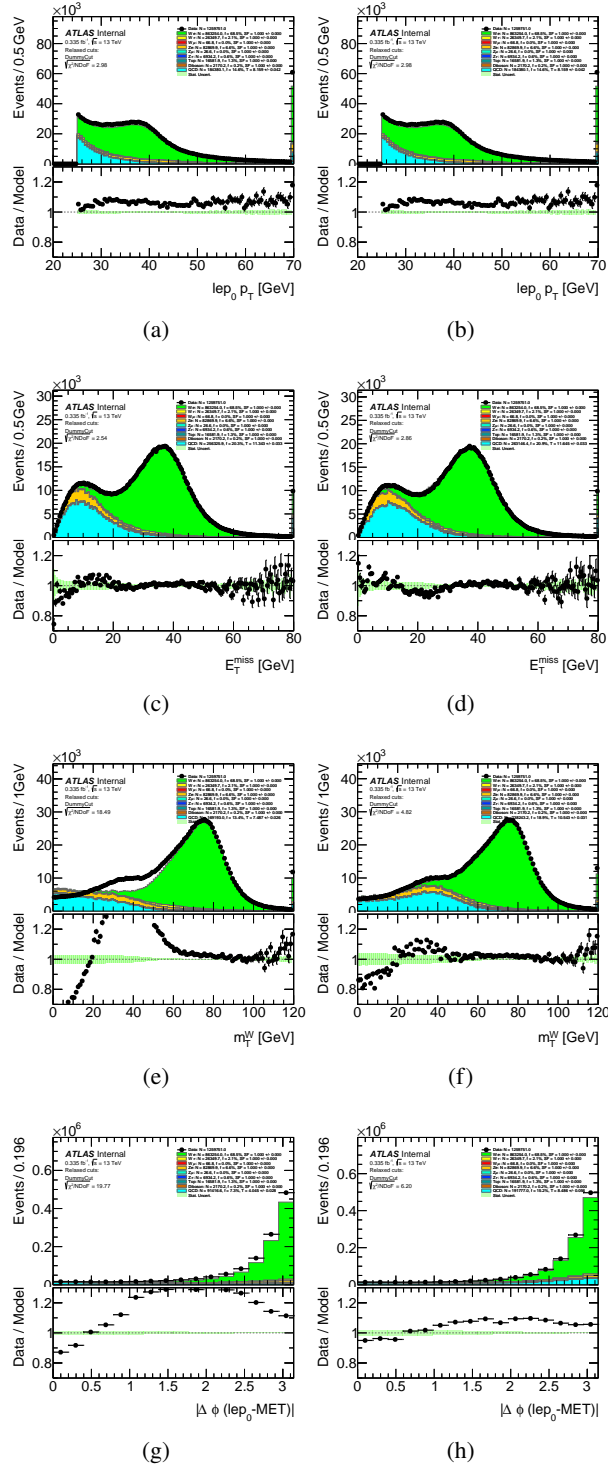


Figure 115: Multi-jet background template fits performed in distributions of $p_T^{e^-}$ (top), E_T^{miss} (middle) and m_T (bottom) with the MJ template obtained from isolation slice $0.2 < \text{topoetcone20}/p_T < 0.25$. The fits are presented in $W^- \rightarrow e^- \bar{\nu}$ channel before (left) and after (right) recoil correction.

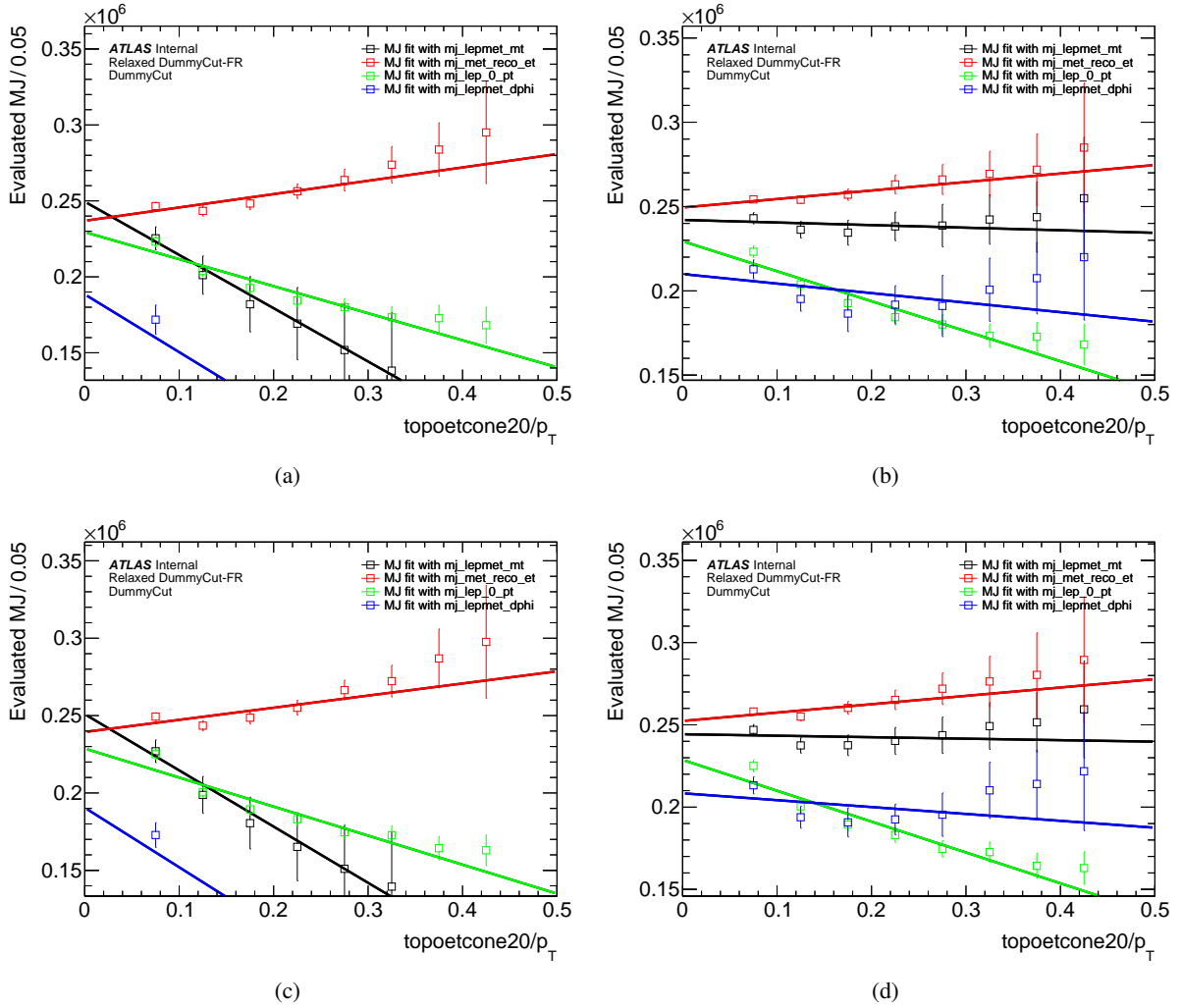


Figure 116: The estimated total number of the multijet background in SR scanned as a function of the isolation slice. Isolation scan in $W^- \rightarrow e^- \bar{\nu}$ (top) and $W^+ \rightarrow e^+ \nu$ (bottom) channels without (left) and with (right) recoil correction. Each point is plotted as average $topoetcone20/p_T$ of each dlice in CR2.

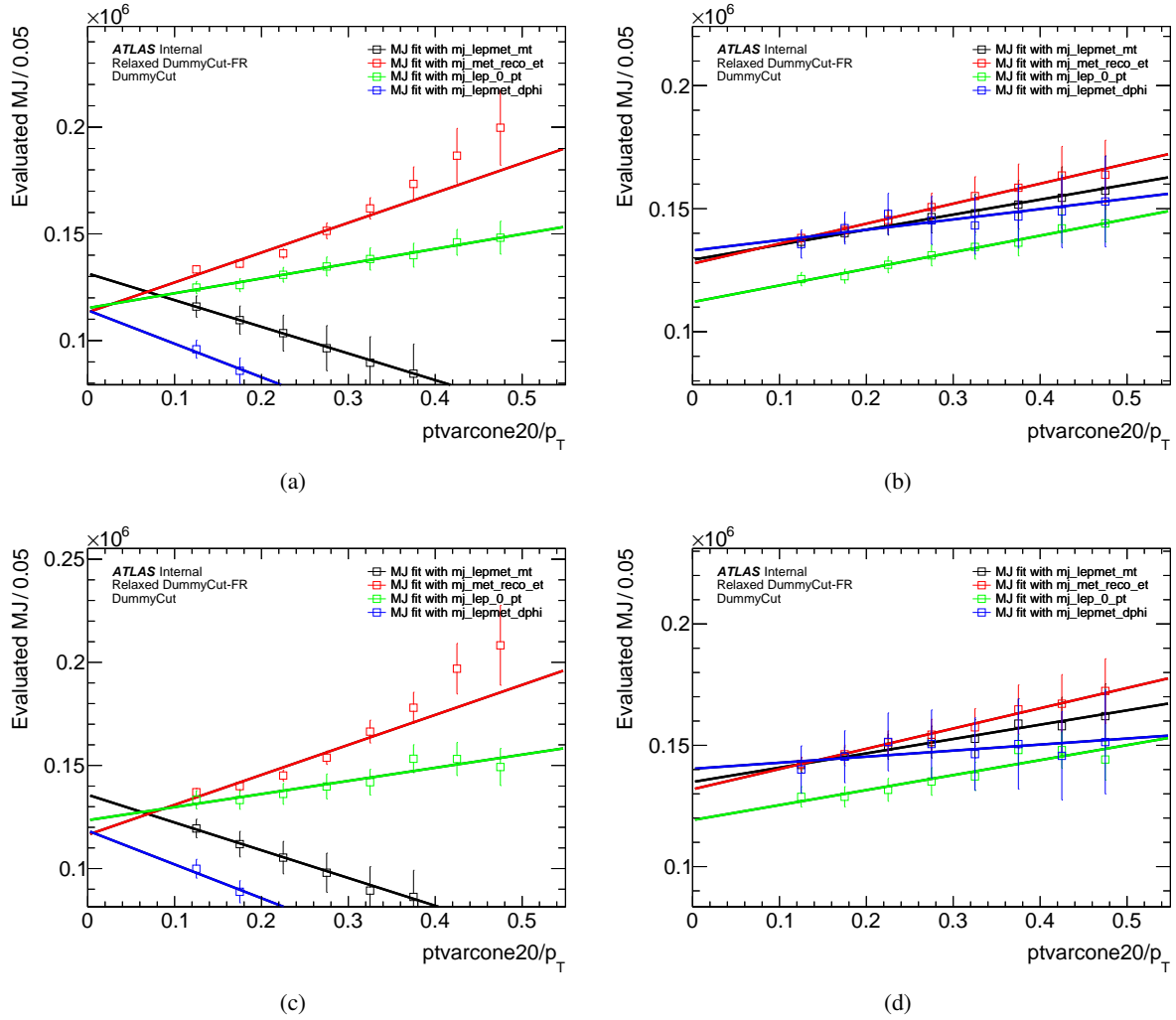


Figure 117: The estimated total number of the multijet background in SR scanned as a function of the isolation slice. Isolation scan in $W^- \rightarrow \mu^- \bar{\nu}$ (top) and $W^+ \rightarrow \mu^+ \nu$ (bottom) channels without (left) and with (right) recoil correction. Each point is plotted as average $ptvarcone20/p_T$ of each slice in CR2.

1215 E.2 Choice of the track-based and calo-based isolation variables scans

1216 This analysis uses different isolation variables for e^\pm and μ^\pm channels:

- 1217 • Slicing $ptvarcone20/pt$ for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$: 8 slicing bins from 0.1 to 0.5
- 1218 • Slicing in $topoetcone20/pt$ for $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$: 8 slicing bins from 0.05 to 0.45

1219 In this chapter highlights why this decision has been made.

1220 This analysis is based on multijet background estimation methodology same as used by WpT analysis
1221 (see [44]). Early multijet studies were focused on the track-based isolation variable. However, data/MC
1222 discrepancies for $\cos \theta_{CS}$ distributions in the $W^- \rightarrow e^- \bar{\nu}$ and $W^+ \rightarrow e^+ \nu$ channels has been observed. In
1223 case of electrons it is shown on Figure 118(a). Effectively it had pointed to the problems in tails of the
1224 lepton η distributions. There are no such data/MC discrepancies observed for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$
1225 channels. Example for muon channel is shown on Figure 118(b).

1226 This issue comes from the multijet template shape estimation. Replacing track-based isolation variable
1227 ($ptvarcone20/p_T$) using same cone sized calo-based isolation variable ($topoetcone20/p_T$) fixes this.
1228 Results on this are shown on Figures 118(c) and 118(d) for $W^- \rightarrow e^- \bar{\nu}$ and $W^- \rightarrow \mu^- \bar{\nu}$ channels
1229 respectively.

1230 Impact on the evaluated normalized MJ shapes in anti-isolated slices of CR1 for η distribution for
1231 $W^- \rightarrow e^- \bar{\nu}$ as a function of track-based and calo-based isolation variables on Figure 119. This might
1232 happen due to tight cut on $topoetcone20/p_T$ during the $ptvarcone20/p_T$ anti-isolation slice scan.

1233 Natural step would be to use same $topoetcone20/p_T$ isolation variable for all channels. However it
1234 was discovered that $topoetcone20$ is not properly modelled in the signal samples for $W^- \rightarrow \mu^- \bar{\nu}$ and
1235 $W^+ \rightarrow \mu^+ \nu$ channels. Figure 120 shows $topoetcone20/p_T$ distributions for $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$
1236 selections. The 20% data/background discrepancy for central rapidity in the $Z \rightarrow \mu\mu$ is shown. There are
1237 no this feature for $Z \rightarrow ee$ channel.

1238 Same effect is observed for the first anti-isolated slice for the $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels.
1239 Figure 121 shows evaluation of the multi-jet background template fit in the $W^- \rightarrow \mu^- \bar{\nu}$ for the first two
1240 anti-isolated slices: $0.05 < topoetcone20/p_T < 0.1$ and $0.1 < topoetcone20/p_T < 0.15$. There are
1241 missing signal MC to subtract in the first anti-isolation slice that brings same 20% discrepancy. Impact on
1242 other anti-isolated slices is not that big as signal contribution to the CR1 drops. Linear extrapolations from
1243 anti-isolated slices for $W^- \rightarrow \mu^- \bar{\nu}$ channel are shown on Figure 122(a).

1244 An overall 20% scaling of signal contamination helps to fix linear extrapolations as it is shown on
1245 Figure 122(b). However it is not best solution as we got negative shape in m_T^W as it is shown on
1246 Figure 123(b).

1247 To get it right we should correct the MC mismodelling of $topoetcone20$, but in a meantime decided to use
1248 $ptvarcone20/p_T$ anti-isolation for muons and anti-muons channels.

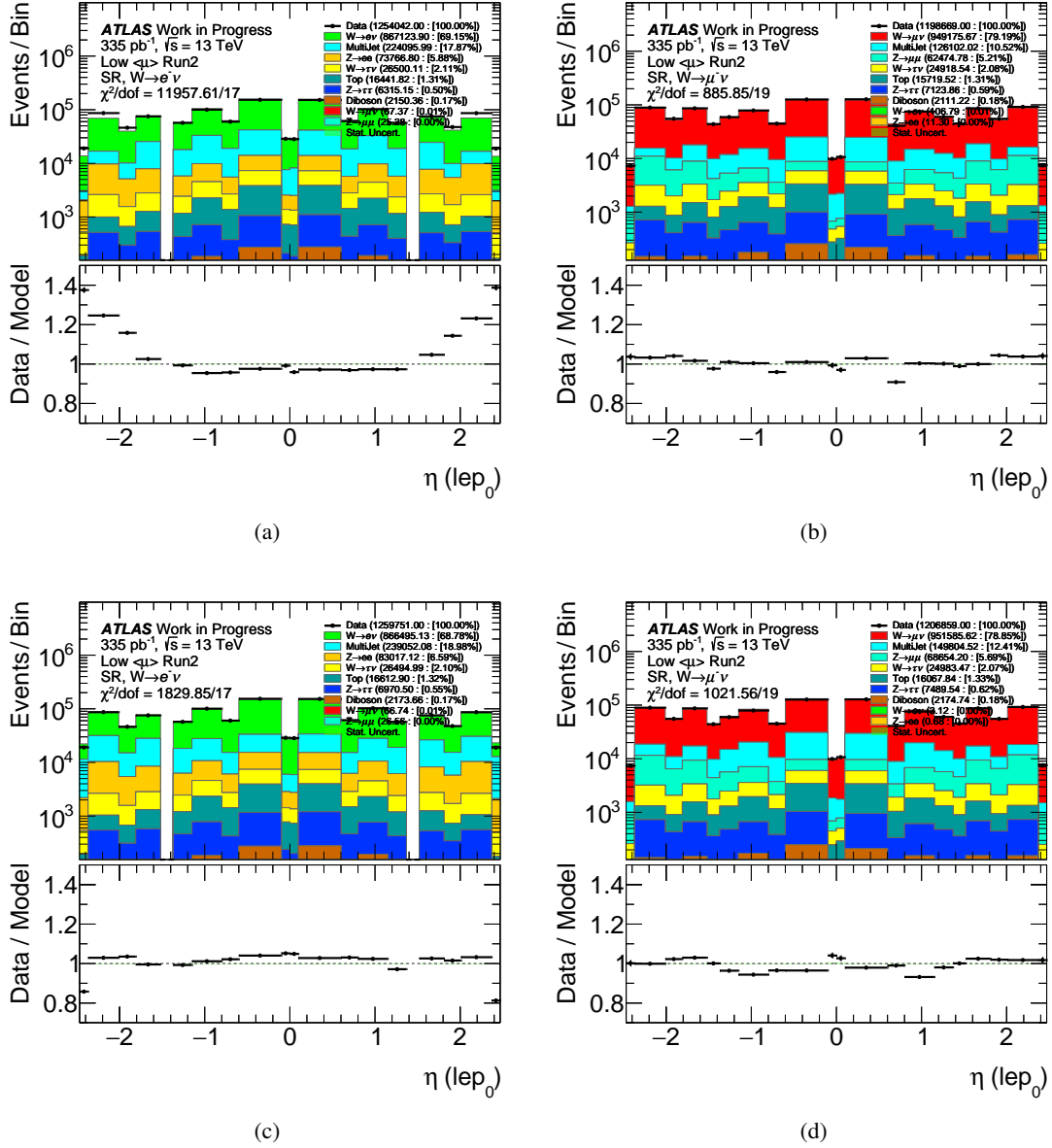


Figure 118: The η distributions for $W^- \rightarrow e^-\bar{\nu}$ (left) and $W^- \rightarrow \mu^-\bar{\nu}$ (right) channels using multijet estimation slicing over $ptvarcone20/p_T$ (top) and $topoetcone20/p_T$ (bottom).

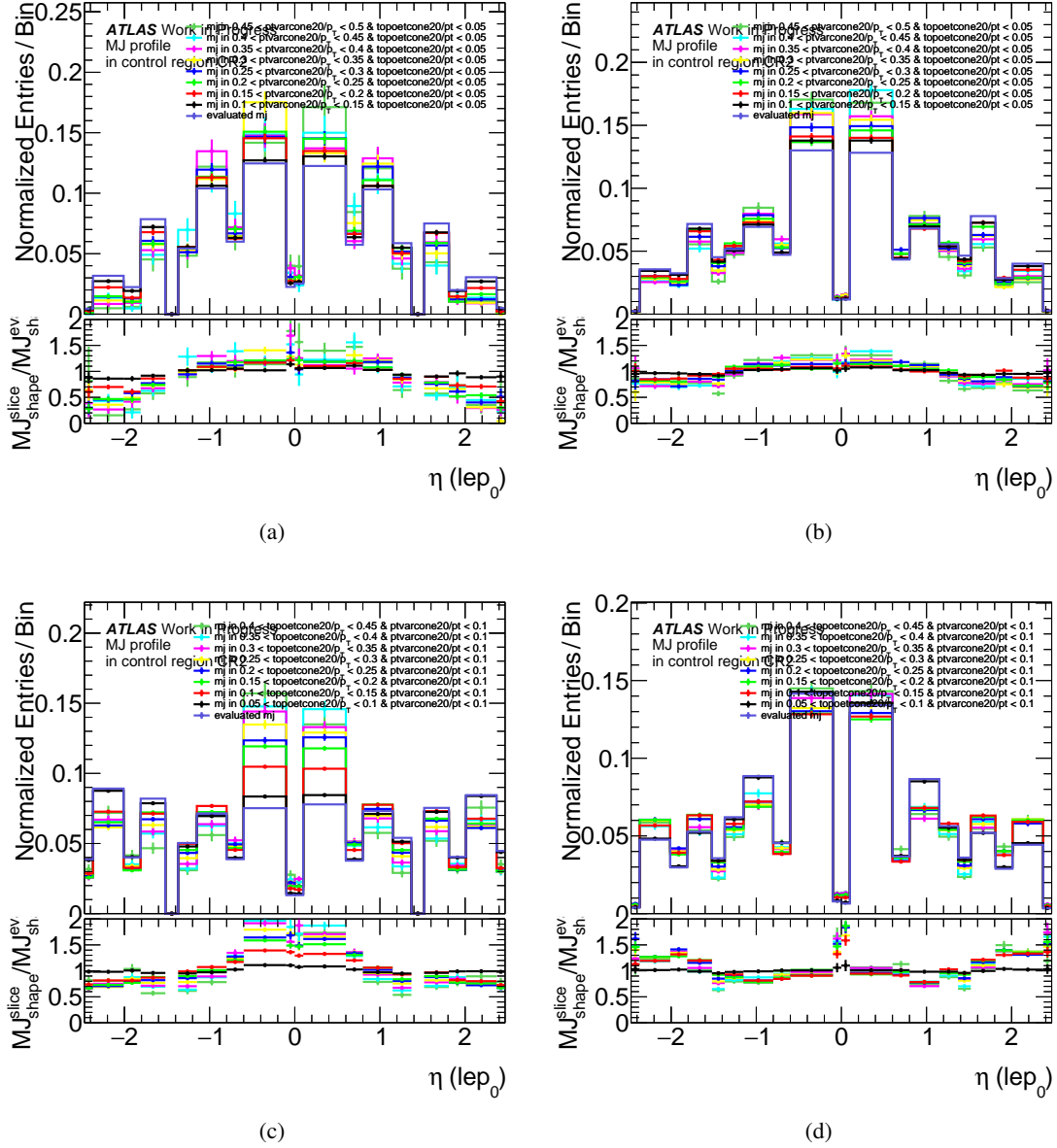


Figure 119: Normalized MJ shapes in anti-isolated slices of CR1 for η distribution for $W^- \rightarrow e^- \bar{\nu}$ (left) and $W^- \rightarrow \mu^- \bar{\nu}$ (right) channels using multijet estimation slicing over $ptvarcone20/p_T$ (top) and $topoetcone20/p_T$ (bottom).

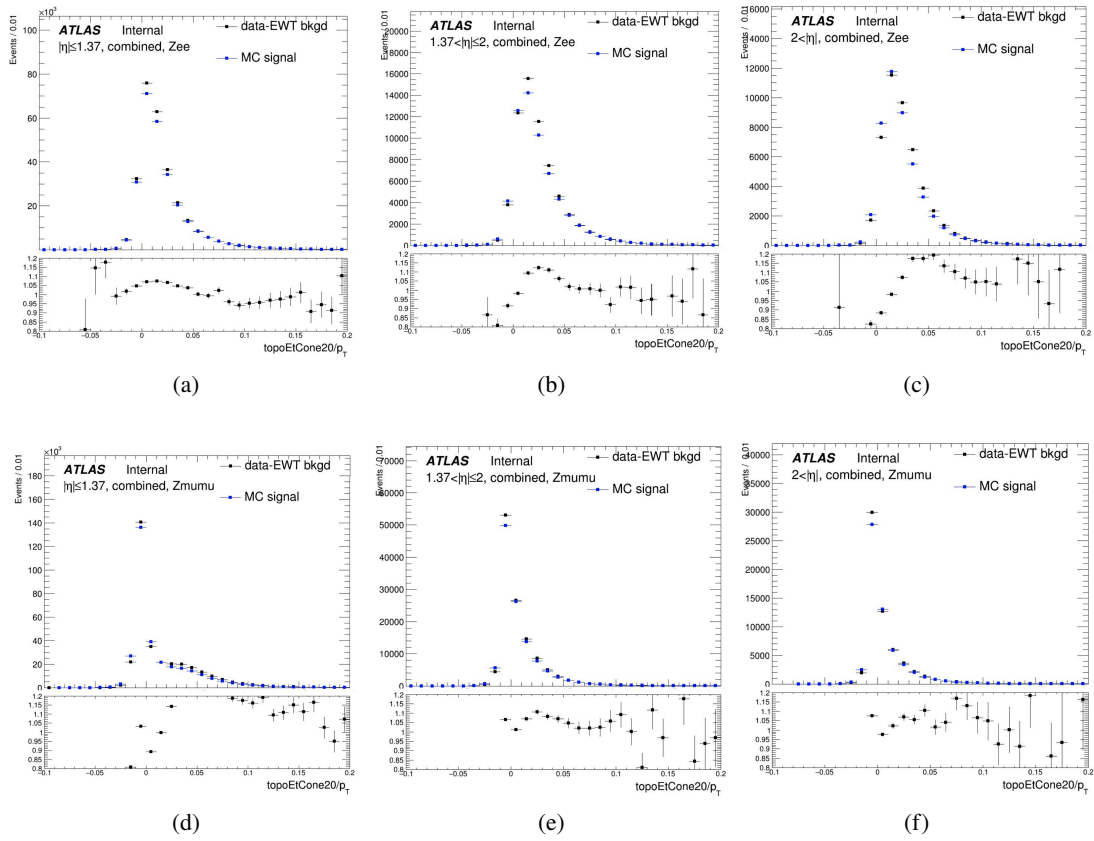


Figure 120: The relative $topoEtCone20/p_T$ distributions for $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ selections.

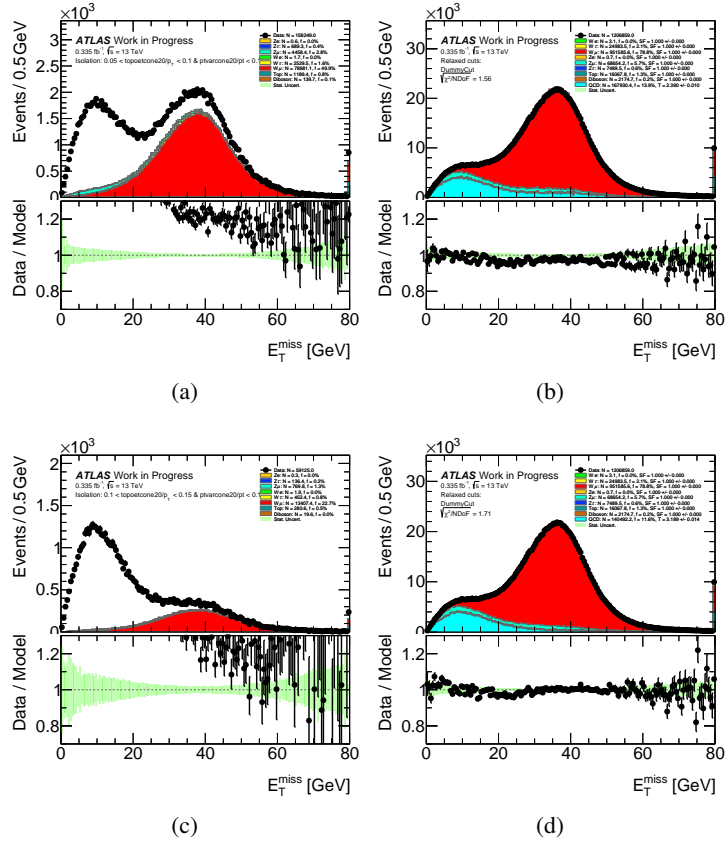


Figure 121: The multi-jet background template fit in the $W^- \rightarrow \mu^- \bar{\nu}$ for the anti-isolated slices $0.05 < \text{topoetcone20}/p_T < 0.1$ (top) and $0.1 < \text{topoetcone20}/p_T < 0.15$ (bottom). Two regions are shown: CR1 (left) and fit region (right).

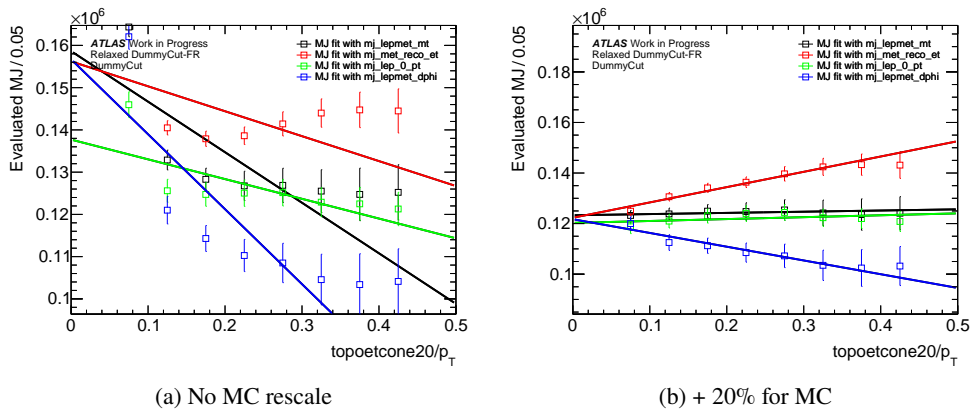


Figure 122: Linear extrapolations from anti-isolated slices for $W^- \rightarrow \mu^- \bar{\nu}$ channel before (left) and after (right) 20% rescale of the MC contribution to the CR1.

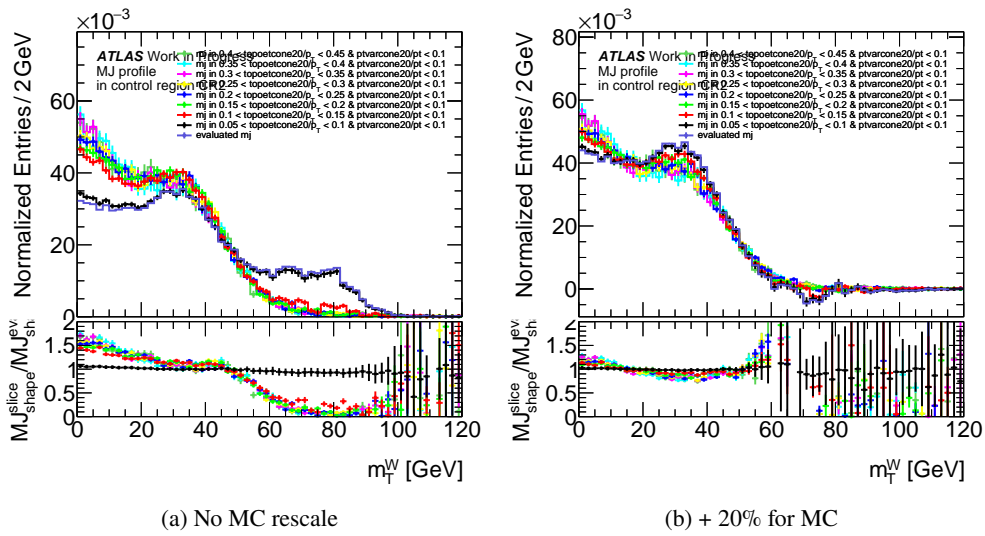


Figure 123: Normalized MJ shapes in anti-isolated slices of CR1 for $W^- \rightarrow \mu^- \bar{\nu}$ channel before (left) and after (right) 20% rescale of the MC contribution to the CR1.

1249 E.3 Multi-jet background template fits

1250 Despite the fact in this analysis we use relaxed kinematic cuts for the signal region comparing to the
 1251 WpT analysis [44], we keeping same naming convention for several samples are defined, corresponding
 1252 to different regions in phase space and isolation. The cuts definitions for control regions are listed in
 1253 Section 5.1.

1254 E.3.1 Electron channel multi-jet background template fits

u_T Cut	m_T^W fits b	m_T^W fits k	E_T^{miss} fits b	E_T^{miss} fits k	p_T fits b	p_T fits k	$ \Delta\phi(\ell, E_T^{miss}) $ fits b	$ \Delta\phi(\ell, E_T^{miss}) $ fits k
–	242061 ± 4801	-15289 ± 32494	249423 ± 2886	50429 ± 22195	229429 ± 3690	-177953 ± 18533	209957 ± 7185	-56528 ± 43084

Table 8: k and b are parameters for the electron extrapolations on 13 TeV. The uncertainty of these two parameters are provided by the extrapolation, from the statistical uncertainty of the parameter T and the number of events in CR of each slices and multiplied by $\sqrt{\chi^2/NDoF}$.

u_T Cut	Slice No.	$Isom_{min}$	$Isom_{max}$	N_{Data}^{FR}	N_{FR}^{EW}	N_{CR1}^{Data}	N_{CR1}^{EW}	N_{CR1}^{MJ}	T	α	χ^2	NDoF	$\sqrt{\chi^2/NDoF}$	N_{CR2}^{Data}	N_{CR2}^{EW}	N_{CR2}^{MJ}	N_{CR2}^{MJ}
–	0	0.05	0.1	1259751 ± 1122	998253 ± 330	222831 ± 472	83732 ± 97	137099 ± 482	1.638 ± 0.008	1.000 ± 0.000	561.53	90	2.50	222831 ± 472	83732 ± 97	137099 ± 482	223194 ± 3475
–	1	0.1	0.15	1259751 ± 1122	998253 ± 330	83148 ± 288	11023 ± 35	72125 ± 291	2.826 ± 0.015	1.000 ± 0.000	611.78	90	2.61	83148 ± 288	11023 ± 35	72125 ± 291	203849 ± 3476
–	2	0.15	0.2	1259751 ± 1122	998253 ± 330	41682 ± 204	2230 ± 16	39452 ± 205	4.885 ± 0.025	1.000 ± 0.000	733.81	90	2.86	41682 ± 204	2230 ± 16	39452 ± 205	192710 ± 4028
–	3	0.2	0.25	1259751 ± 1122	998253 ± 330	23298 ± 153	700 ± 9	22598 ± 153	8.159 ± 0.042	1.000 ± 0.000	797.02	90	2.98	23298 ± 153	700 ± 9	22598 ± 153	184380 ± 4672
–	4	0.25	0.3	1259751 ± 1122	998253 ± 330	13388 ± 116	340 ± 6	13048 ± 116	13.797 ± 0.071	1.000 ± 0.000	858.68	90	3.09	13388 ± 116	340 ± 6	13048 ± 116	180023 ± 5713
–	5	0.3	0.35	1259751 ± 1122	998253 ± 330	7932 ± 89	194 ± 5	7738 ± 89	22.406 ± 0.116	1.000 ± 0.000	887.39	90	3.14	7932 ± 89	194 ± 5	7738 ± 89	173364 ± 6882
–	6	0.35	0.4	1259751 ± 1122	998253 ± 330	4688 ± 68	128 ± 4	4560 ± 69	37.883 ± 0.196	1.000 ± 0.000	867.52	90	3.10	4688 ± 68	128 ± 4	4560 ± 69	172758 ± 8526
–	7	0.4	0.45	1259751 ± 1122	998253 ± 330	2998 ± 55	88 ± 3	2910 ± 55	57.770 ± 0.301	1.000 ± 0.000	1208.73	90	3.66	2998 ± 55	88 ± 3	2910 ± 55	168140 ± 12043

Table 9: Numbers for the electron p_T fits on 13 TeV. Numbers in FR are not affected by $topoetcone20/p_T$ Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDoF}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

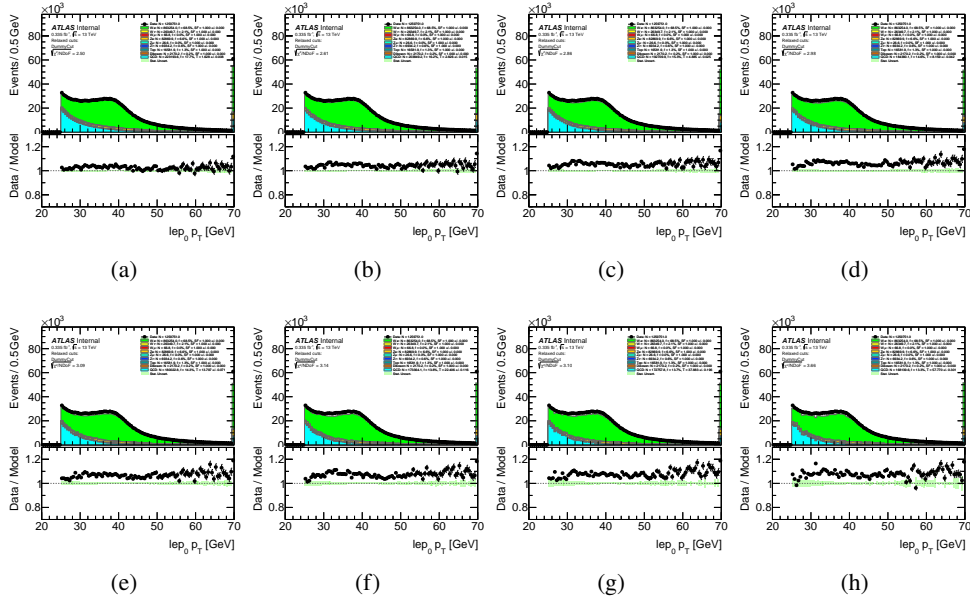


Figure 124: Multi-jet background template fits performed in distributions of p_T^ℓ of the electron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < topoetcone20/p_T < 0.1$ to $0.4 < topoetcone20/p_T < 0.45$.

trj Cut	Slice No.	Isomin	Isomax	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	T	α	χ^2	NDoF	$\sqrt{\chi^2}/NDoF$	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	N_{MJ}^{FR}
-	0	0.05	0.1	1259751 ± 1122	998253 ± 330	222831 ± 472	85732 ± 97	137099 ± 482	1.854 ± 0.005	1.000 ± 0.000	369.87	160	1.52	222831 ± 472	85732 ± 97	137099 ± 482	254235 ± 1760
-	1	0.1	0.15	1259751 ± 1122	998253 ± 330	83148 ± 288	11023 ± 35	72125 ± 291	3.522 ± 0.010	1.000 ± 0.000	455.10	160	1.69	83148 ± 288	11023 ± 35	72125 ± 291	254022 ± 2123
-	2	0.15	0.2	1259751 ± 1122	998253 ± 330	41682 ± 204	2230 ± 16	39452 ± 205	6.515 ± 0.019	1.000 ± 0.000	790.71	160	2.22	41682 ± 204	2230 ± 16	39452 ± 205	257045 ± 3392
-	3	0.2	0.25	1259751 ± 1122	998253 ± 330	23298 ± 153	700 ± 9	22598 ± 153	11.645 ± 0.033	1.000 ± 0.000	1304.64	160	2.86	23298 ± 153	700 ± 9	22598 ± 153	263146 ± 5523
-	4	0.25	0.3	1259751 ± 1122	998253 ± 330	13388 ± 116	340 ± 6	13048 ± 116	20.389 ± 0.059	1.000 ± 0.000	2030.98	160	3.56	13388 ± 116	340 ± 6	13048 ± 116	266041 ± 8847
-	5	0.3	0.35	1259751 ± 1122	998253 ± 330	7932 ± 89	194 ± 5	7738 ± 89	34.814 ± 0.100	1.000 ± 0.000	2789.12	160	4.18	7932 ± 89	194 ± 5	7738 ± 89	269573 ± 13364
-	6	0.35	0.4	1259751 ± 1122	998253 ± 330	4688 ± 68	128 ± 4	4560 ± 69	59.614 ± 0.172	1.000 ± 0.000	4198.29	160	5.12	4688 ± 68	128 ± 4	4560 ± 69	271856 ± 21318
-	7	0.4	0.45	1259751 ± 1122	998253 ± 330	2998 ± 55	88 ± 3	2910 ± 55	97.910 ± 0.281	1.000 ± 0.000	7981.09	160	7.06	2998 ± 55	88 ± 3	2910 ± 55	284964 ± 38353

Table 10: Numbers for the electron E_T^{miss} fits on 13 TeV. Numbers in FR are not affected by $topoetcone20/p_T$ Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

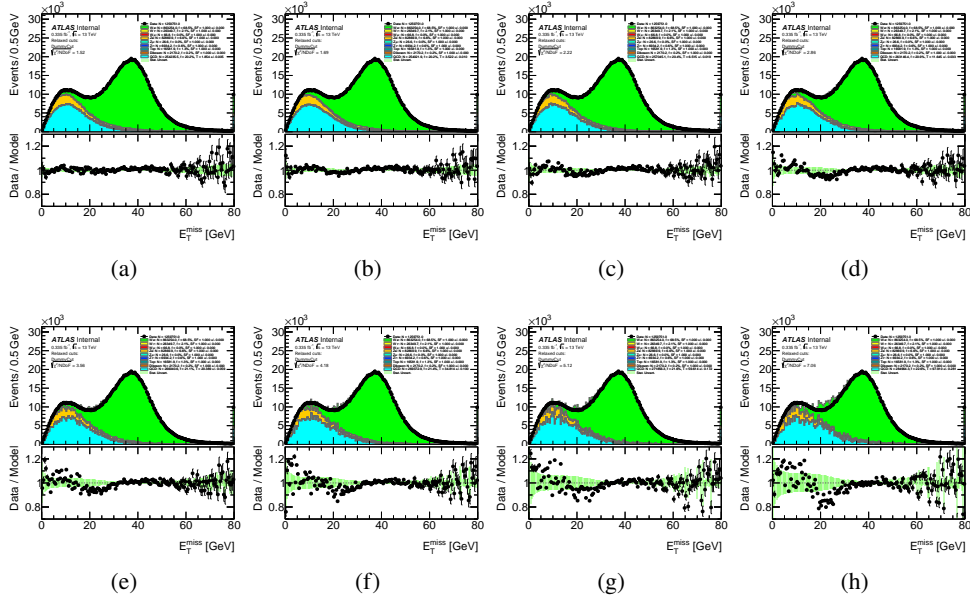


Figure 125: Multi-jet background template fits performed in distributions of E_T^{miss} of the electron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < topoetcone20/p_T < 0.1$ to $0.4 < topoetcone20/p_T < 0.45$.

trj Cut	Slice No.	Isomin	Isomax	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	T	α	χ^2	NDoF	$\sqrt{\chi^2}/NDoF$	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	N_{MJ}^{FR}
-	0	0.05	0.1	1259751 ± 1122	998253 ± 330	222831 ± 472	85732 ± 97	137099 ± 482	1.773 ± 0.005	1.000 ± 0.000	1080.63	120	3.00	222831 ± 472	85732 ± 97	137099 ± 482	243051 ± 3359
-	1	0.1	0.15	1259751 ± 1122	998253 ± 330	83148 ± 288	11023 ± 35	72125 ± 291	3.276 ± 0.010	1.000 ± 0.000	2100.72	120	4.18	83148 ± 288	11023 ± 35	72125 ± 291	236284 ± 4955
-	2	0.15	0.2	1259751 ± 1122	998253 ± 330	41682 ± 204	2230 ± 16	39452 ± 205	5.945 ± 0.018	1.000 ± 0.000	3325.20	120	5.26	41682 ± 204	2230 ± 16	39452 ± 205	234335 ± 7397
-	3	0.2	0.25	1259751 ± 1122	998253 ± 330	23298 ± 153	700 ± 9	22598 ± 153	10.543 ± 0.031	1.000 ± 0.000	2783.77	120	4.82	23298 ± 153	700 ± 9	22598 ± 153	238243 ± 8483
-	4	0.25	0.3	1259751 ± 1122	998253 ± 330	13388 ± 116	340 ± 6	13048 ± 116	18.296 ± 0.055	1.000 ± 0.000	3789.75	120	5.62	13388 ± 116	340 ± 6	13048 ± 116	238726 ± 12569
-	5	0.3	0.35	1259751 ± 1122	998253 ± 330	7932 ± 89	194 ± 5	7738 ± 89	31.318 ± 0.093	1.000 ± 0.000	3077.29	120	5.06	7932 ± 89	194 ± 5	7738 ± 89	242328 ± 14612
-	6	0.35	0.4	1259751 ± 1122	998253 ± 330	4688 ± 68	128 ± 4	4560 ± 69	53.445 ± 0.159	1.000 ± 0.000	3731.93	120	5.58	4688 ± 68	128 ± 4	4560 ± 69	243725 ± 20831
-	7	0.4	0.45	1259751 ± 1122	998253 ± 330	2998 ± 55	88 ± 3	2910 ± 55	87.588 ± 0.261	1.000 ± 0.000	6642.97	120	7.44	2998 ± 55	88 ± 3	2910 ± 55	254023 ± 36174

Table 11: Numbers for the electron m_T^W fits on 13 TeV. Numbers in FR are not affected by $topoetcone20/p_T$ Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

trj Cut	Slice No.	Isomin	Isomax	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	T	α	χ^2	NDoF	$\sqrt{\chi^2}/NDoF$	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{FR}	N_{MJ}^{FR}
-	0	0.05	0.1	1259751 ± 1122	998253 ± 330	222831 ± 472	85732 ± 97	137099 ± 482	1.552 ± 0.010	1.000 ± 0.000	205.30	16	3.58	222831 ± 472	85732 ± 97	137099 ± 482	212777 ± 5617
-	1	0.1	0.15	1259751 ± 1122	998253 ± 330	83148 ± 288	11023 ± 35	72125 ± 291	2.706 ± 0.018	1.000 ± 0.000	378.99	16	4.87	83148 ± 288	11023 ± 35	72125 ± 291	195197 ± 7294
-	2	0.15	0.2	1259751 ± 1122	998253 ± 330	41682 ± 204	2230 ± 16	39452 ± 205	4.728 ± 0.031	1.000 ± 0.000	786.39	16	6.83	41682 ± 204	2230 ± 16	39452 ± 205	186521 ± 10723
-	3	0.2	0.25	1259751 ± 1122	998253 ± 330	23298 ± 153	700 ± 9	22598 ± 153	8.406 ± 0.056	1.000 ± 0.000	615.89	16	6.20	23298 ± 153	700 ± 9	22598 ± 153	191777 ± 11242
-	4	0.25	0.3	1259751 ± 1122	998253 ± 330	13388 ± 116	340 ± 6	13048 ± 116	14.644 ± 0.098	1.000 ± 0.000	1167.58	16	8.54	13388 ± 116	340 ± 6	13048 ± 116	191069 ± 18159
-	5	0.3	0.35	1259751 ± 1122	998253 ± 330	7932 ± 89	194 ± 5	7738 ± 89	25.933 ± 0.172	1.000 ± 0.000	789.65	16	7.03	7932 ± 89	194 ± 5	7738 ± 89	200655 ± 18745
-	6	0.35	0.4	1259751 ± 1122	998253 ± 330	4688 ± 68	128 ± 4	4560 ± 69	45.901 ± 0.299	1.000 ± 0.000	619.39	16	6.22	4688 ± 68	128 ± 4	4560 ± 69	207497 ± 21185
-	7	0.4	0.45	1259751 ± 1122	998253 ± 330	2998 ± 55	88 ± 3	2910 ± 55	75.584 ± 0.503	1.000 ± 0.000	1155.66	16	8.50	2998 ± 55	88 ± 3	2910 ± 55	219986 ± 37356

Table 12: Numbers for the electron $|\Delta\phi(lep_0 - MET)|$ fits on 13 TeV. Numbers in FR are not affected by $topoetcone20/p_T$ Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

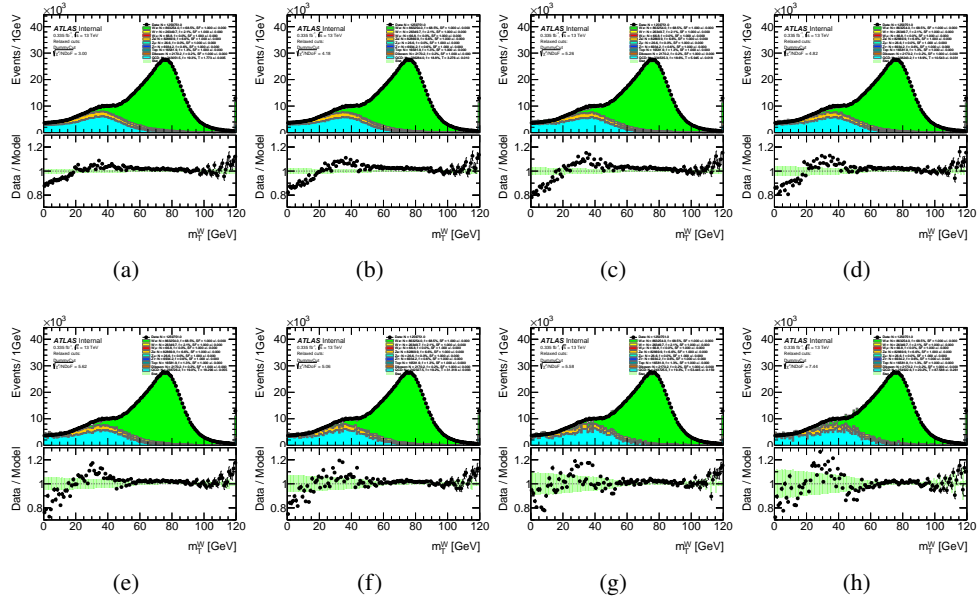


Figure 126: Multi-jet background template fits performed in distributions of m_T^W of the electron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < \text{topocone20}/p_T < 0.1$ to $0.4 < \text{topocone20}/p_T < 0.45$.

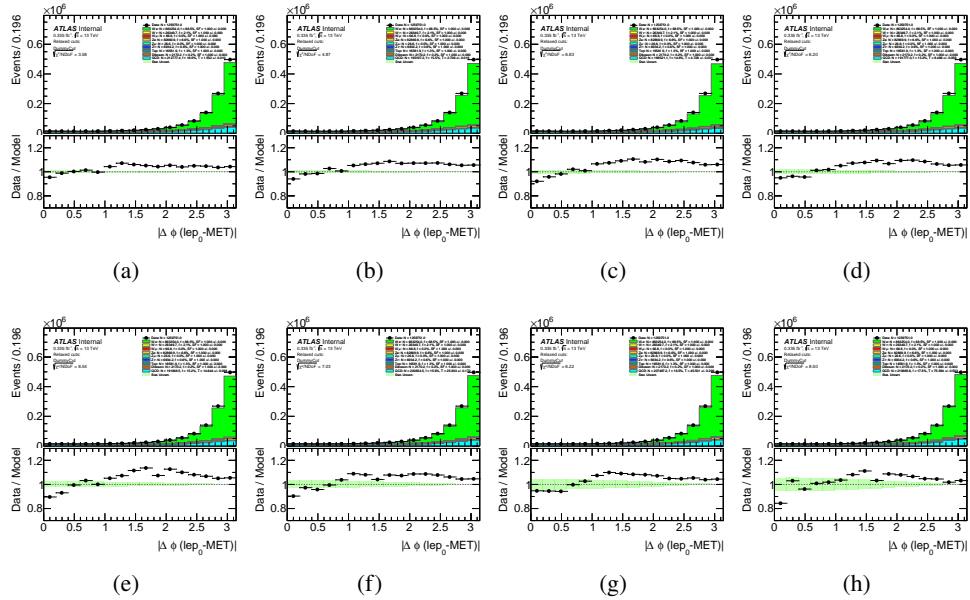


Figure 127: Multi-jet background template fits performed in distributions of $|\Delta\phi(\text{lep}_0 - \text{MET})|$ of the electron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < \text{topocone20}/p_T < 0.1$ to $0.4 < \text{topocone20}/p_T < 0.45$.

E.3.2 Positron channel multi-jet background template fits

u_T Cut	m_T^W fits b	m_T^W fits k	E_T^{miss} fits b	E_T^{miss} fits k	p_T fits b	p_T fits k	$ \Delta\phi(\ell, E_T^{miss}) $ fits b	$ \Delta\phi(\ell, E_T^{miss}) $ fits k
–	244293 ± 4561	-9016 ± 29922	252361 ± 3305	50869 ± 24965	228703 ± 3598	-187574 ± 17195	208405 ± 6582	-41852 ± 38595

Table 13: k and b are parameters for the electron extrapolations on 13 TeV. The uncertainty of these two parameters are provided by the extrapolation, from the statistical uncertainty of the parameter T and the number of events in CR of each slices and multiplied by $\sqrt{\chi^2/NDof}$.

u_T Cut	Slice No.	Iso_{min}	Iso_{max}	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR}	N_{EW}^{CR}	N_{MJ}	T	α	χ^2	NDoF	$\sqrt{\chi^2/NDof}$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}	N_{SR}^{MJ}
–	0	0.05	0.1	1531246 ± 1237	1258999 ± 370	252043 ± 302	112314 ± 111	139729 ± 514	1.611 ± 0.010	1.000 ± 0.000	438.18	90	2.26	252043 ± 302	112314 ± 111	139729 ± 514	252070 ± 3638
–	1	0.1	0.15	1531246 ± 1237	1258999 ± 370	86953 ± 295	14207 ± 40	72746 ± 298	2.755 ± 0.017	1.000 ± 0.000	438.53	90	2.26	86953 ± 295	14207 ± 40	72746 ± 298	200418 ± 3342
–	2	0.15	0.2	1531246 ± 1237	1258999 ± 370	43013 ± 207	2802 ± 18	40211 ± 208	4.711 ± 0.029	1.000 ± 0.000	546.78	90	2.46	43013 ± 207	2802 ± 18	40211 ± 208	189418 ± 3758
–	3	0.2	0.25	1531246 ± 1237	1258999 ± 370	23488 ± 153	859 ± 10	22629 ± 154	8.088 ± 0.050	1.000 ± 0.000	631.92	90	2.65	23488 ± 153	859 ± 10	22629 ± 154	183019 ± 4451
–	4	0.25	0.3	1531246 ± 1237	1258999 ± 370	13584 ± 117	400 ± 6	13184 ± 117	13.245 ± 0.082	1.000 ± 0.000	638.51	90	2.66	13584 ± 117	400 ± 6	13184 ± 117	174614 ± 5024
–	5	0.3	0.35	1531246 ± 1237	1258999 ± 370	7999 ± 89	240 ± 5	7759 ± 90	22.251 ± 0.138	1.000 ± 0.000	638.04	90	2.70	7999 ± 89	240 ± 5	7759 ± 90	172854 ± 6113
–	6	0.35	0.4	1531246 ± 1237	1258999 ± 370	4895 ± 70	163 ± 4	4732 ± 70	34.723 ± 0.215	1.000 ± 0.000	741.46	90	2.87	4895 ± 70	163 ± 4	4732 ± 70	164317 ± 7574
–	7	0.4	0.45	1531246 ± 1237	1258999 ± 370	3073 ± 55	108 ± 4	2965 ± 56	54.959 ± 0.342	1.000 ± 0.000	857.41	90	3.09	3073 ± 55	108 ± 4	2965 ± 56	162955 ± 9930

Table 14: Numbers for the positron p_T fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

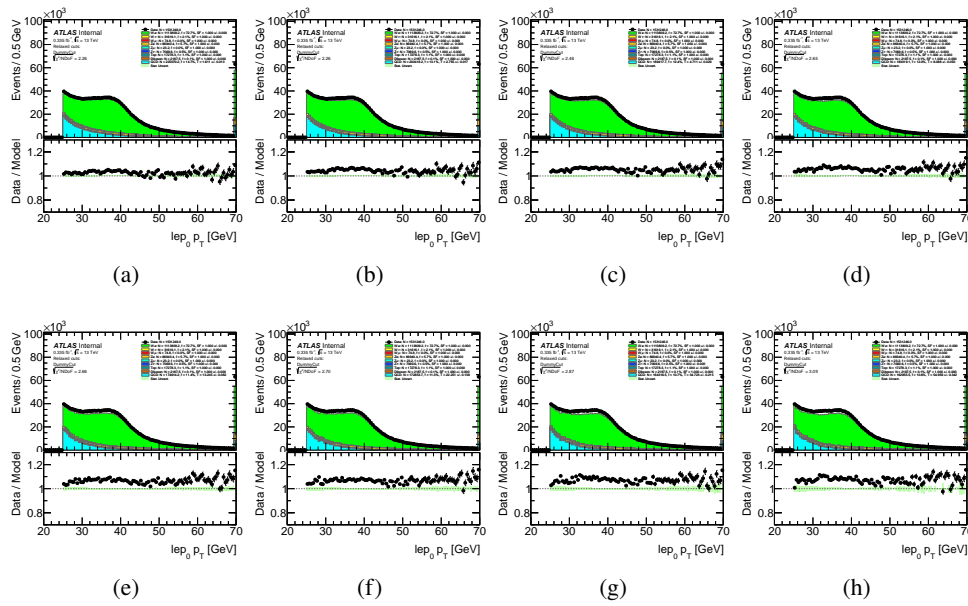


Figure 128: Multi-jet background template fits performed in distributions of p_T^l of the positron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < topoetcone20/p_T < 0.1$ to $0.4 < topoetcone20/p_T < 0.45$.

ur_Cut	Slice No.	Iso _{min}	Iso _{max}	N _{Data}	N _{EW}	N _{Data}	N _{EW}	N _{MJ}	T	α	χ ²	NDoF	√χ ² /NDoF	N _{Data}	N _{EW}	N _{MJ}	N _{MJ}
-	0	0.05	0.1	1531246 ± 1237	1258999 ± 370	252043 ± 502	112314 ± 111	139729 ± 514	1.847 ± 0.006	1.000 ± 0.000	448.96	160	1.68	252043 ± 502	112314 ± 111	139729 ± 514	258116 ± 2050
-	1	0.1	0.15	1531246 ± 1237	1258999 ± 370	86953 ± 295	14207 ± 40	72746 ± 298	5.506 ± 0.010	1.000 ± 0.000	638.82	160	2.00	86953 ± 295	14207 ± 40	72746 ± 298	255021 ± 2378
-	2	0.15	0.2	1531246 ± 1237	1258999 ± 370	43013 ± 207	2802 ± 18	40211 ± 208	6.473 ± 0.019	1.000 ± 0.000	1018.59	160	2.52	43013 ± 207	2802 ± 18	40211 ± 208	260308 ± 3920
-	3	0.2	0.25	1531246 ± 1237	1258999 ± 370	23488 ± 153	859 ± 10	22629 ± 154	11.719 ± 0.035	1.000 ± 0.000	1448.05	160	3.01	23488 ± 153	859 ± 10	22629 ± 154	265184 ± 5909
-	4	0.25	0.3	1531246 ± 1237	1258999 ± 370	13584 ± 117	400 ± 6	13184 ± 117	20.629 ± 0.161	1.000 ± 0.000	2264.90	160	3.76	13584 ± 117	400 ± 6	13184 ± 117	271975 ± 9553
-	5	0.3	0.35	1531246 ± 1237	1258999 ± 370	7999 ± 89	240 ± 5	7759 ± 90	35.616 ± 0.106	1.000 ± 0.000	3439.50	160	4.64	7999 ± 89	240 ± 5	7759 ± 90	276363 ± 15272
-	6	0.35	0.4	1531246 ± 1237	1258999 ± 370	4895 ± 70	163 ± 4	4732 ± 70	59.246 ± 0.177	1.000 ± 0.000	5828.31	160	6.04	4895 ± 70	163 ± 4	4732 ± 70	280365 ± 25567
-	7	0.4	0.45	1531246 ± 1237	1258999 ± 370	3073 ± 55	108 ± 4	2965 ± 56	97.624 ± 0.250	1.000 ± 0.000	7773.44	160	6.97	3073 ± 55	108 ± 4	2965 ± 56	289460 ± 38274

Table 15: Numbers for the positron E_T^{miss} fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDoF}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

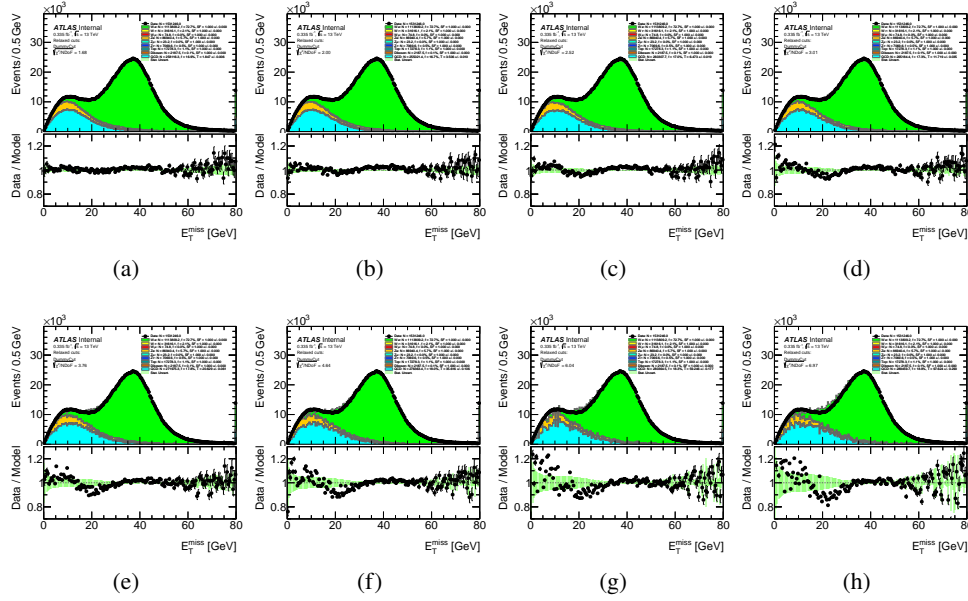


Figure 129: Multi-jet background template fits performed in distributions of E_T^{miss} of the positron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < \text{topocone20}/p_T < 0.1$ to $0.4 < \text{topocone20}/p_T < 0.45$.

ur_Cut	Slice No.	Iso _{min}	Iso _{max}	N _{Data}	N _{EW}	N _{Data}	N _{EW}	N _{MJ}	T	α	χ ²	NDoF	√χ ² /NDoF	N _{Data}	N _{EW}	N _{MJ}	N _{MJ}
-	0	0.05	0.1	1531246 ± 1237	1258999 ± 370	252043 ± 502	112314 ± 111	139729 ± 514	1.767 ± 0.005	1.000 ± 0.000	912.86	120	2.76	252043 ± 502	112314 ± 111	139729 ± 514	246931 ± 3277
-	1	0.1	0.15	1531246 ± 1237	1258999 ± 370	86953 ± 295	14207 ± 40	72746 ± 298	3.265 ± 0.010	1.000 ± 0.000	1766.95	120	3.84	86953 ± 295	14207 ± 40	72746 ± 298	237494 ± 4683
-	2	0.15	0.2	1531246 ± 1237	1258999 ± 370	43013 ± 207	2802 ± 18	40211 ± 208	5.909 ± 0.018	1.000 ± 0.000	2302.44	120	4.38	43013 ± 207	2802 ± 18	40211 ± 208	237609 ± 6287
-	3	0.2	0.25	1531246 ± 1237	1258999 ± 370	23488 ± 153	859 ± 10	22629 ± 154	10.618 ± 0.033	1.000 ± 0.000	2469.89	120	4.54	23488 ± 153	859 ± 10	22629 ± 154	240281 ± 8138
-	4	0.25	0.3	1531246 ± 1237	1258999 ± 370	13584 ± 117	400 ± 6	13184 ± 117	18.844 ± 0.057	1.000 ± 0.000	2776.26	120	4.81	13584 ± 117	400 ± 6	13184 ± 117	243694 ± 10997
-	5	0.3	0.35	1531246 ± 1237	1258999 ± 370	7999 ± 89	240 ± 5	7759 ± 90	32.119 ± 0.099	1.000 ± 0.000	2713.78	120	4.76	7999 ± 89	240 ± 5	7759 ± 90	239226 ± 14165
-	6	0.35	0.4	1531246 ± 1237	1258999 ± 370	4895 ± 70	163 ± 4	4732 ± 70	53.157 ± 0.164	1.000 ± 0.000	2698.31	120	4.74	4895 ± 70	163 ± 4	4732 ± 70	251550 ± 18049
-	7	0.4	0.45	1531246 ± 1237	1258999 ± 370	3073 ± 55	108 ± 4	2965 ± 56	87.492 ± 0.271	1.000 ± 0.000	4295.06	120	5.98	3073 ± 55	108 ± 4	2965 ± 56	259417 ± 29473

Table 16: Numbers for the positron m_T^W fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDoF}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

ur_Cut	Slice No.	Iso _{min}	Iso _{max}	N _{Data}	N _{EW}	N _{Data}	N _{EW}	N _{MJ}	T	α	χ ²	NDoF	√χ ² /NDoF	N _{Data}	N _{EW}	N _{MJ}	N _{MJ}
-	0	0.05	0.1	1531246 ± 1237	1258999 ± 370	252043 ± 502	112314 ± 111	139729 ± 514	1.526 ± 0.010	1.000 ± 0.000	154.72	16	3.11	252043 ± 502	112314 ± 111	139729 ± 514	213369 ± 5168
-	1	0.1	0.15	1531246 ± 1237	1258999 ± 370	86953 ± 295	14207 ± 40	72746 ± 298	2.665 ± 0.018	1.000 ± 0.000	291.91	16	5.34	86953 ± 295	14207 ± 40	72746 ± 298	193837 ± 6656
-	2	0.15	0.2	1531246 ± 1237	1258999 ± 370	43013 ± 207	2802 ± 18	40211 ± 208	4.743 ± 0.033	1.000 ± 0.000	455.81	16	4.95	43013 ± 207	2802 ± 18	40211 ± 208	190724 ± 8847
-	3	0.2	0.25	1531246 ± 1237	1258999 ± 370	23488 ± 153	859 ± 10	22629 ± 154	8.507 ± 0.059	1.000 ± 0.000	392.61	16	6.02	23488 ± 153	859 ± 10	22629 ± 154	192507 ± 9265
-	4	0.25	0.3	1531246 ± 1237	1258999 ± 370	13584 ± 117	400 ± 6	13184 ± 117	14.821 ± 0.104	1.000 ± 0.000	572.38	16	5.98	13584 ± 117	400 ± 6	13184 ± 117	195402 ± 13190
-	5	0.3	0.35	1531246 ± 1237	1258999 ± 370	7999 ± 89	240 ± 5	7759 ± 90	27.089 ± 0.189	1.000 ± 0.000	580.01	16	6.02	7999 ± 89	240 ± 5	7759 ± 90	210194 ± 17074
-	6	0.35	0.4	1531246 ± 1237	1258999 ± 370	4895 ± 70	163 ± 4	4732 ± 70	45.239 ± 0.315	1.000 ± 0.000	547.68	16	5.85	4895 ± 70	163 ± 4	4732 ± 70	214078 ± 20564
-	7	0.4	0.45	1531246 ± 1237	1258999 ± 370	3073 ± 55	108 ± 4	2965 ± 56	74.805 ± 0.527	1.000 ± 0.000	1058.88	16	8.14	3073 ± 55	108 ± 4	2965 ± 56	221900 ± 36119

Table 17: Numbers for the positron $|\Delta\phi(\text{lep}_0 - MET)|$ fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDoF}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

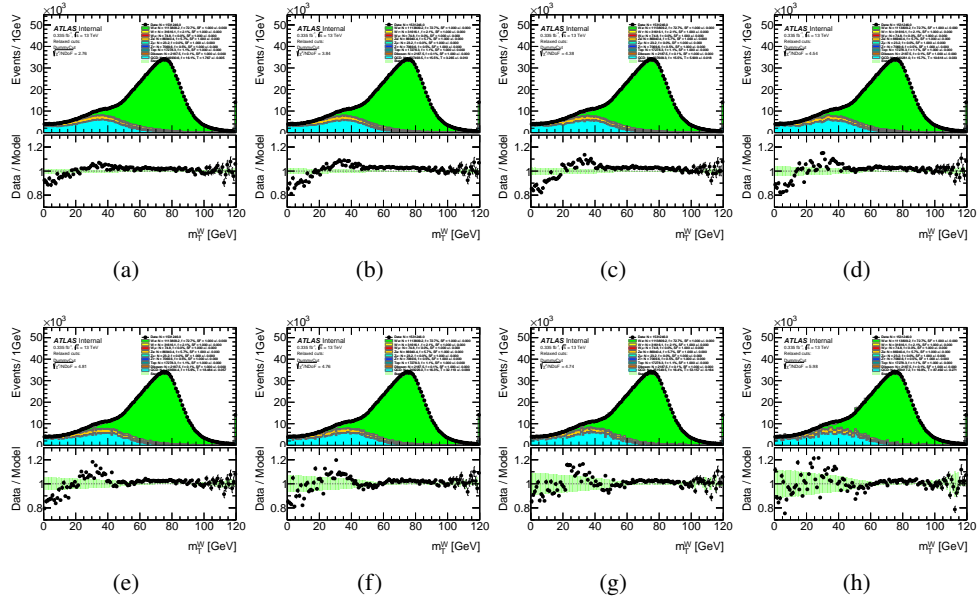


Figure 130: Multi-jet background template fits performed in distributions of m_T^W of the positron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < \text{topocone20}/p_T < 0.1$ to $0.4 < \text{topocone20}/p_T < 0.45$.

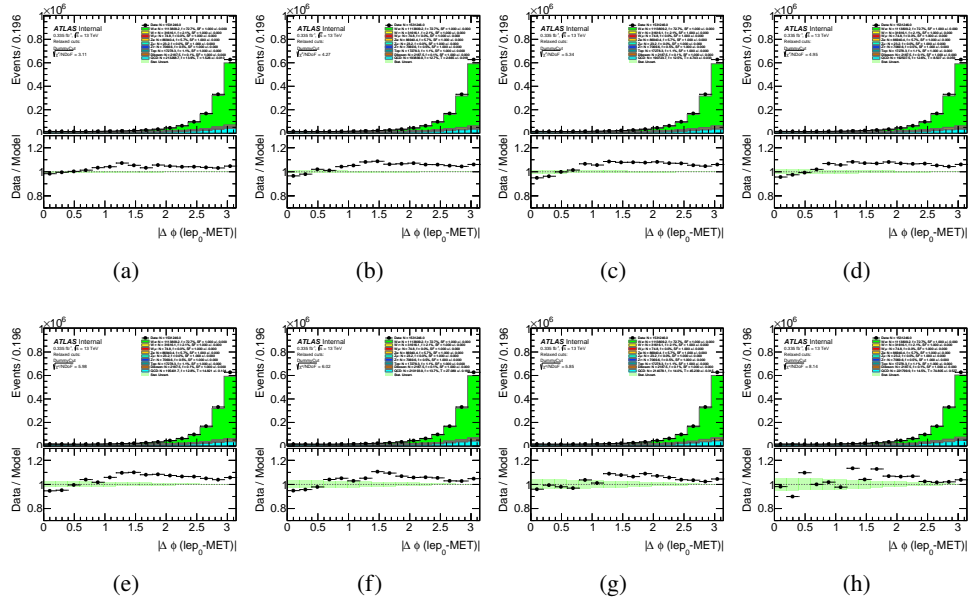


Figure 131: Multi-jet background template fits performed in distributions of $|\Delta\phi(\text{lep}_0 - \text{MET})|$ of the positron channel, columns from (a) to (h) are the fits with the isolation slices from $0.05 < \text{topocone20}/p_T < 0.1$ to $0.4 < \text{topocone20}/p_T < 0.45$.

E.3.3 Muons channel multi-jet background template fits

u_T Cut	m_T^W fits b	m_T^W fits k	E_T^{miss} fits b	E_T^{miss} fits k	p_T fits b	p_T fits k	$ \Delta\phi(\ell, E_T^{miss}) $ fits b	$ \Delta\phi(\ell, E_T^{miss}) $ fits k
–	129226 ± 4417	61172 ± 21139	127653 ± 4021	81262 ± 19825	112057 ± 3462	67441 ± 13814	132988 ± 7780	42104 ± 32182

Table 18: k and b are parameters for the electron extrapolations on 13 TeV. The uncertainty of these two parameters are provided by the extrapolation, from the statistical uncertainty of the parameter T and the number of events in CR of each slices and multiplied by $\sqrt{\chi^2/NDof}$.

u_T Cut	Slice No.	Iso_{min}	Iso_{max}	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR}	N_{EW}^{CR}	N_{MJ}^{CR}	T	α	χ^2	NDof	$\sqrt{\chi^2/NDof}$	N_{Data}^{FR}	N_{EW}^{FR}	N_{MJ}^{CR}	N_{MJ}^{SR}
–	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	23759 ± 160	5.109 ± 0.042	1.000 ± 0.000	403.40	90	2.12	25514 ± 160	1755 ± 14	23759 ± 160	121382 ± 2717
–	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	19209 ± 141	6.380 ± 0.052	1.000 ± 0.000	407.30	90	2.13	19741 ± 141	532 ± 7	19209 ± 141	122561 ± 2858
–	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	15194 ± 124	8.374 ± 0.068	1.000 ± 0.000	451.98	90	2.24	15432 ± 124	238 ± 5	15194 ± 124	127230 ± 3590
–	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	11665 ± 109	11.232 ± 0.092	1.000 ± 0.000	621.43	90	2.63	11810 ± 109	145 ± 5	11665 ± 109	131022 ± 4265
–	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9113 ± 96	14.755 ± 0.120	1.000 ± 0.000	695.62	90	2.78	9202 ± 96	89 ± 3	9113 ± 96	134456 ± 4981
–	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	7331 ± 86	18.576 ± 0.152	1.000 ± 0.000	683.06	90	2.75	7395 ± 86	64 ± 2	7331 ± 86	136187 ± 5364
–	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5859 ± 77	24.235 ± 0.198	1.000 ± 0.000	637.44	90	2.66	5904 ± 77	45 ± 2	5859 ± 77	141990 ± 5837
–	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4823 ± 70	29.861 ± 0.244	1.000 ± 0.000	897.62	90	3.16	4854 ± 70	31 ± 2	4823 ± 70	144019 ± 7550

Table 19: Numbers for the muon p_T fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{MJ}^{SR} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

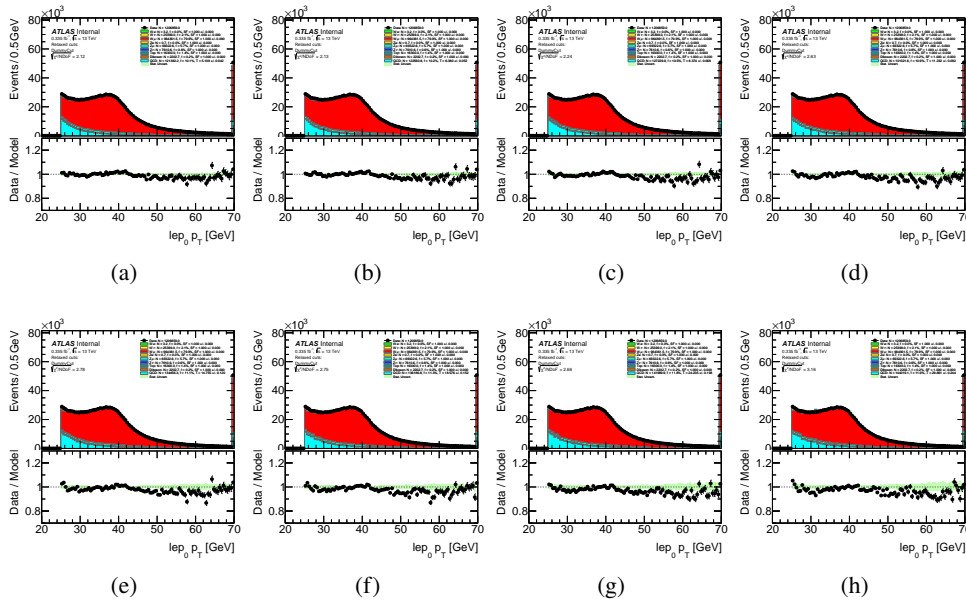


Figure 132: Multi-jet background template fits performed in distributions of p_T^ℓ of the muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

tr_j Cut	Slice No.	$Isom_{min}$	$Isom_{max}$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDof$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	23759 ± 160	5.812 ± 0.025	1.000 ± 0.000	728.03	160	2.13	25514 ± 160	1755 ± 14	23759 ± 160	138087 ± 2355
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	19209 ± 141	7.355 ± 0.031	1.000 ± 0.000	1116.55	160	2.64	19741 ± 141	532 ± 7	19209 ± 141	141284 ± 3166
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	15194 ± 124	9.571 ± 0.041	1.000 ± 0.000	1621.56	160	3.18	15432 ± 124	238 ± 5	15194 ± 124	145418 ± 4272
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	11665 ± 109	12.914 ± 0.055	1.000 ± 0.000	2158.98	160	3.67	11810 ± 109	145 ± 5	11665 ± 109	150635 ± 5674
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9113 ± 96	17.023 ± 0.073	1.000 ± 0.000	3193.11	160	4.47	9202 ± 96	89 ± 3	9113 ± 96	155119 ± 7874
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	7331 ± 86	21.620 ± 0.092	1.000 ± 0.000	3785.65	160	4.86	7395 ± 86	64 ± 2	7331 ± 86	158508 ± 9626
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5859 ± 77	27.904 ± 0.119	1.000 ± 0.000	4387.16	160	5.24	5904 ± 77	45 ± 2	5859 ± 77	163486 ± 11809
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4823 ± 70	33.961 ± 0.145	1.000 ± 0.000	5154.56	160	5.68	4854 ± 70	31 ± 2	4823 ± 70	163794 ± 14009

Table 20: Numbers for the muon E_T^{miss} fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDof$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

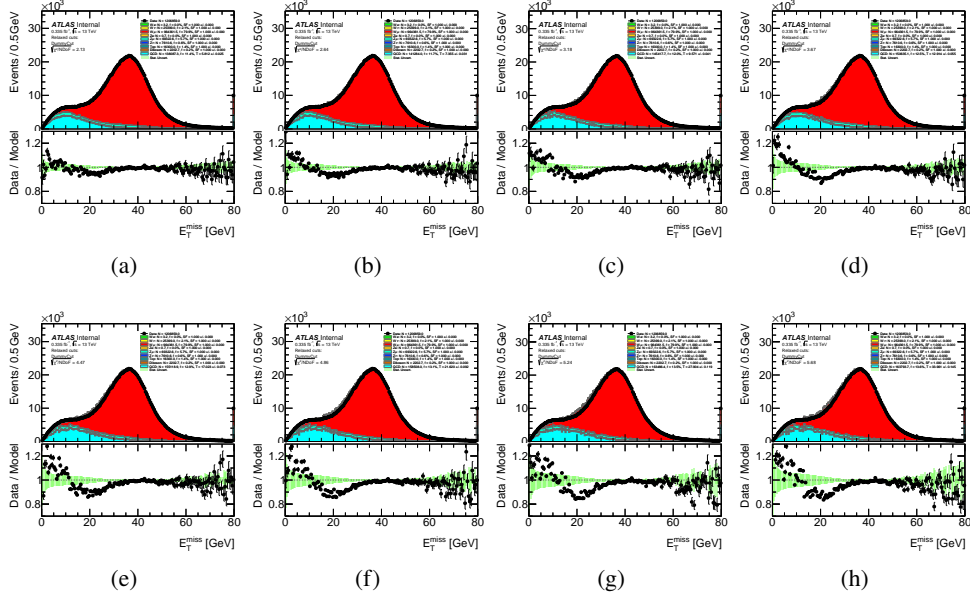


Figure 133: Multi-jet background template fits performed in distributions of E_T^{miss} of the muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/pt < 0.1$ to $0.4 < ptvarcone20/pt < 0.5$.

tr_j Cut	Slice No.	$Isom_{min}$	$Isom_{max}$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDof$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	23759 ± 160	5.747 ± 0.023	1.000 ± 0.000	752.74	120	2.50	25514 ± 160	1755 ± 14	23759 ± 160	136539 ± 2680
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	19209 ± 141	7.294 ± 0.029	1.000 ± 0.000	1208.27	120	3.17	19741 ± 141	532 ± 7	19209 ± 141	140107 ± 3705
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	15194 ± 124	9.501 ± 0.038	1.000 ± 0.000	1800.53	120	3.87	15432 ± 124	238 ± 5	15194 ± 124	144348 ± 5085
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	11665 ± 109	12.564 ± 0.050	1.000 ± 0.000	2226.26	120	4.31	11810 ± 109	145 ± 5	11665 ± 109	146555 ± 6000
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9113 ± 96	16.256 ± 0.065	1.000 ± 0.000	2616.40	120	4.67	9202 ± 96	89 ± 3	9113 ± 96	148133 ± 7788
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	7331 ± 86	20.686 ± 0.082	1.000 ± 0.000	3288.24	120	5.23	7395 ± 86	64 ± 2	7331 ± 86	151657 ± 9838
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5859 ± 77	26.359 ± 0.105	1.000 ± 0.000	4232.73	120	5.94	5904 ± 77	45 ± 2	5859 ± 77	154431 ± 12580
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4823 ± 70	32.615 ± 0.130	1.000 ± 0.000	4307.65	120	5.99	4854 ± 70	31 ± 2	4823 ± 70	157304 ± 14129

Table 21: Numbers for the muon m_T^W fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDof$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

tr_j Cut	Slice No.	$Isom_{min}$	$Isom_{max}$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDof$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	23759 ± 160	5.709 ± 0.037	1.000 ± 0.000	325.93	16	4.51	25514 ± 160	1755 ± 14	23759 ± 160	135637 ± 5741
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	19209 ± 141	7.397 ± 0.048	1.000 ± 0.000	343.69	16	4.63	19741 ± 141	532 ± 7	19209 ± 141	142088 ± 6457
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	15194 ± 124	9.732 ± 0.064	1.000 ± 0.000	466.29	16	5.40	15432 ± 124	238 ± 5	15194 ± 124	147866 ± 8355
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	11665 ± 109	12.451 ± 0.082	1.000 ± 0.000	551.40	16	5.87	11810 ± 109	145 ± 5	11665 ± 109	145240 ± 9750
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9113 ± 96	15.713 ± 0.085	1.000 ± 0.000	685.41	16	6.55	9202 ± 96	89 ± 3	9113 ± 96	143184 ± 11681
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	7331 ± 86	20.025 ± 0.133	1.000 ± 0.000	671.89	16	6.48	7395 ± 86	64 ± 2	7331 ± 86	146813 ± 12836
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5859 ± 77	25.818 ± 0.169	1.000 ± 0.000	724.86	16	6.73	5904 ± 77	45 ± 2	5859 ± 77	148920 ± 14752
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4823 ± 70	31.702 ± 0.213	1.000 ± 0.000	916.32	16	7.57	4854 ± 70	31 ± 2	4823 ± 70	152903 ± 18434

Table 22: Numbers for the muon $|\Delta\phi(lep_0 - MET)|$ fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDof$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

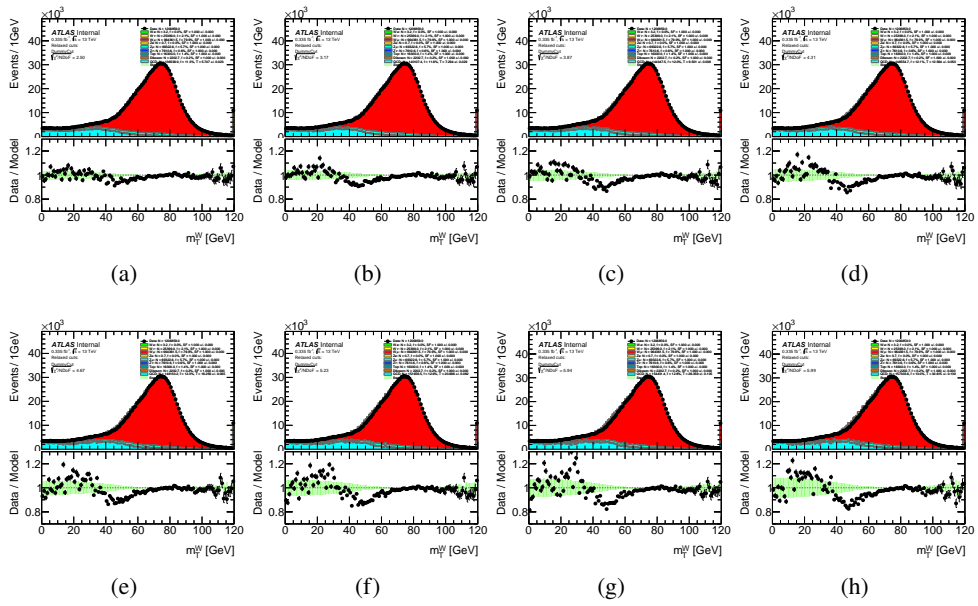


Figure 134: Multi-jet background template fits performed in distributions of m_T^W of the muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

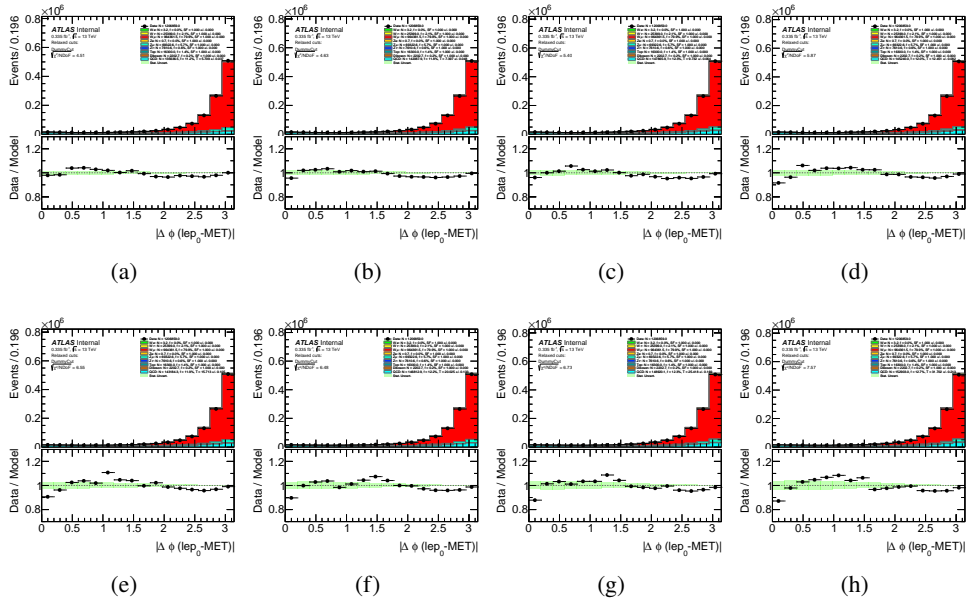


Figure 135: Multi-jet background template fits performed in distributions of $|\Delta\phi(lep_0 - MET)|$ of the muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

E.3.4 Antimuon channel multi-jet background template fits

u_T Cut	m_T^W fits b	m_T^W fits k	E_T^{miss} fits b	E_T^{miss} fits k	p_T fits b	p_T fits k	$ \Delta\phi(\ell, E_T^{miss}) $ fits b	$ \Delta\phi(\ell, E_T^{miss}) $ fits k
–	134884 ± 4658	59025 ± 21545	131867 ± 4410	83510 ± 20832	119174 ± 4895	61803 ± 18093	140323 ± 11942	24939 ± 45036

Table 23: k and b are parameters for the electron extrapolations on 13 TeV. The uncertainty of these two parameters are provided by the extrapolation, from the statistical uncertainty of the parameter T and the number of events in CR of each slices and multiplied by $\sqrt{\chi^2/NDof}$.

u_T Cut	Slice No.	$Is_{o,min}$	$Is_{o,max}$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR}	N_{EW}^{CR}	N_{MJ}^{CR}	T	α	χ^2	NDof	$\sqrt{\chi^2/NDof}$	N_{Data}^{CR}	N_{EW}^{CR}	N_{MJ}^{CR}	N_{MJ}^{SR}
–	0	0.1	0.15	1533100 ± 1238	1402422 ± 387	27011 ± 164	2334 ± 15	24671 ± 165	5.217 ± 0.050	1.000 ± 0.000	694.21	90	2.78	27011 ± 164	2334 ± 15	24671 ± 165	128739 ± 4194
–	1	0.15	0.2	1533100 ± 1238	1402422 ± 387	20028 ± 142	694 ± 9	19334 ± 142	6.661 ± 0.064	1.000 ± 0.000	649.00	90	2.69	20028 ± 142	694 ± 9	19334 ± 142	128789 ± 4187
–	2	0.2	0.25	1533100 ± 1238	1402422 ± 387	15718 ± 125	305 ± 7	15413 ± 126	8.540 ± 0.082	1.000 ± 0.000	721.78	90	2.83	15718 ± 125	305 ± 7	15413 ± 126	131631 ± 4710
–	3	0.25	0.3	1533100 ± 1238	1402422 ± 387	12307 ± 111	177 ± 4	12130 ± 111	11.136 ± 0.108	1.000 ± 0.000	897.57	90	3.16	12307 ± 111	177 ± 4	12130 ± 111	138075 ± 5690
–	4	0.3	0.35	1533100 ± 1238	1402422 ± 387	9514 ± 98	115 ± 4	9399 ± 98	14.595 ± 0.141	1.000 ± 0.000	798.60	90	2.98	9514 ± 98	115 ± 4	9399 ± 98	137182 ± 5801
–	5	0.35	0.4	1533100 ± 1238	1402422 ± 387	7668 ± 88	71 ± 3	7597 ± 88	19.490 ± 0.188	1.000 ± 0.000	742.33	90	2.87	7668 ± 88	71 ± 3	7597 ± 88	148066 ± 6393
–	6	0.4	0.45	1533100 ± 1238	1402422 ± 387	5912 ± 77	54 ± 3	5858 ± 77	25.266 ± 0.244	1.000 ± 0.000	865.38	90	3.10	5912 ± 77	54 ± 3	5858 ± 77	148004 ± 7486
–	7	0.45	0.5	1533100 ± 1238	1402422 ± 387	4811 ± 69	38 ± 2	4773 ± 69	30.199 ± 0.293	1.000 ± 0.000	1020.93	90	3.37	4811 ± 69	38 ± 2	4773 ± 69	144141 ± 8488

Table 24: Numbers for the antimuon p_T fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{MJ}^{SR} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

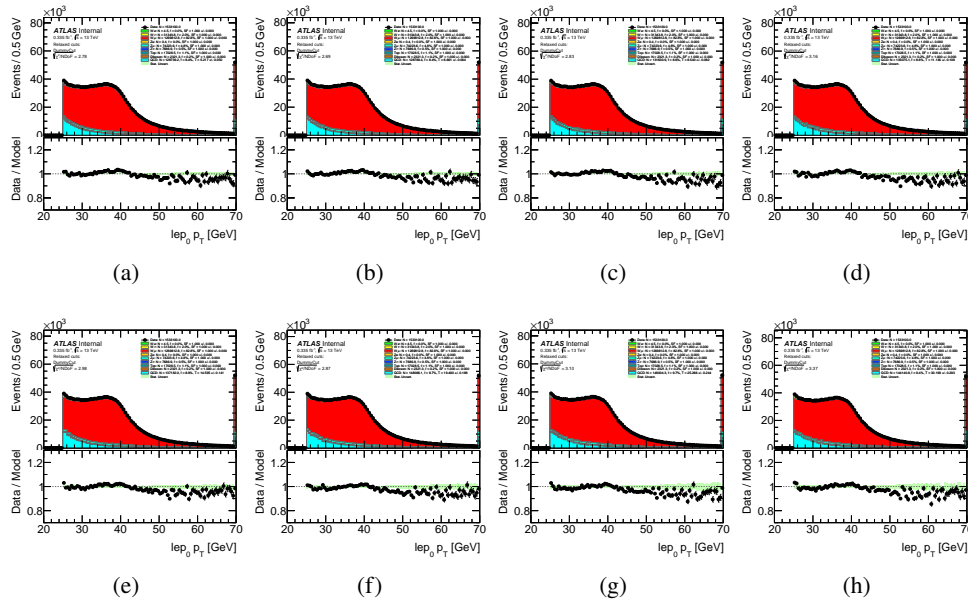


Figure 136: Multi-jet background template fits performed in distributions of p_T^ℓ of the anti-muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

ur Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2/NDof}$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1533100 ± 1238	1402422 ± 387	27011 ± 164	2334 ± 15	24677 ± 165	5.769 ± 0.025	1.000 ± 0.000	910.32	160	2.39	27011 ± 164	2334 ± 15	24677 ± 165	142356 ± 2719
-	1	0.15	0.2	1533100 ± 1238	1402422 ± 387	20028 ± 142	694 ± 9	19334 ± 142	7.567 ± 0.033	1.000 ± 0.000	1436.73	160	3.00	20028 ± 142	694 ± 9	19334 ± 142	146299 ± 3747
-	2	0.2	0.25	1533100 ± 1238	1402422 ± 387	15718 ± 125	305 ± 7	15413 ± 126	9.810 ± 0.043	1.000 ± 0.000	1829.77	160	3.38	15718 ± 125	305 ± 7	15413 ± 126	151214 ± 4732
-	3	0.25	0.3	1533100 ± 1238	1402422 ± 387	12307 ± 111	177 ± 4	12130 ± 111	12.728 ± 0.056	1.000 ± 0.000	2588.39	160	4.02	12307 ± 111	177 ± 4	12130 ± 111	154389 ± 6302
-	4	0.3	0.35	1533100 ± 1238	1402422 ± 387	9514 ± 98	115 ± 4	9399 ± 98	16.747 ± 0.074	1.000 ± 0.000	3037.60	160	4.36	9514 ± 98	115 ± 4	9399 ± 98	157411 ± 7733
-	5	0.35	0.4	1533100 ± 1238	1402422 ± 387	7668 ± 88	71 ± 3	7597 ± 88	21.690 ± 0.095	1.000 ± 0.000	3949.30	160	4.97	7668 ± 88	71 ± 3	7597 ± 88	164793 ± 10101
-	6	0.4	0.45	1533100 ± 1238	1402422 ± 387	5912 ± 77	54 ± 3	5858 ± 77	28.532 ± 0.125	1.000 ± 0.000	4297.04	160	5.18	5912 ± 77	54 ± 3	5858 ± 77	167138 ± 11997
-	7	0.45	0.5	1533100 ± 1238	1402422 ± 387	4811 ± 69	38 ± 2	4773 ± 69	36.118 ± 0.159	1.000 ± 0.000	4115.95	160	5.07	4811 ± 69	38 ± 2	4773 ± 69	172392 ± 13280

Table 25: Numbers for the antimuon E_T^{miss} fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

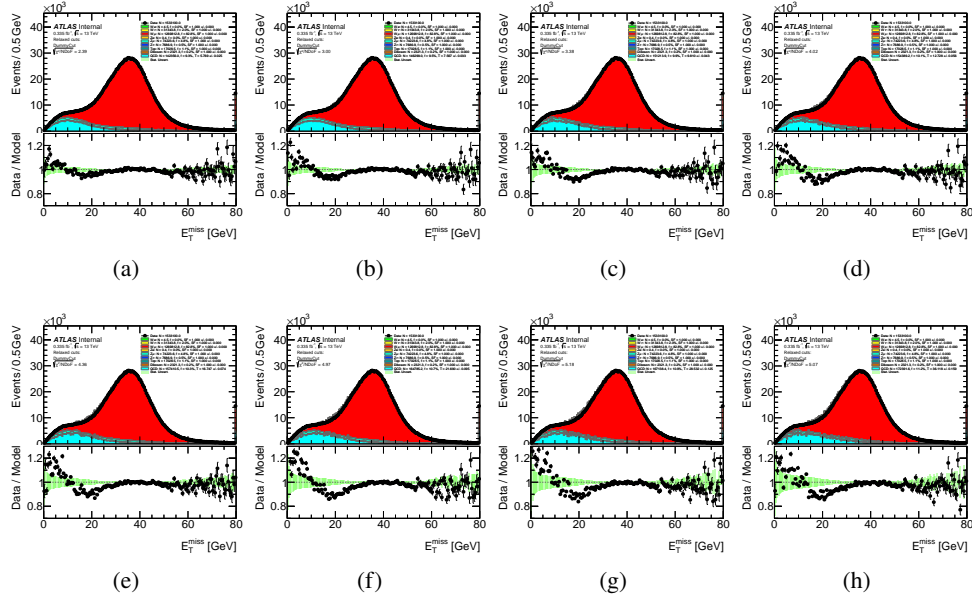


Figure 137: Multi-jet background template fits performed in distributions of E_T^{miss} of the anti-muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

ur Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2/NDof}$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1533100 ± 1238	1402422 ± 387	27011 ± 164	2334 ± 15	24677 ± 165	5.750 ± 0.024	1.000 ± 0.000	851.76	120	2.66	27011 ± 164	2334 ± 15	24677 ± 165	141889 ± 2967
-	1	0.15	0.2	1533100 ± 1238	1402422 ± 387	20028 ± 142	694 ± 9	19334 ± 142	7.533 ± 0.031	1.000 ± 0.000	1220.25	120	3.19	20028 ± 142	694 ± 9	19334 ± 142	145455 ± 3897
-	2	0.2	0.25	1533100 ± 1238	1402422 ± 387	15718 ± 125	305 ± 7	15413 ± 126	9.691 ± 0.040	1.000 ± 0.000	1729.32	120	3.80	15718 ± 125	305 ± 7	15413 ± 126	149370 ± 5170
-	3	0.25	0.3	1533100 ± 1238	1402422 ± 387	12307 ± 111	177 ± 4	12130 ± 111	12.476 ± 0.051	1.000 ± 0.000	2247.84	120	4.33	12307 ± 111	177 ± 4	12130 ± 111	151338 ± 6568
-	4	0.3	0.35	1533100 ± 1238	1402422 ± 387	9514 ± 98	115 ± 4	9399 ± 98	16.244 ± 0.067	1.000 ± 0.000	2571.70	120	4.63	9514 ± 98	115 ± 4	9399 ± 98	152684 ± 7895
-	5	0.35	0.4	1533100 ± 1238	1402422 ± 387	7668 ± 88	71 ± 3	7597 ± 88	20.921 ± 0.086	1.000 ± 0.000	3365.05	120	5.30	7668 ± 88	71 ± 3	7597 ± 88	158938 ± 10306
-	6	0.4	0.45	1533100 ± 1238	1402422 ± 387	5912 ± 77	54 ± 3	5858 ± 77	26.951 ± 0.112	1.000 ± 0.000	3627.03	120	5.50	5912 ± 77	54 ± 3	5858 ± 77	157876 ± 11953
-	7	0.45	0.5	1533100 ± 1238	1402422 ± 387	4811 ± 69	38 ± 2	4773 ± 69	33.948 ± 0.140	1.000 ± 0.000	3527.45	120	5.42	4811 ± 69	38 ± 2	4773 ± 69	162034 ± 13276

Table 26: Numbers for the antimuon m_T^W fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

ur Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2/NDof}$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1533100 ± 1238	1402422 ± 387	27011 ± 164	2334 ± 15	24677 ± 165	5.674 ± 0.038	1.000 ± 0.000	855.16	16	7.31	27011 ± 164	2334 ± 15	24677 ± 165	140028 ± 9732
-	1	0.15	0.2	1533100 ± 1238	1402422 ± 387	20028 ± 142	694 ± 9	19334 ± 142	7.514 ± 0.051	1.000 ± 0.000	873.16	16	7.39	20028 ± 142	694 ± 9	19334 ± 142	145285 ± 10718
-	2	0.2	0.25	1533100 ± 1238	1402422 ± 387	15718 ± 125	305 ± 7	15413 ± 126	9.822 ± 0.067	1.000 ± 0.000	884.27	16	7.43	15718 ± 125	305 ± 7	15413 ± 126	151393 ± 11931
-	3	0.25	0.3	1533100 ± 1238	1402422 ± 387	12307 ± 111	177 ± 4	12130 ± 111	12.410 ± 0.085	1.000 ± 0.000	1065.86	16	8.16	12307 ± 111	177 ± 4	12130 ± 111	150533 ± 14032
-	4	0.3	0.35	1533100 ± 1238	1402422 ± 387	9514 ± 98	115 ± 4	9399 ± 98	15.572 ± 0.107	1.000 ± 0.000	1071.94	16	8.19	9514 ± 98	115 ± 4	9399 ± 98	146368 ± 14933
-	5	0.35	0.4	1533100 ± 1238	1402422 ± 387	7668 ± 88	71 ± 3	7597 ± 88	19.811 ± 0.138	1.000 ± 0.000	1347.90	16	9.18	7668 ± 88	71 ± 3	7597 ± 88	146507 ± 18608
-	6	0.4	0.45	1533100 ± 1238	1402422 ± 387	5912 ± 77	54 ± 3	5858 ± 77	24.866 ± 0.173	1.000 ± 0.000	1128.35	16	8.40	5912 ± 77	54 ± 3	5858 ± 77	145663 ± 18190
-	7	0.45	0.5	1533100 ± 1238	1402422 ± 387	4811 ± 69	38 ± 2	4773 ± 69	31.708 ± 0.220	1.000 ± 0.000	1230.97	16	8.77	4811 ± 69	38 ± 2	4773 ± 69	151342 ± 21388

Table 27: Numbers for the antimuon $|\Delta\phi(lep_0 - MET)|$ fits on 13 TeV. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2/NDof}$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

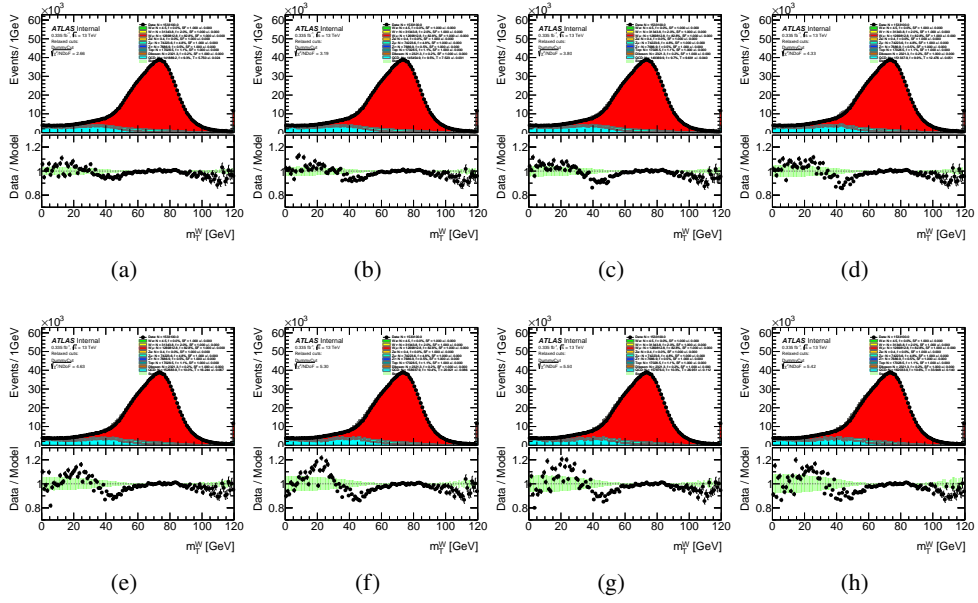


Figure 138: Multi-jet background template fits performed in distributions of m_T^W of the anti-muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

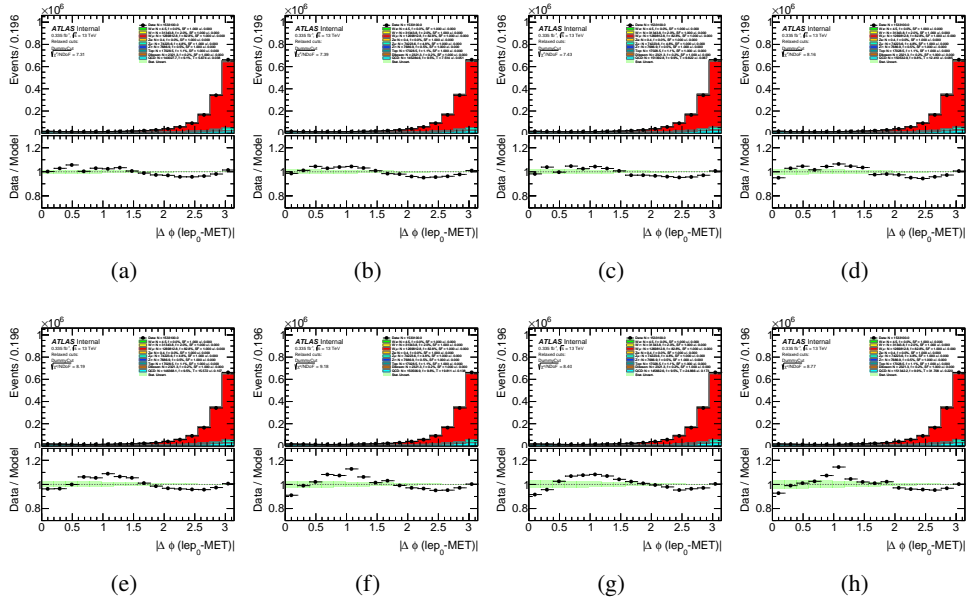


Figure 139: Multi-jet background template fits performed in distributions of $|\Delta\phi(lep_0 - MET)|$ of the anti-muon channel, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.1$ to $0.4 < ptvarcone20/p_T < 0.5$.

1258 E.4 Closure tests for data-driven MJ estimation algorithm

1259 To make sure anti-isolated slices extrapolation method works properly number of closure tests has been
 1260 performed. We used $b\bar{b} + c\bar{c}$ MC samples for $W^- \rightarrow \mu^- \bar{\nu}$ and $W^+ \rightarrow \mu^+ \nu$ channels.

1261 The properties of the $b\bar{b}$ and $c\bar{c}$ samples at $\sqrt{s} = 13$ TeV are given in Table 28. The heavy-flavour production
 1262 cross sections are higher than signal and other backgrounds, but their calculation carries significant
 1263 uncertainties from the dijet production cross section (calculated at leading order), and the $b/c \rightarrow \mu$ decay
 1264 fractions and fragmentation functions.

Process	Data set	Generator	$\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}}$ [nb]
$b\bar{b} \rightarrow \mu$	361250	Pythia8B+A14+NNPDF23LO	186
$c\bar{c} \rightarrow \mu$	361251	Pythia8B+A14+NNPDF23LO	58

Table 28: MC simulation of $b\bar{b}$ and $c\bar{c}$ decay to muon at $\sqrt{s} = 13$ TeV.

1265 This MC samples were used to derive MJ templates in the CR1 (CR2) anti-isolated slices. Linear
 1266 extrapolations from anti-isolated slices are shown on Fig. 140 and listed in the Tables. 29. It is clear to see
 1267 impact from limited MC samples statistics to the anti-isolated templates. Results are compatible with liner
 1268 extrapolations shown at Fig. 13 that are derived by data-driven method.

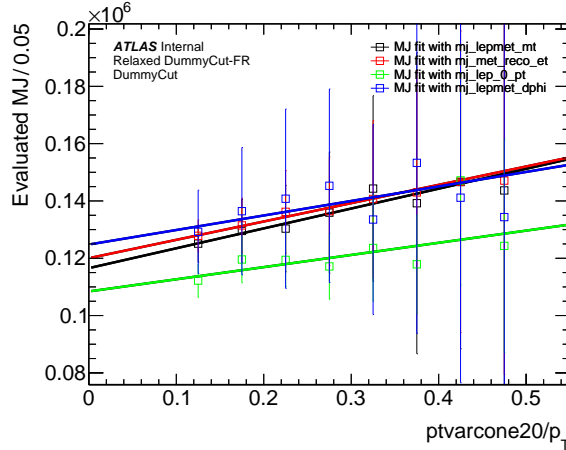


Figure 140: Linear extrapolations from anti-isolated slices for $W^- \rightarrow \mu^- \bar{\nu}$ channels using $b\bar{b} + c\bar{c}$ MC samples as a source of templates in anti-isolated region.

u_T Cut	m_T^W fits b	m_T^W fits k	E_T^{miss} fits b	E_T^{miss} fits k	p_T fits b	p_T fits k	$ \Delta\phi(l_e p_0 - MET) $ fits b	$ \Delta\phi(l_e p_0 - MET) $ fits k
-	116548 ± 13931	69343 ± 78742	120025 ± 12112	64022 ± 69345	108412 ± 9743	42472 ± 46358	124791 ± 24369	50671 ± 115205

Table 29: k and b are parameters for the muon extrapolations on 13 TeV using $b\bar{b} + c\bar{c}$ MC sample as a source of templates in anti-isolated region. The uncertainty of these two parameters are provided by the extrapolation, from the statistical uncertainty of the parameter T and the number of events in CR2 of each slices and multiplied by $\sqrt{\chi^2/NDoF}$.

u_T Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDoF$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	29180 ± 592	3.845 ± 0.031	1.000 ± 0.000	503.42	90	2.37	25514 ± 160	1755 ± 14	29180 ± 592	112300 ± 5807
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	23045 ± 524	5.189 ± 0.042	1.000 ± 0.000	714.02	90	2.82	19741 ± 141	532 ± 7	23045 ± 524	119473 ± 8146
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	17122 ± 451	6.976 ± 0.057	1.000 ± 0.000	723.30	90	2.83	15432 ± 124	238 ± 5	17122 ± 451	119447 ± 9343
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	13481 ± 400	8.690 ± 0.072	1.000 ± 0.000	910.74	90	3.18	11810 ± 109	145 ± 5	13481 ± 400	117147 ± 11489
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9429 ± 324	13.108 ± 0.110	1.000 ± 0.000	1644.71	90	4.27	9202 ± 96	89 ± 3	9429 ± 324	123587 ± 18688
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	6993 ± 277	16.859 ± 0.139	1.000 ± 0.000	1153.85	90	3.58	7395 ± 86	64 ± 2	6993 ± 277	117897 ± 17101
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5174 ± 235	28.443 ± 0.242	1.000 ± 0.000	2606.78	90	5.38	5904 ± 77	45 ± 2	5174 ± 235	147169 ± 36579
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4409 ± 212	28.196 ± 0.247	1.000 ± 0.000	3394.43	90	6.14	4854 ± 70	31 ± 2	4409 ± 212	124324 ± 37238

Table 30: Numbers for the muon p_T fits on 13 TeV using $b\bar{b} + c\bar{c}$ MC samples. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

u_T Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDoF$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	29180 ± 592	4.381 ± 0.027	1.000 ± 0.000	672.90	160	2.05	25514 ± 160	1755 ± 14	29180 ± 592	127834 ± 5356
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	23045 ± 524	5.705 ± 0.030	1.000 ± 0.000	1510.58	160	3.07	19741 ± 141	532 ± 7	23045 ± 524	131469 ± 9434
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	17122 ± 451	7.955 ± 0.038	1.000 ± 0.000	2503.54	160	3.96	15432 ± 124	238 ± 5	17122 ± 451	136211 ± 14421
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	13481 ± 400	10.121 ± 0.046	1.000 ± 0.000	3417.58	160	4.62	11810 ± 109	145 ± 5	13481 ± 400	136444 ± 18950
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9429 ± 324	14.922 ± 0.067	1.000 ± 0.000	5045.71	160	5.62	9202 ± 96	89 ± 3	9429 ± 324	140692 ± 27390
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	6993 ± 277	20.446 ± 0.093	1.000 ± 0.000	8781.68	160	7.41	7395 ± 86	64 ± 2	6993 ± 277	142980 ± 42285
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5174 ± 235	28.353 ± 0.128	1.000 ± 0.000	9889.89	160	7.86	5904 ± 77	45 ± 2	5174 ± 235	146704 ± 52612
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4409 ± 212	33.362 ± 0.153	1.000 ± 0.000	14594.91	160	9.55	4854 ± 70	31 ± 2	4409 ± 212	147099 ± 67716

Table 31: Numbers for the muon E_T^{miss} fits on 13 TeV using $b\bar{b} + c\bar{c}$ MC samples. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

u_T Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDoF$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	29180 ± 592	4.287 ± 0.026	1.000 ± 0.000	719.77	120	2.45	25514 ± 160	1755 ± 14	29180 ± 592	125106 ± 6493
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	23045 ± 524	5.621 ± 0.032	1.000 ± 0.000	1571.79	120	3.62	19741 ± 141	532 ± 7	23045 ± 524	129538 ± 10900
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	17122 ± 451	7.613 ± 0.042	1.000 ± 0.000	2222.31	120	4.30	15432 ± 124	238 ± 5	17122 ± 451	130349 ± 15092
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	13481 ± 400	10.082 ± 0.050	1.000 ± 0.000	3202.30	120	5.17	11810 ± 109	145 ± 5	13481 ± 400	135919 ± 21143
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9429 ± 324	15.306 ± 0.076	1.000 ± 0.000	5017.34	120	6.47	9202 ± 96	89 ± 3	9429 ± 324	144317 ± 32412
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	6993 ± 277	19.960 ± 0.089	1.000 ± 0.000	10730.66	120	9.46	7395 ± 86	64 ± 2	6993 ± 277	139207 ± 52539
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5174 ± 235	28.333 ± 0.123	1.000 ± 0.000	9086.66	120	8.70	5904 ± 77	45 ± 2	5174 ± 235	146599 ± 58169
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4409 ± 212	32.584 ± 0.140	1.000 ± 0.000	12470.62	120	10.19	4854 ± 70	31 ± 2	4409 ± 212	143669 ± 70351

Table 32: Numbers for the muon m_T^W fits on 13 TeV using $b\bar{b} + c\bar{c}$ MC samples. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

u_T Cut	Slice No.	$Isomin$	$Isomax$	N_{Data}^{FR}	N_{EW}^{FR}	N_{Data}^{CR1}	N_{EW}^{CR1}	N_{MJ}^{CR1}	T	α	χ^2	NDof	$\sqrt{\chi^2}/NDoF$	N_{Data}^{CR2}	N_{EW}^{CR2}	N_{MJ}^{CR2}	N_{MJ}^{SR}
-	0	0.1	0.15	1206859 ± 1099	1085200 ± 343	25514 ± 160	1755 ± 14	29180 ± 592	4.428 ± 0.029	1.000 ± 0.000	446.29	16	5.38	25514 ± 160	1755 ± 14	29180 ± 592	129215 ± 14561
-	1	0.15	0.2	1206859 ± 1099	1085200 ± 343	19741 ± 141	532 ± 7	23045 ± 524	5.920 ± 0.039	1.000 ± 0.000	757.45	16	6.88	19741 ± 141	532 ± 7	23045 ± 524	136427 ± 22225
-	2	0.2	0.25	1206859 ± 1099	1085200 ± 343	15432 ± 124	238 ± 5	17122 ± 451	8.222 ± 0.054	1.000 ± 0.000	1072.60	16	8.19	15432 ± 124	238 ± 5	17122 ± 451	140778 ± 31296
-	3	0.25	0.3	1206859 ± 1099	1085200 ± 343	11810 ± 109	145 ± 5	13481 ± 400	10.778 ± 0.071	1.000 ± 0.000	934.29	16	7.64	11810 ± 109	145 ± 5	13481 ± 400	145295 ± 33774
-	4	0.3	0.35	1206859 ± 1099	1085200 ± 343	9202 ± 96	89 ± 3	9429 ± 324	14.160 ± 0.094	1.000 ± 0.000	805.03	16	7.09	9202 ± 96	89 ± 3	9429 ± 324	133513 ± 33158
-	5	0.35	0.4	1206859 ± 1099	1085200 ± 343	7395 ± 86	64 ± 2	6993 ± 277	21.920 ± 0.146	1.000 ± 0.000	1491.62	16	9.66	7395 ± 86	64 ± 2	6993 ± 277	153289 ± 59519
-	6	0.4	0.45	1206859 ± 1099	1085200 ± 343	5904 ± 77	45 ± 2	5174 ± 235	27.277 ± 0.185	1.000 ± 0.000	1590.97	16	9.97	5904 ± 77	45 ± 2	5174 ± 235	141138 ± 64594
-	7	0.45	0.5	1206859 ± 1099	1085200 ± 343	4854 ± 70	31 ± 2	4409 ± 212	30.474 ± 0.218	1.000 ± 0.000	3818.26	16	15.45	4854 ± 70	31 ± 2	4409 ± 212	134366 ± 100694

Table 33: Numbers for the muon $|\Delta\phi(lep_0 - MET)|$ fits on 13 TeV using $b\bar{b} + c\bar{c}$ MC samples. Numbers in FR are not affected by Slice No. The uncertainty of the numbers of events are statistical only except N_{SR}^{MJ} which is multiplied by $\sqrt{\chi^2}/NDoF$ and the uncertainty of the parameters are from the statistical uncertainty of the fitting.

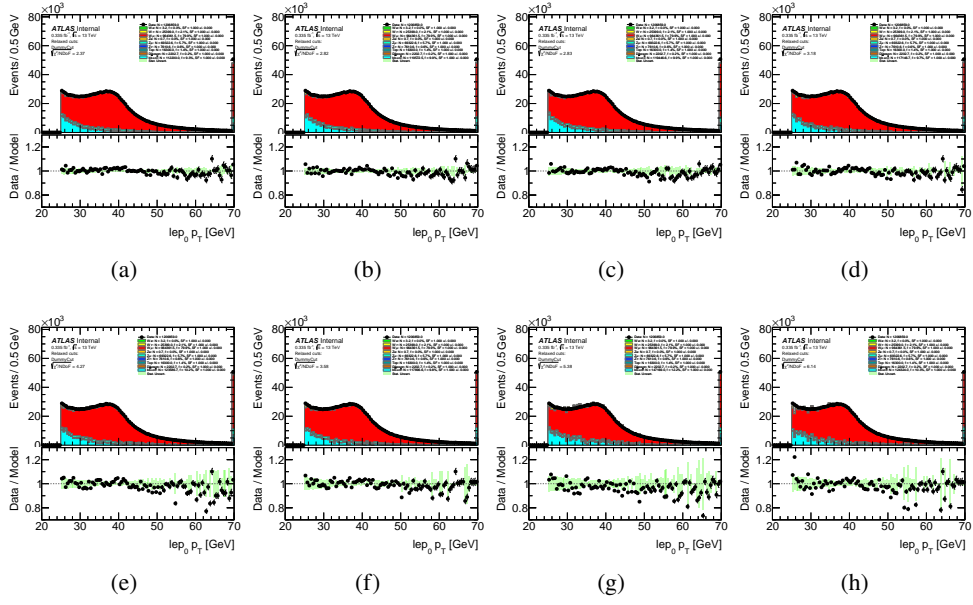


Figure 141: Multi-jet background template fits performed in distributions of p_T^ℓ of the muons channel using $b\bar{b} + c\bar{c}$ MC samples, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.15$ to $0.45 < ptvarcone20/p_T < 0.5$.

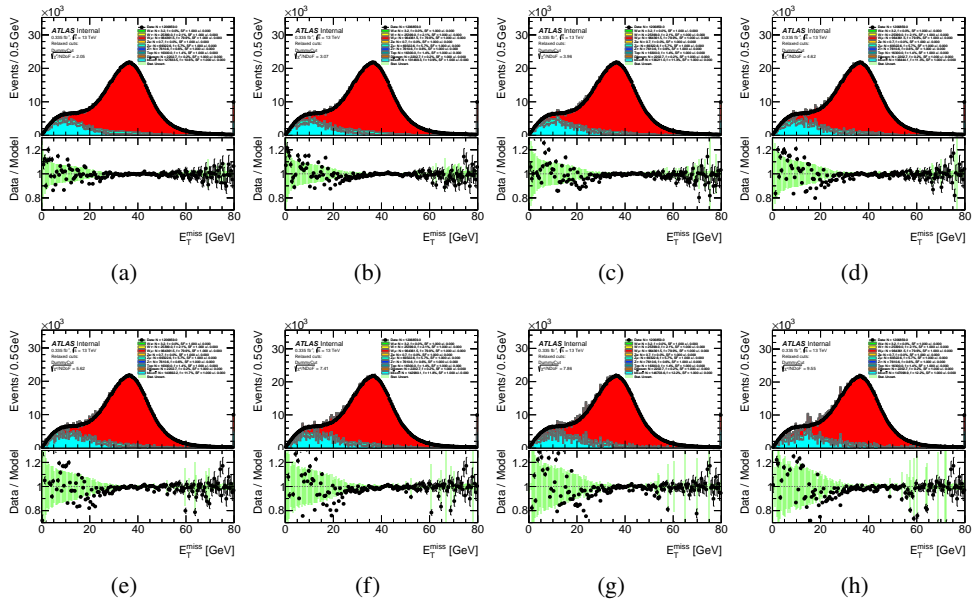


Figure 142: Multi-jet background template fits performed in distributions of E_T^{miss} of the muons channel using $b\bar{b} + c\bar{c}$ MC samples, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.15$ to $0.45 < ptvarcone20/p_T < 0.5$.

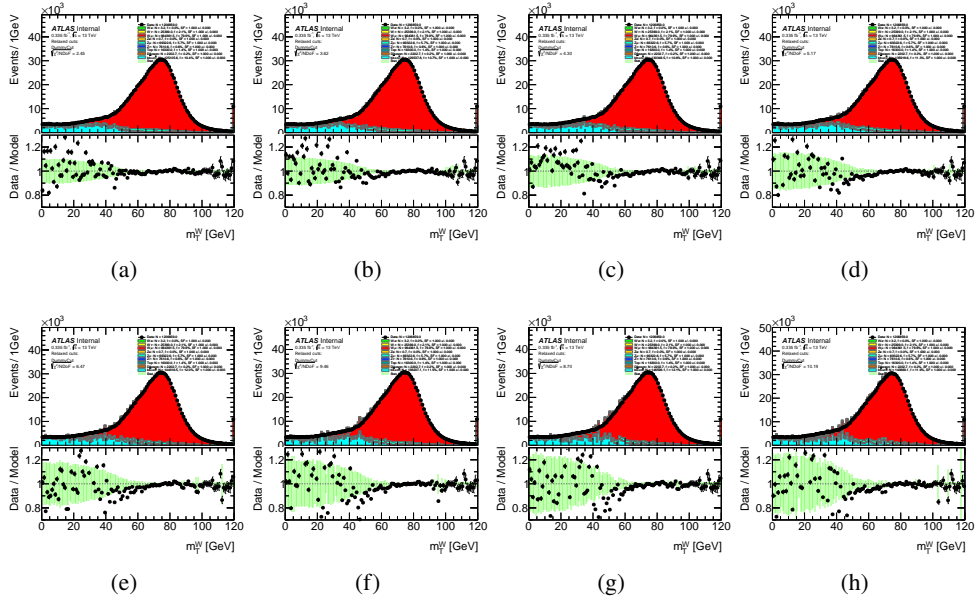


Figure 143: Multi-jet background template fits performed in distributions of m_T^W of the muons channel using $b\bar{b} + c\bar{c}$ MC samples, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.15$ to $0.45 < ptvarcone20/p_T < 0.5$.

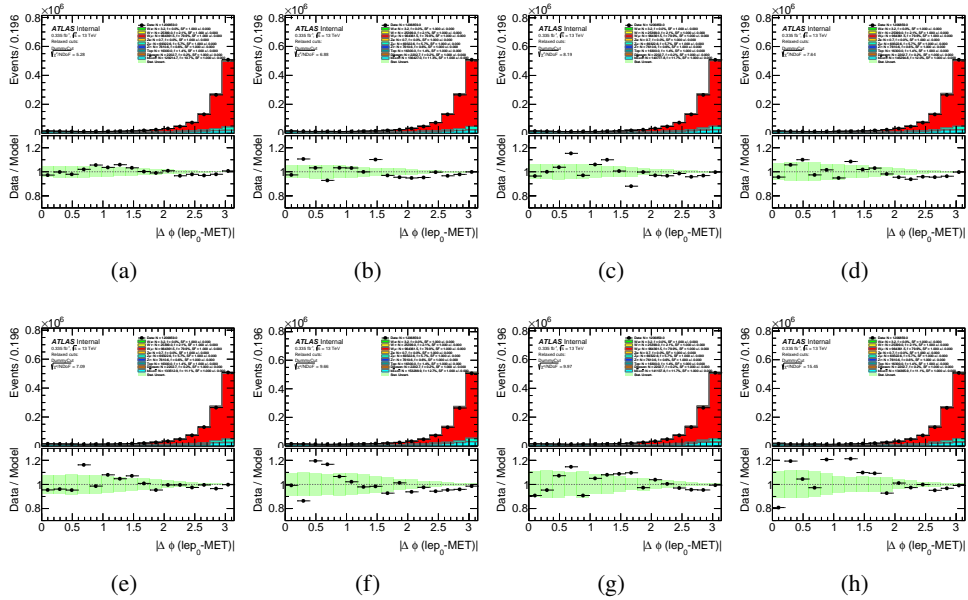


Figure 144: Multi-jet background template fits performed in distributions of $|\Delta\phi(lep_0 - MET)|$ of the muons channel using $b\bar{b} + c\bar{c}$ MC samples, columns from (a) to (h) are the fits with the isolation slices from $0.1 < ptvarcone20/p_T < 0.15$ to $0.45 < ptvarcone20/p_T < 0.5$.

1269 **E.5 Acceptance correction functions for MJ background**

1270 **E.5.1 MC sampels**

1271 To extract the MJ acceptance correction functions the following MC samples are used.

1272 The properties of the $b\bar{b}$ and $c\bar{c}$ samples at $\sqrt{s} = 13$ TeV are given in Table 34. The heavy-flavour production
 1273 cross sections are higher than signal and other backgrounds, but their calculation carries significant
 1274 uncertainties from the dijet production cross section (calculated at leading order), and the $b/c \rightarrow \mu$ decay
 1275 fractions and fragmentation functions.

Process	Data set	Generator	$\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}}$ [nb]
$b\bar{b} \rightarrow \mu$	361250	Pythia8B+A14+NNPDF23LO	186
$c\bar{c} \rightarrow \mu$	361251	Pythia8B+A14+NNPDF23LO	58

Table 34: MC simulation of $b\bar{b}$ and $c\bar{c}$ decay to muon at $\sqrt{s} = 13$ TeV.

1276 For the electron channel multijet production was generated using PYTHIA [8.230] [47] with leading-order
 1277 matrix elements for dijet production which were matched to the parton shower. Table 35 details the
 1278 properties of the JF17 samples used.

Process	Data set	Generator	$\sigma \cdot \text{BR} \cdot \epsilon_{\text{filter}}$ [nb]
<i>JF17</i>	423300	Pythia8B+A14+NNPDF23LO	180×10^3

Table 35: MC simulation of the MJ JF17 sample $\sqrt{s} = 13$ TeV.

1279 **E.5.2 Electron channel multi-jet background 1D acceptance correction functions**

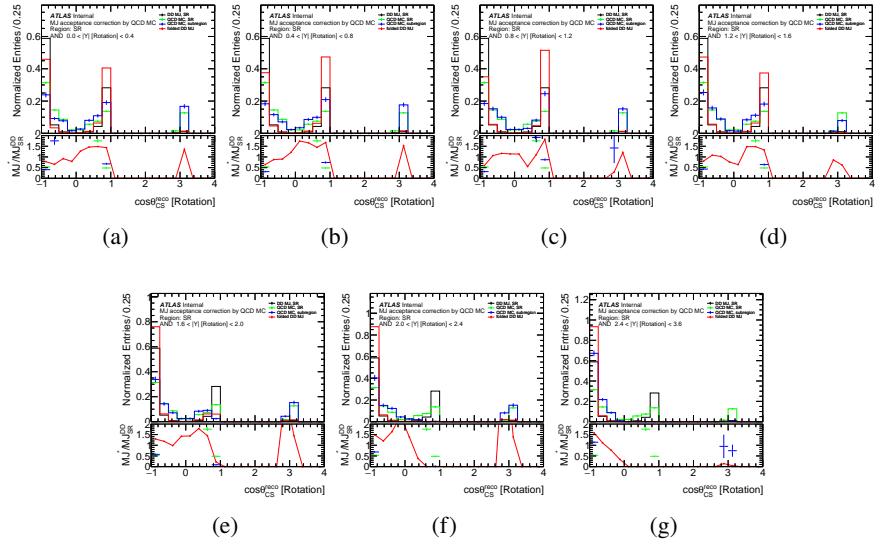


Figure 145: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $|Y|$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

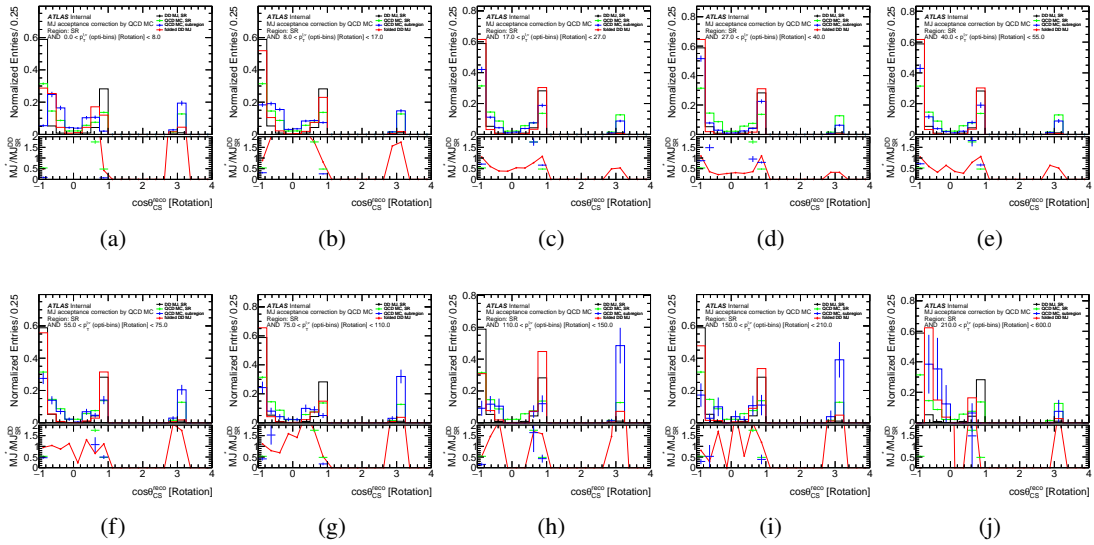


Figure 146: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $p_T^{\ell,\nu}$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

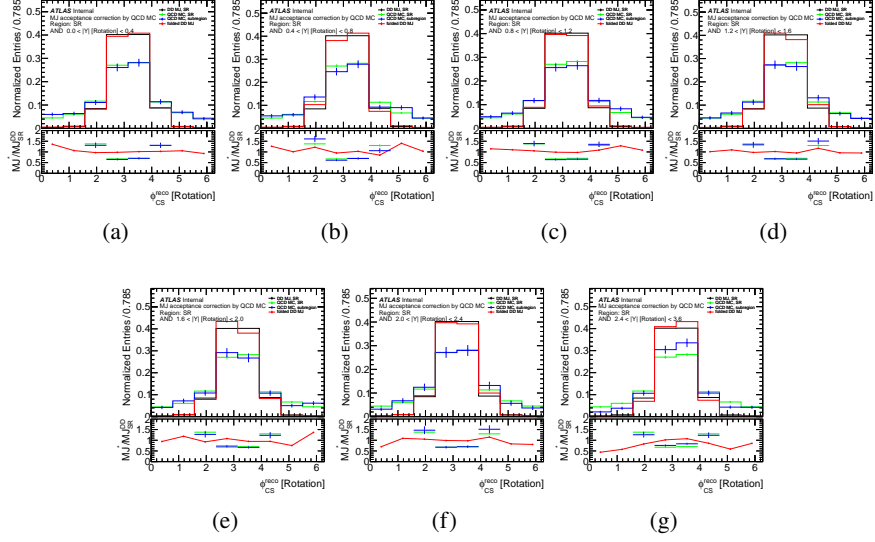


Figure 147: Multi-jet background template acceptance correction functions for ϕ_{CS} for $|Y|$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

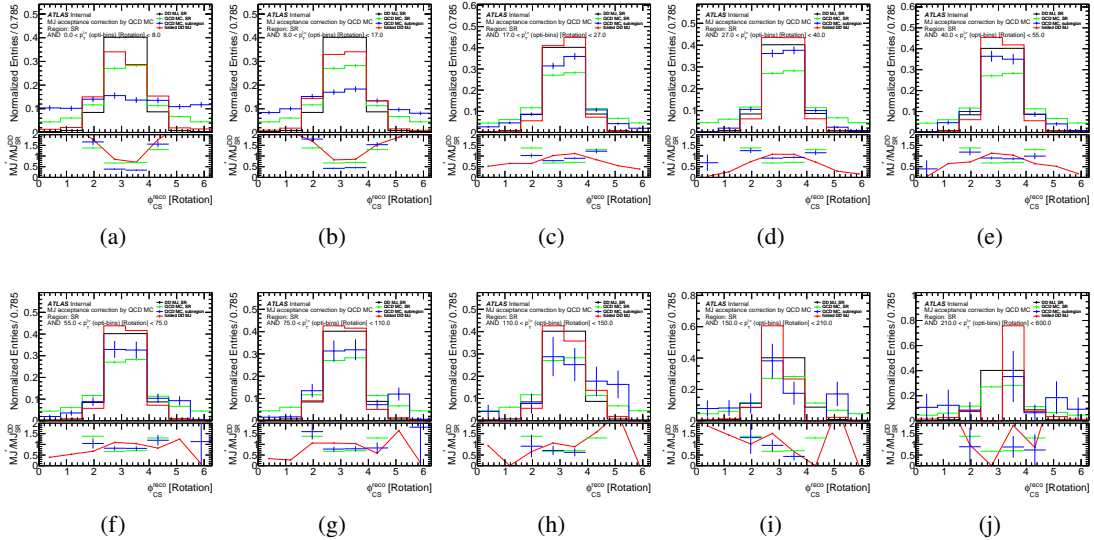


Figure 148: Multi-jet background template acceptance correction functions for $p_T^{\ell, \nu}$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

1280 **E.5.3 Positron channel multi-jet background 1D acceptance correction functions**

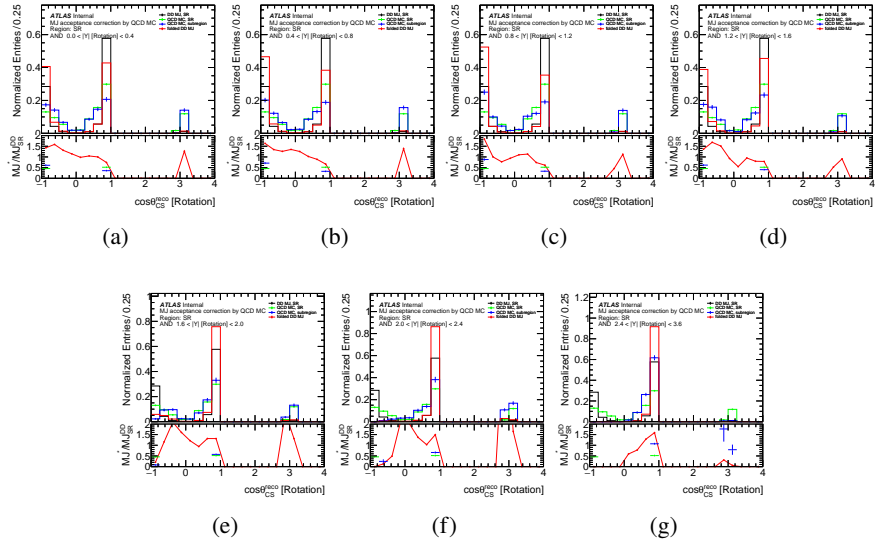


Figure 149: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $|Y|$ bins for $W^+ \rightarrow e^+ \nu$ channel.

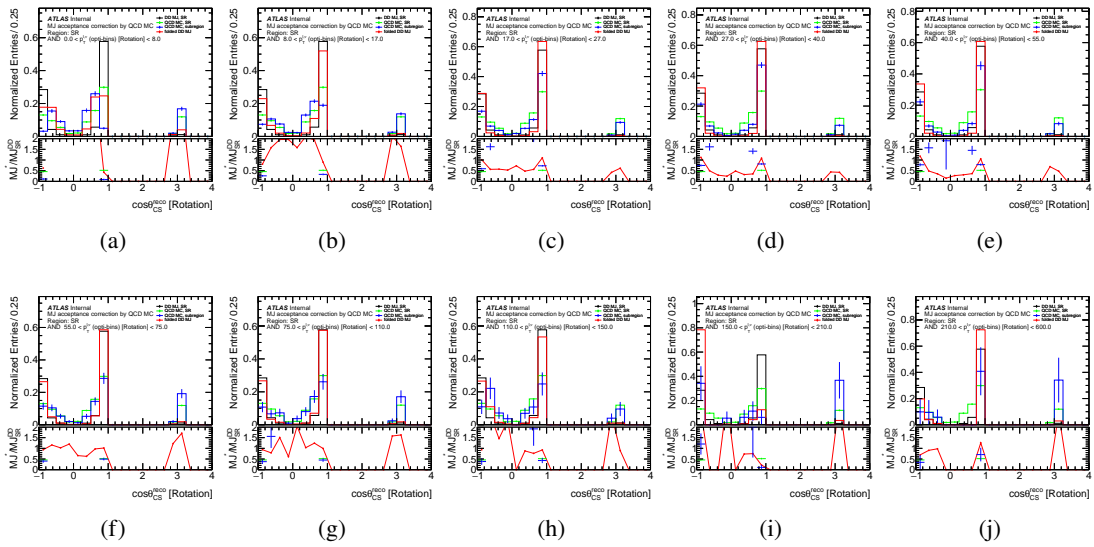


Figure 150: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $p_T^{\ell, \nu}$ bins for $W^+ \rightarrow e^+ \nu$ channel.

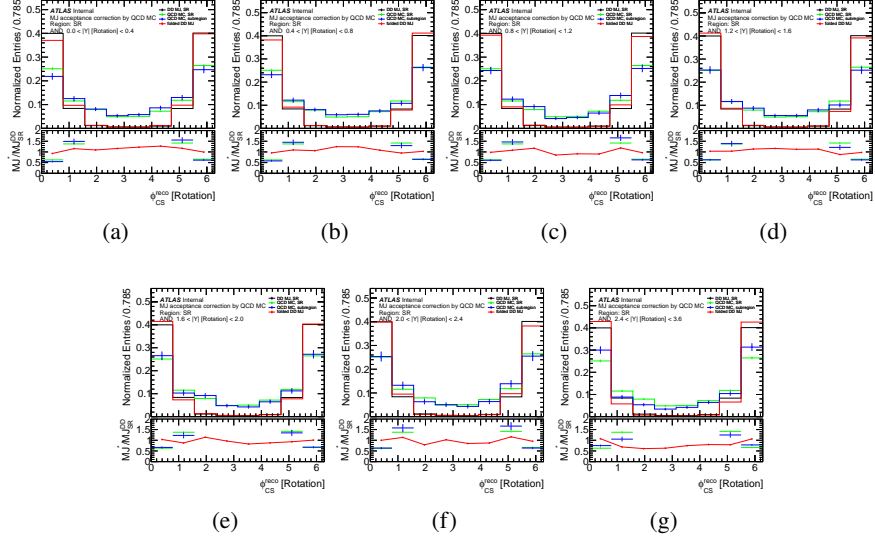


Figure 151: Multi-jet background template acceptance correction functions for ϕ_{CS} for $|Y|$ bins for $W^+ \rightarrow e^+ \nu$ channel.

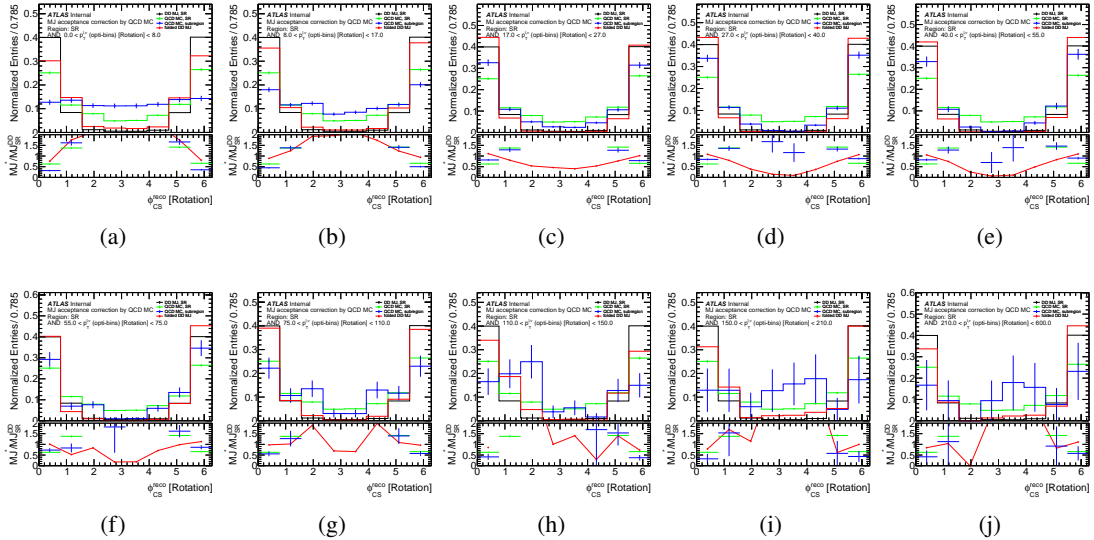


Figure 152: Multi-jet background template acceptance correction functions for $p_T^{\ell, \nu}$ bins for $W^+ \rightarrow e^+ \nu$ channel.

E.5.4 Muon channel multi-jet background 1D acceptance correction functions

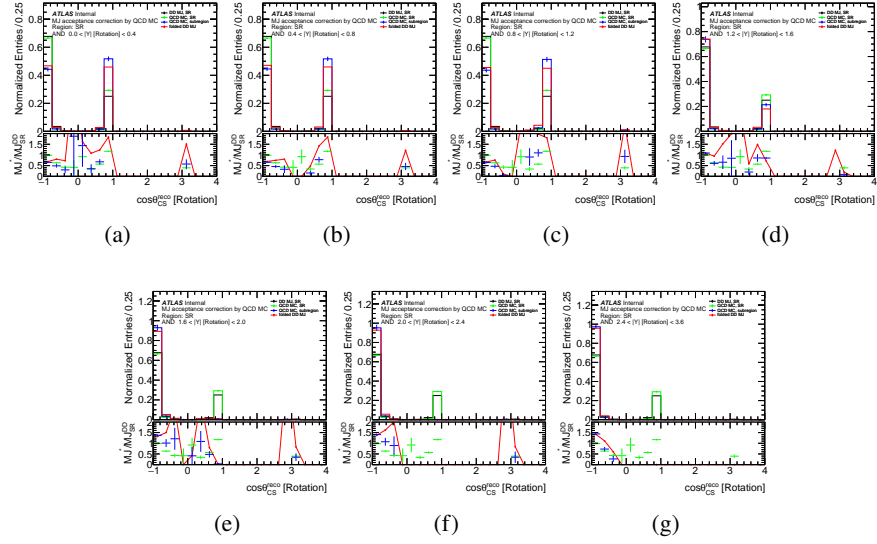


Figure 153: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $|Y|$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

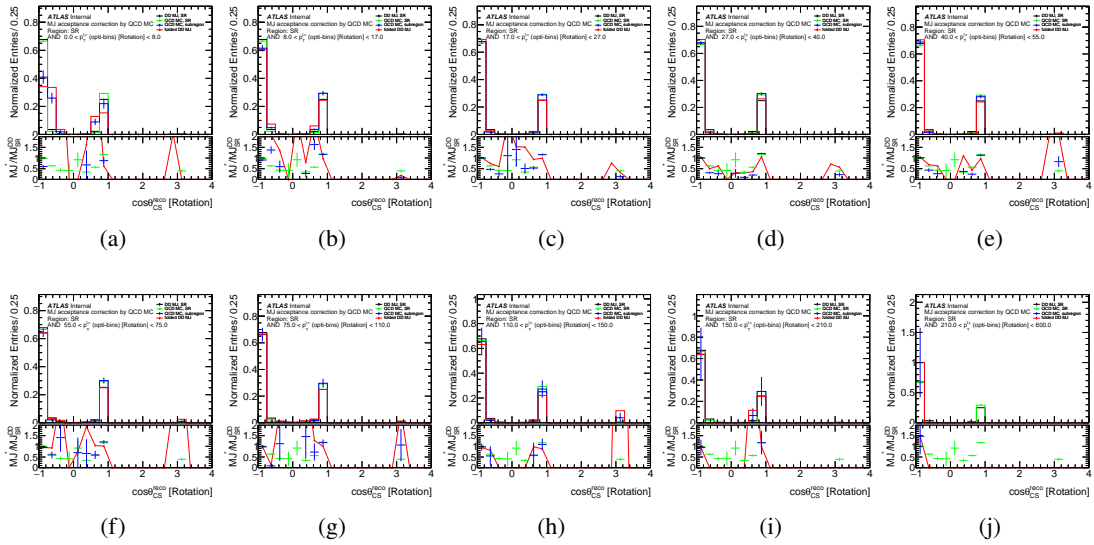


Figure 154: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $p_T^{\ell,\nu}$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

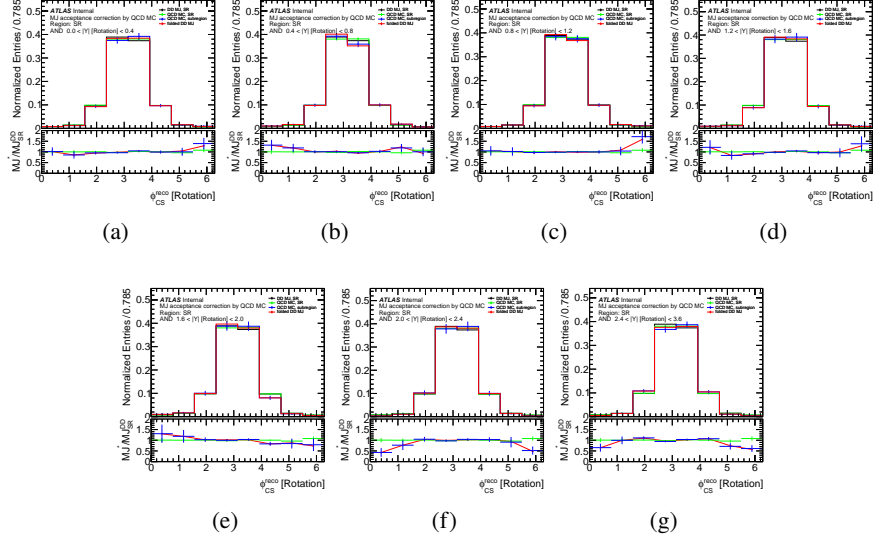


Figure 155: Multi-jet background template acceptance correction functions for ϕ_{CS} for $|Y|$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

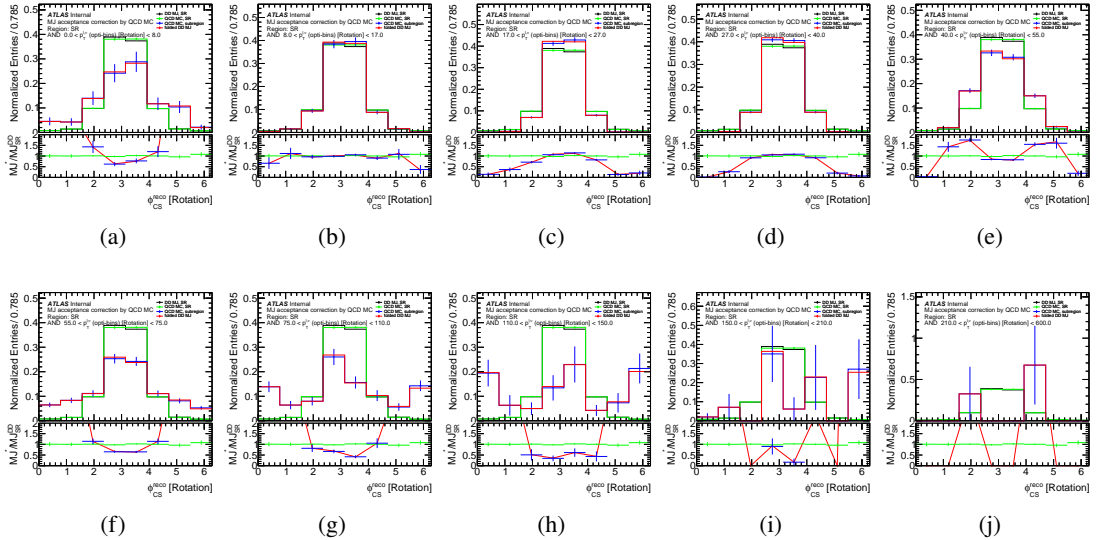


Figure 156: Multi-jet background template acceptance correction functions for $p_T^{\ell, \nu}$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

1282 **E.5.5 Anti-muon channel multi-jet background 1D acceptance correction functions**

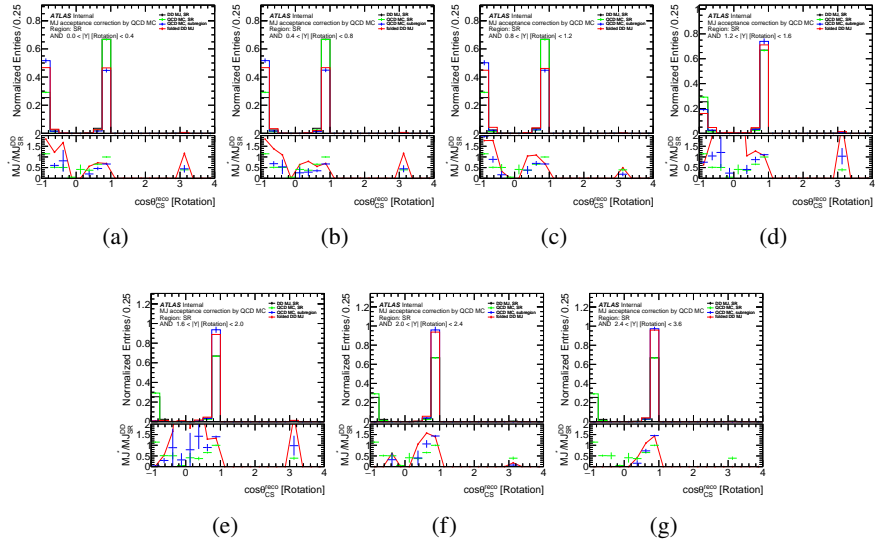


Figure 157: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $|Y|$ bins for $W^+ \rightarrow \mu^+ \nu$ channel.

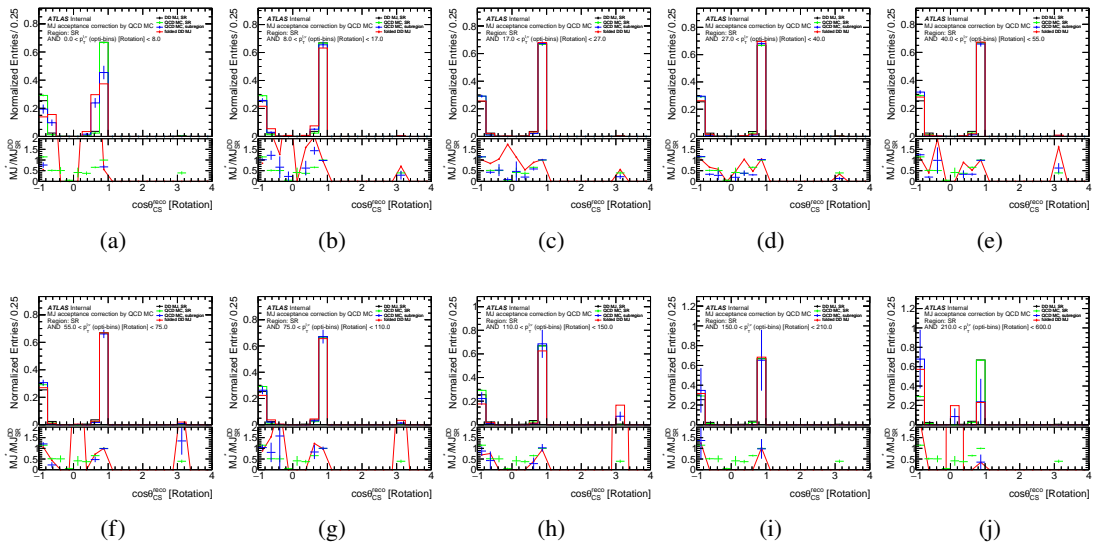


Figure 158: Multi-jet background template acceptance correction functions for $\cos \theta_{CS}$ for $p_T^{\ell, \nu}$ bins for $W^+ \rightarrow \mu^+ \nu$ channel.

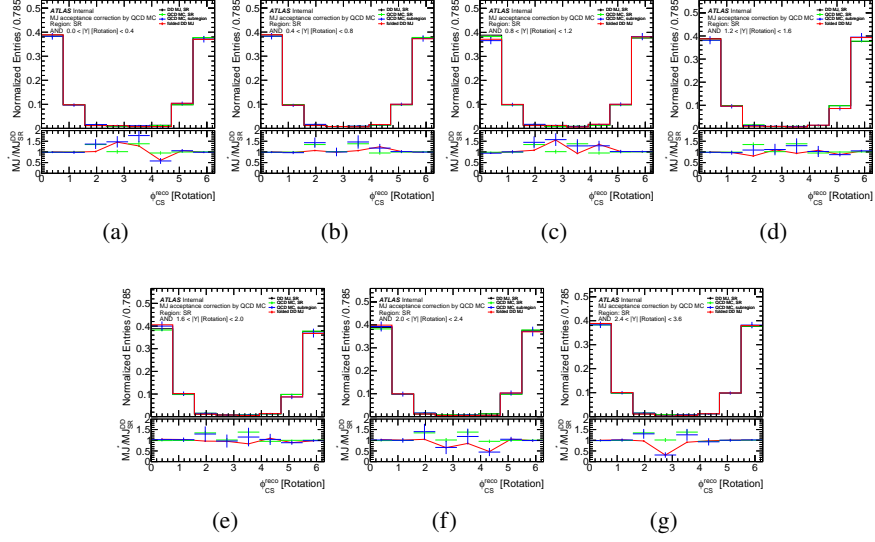


Figure 159: Multi-jet background template acceptance correction functions for ϕ_{CS} for $|Y|$ bins for $W^+ \rightarrow \mu^+ \nu$ channel.

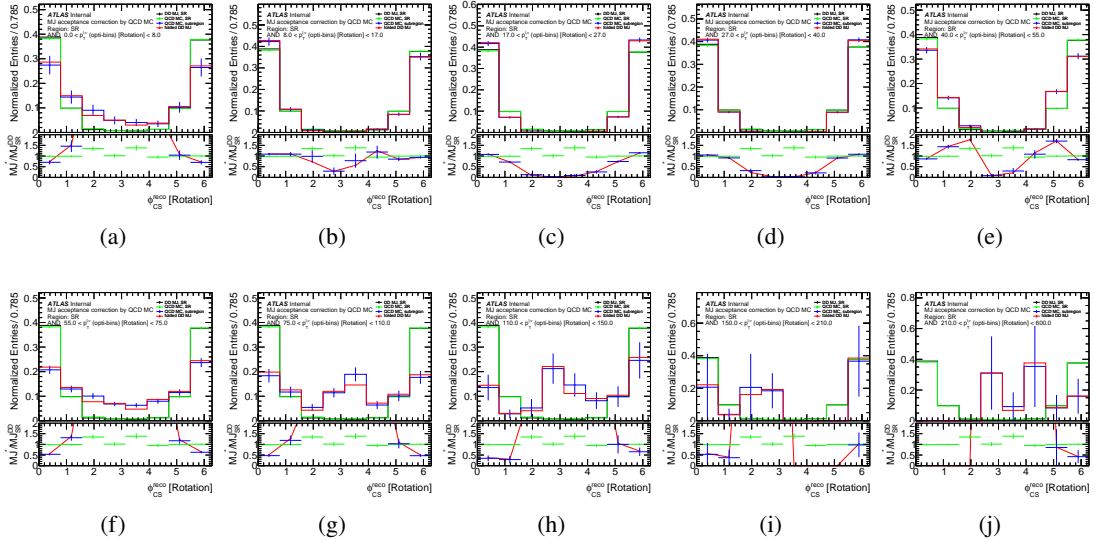


Figure 160: Multi-jet background template acceptance correction functions for $p_T^{\ell, \nu}$ bins for $W^+ \rightarrow \mu^+ \nu$ channel.

E.5.6 Electron channel multi-jet background 2D acceptance correction functions

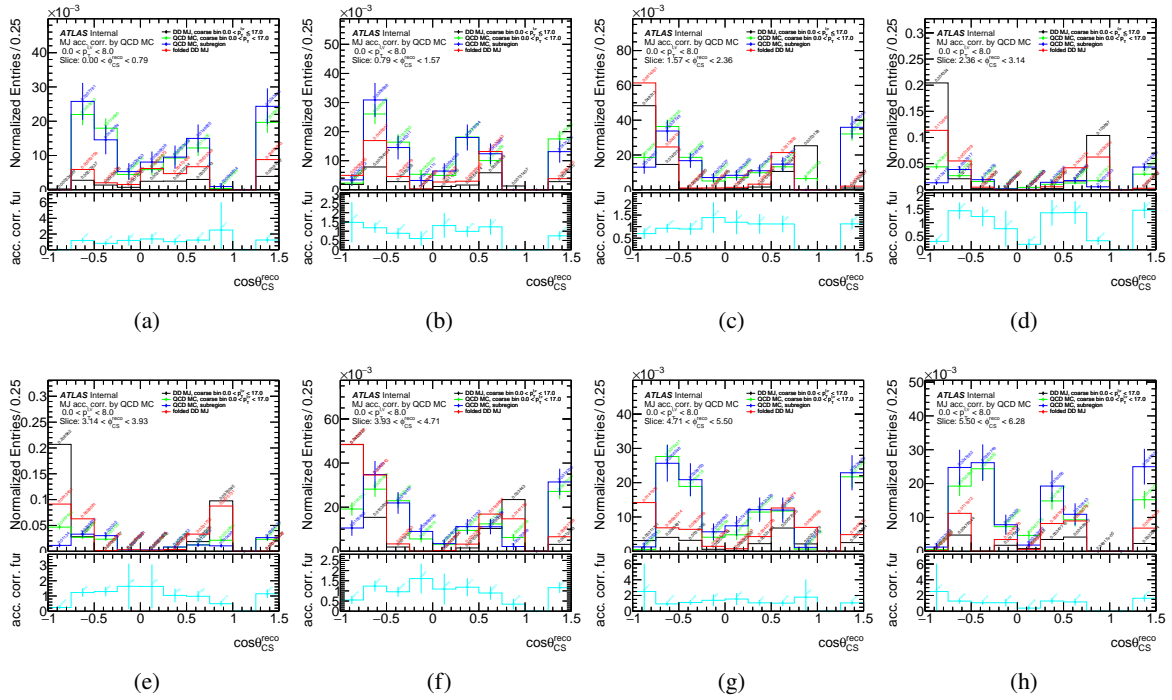


Figure 161: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

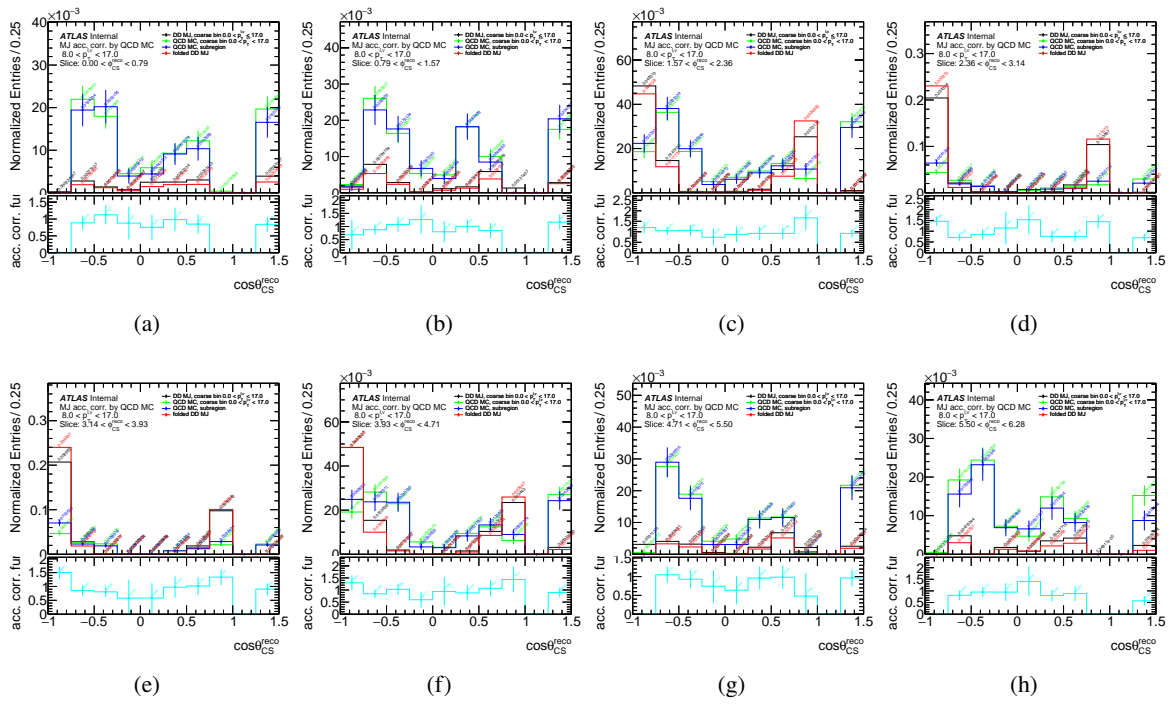


Figure 162: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

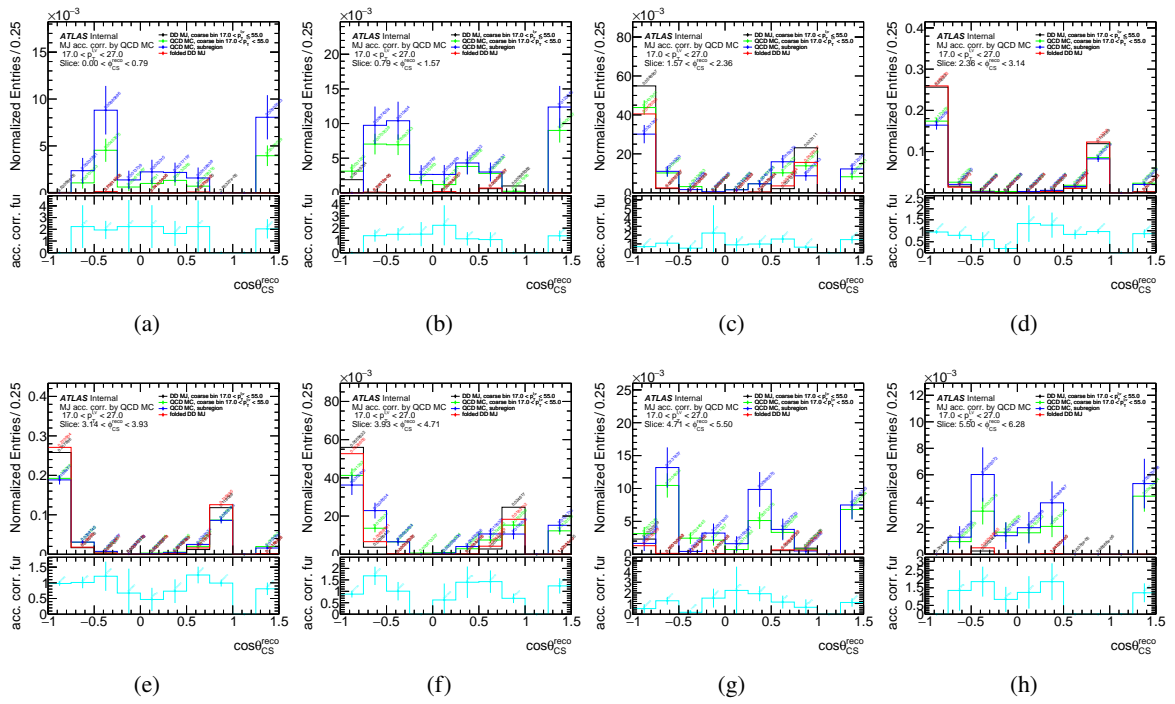


Figure 163: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

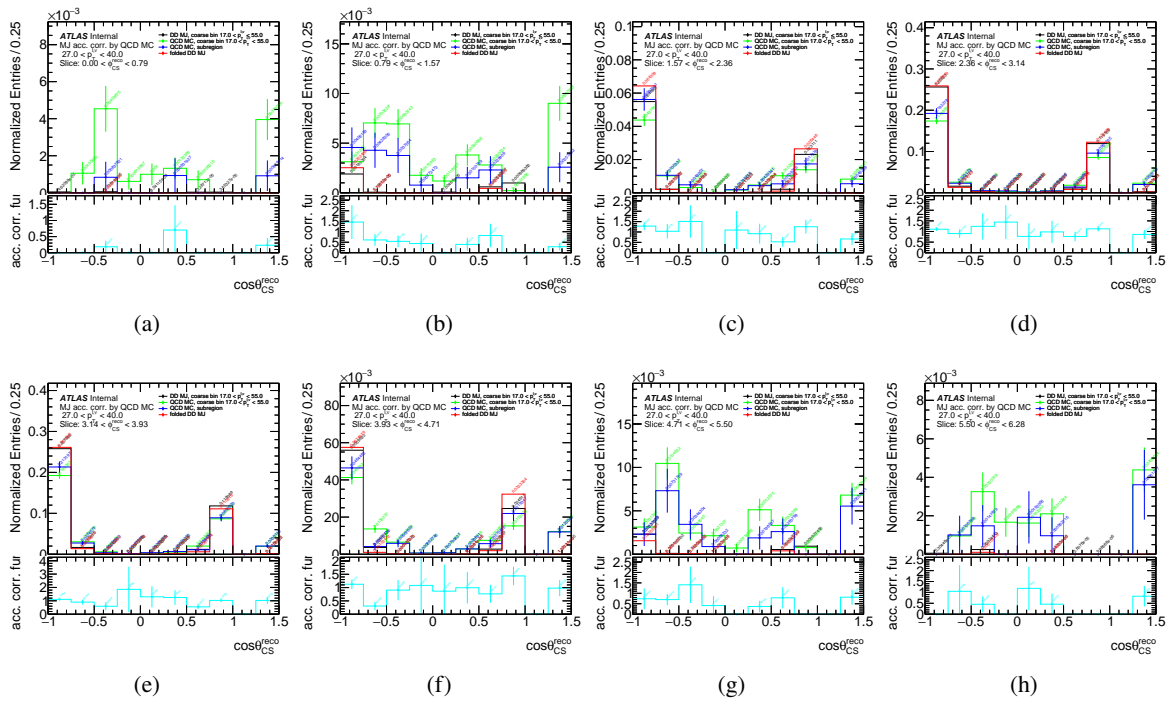


Figure 164: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

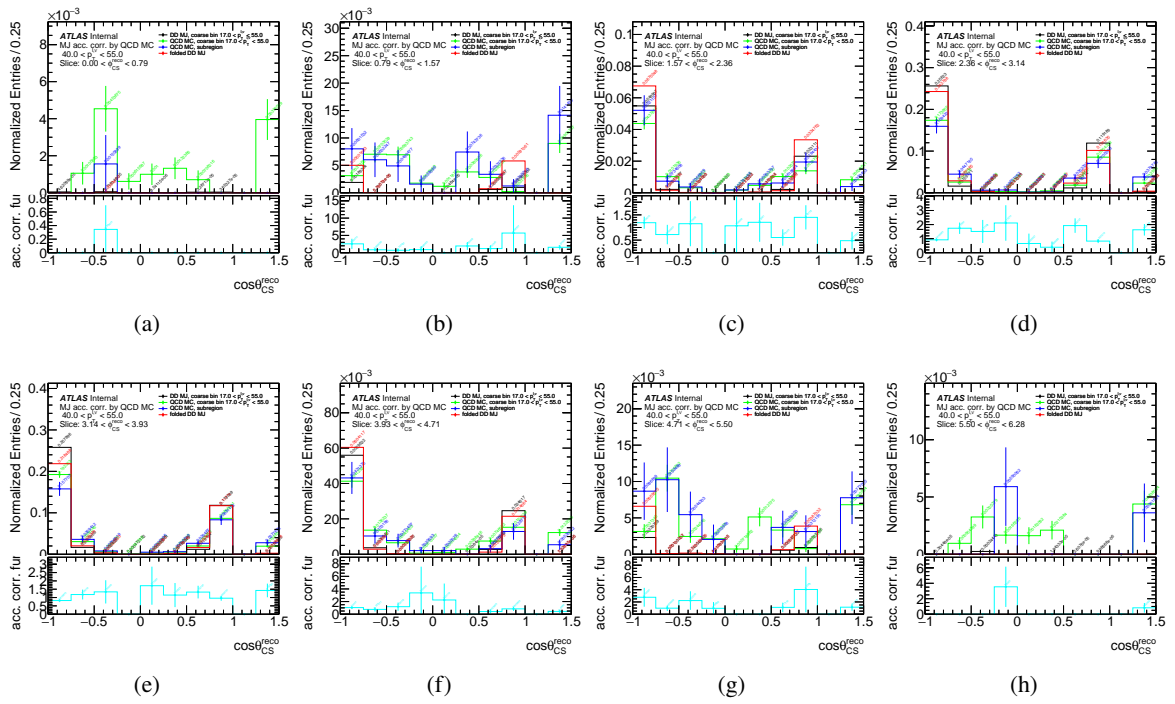


Figure 165: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

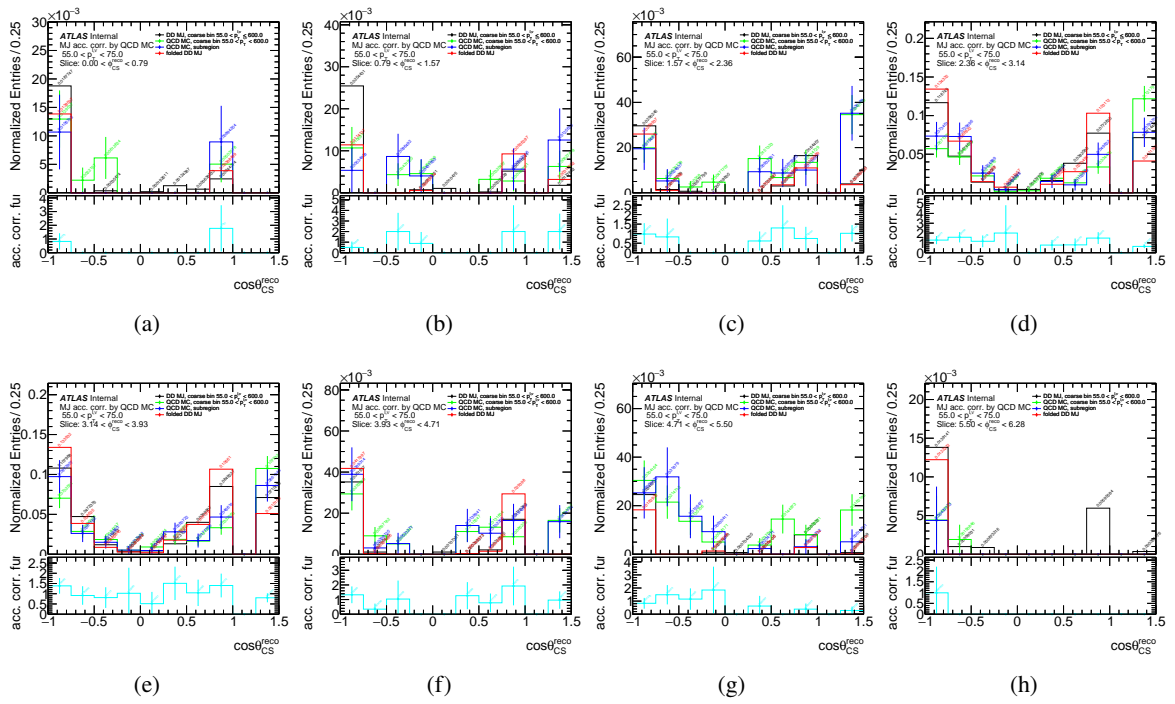


Figure 166: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

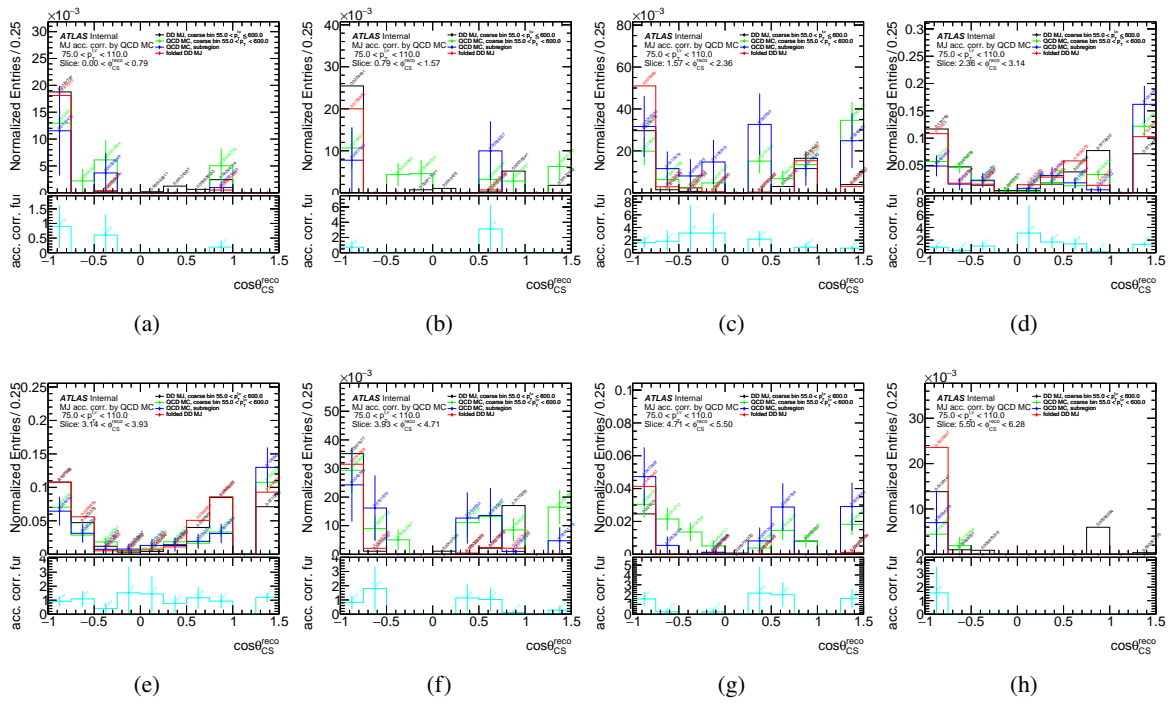


Figure 167: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

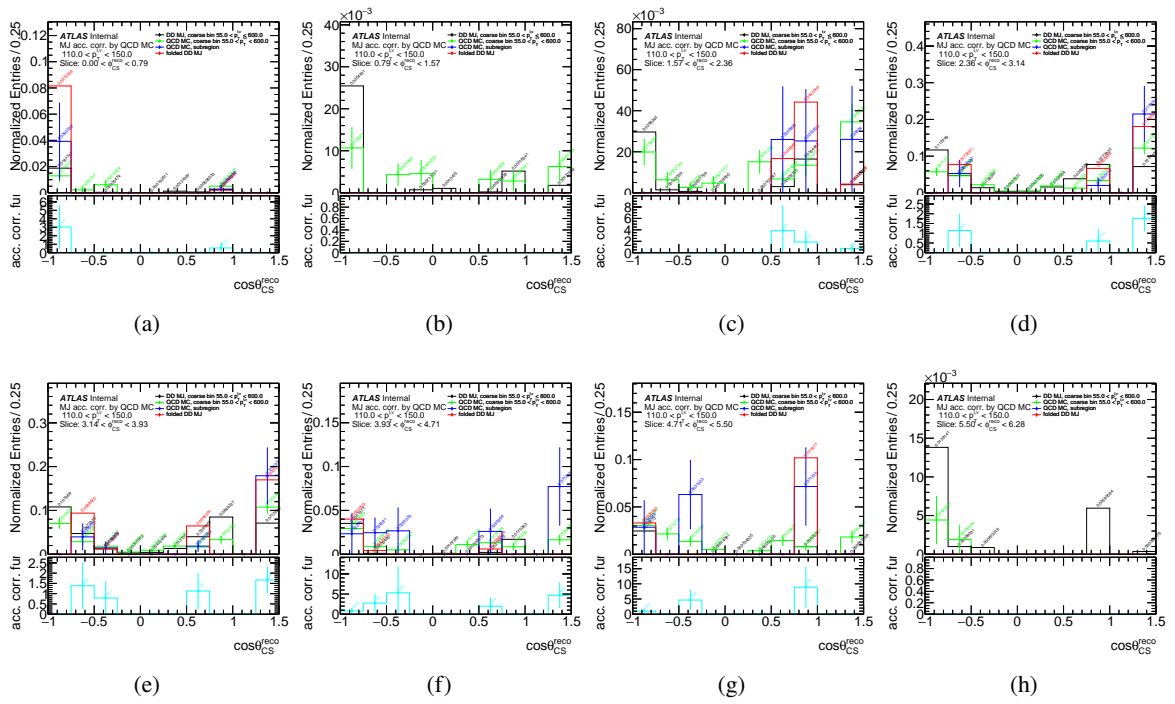


Figure 168: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

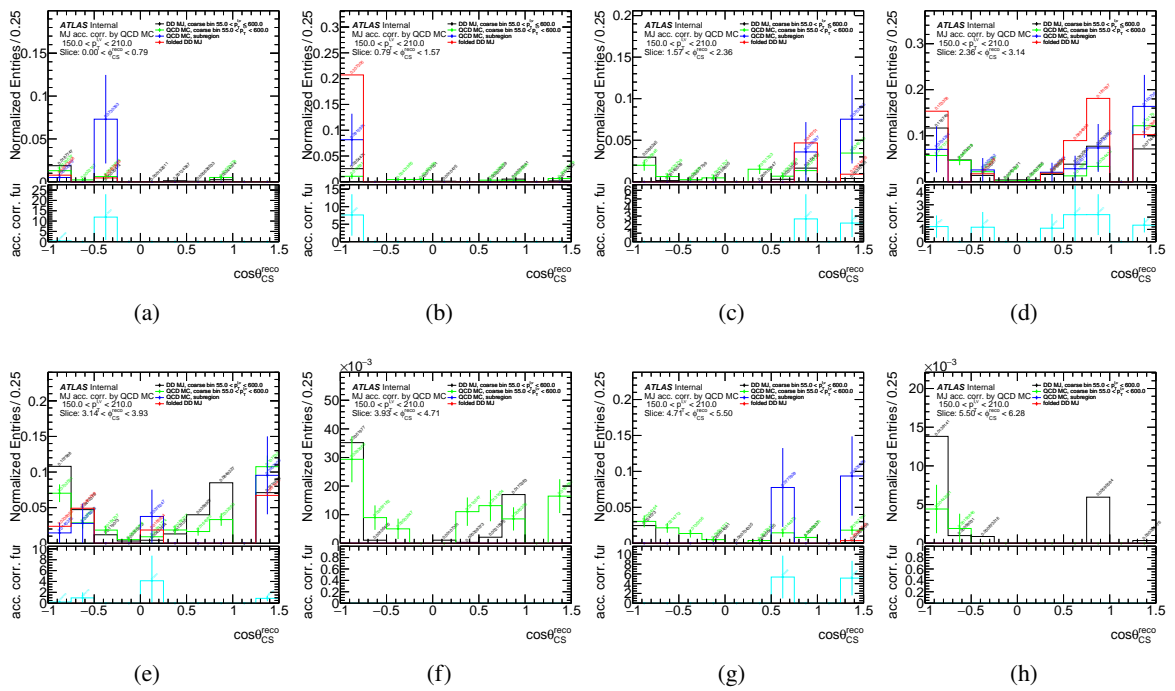


Figure 169: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

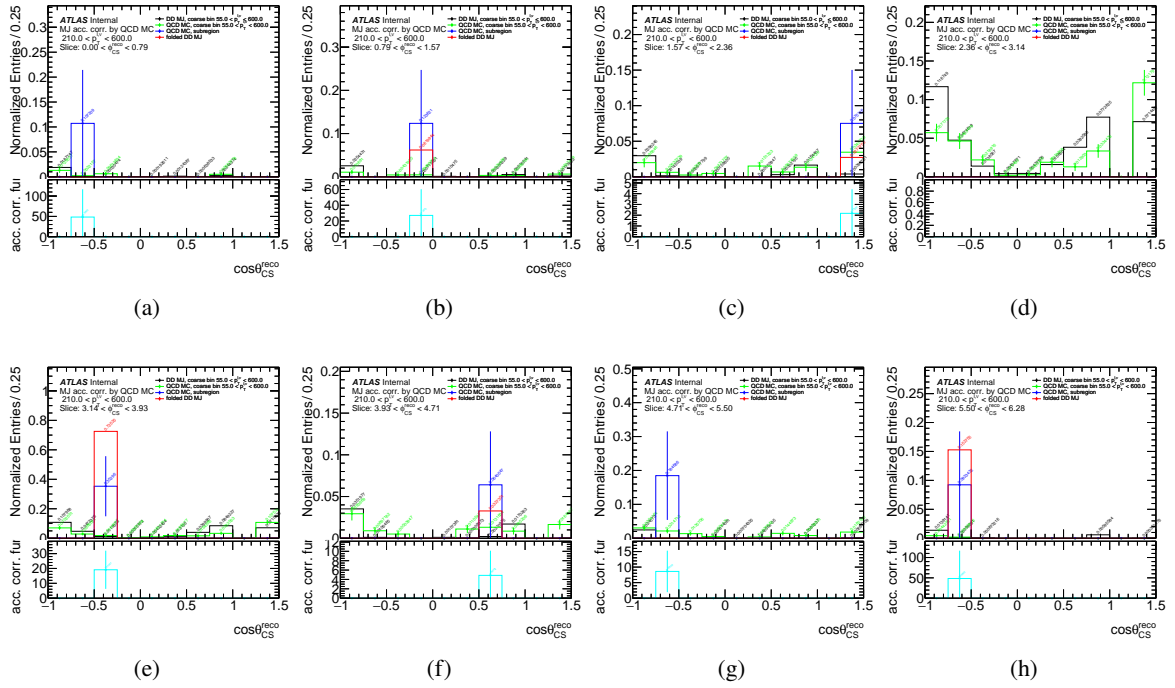


Figure 170: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

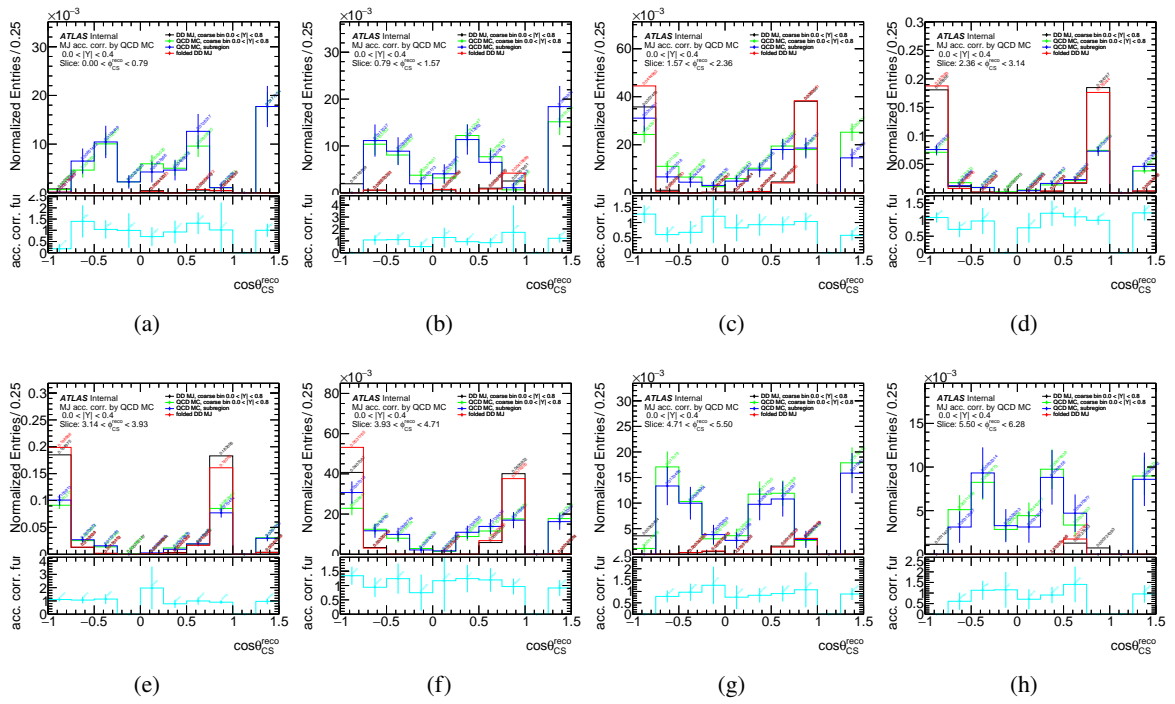


Figure 171: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin in $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represent statistical uncertainties.

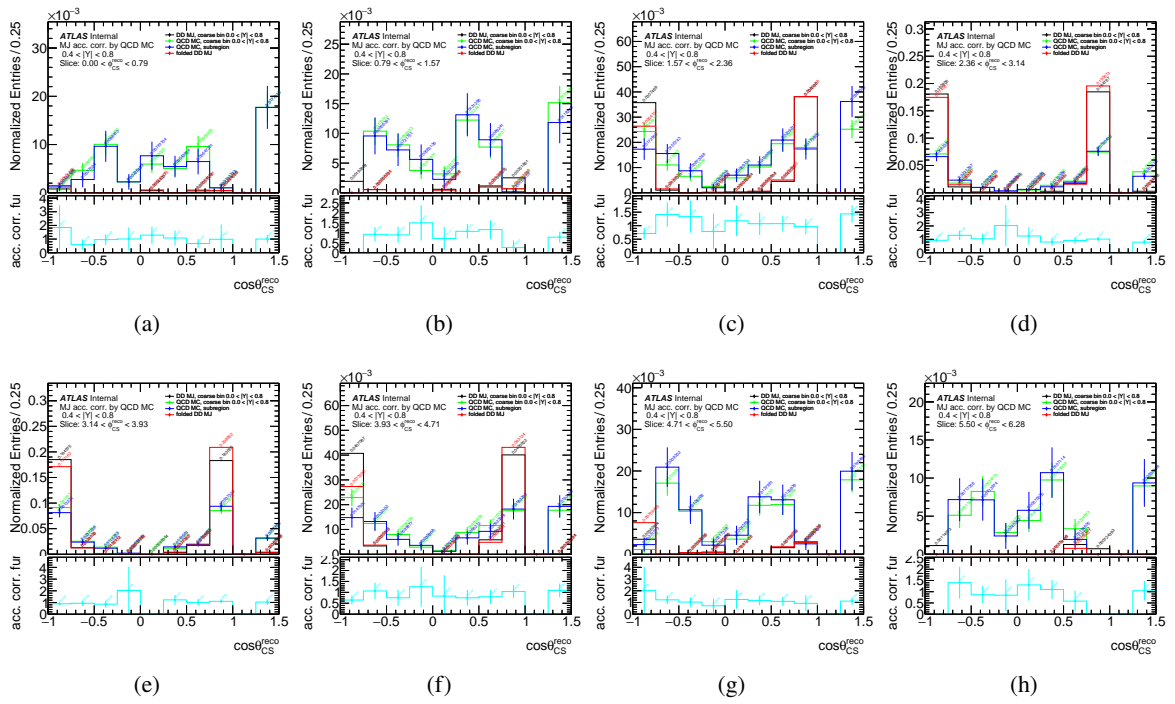


Figure 172: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

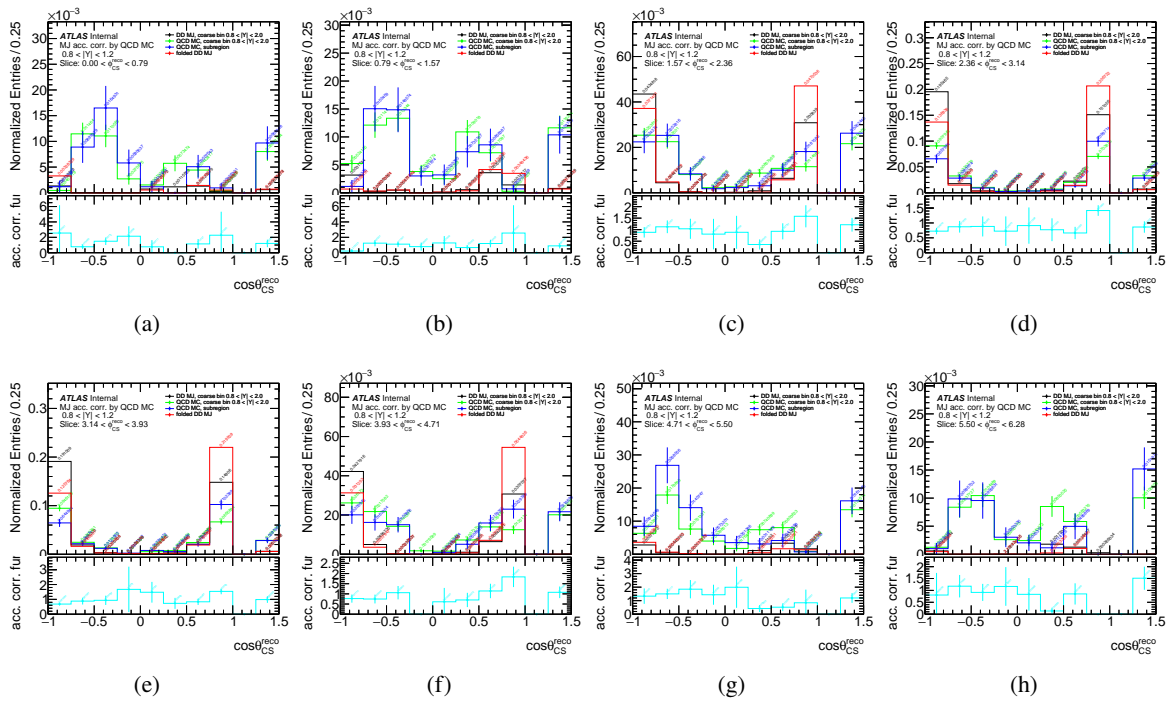


Figure 173: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

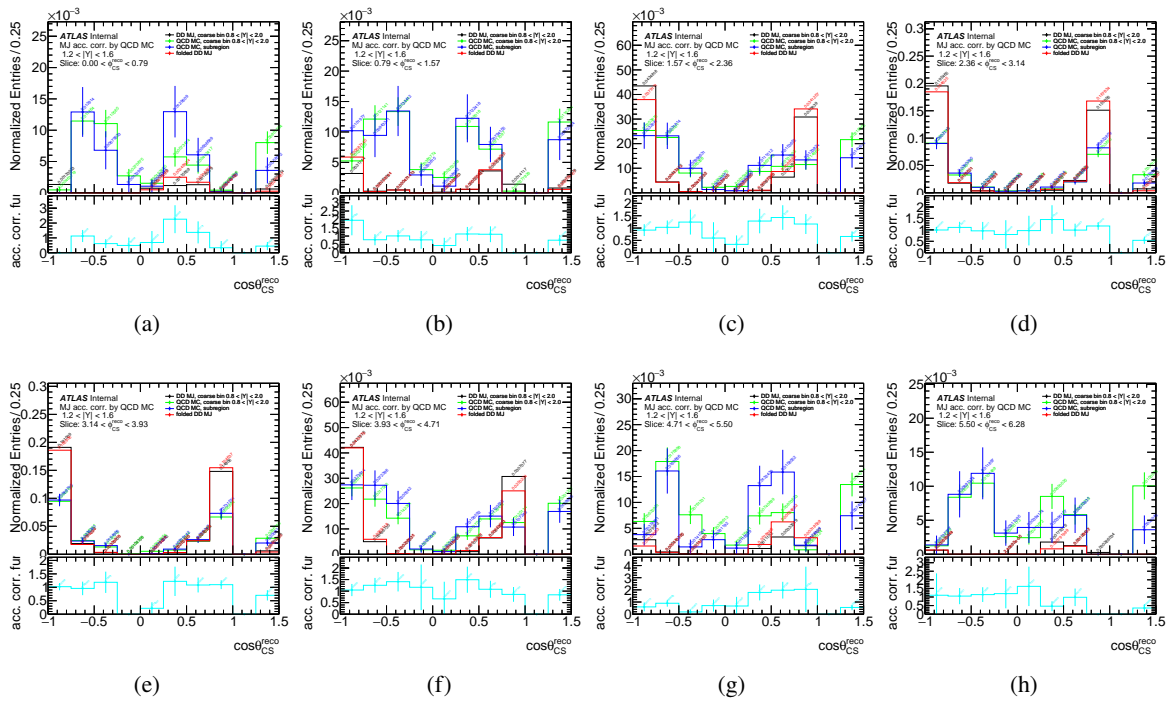


Figure 174: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

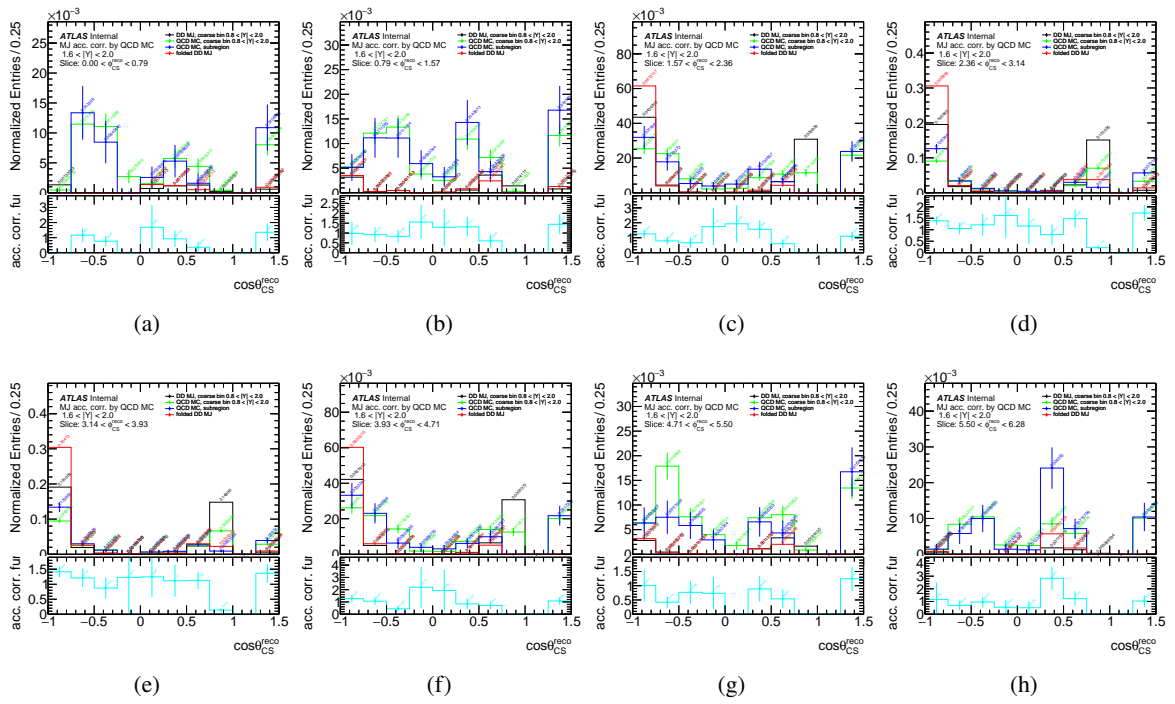


Figure 175: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

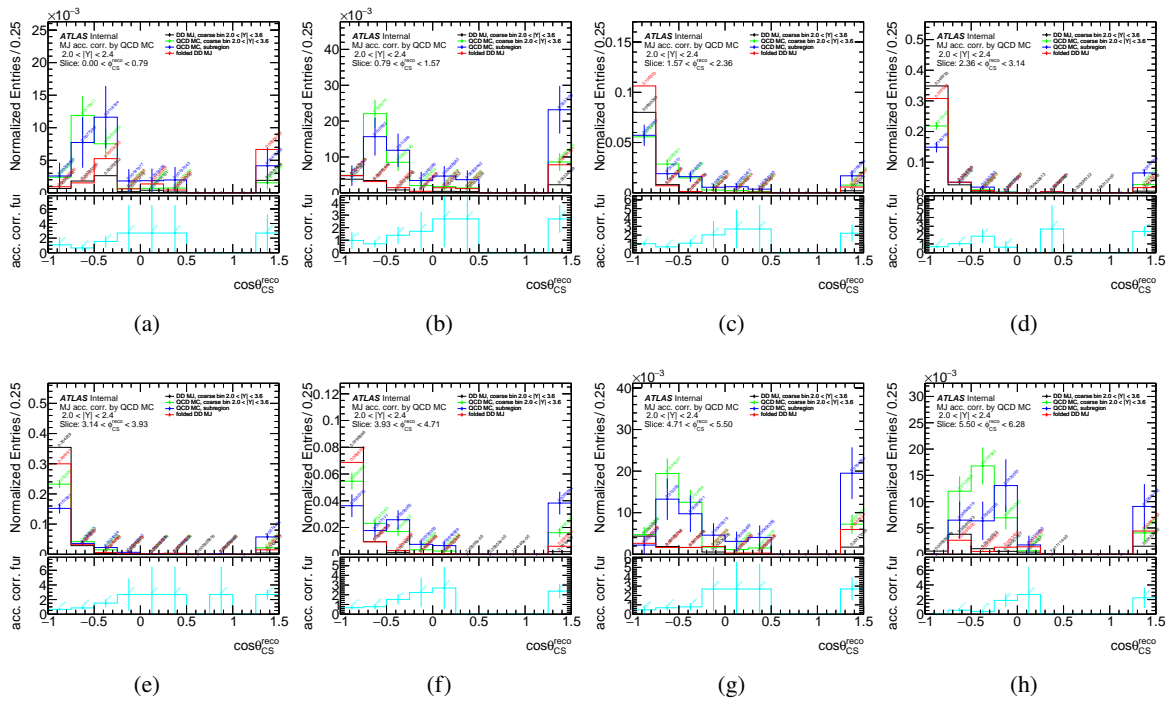


Figure 176: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

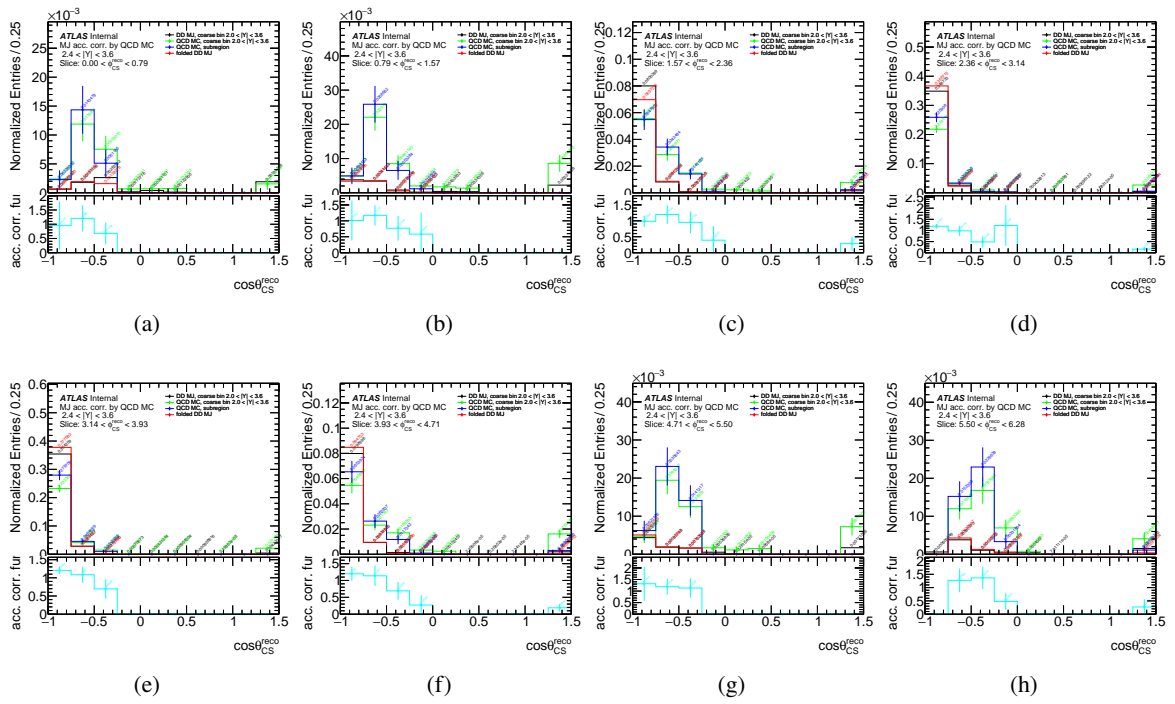


Figure 177: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

1284 **E.5.7 Positron channel multi-jet background 2D acceptance correction functions**

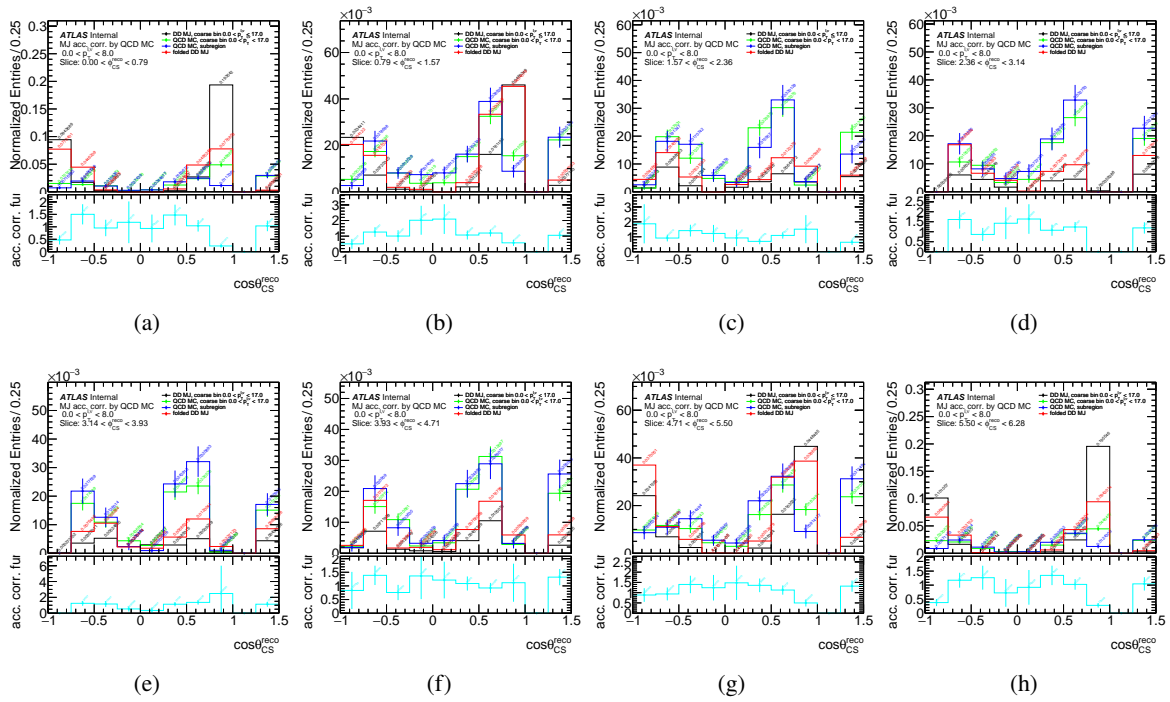


Figure 178: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

1285 **E.5.8 Muon channel multi-jet background 2D acceptance correction functions**

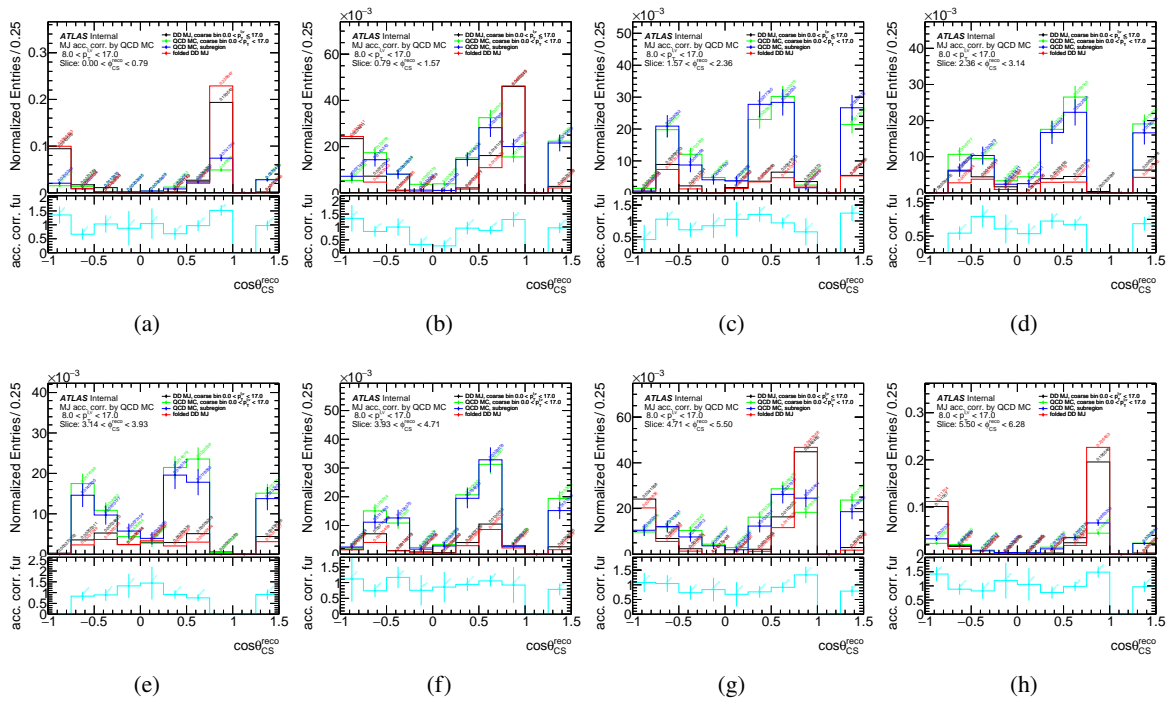


Figure 179: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

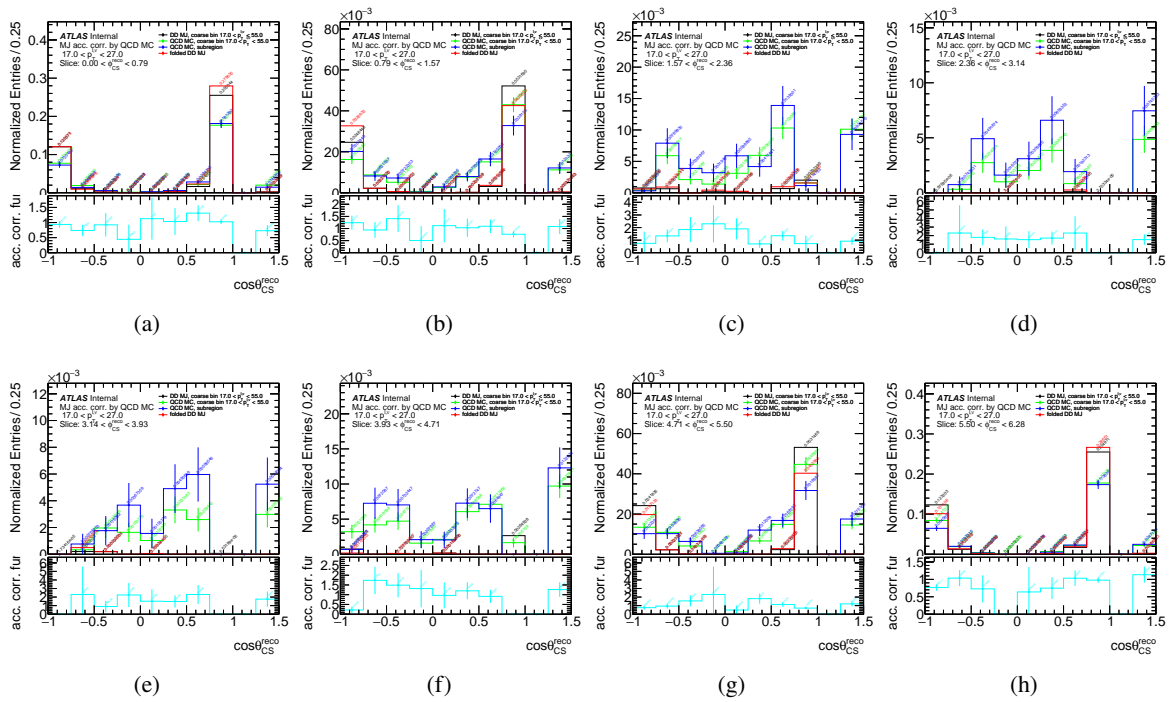


Figure 180: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

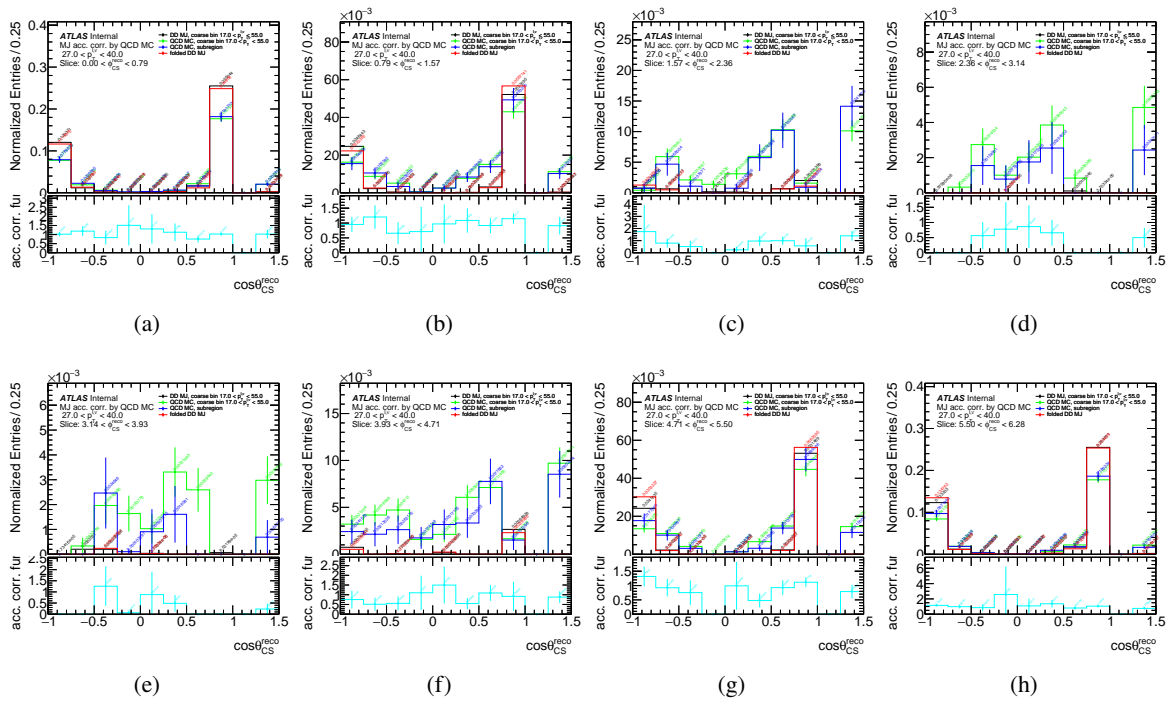


Figure 181: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

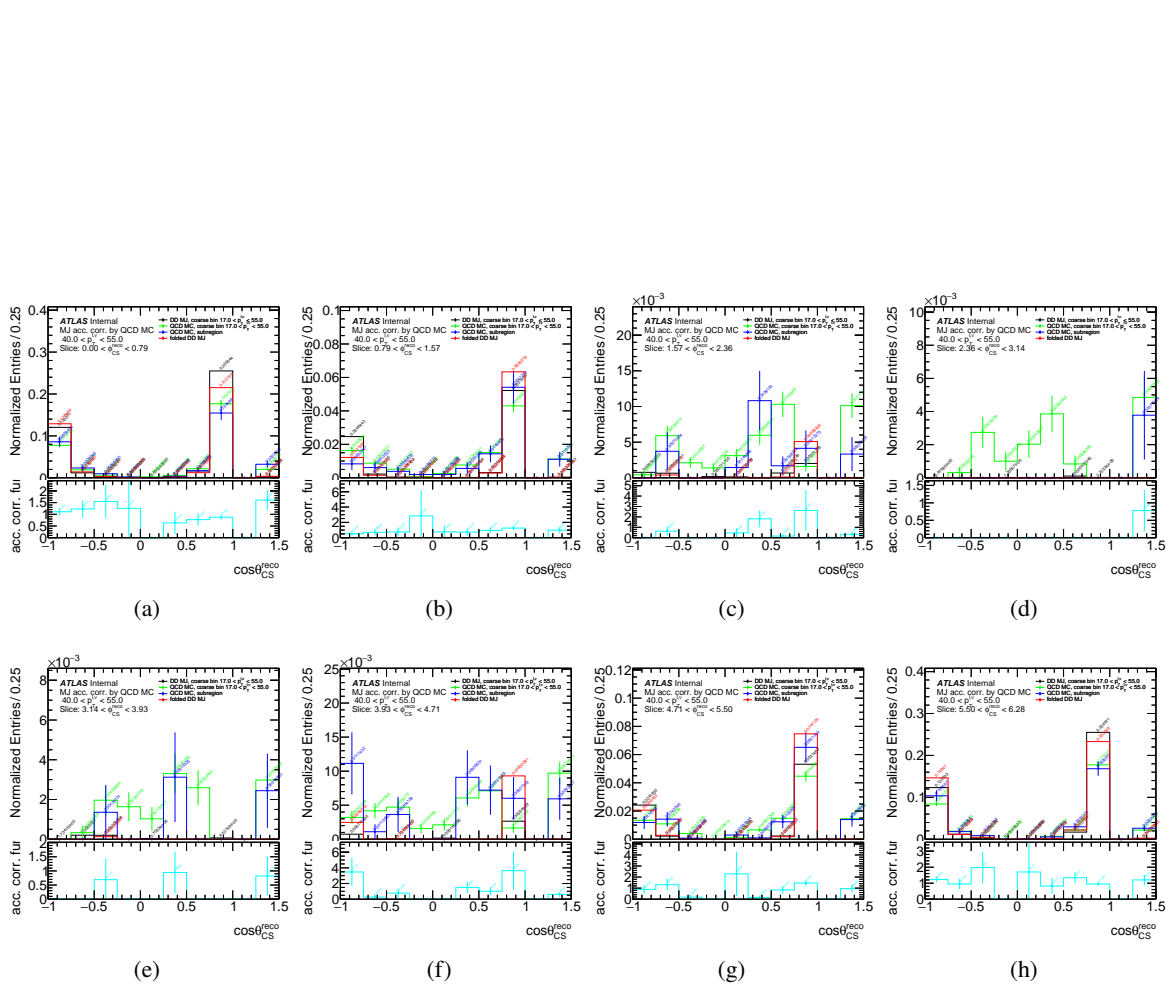


Figure 182: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

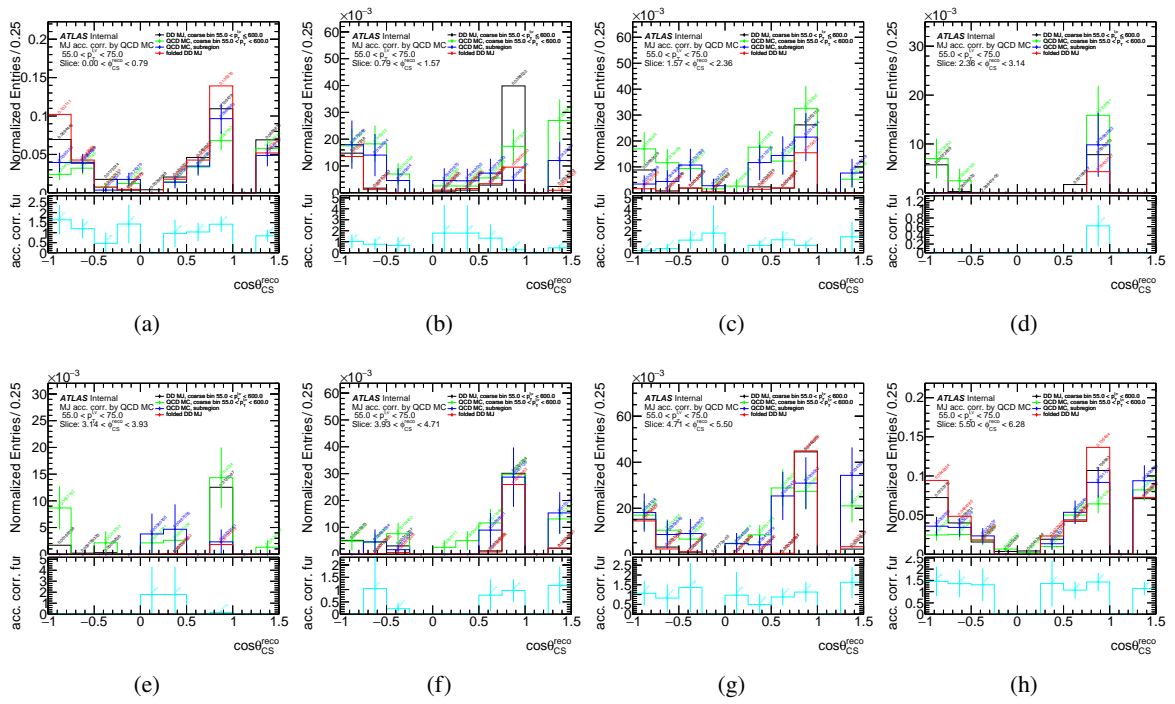


Figure 183: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

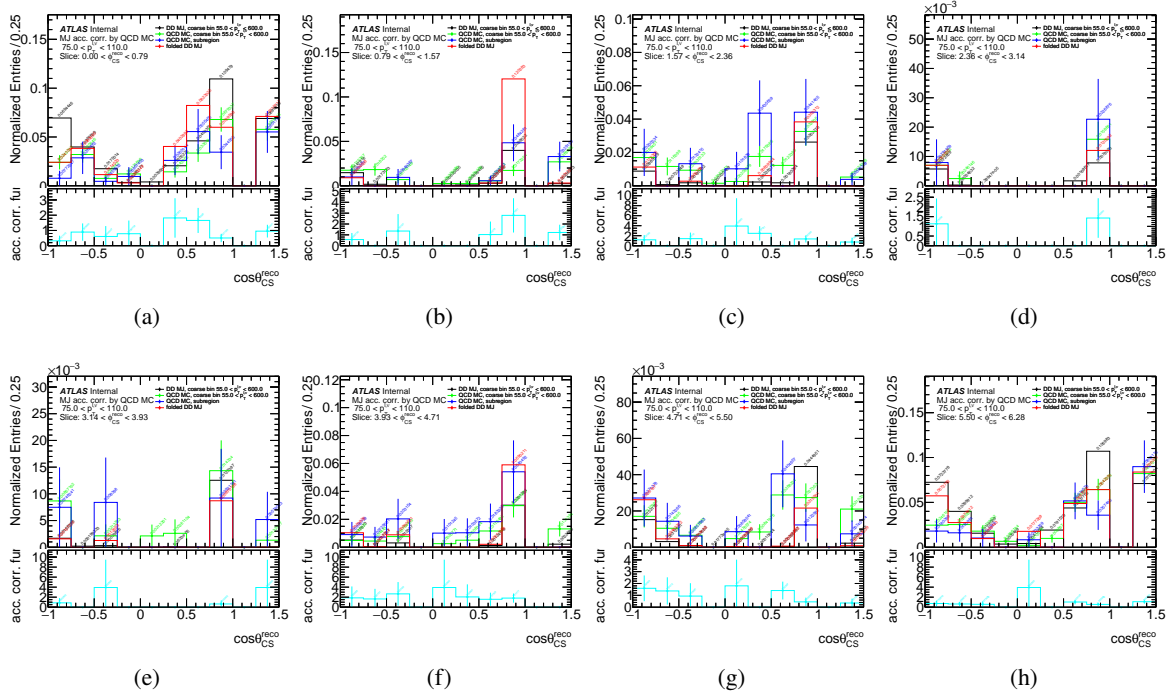


Figure 184: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

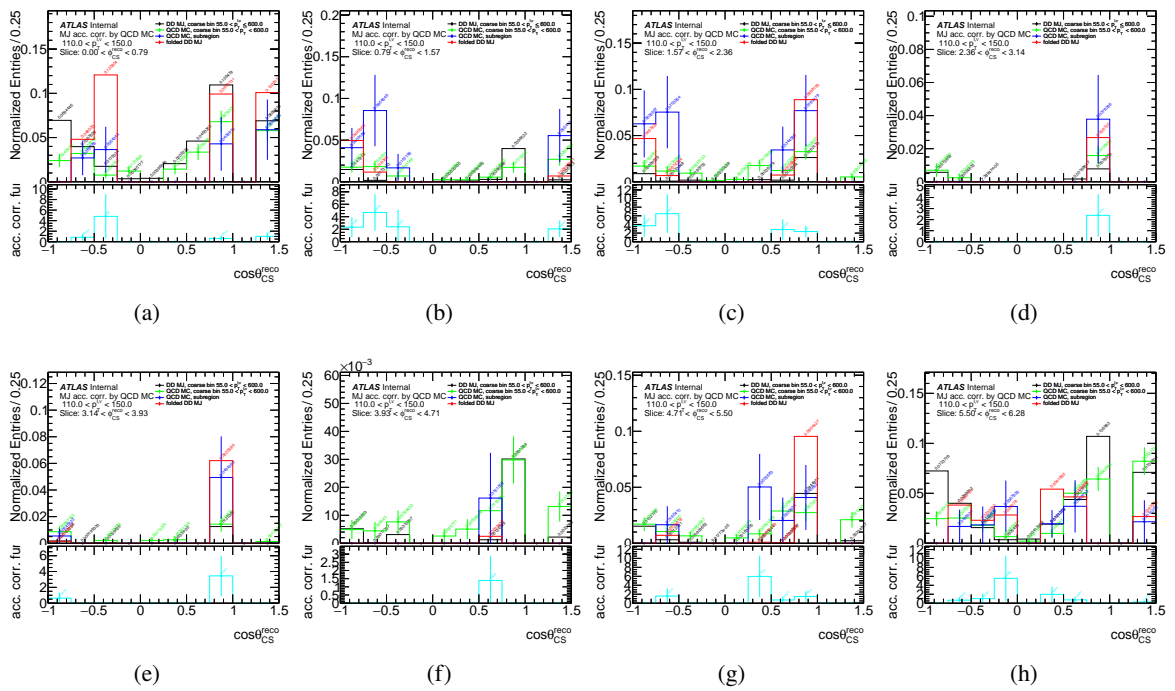


Figure 185: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

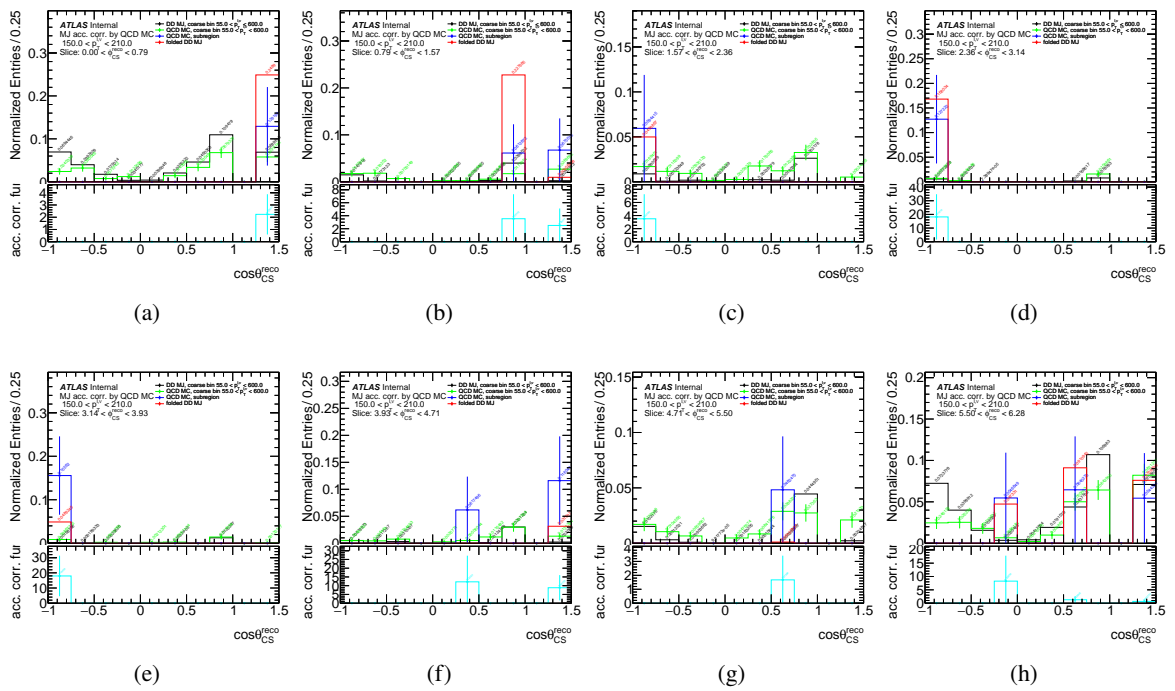


Figure 186: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

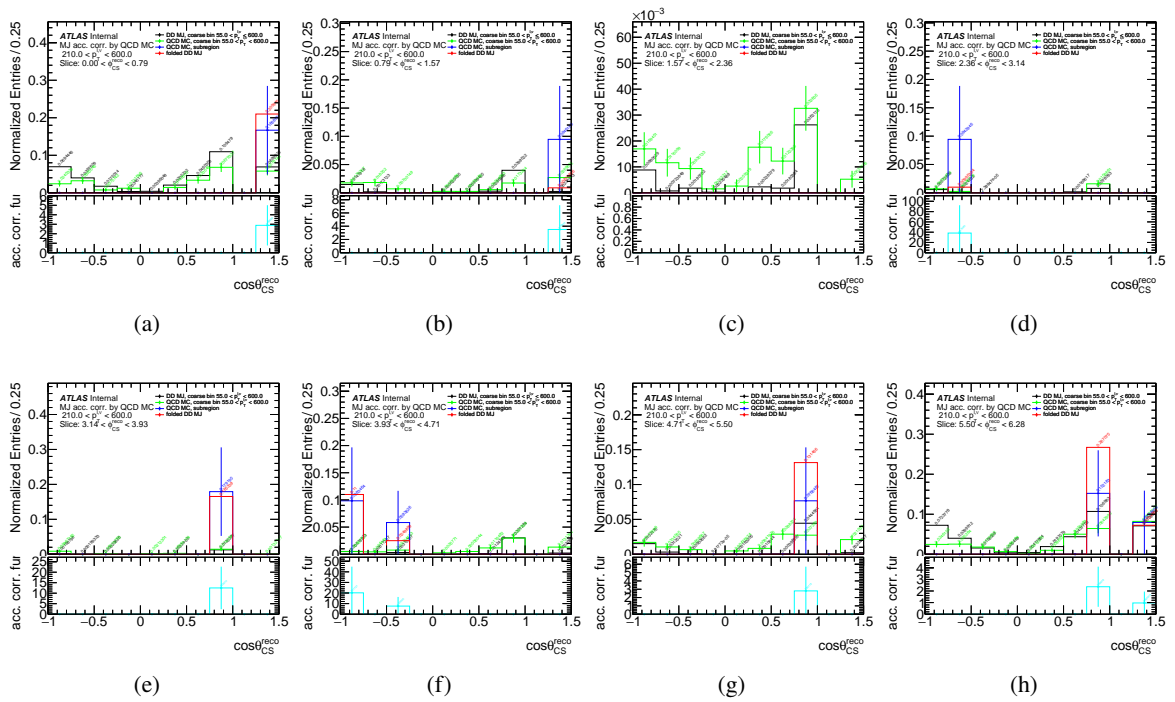


Figure 187: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

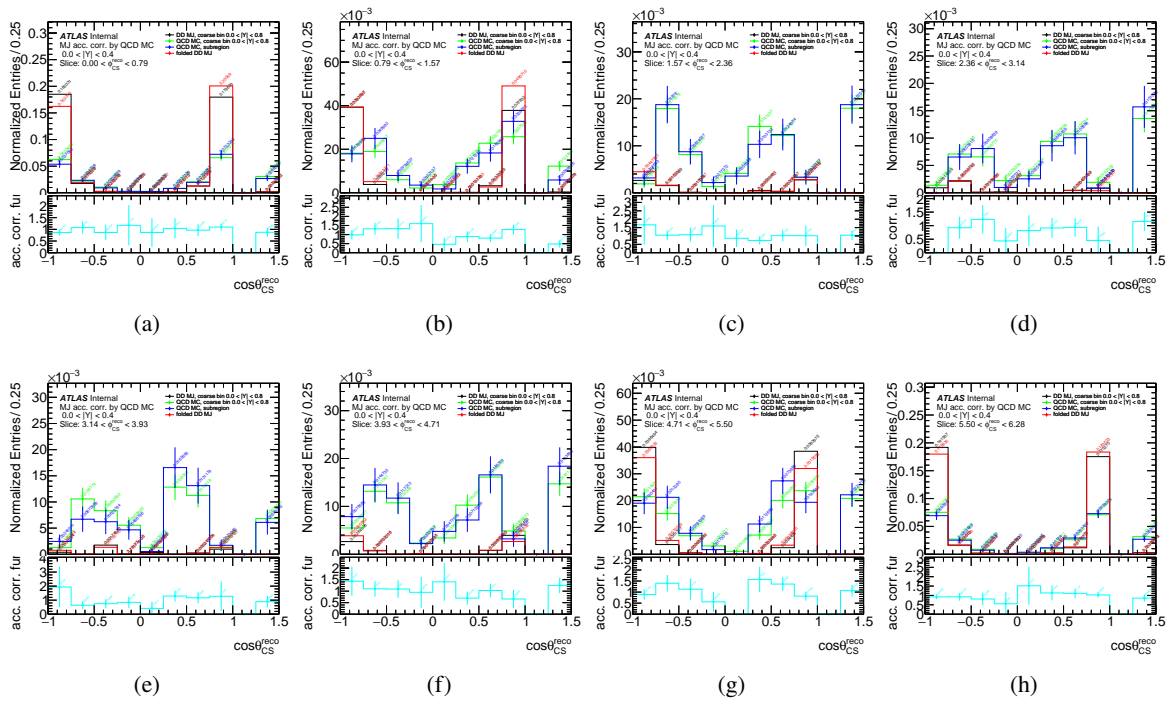


Figure 188: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

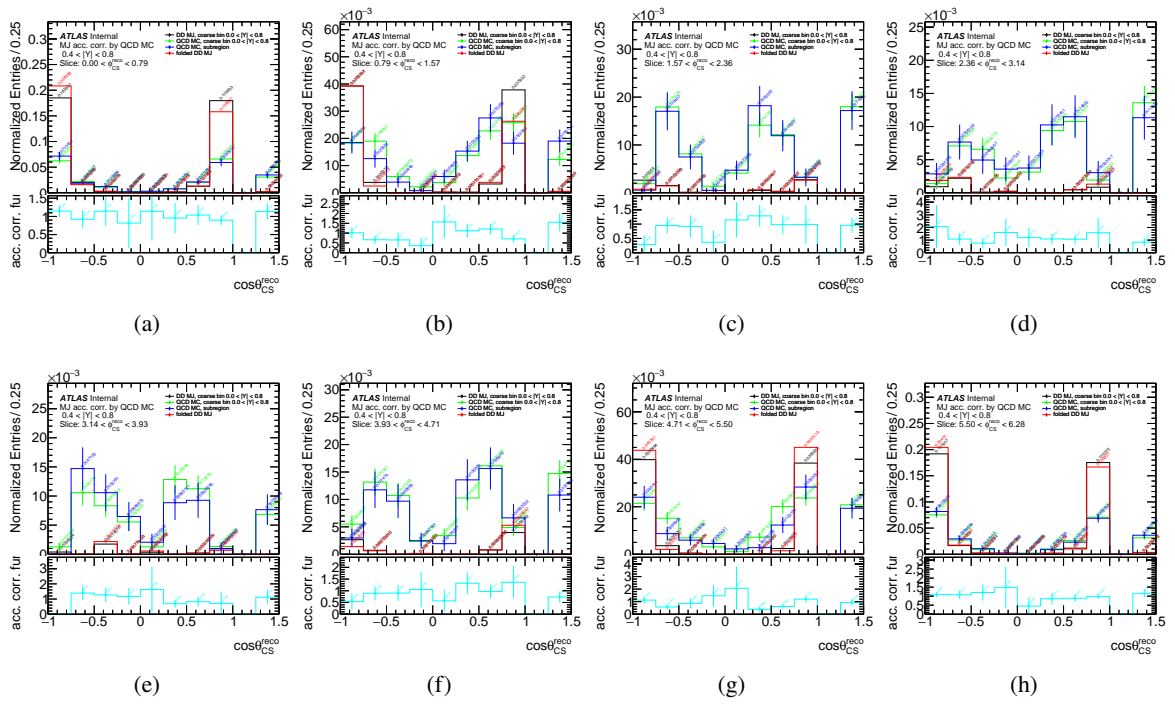


Figure 189: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

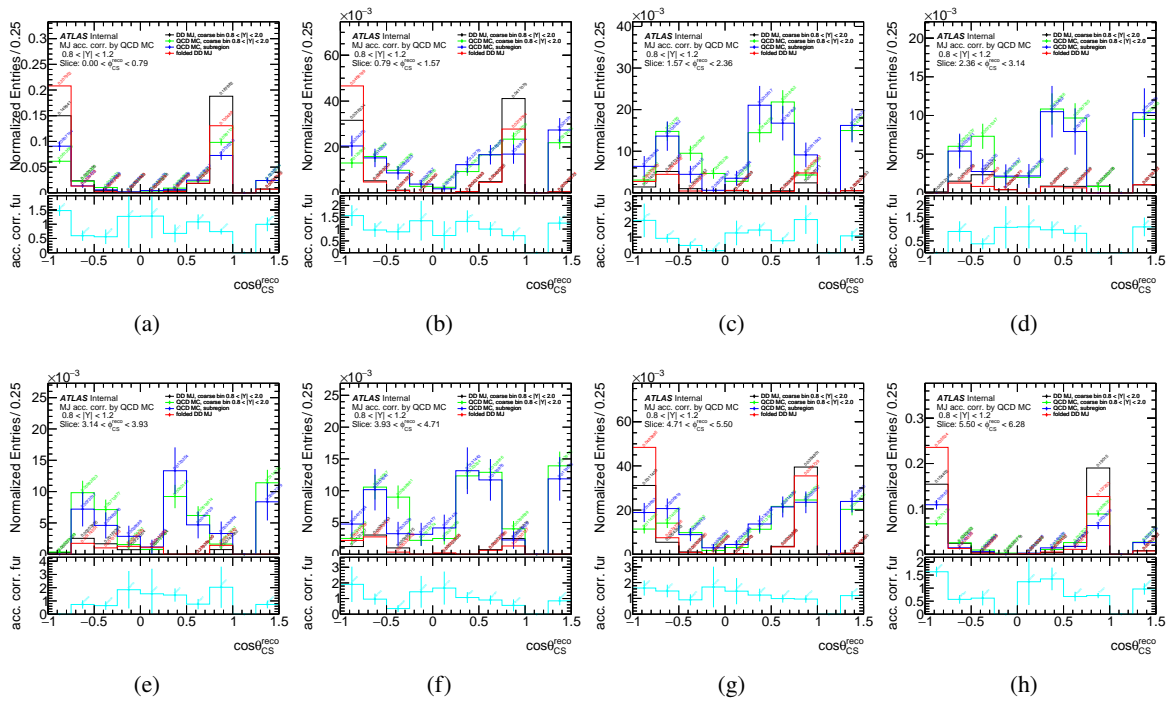


Figure 190: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

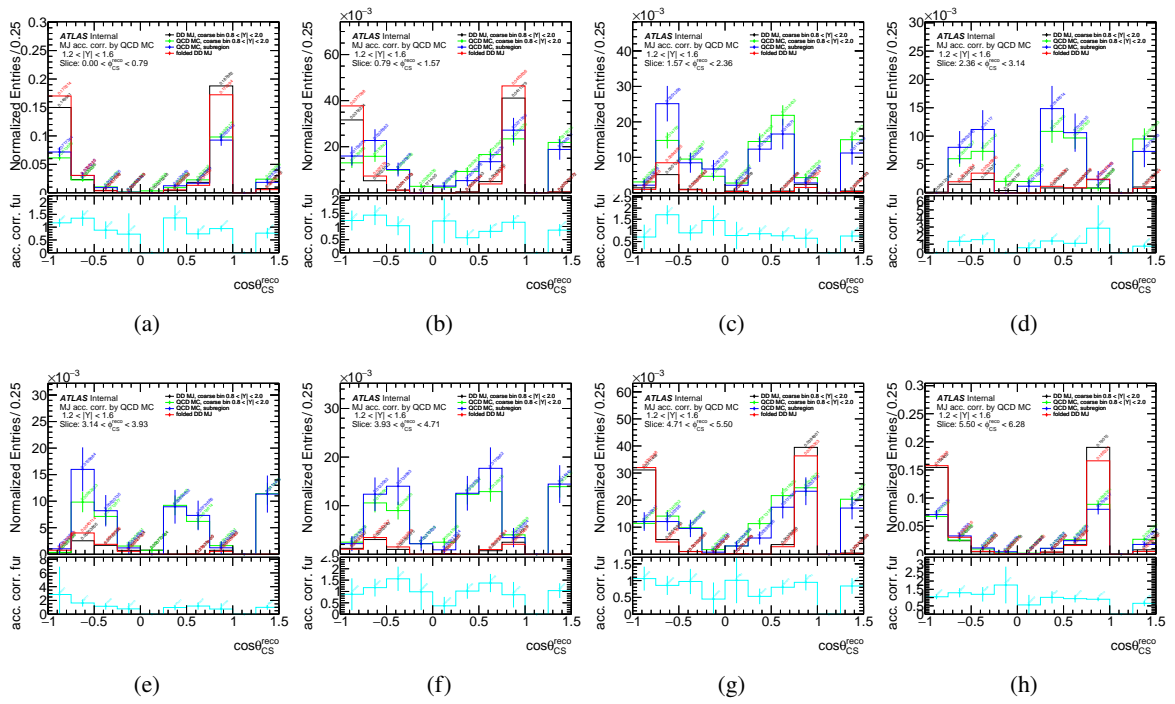


Figure 191: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

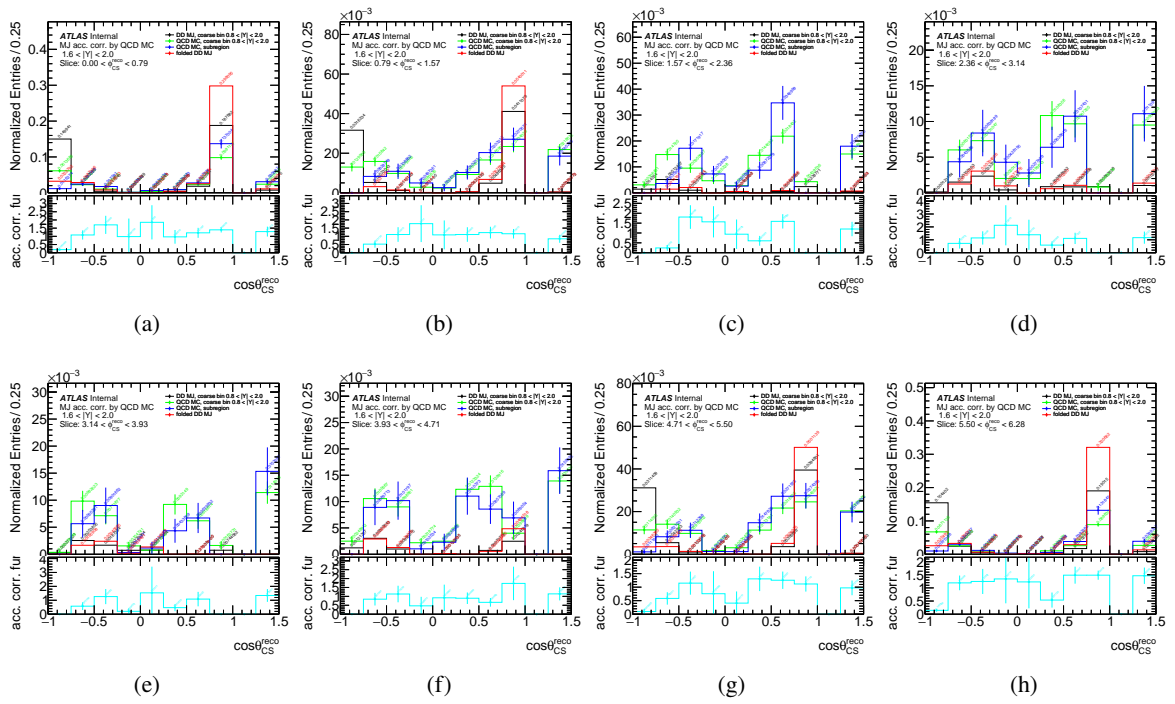


Figure 192: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

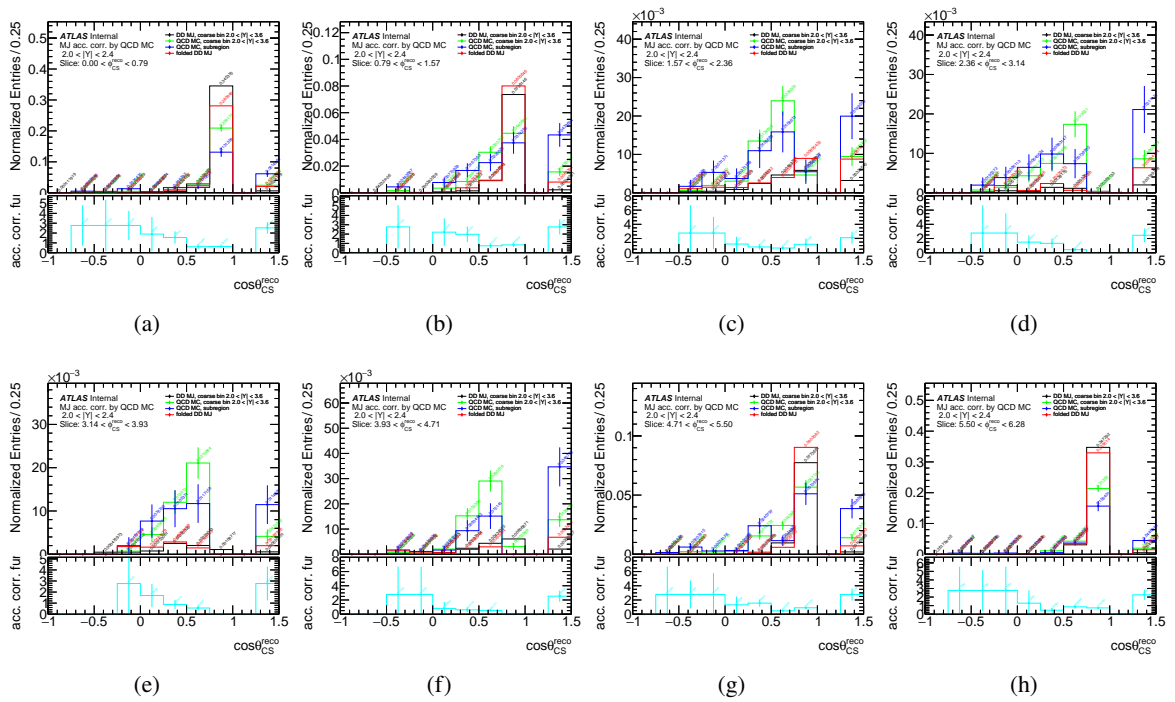


Figure 193: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow e^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

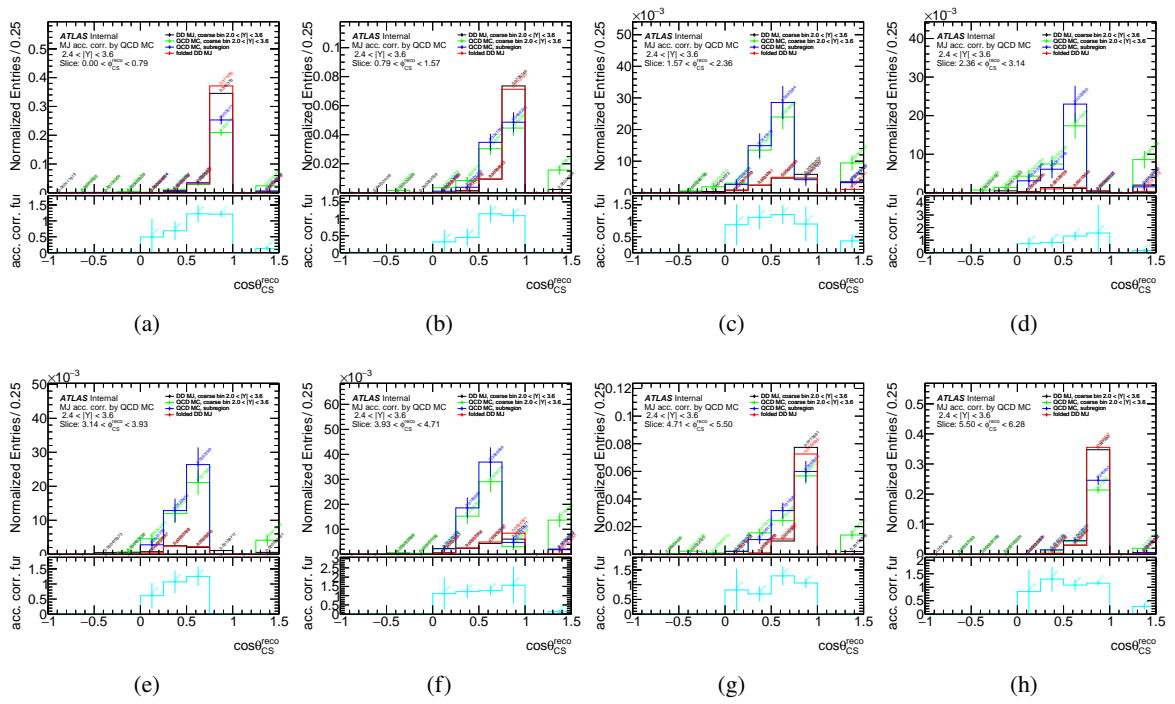


Figure 194: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow e^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

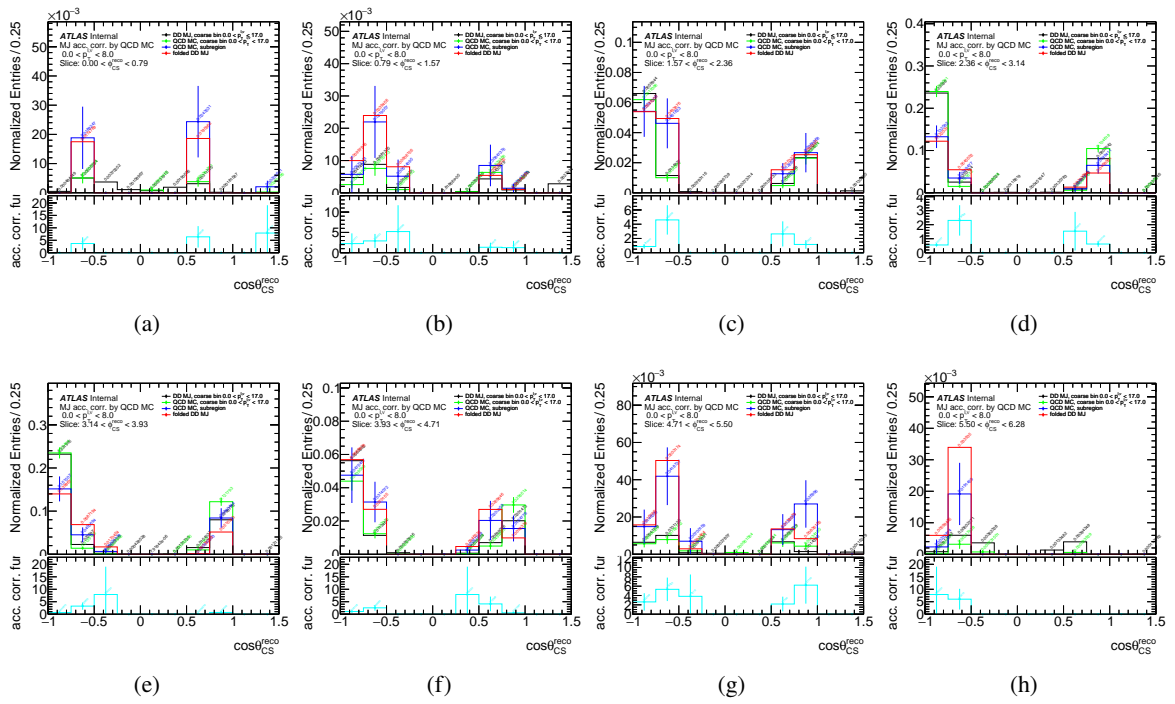


Figure 195: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

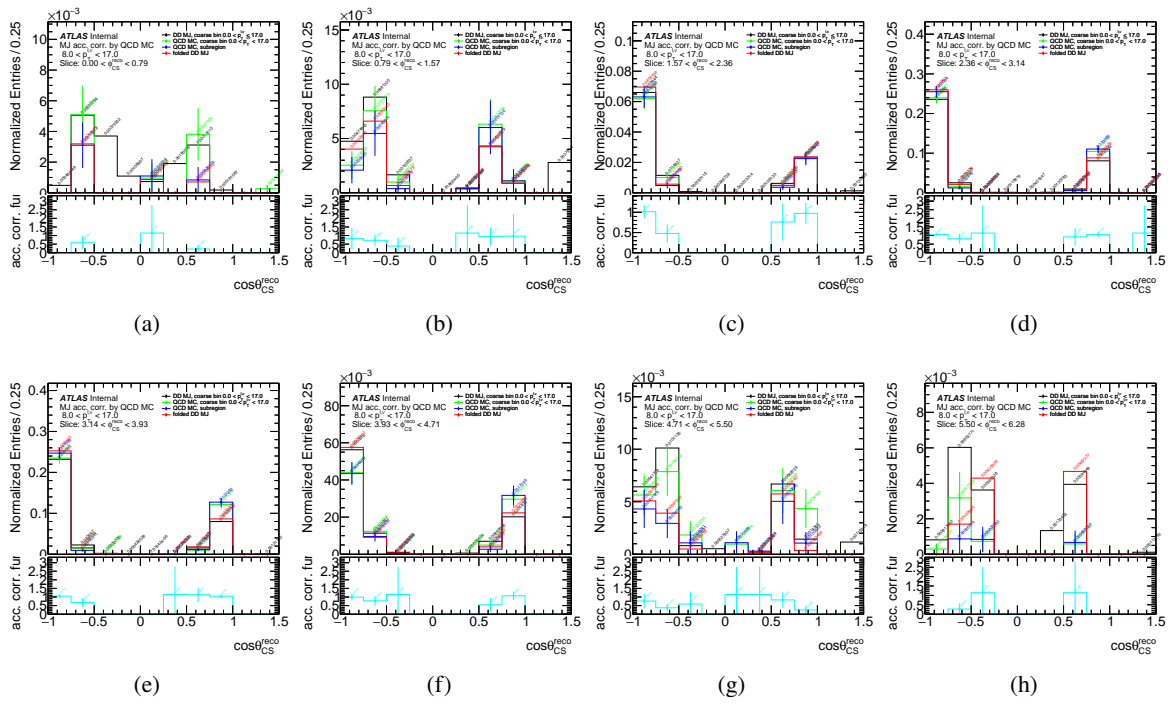


Figure 196: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

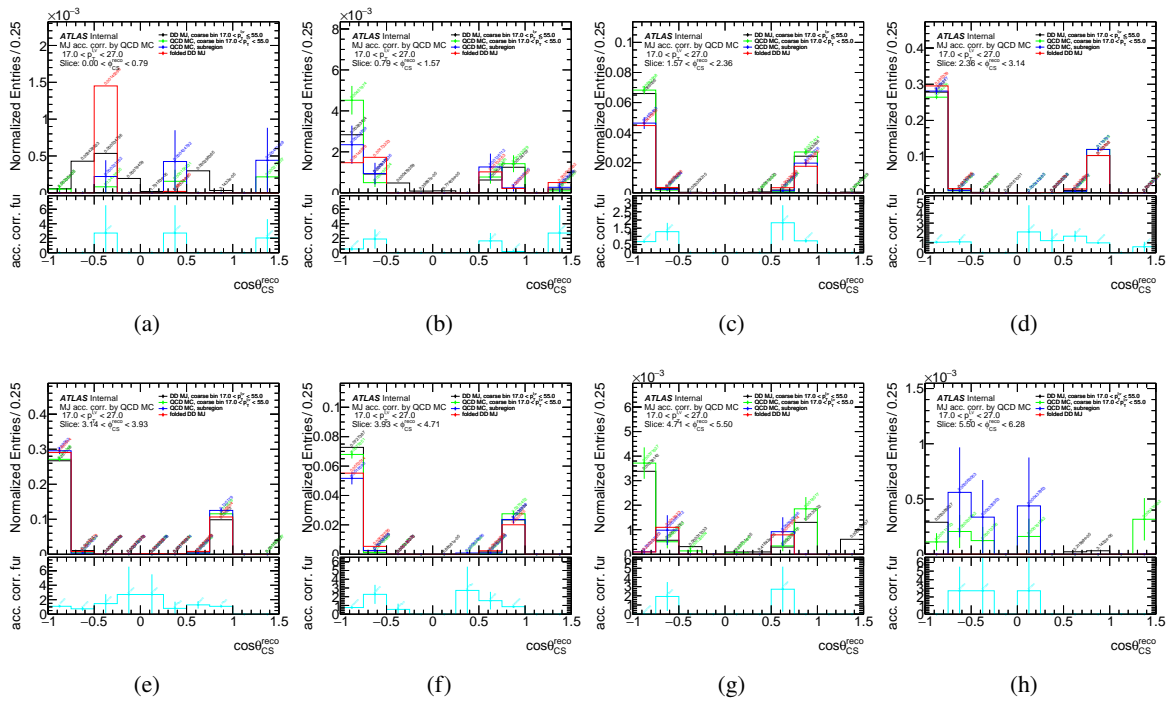


Figure 197: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

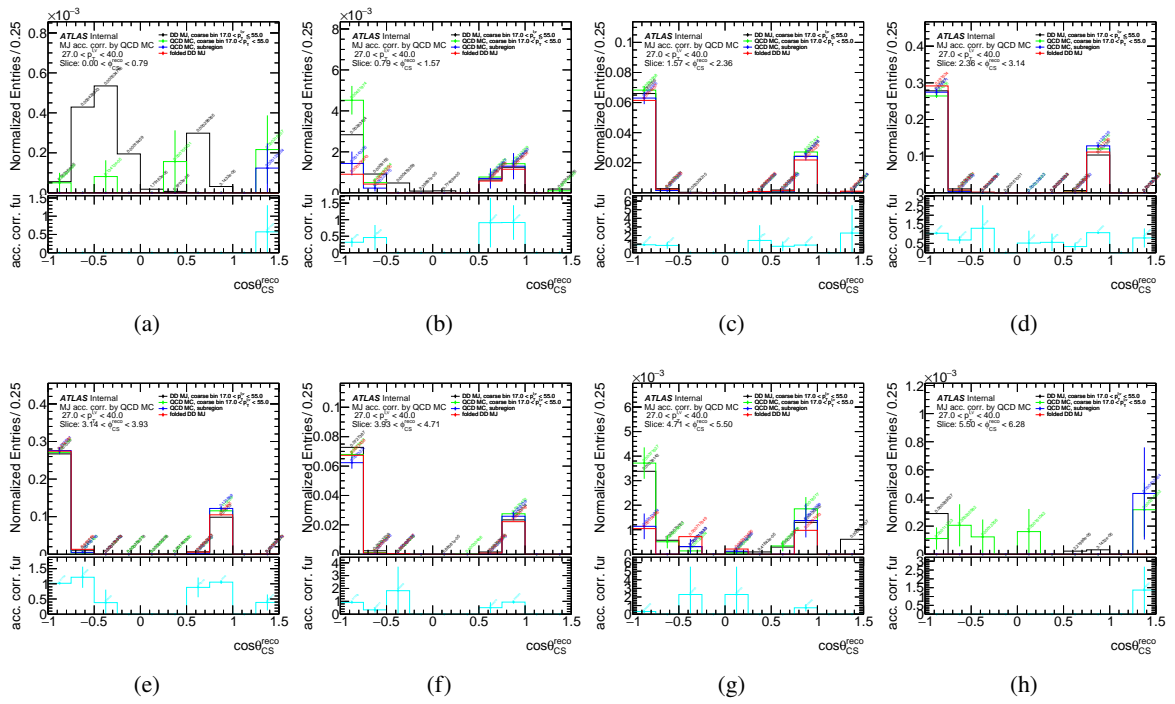


Figure 198: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

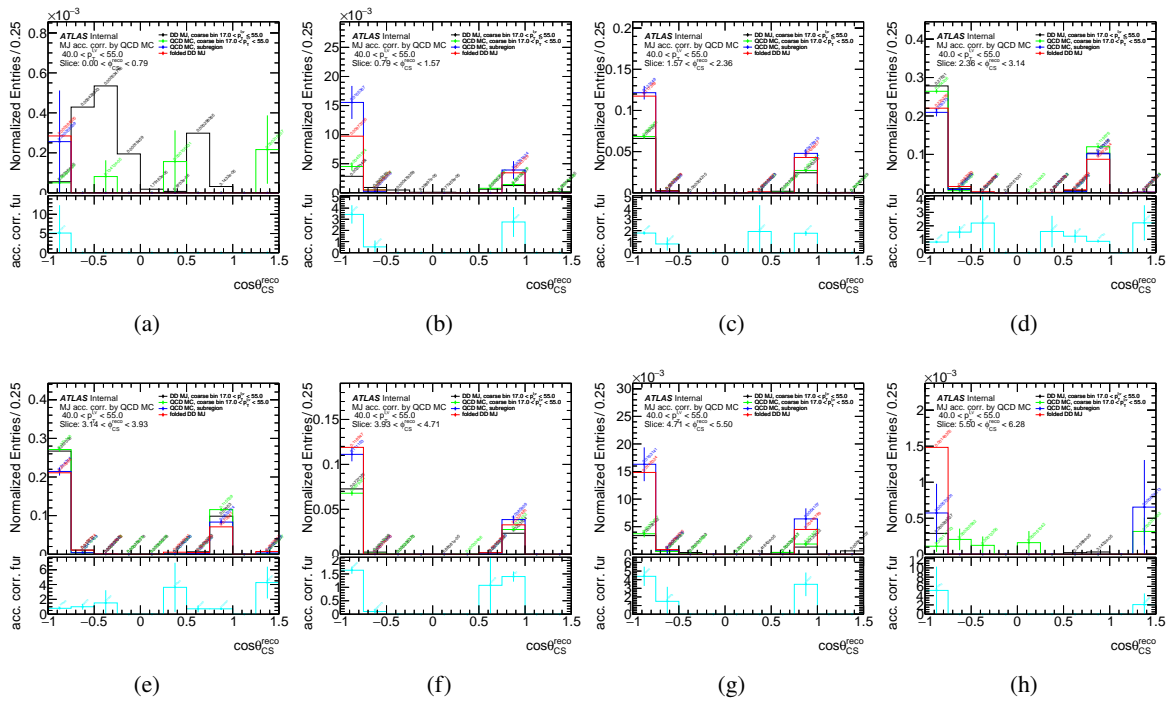


Figure 199: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

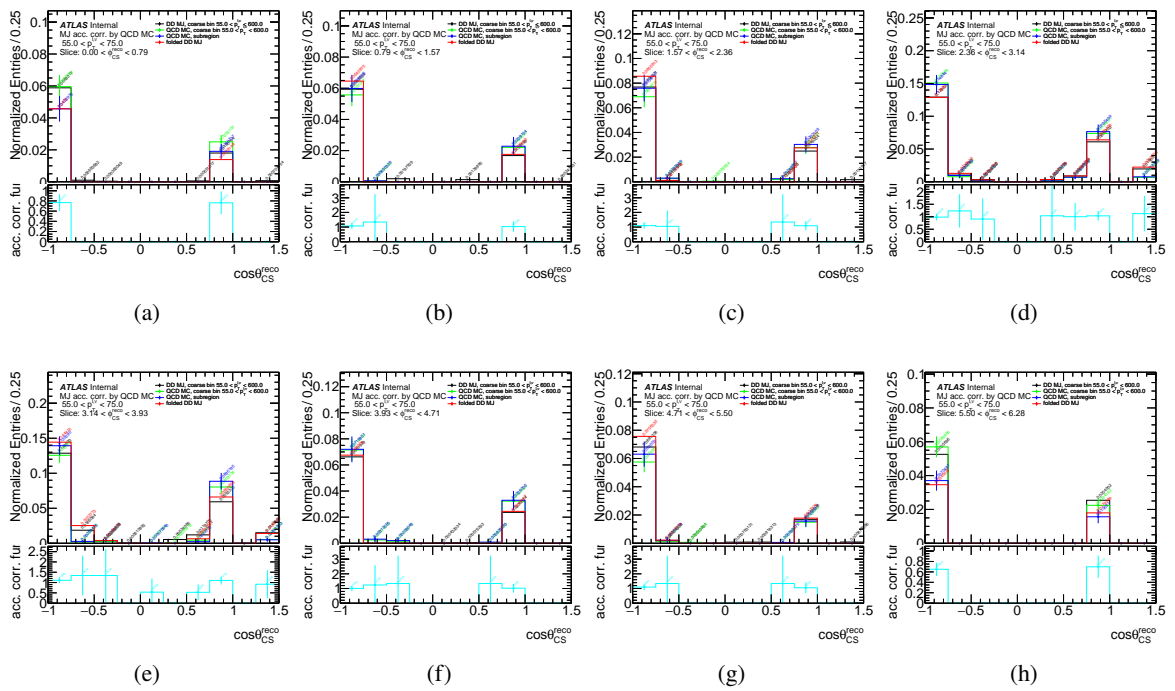


Figure 200: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

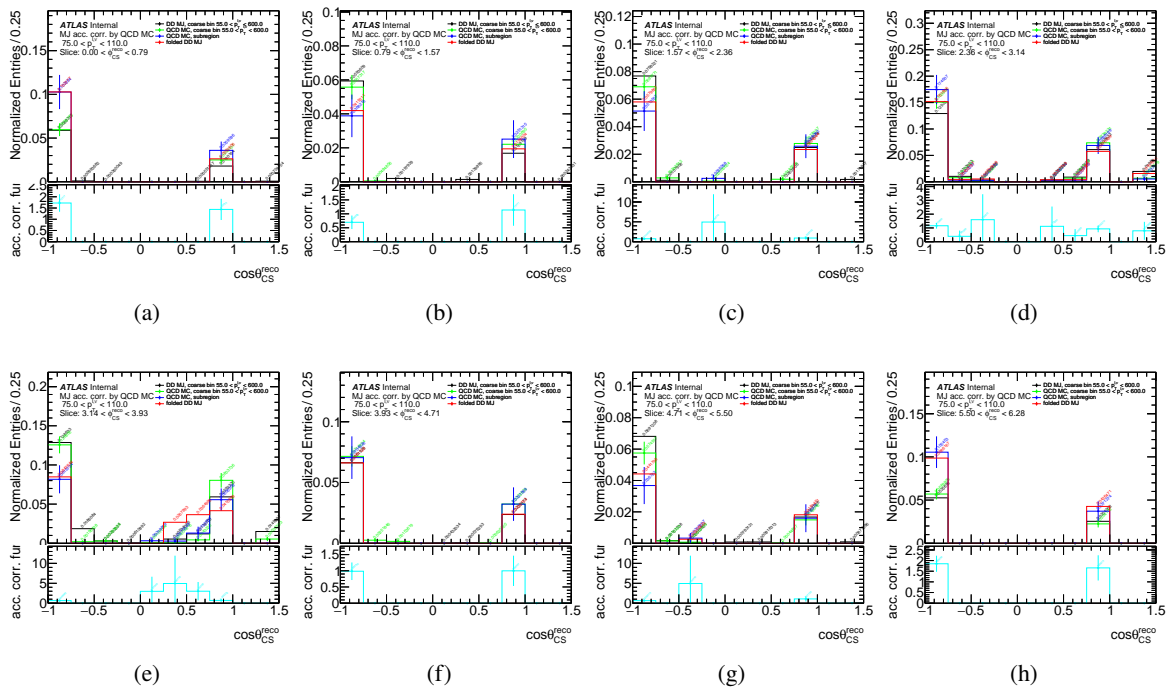


Figure 201: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

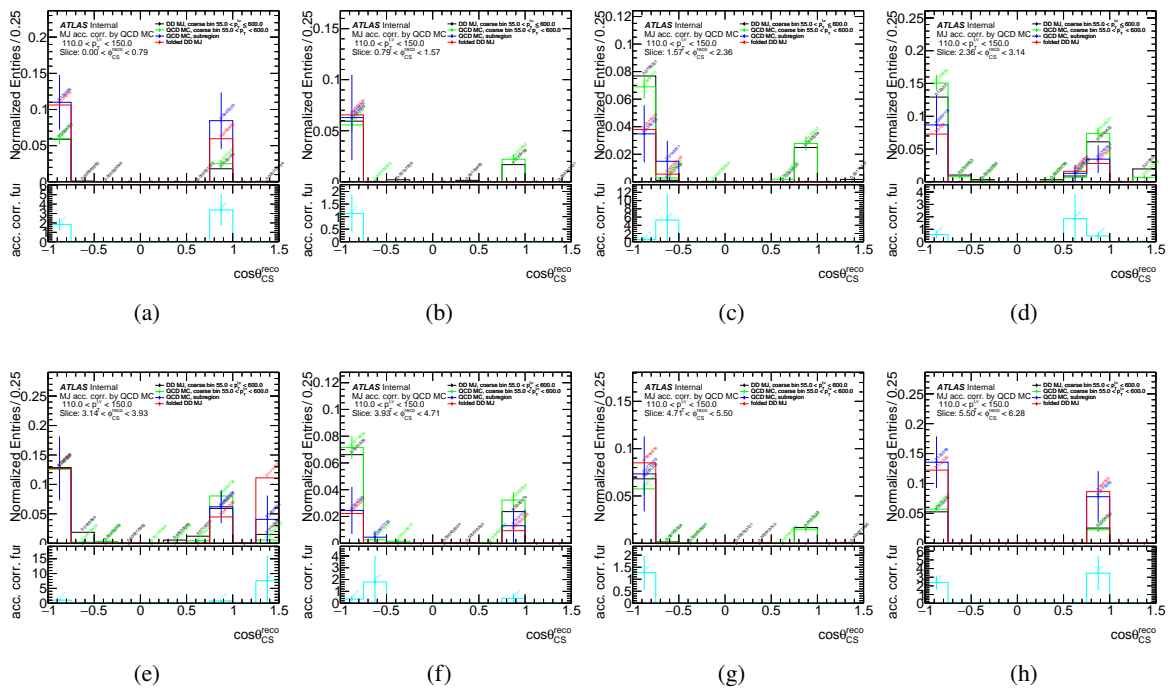


Figure 202: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

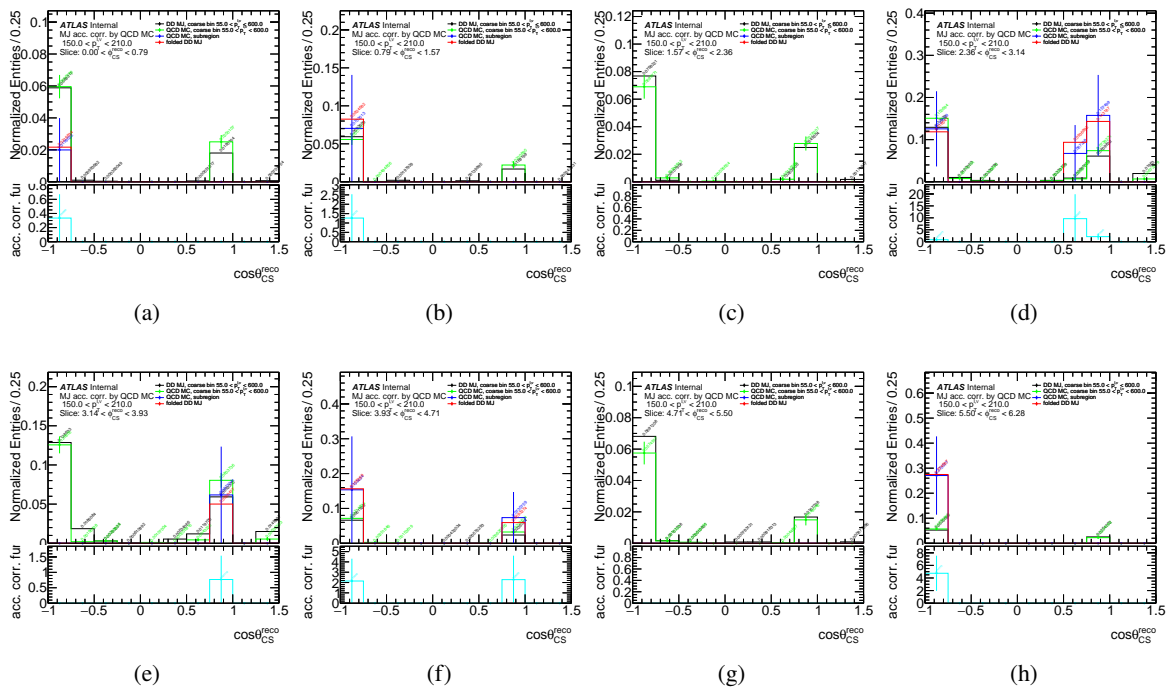


Figure 203: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

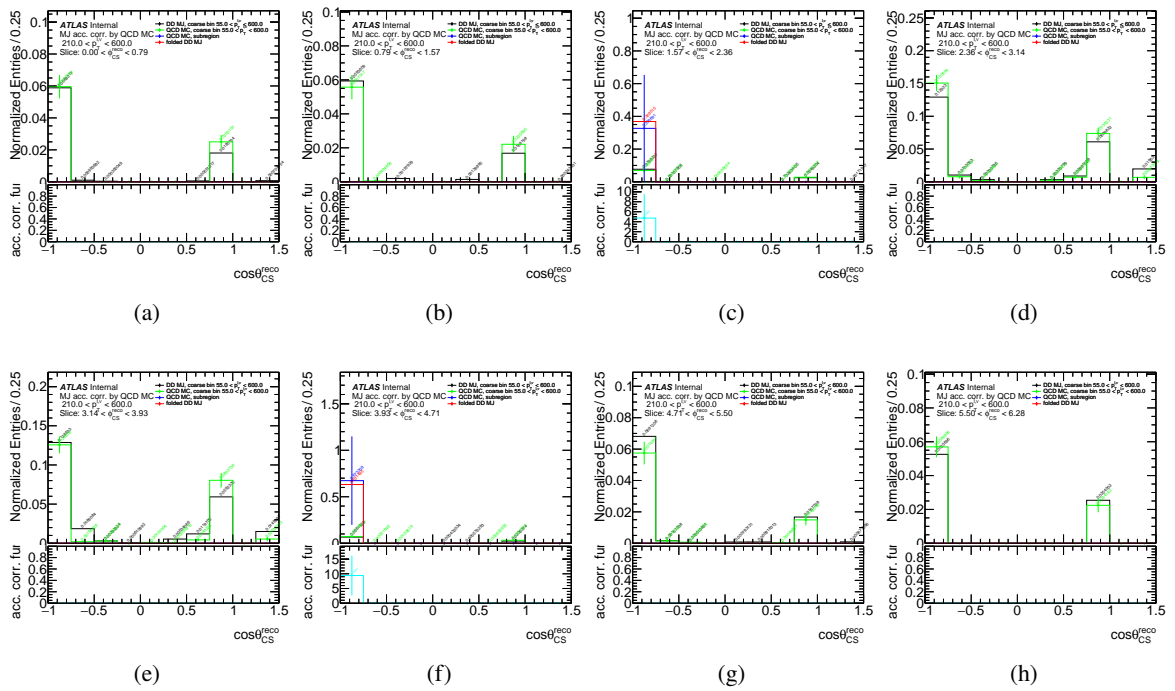


Figure 204: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

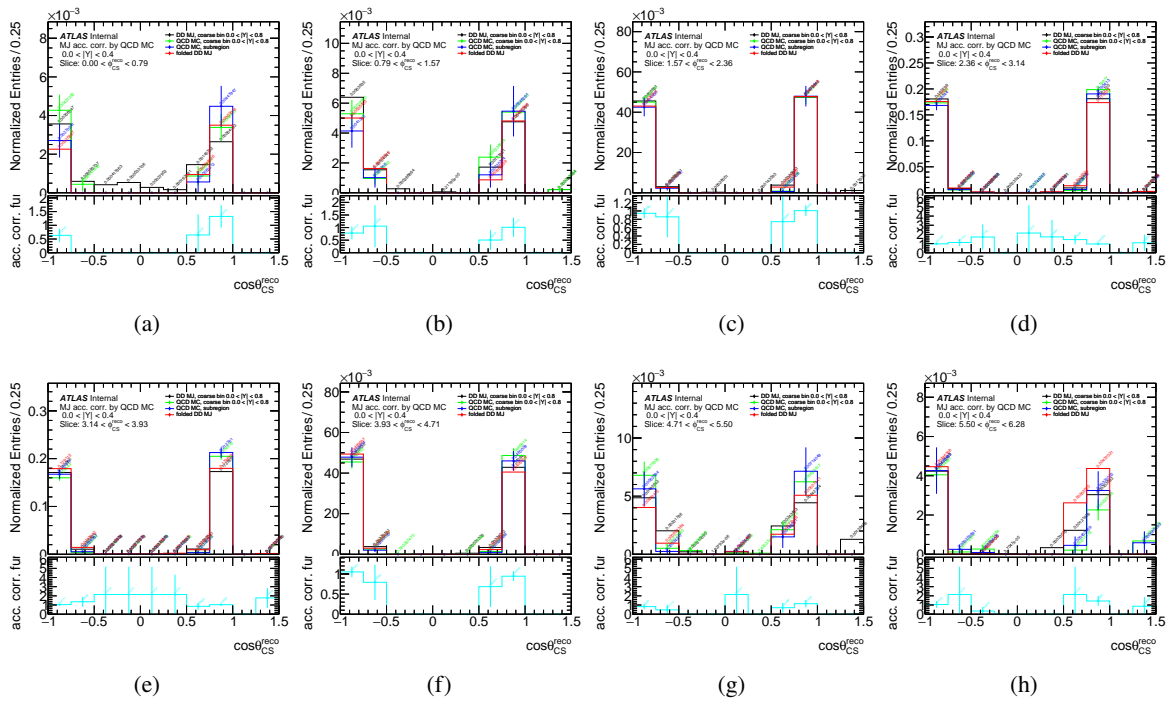


Figure 205: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

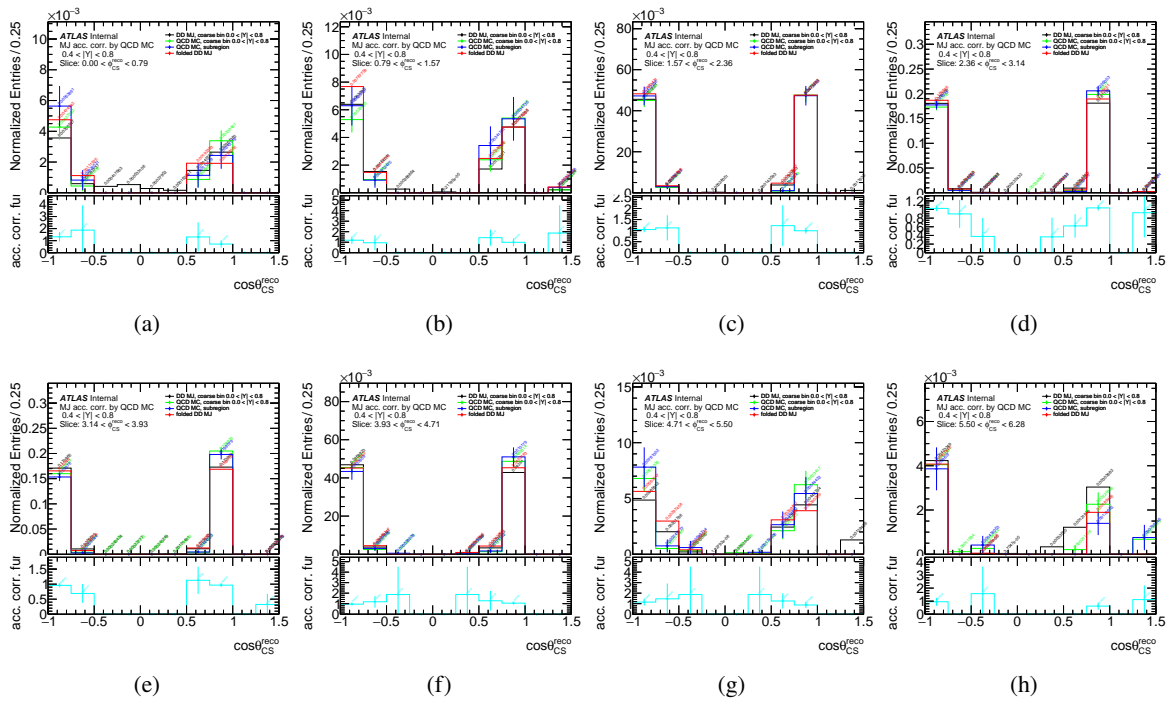


Figure 206: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

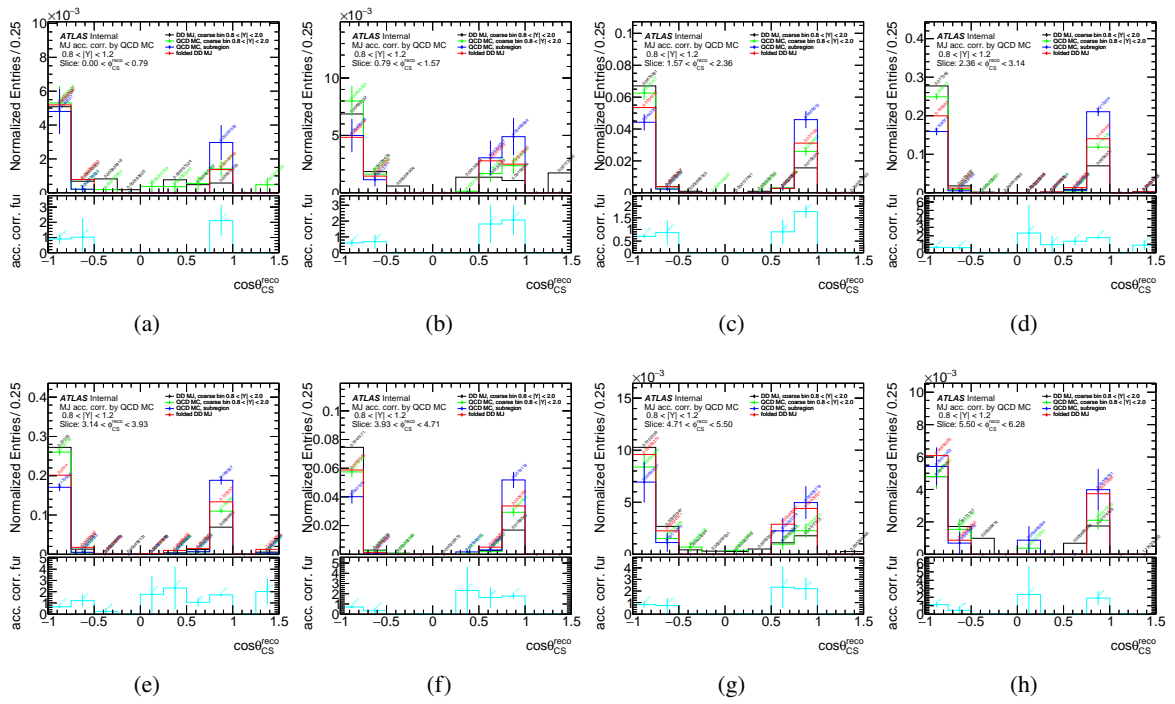


Figure 207: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

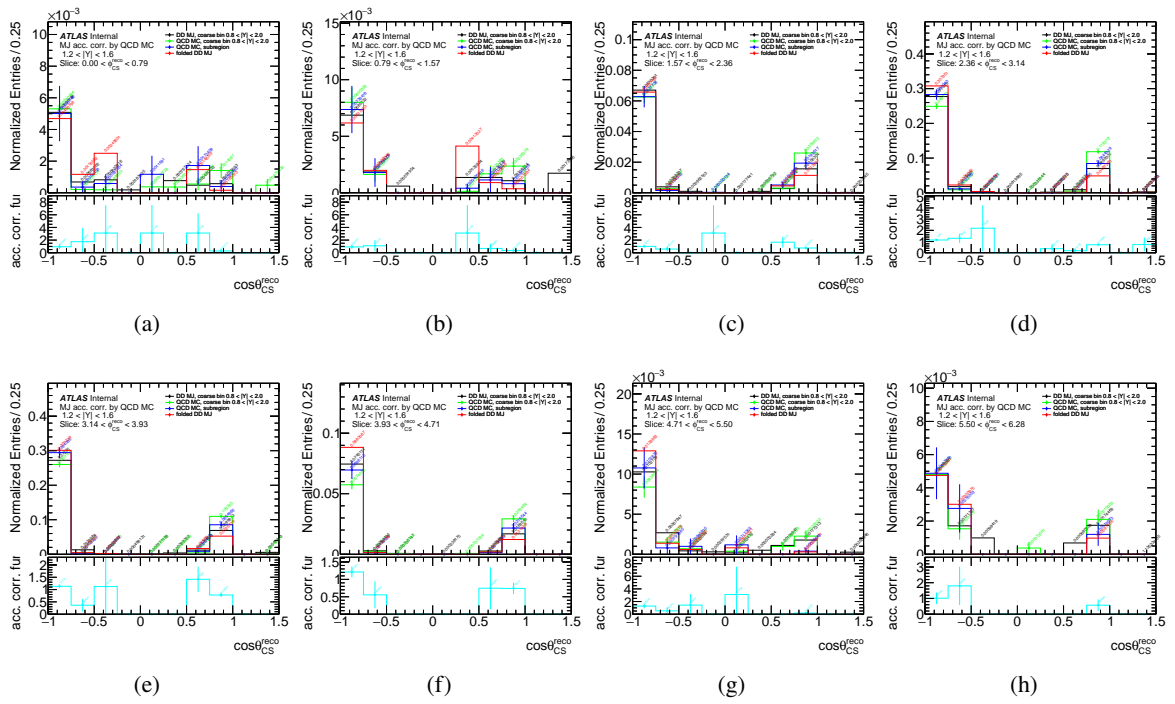


Figure 208: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

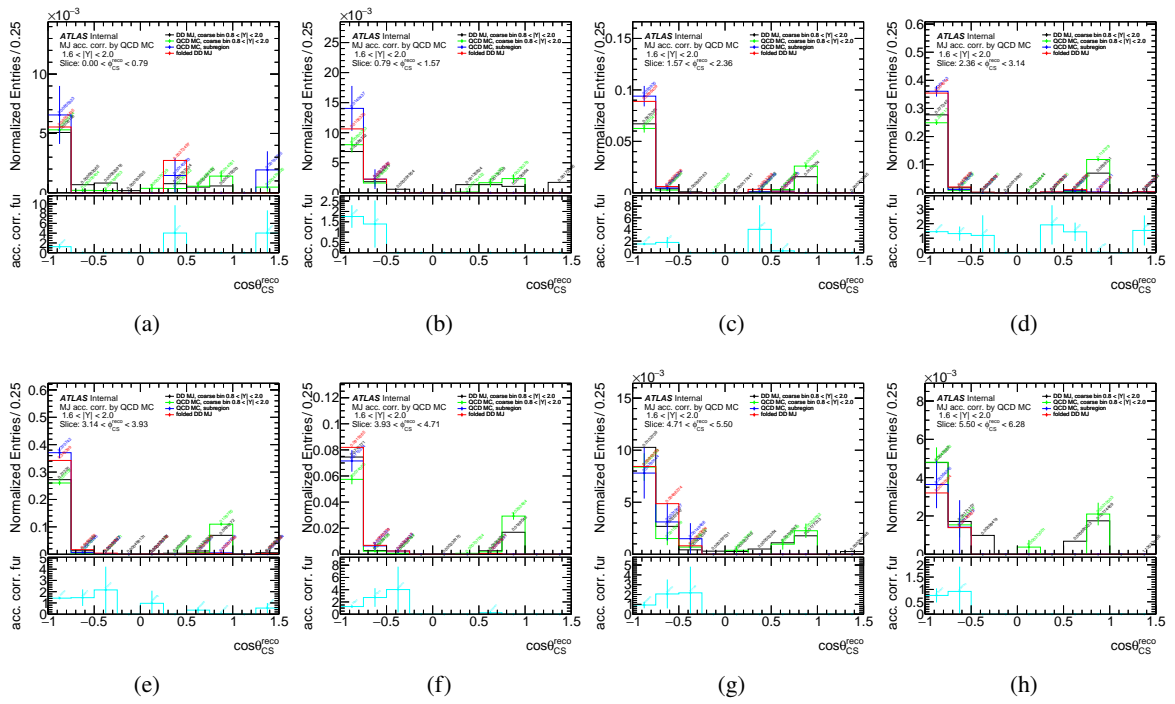


Figure 209: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

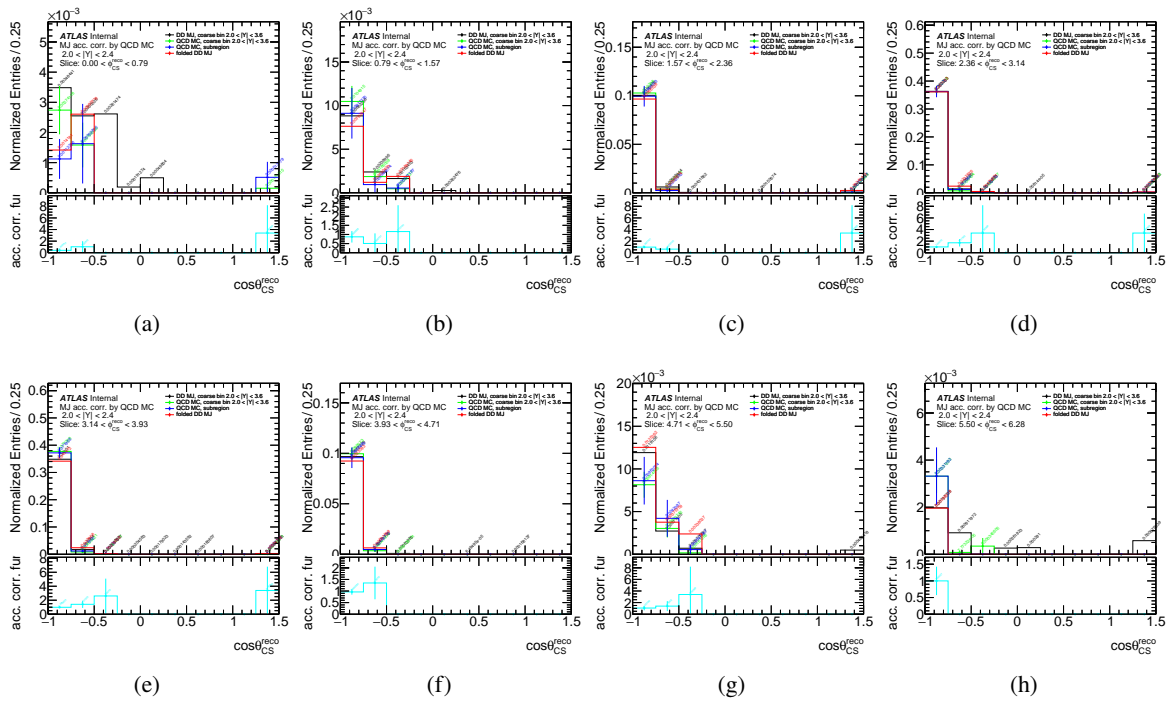


Figure 210: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

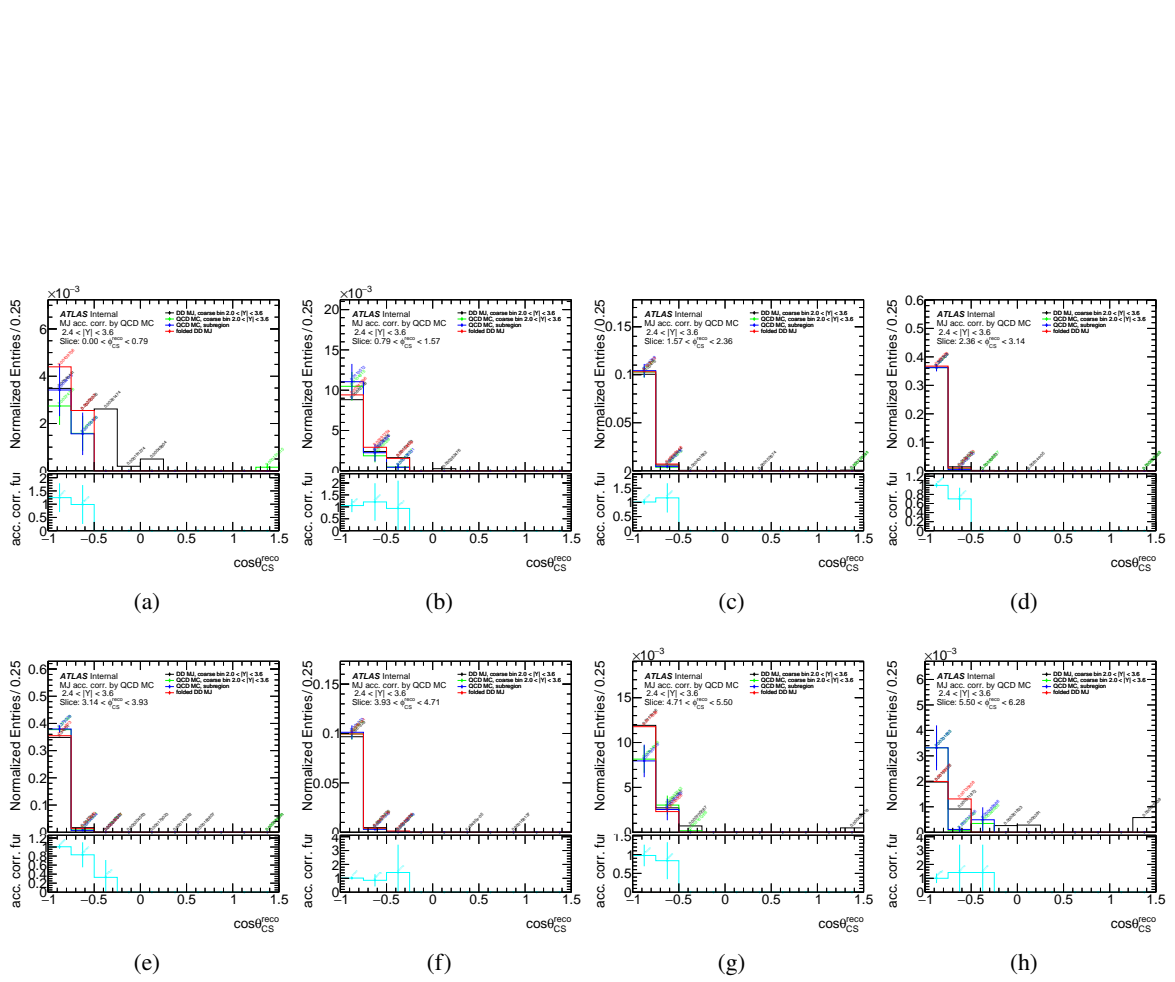


Figure 211: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

E.5.9 Anti-muon channel multi-jet background 2D acceptance correction functions

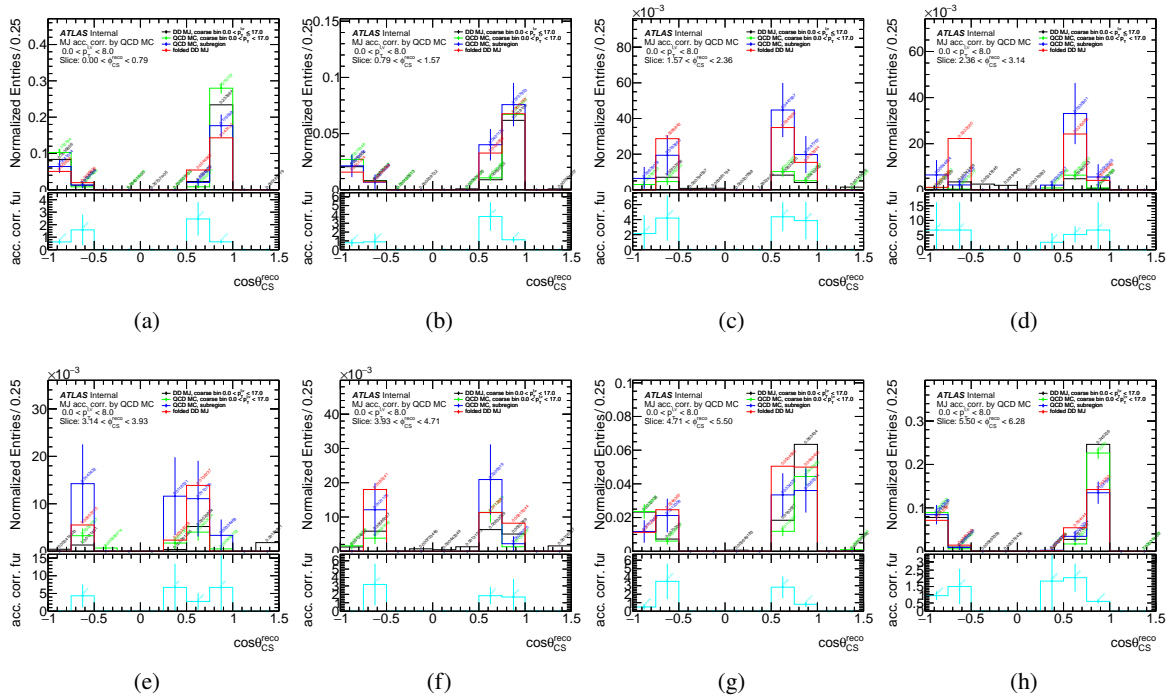


Figure 212: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

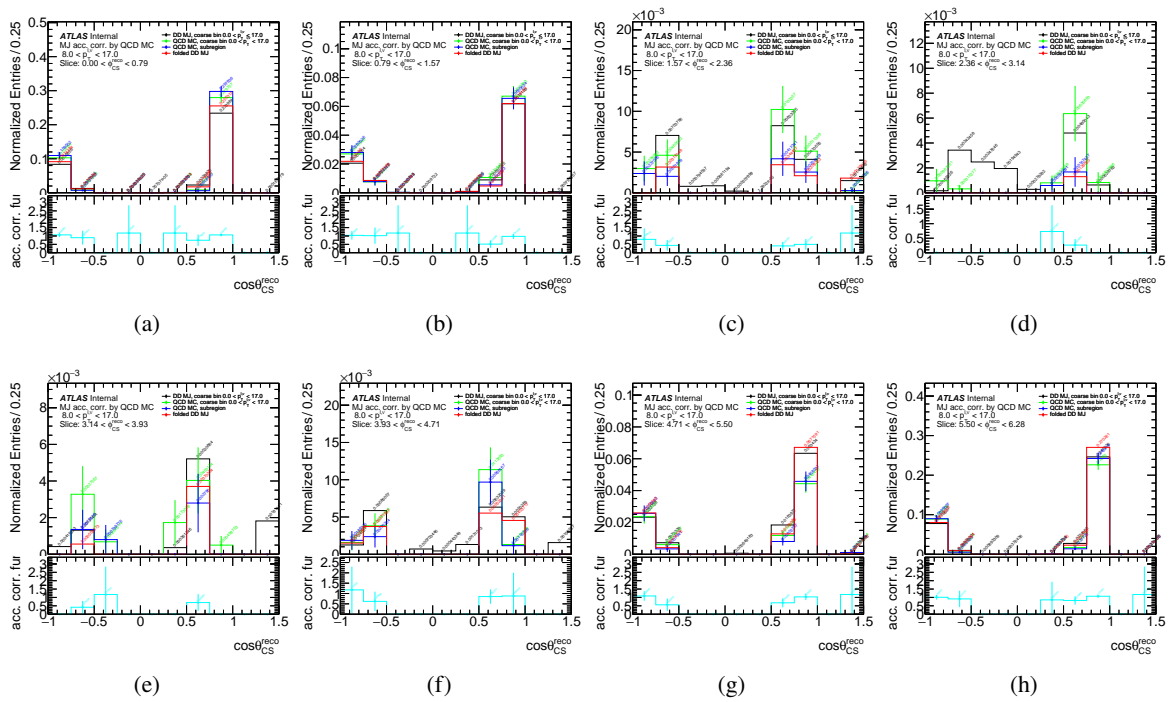


Figure 213: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

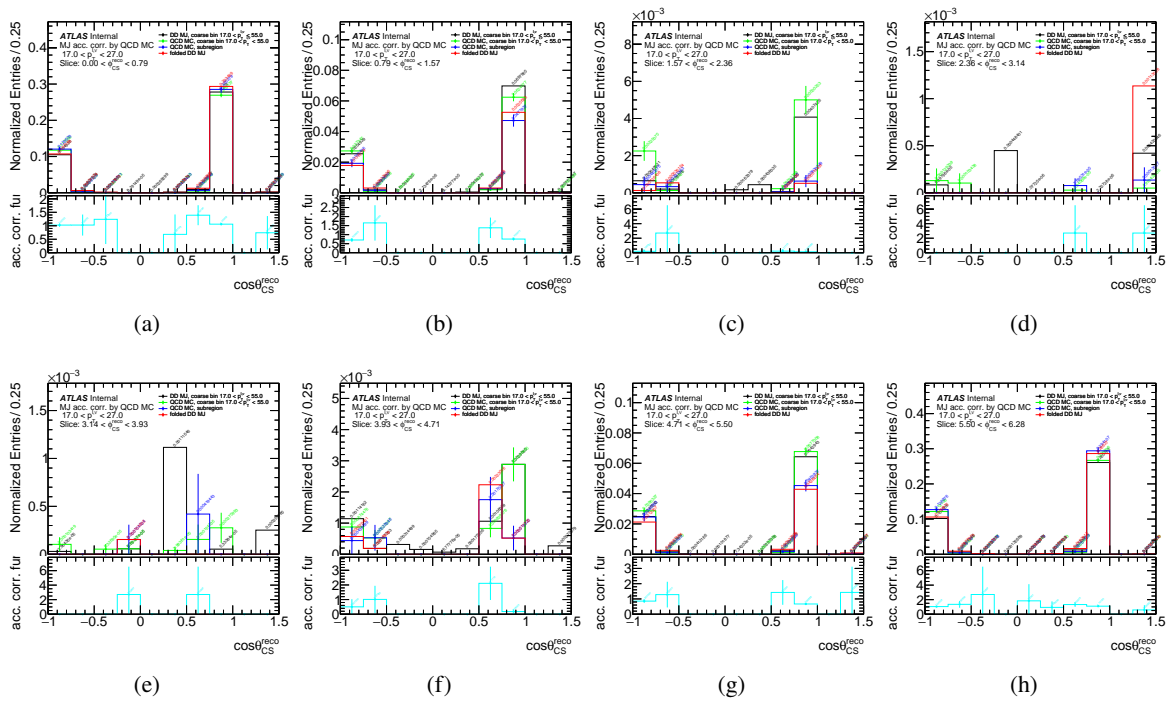


Figure 214: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

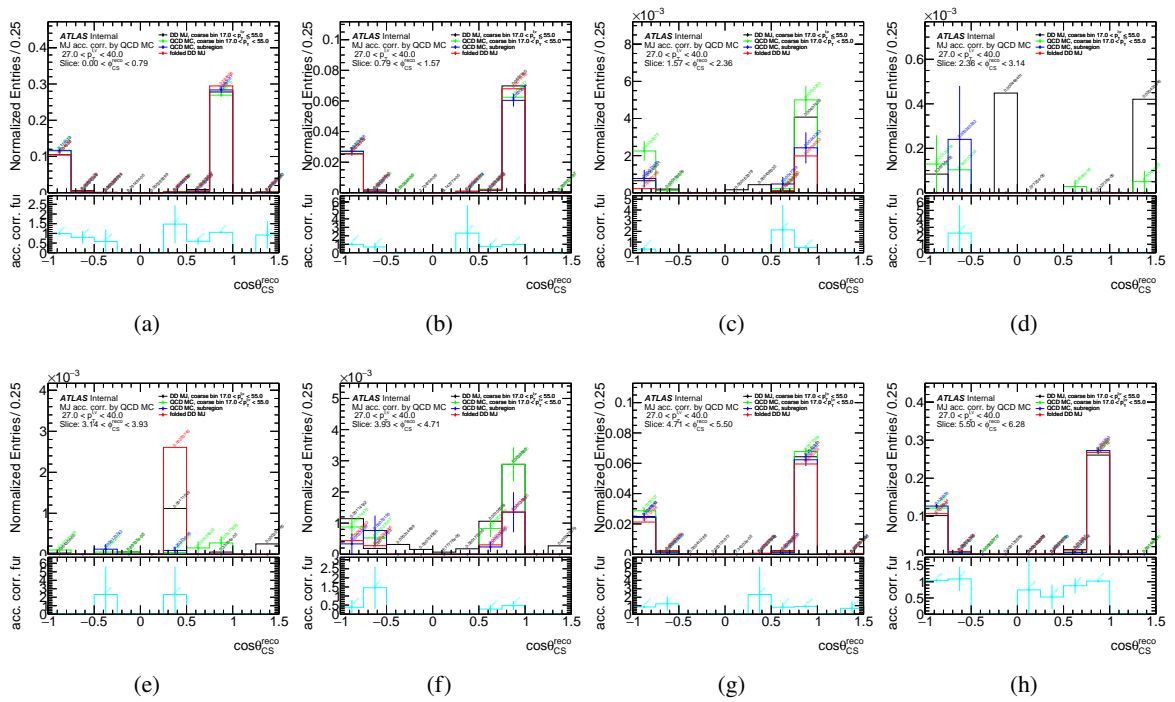


Figure 215: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

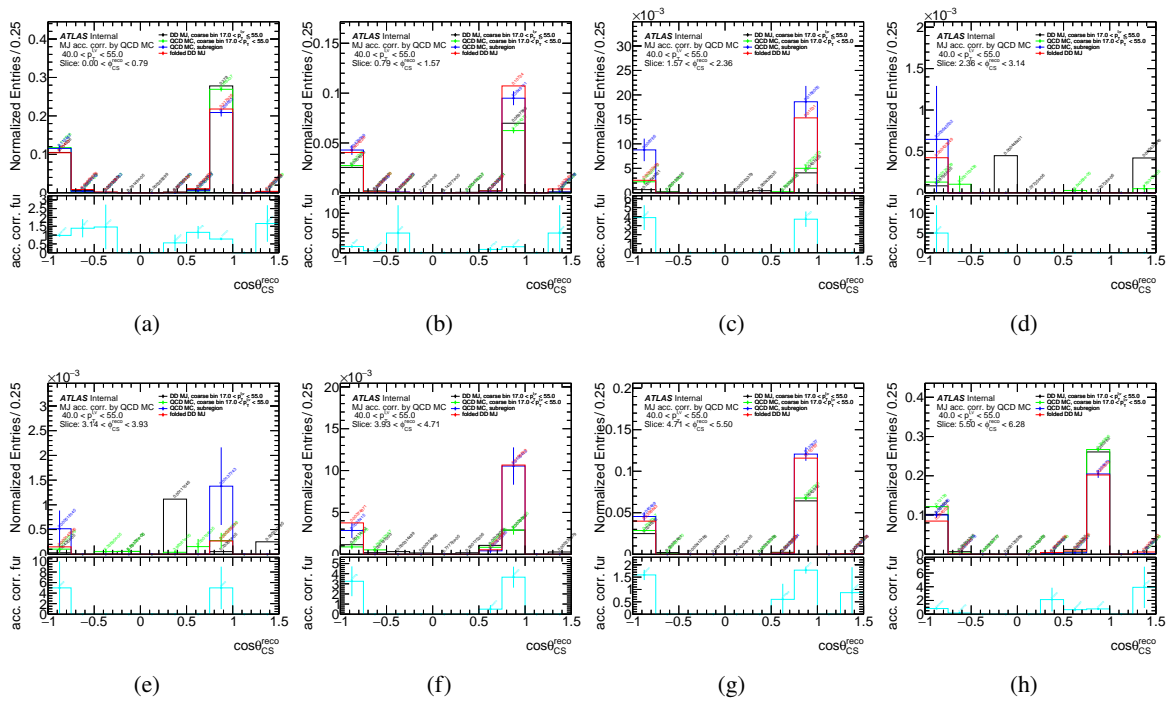


Figure 216: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

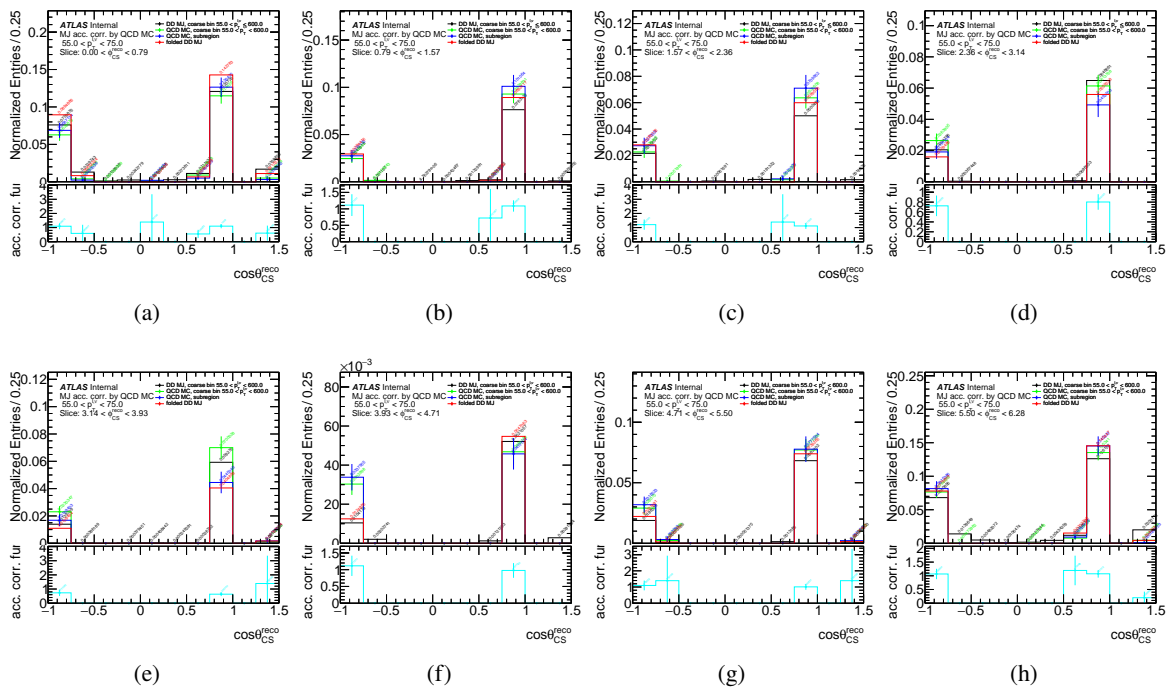


Figure 217: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

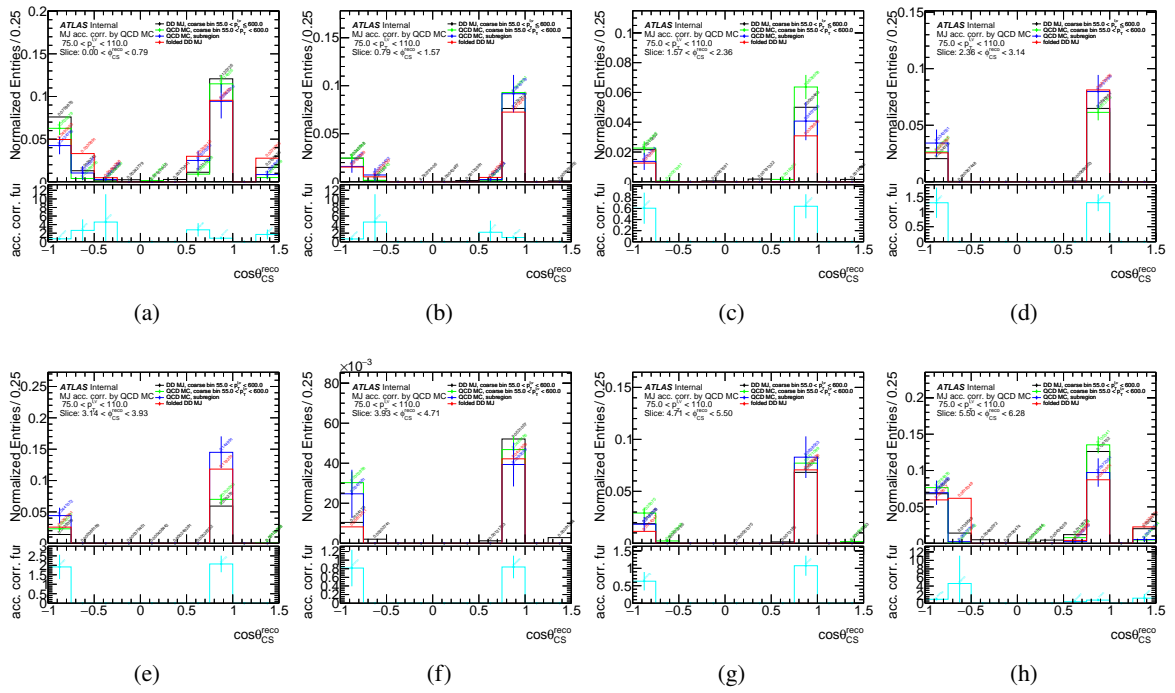


Figure 218: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

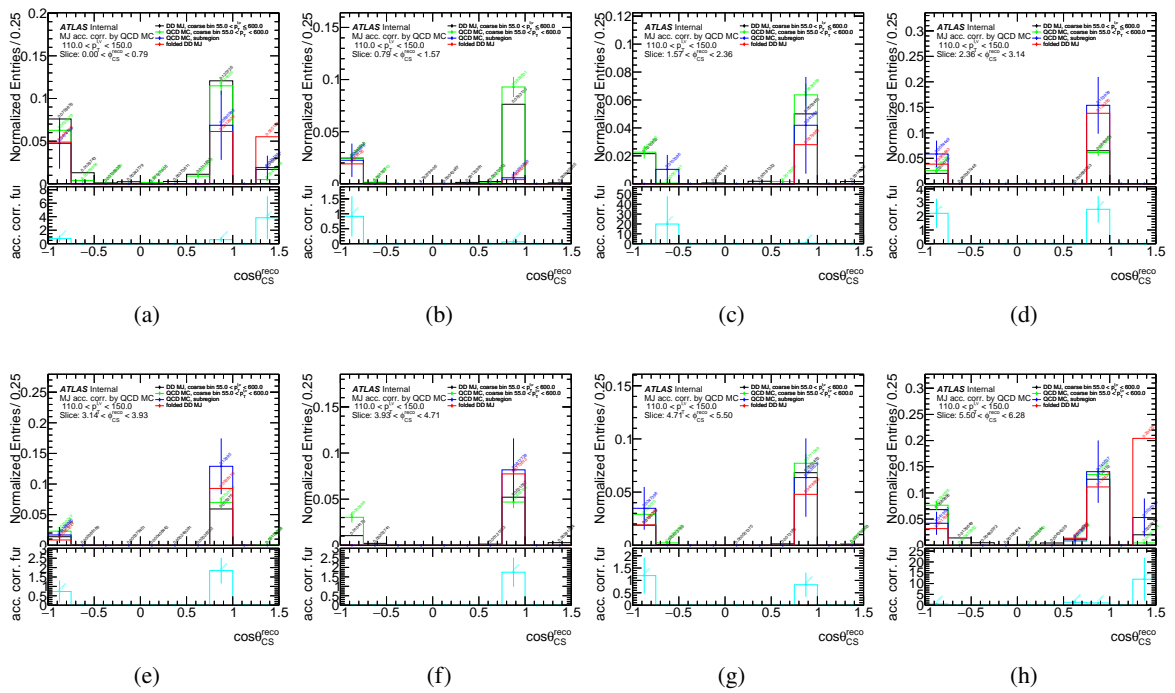


Figure 219: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

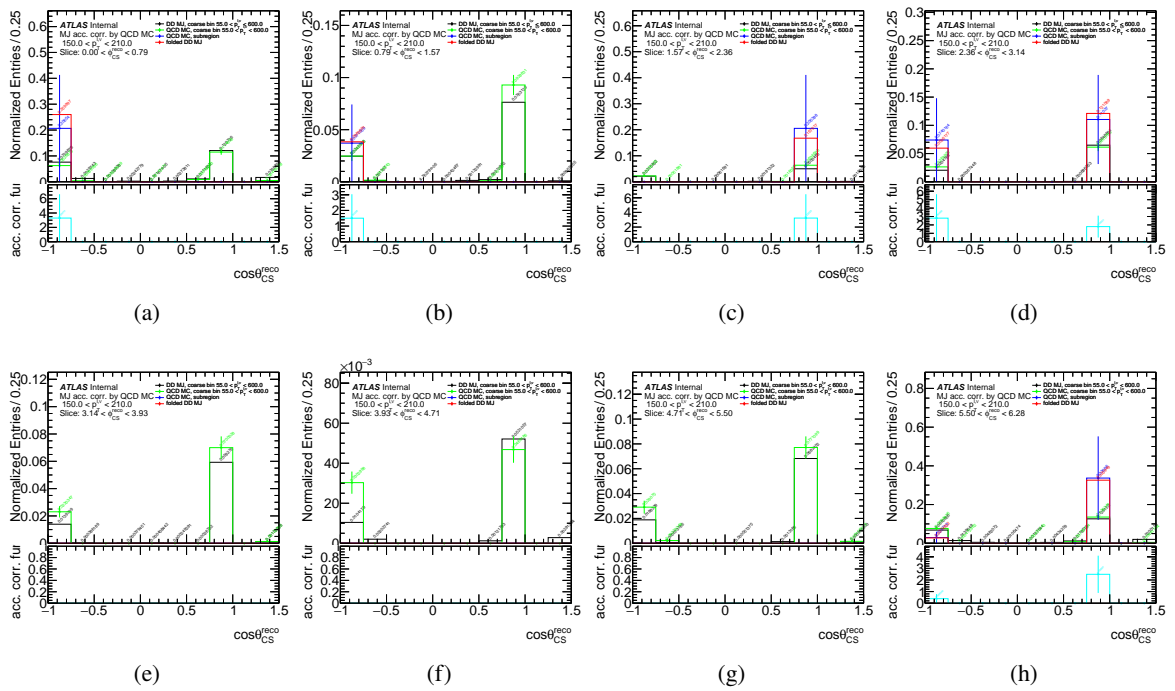


Figure 220: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

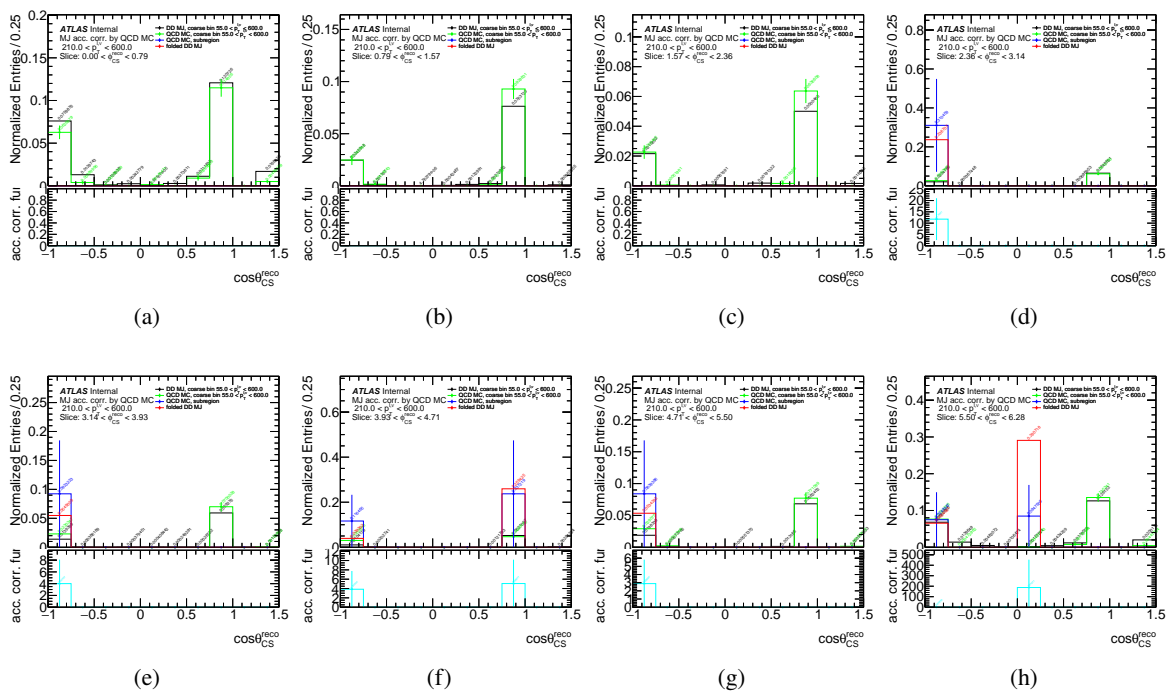


Figure 221: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

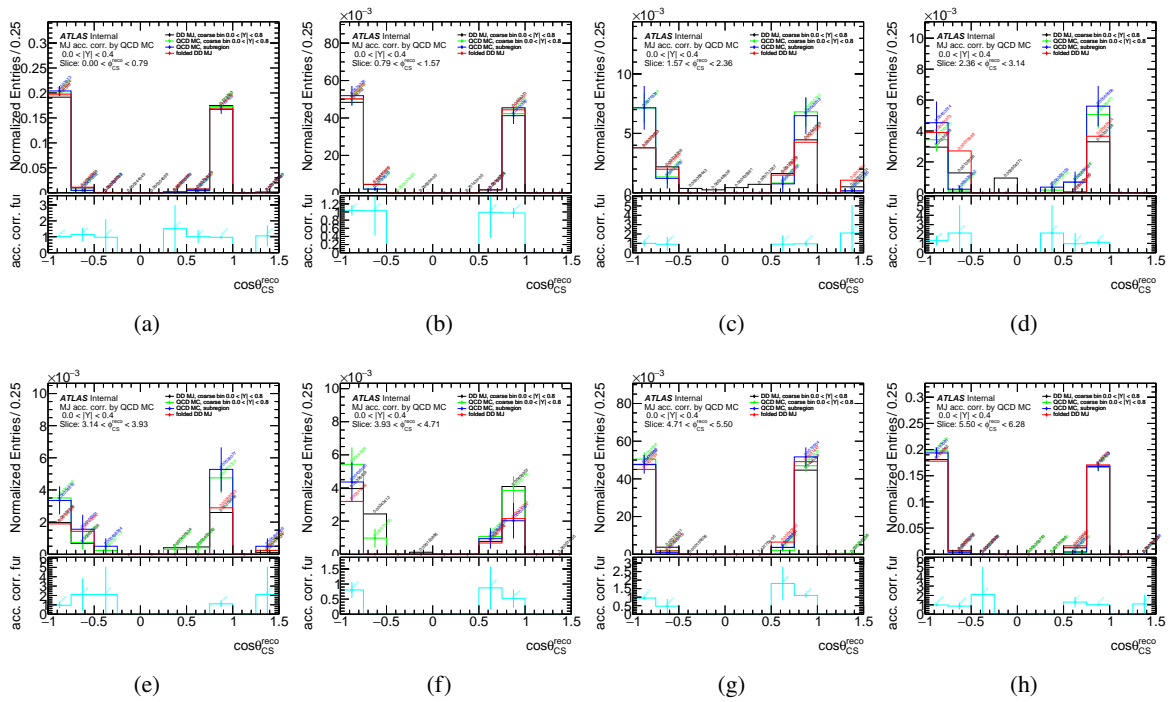


Figure 222: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^+ \rightarrow \mu^+\nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represent statistical uncertainties.

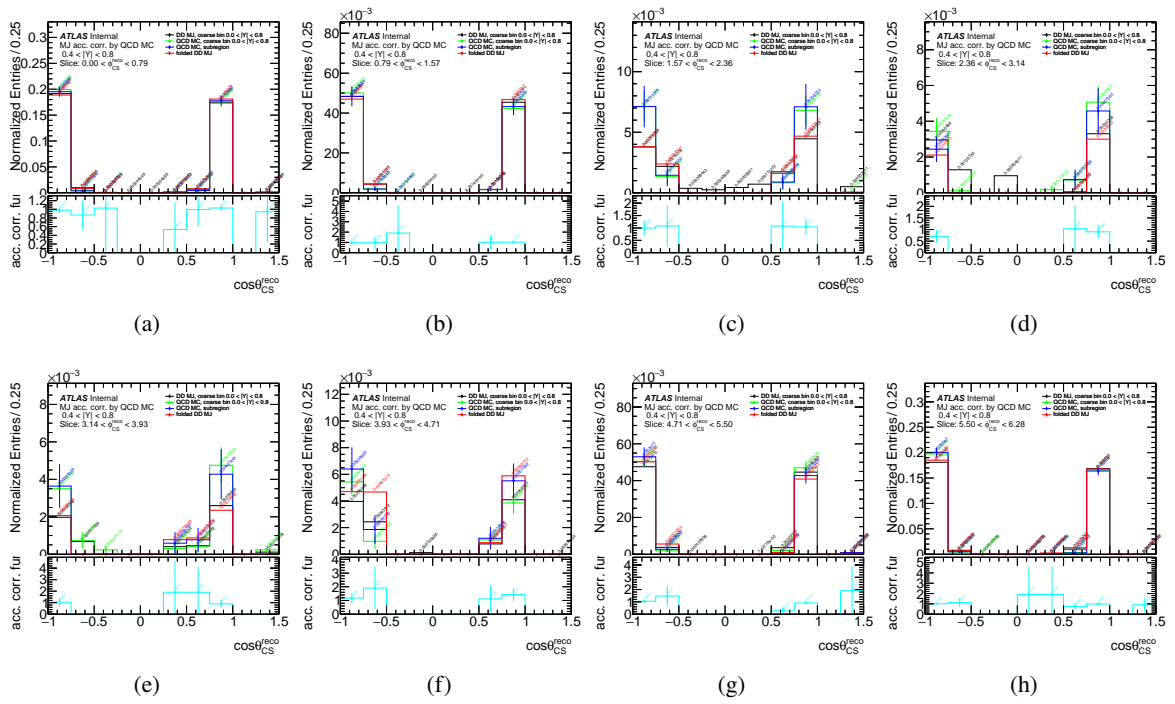


Figure 223: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

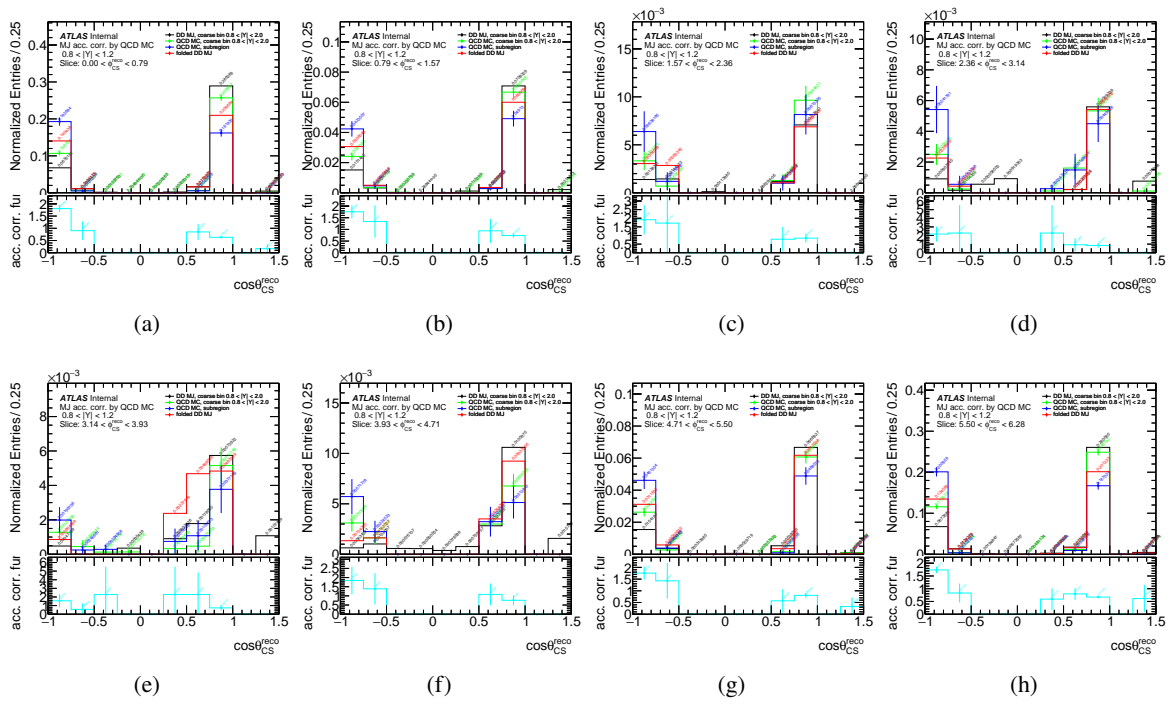


Figure 224: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

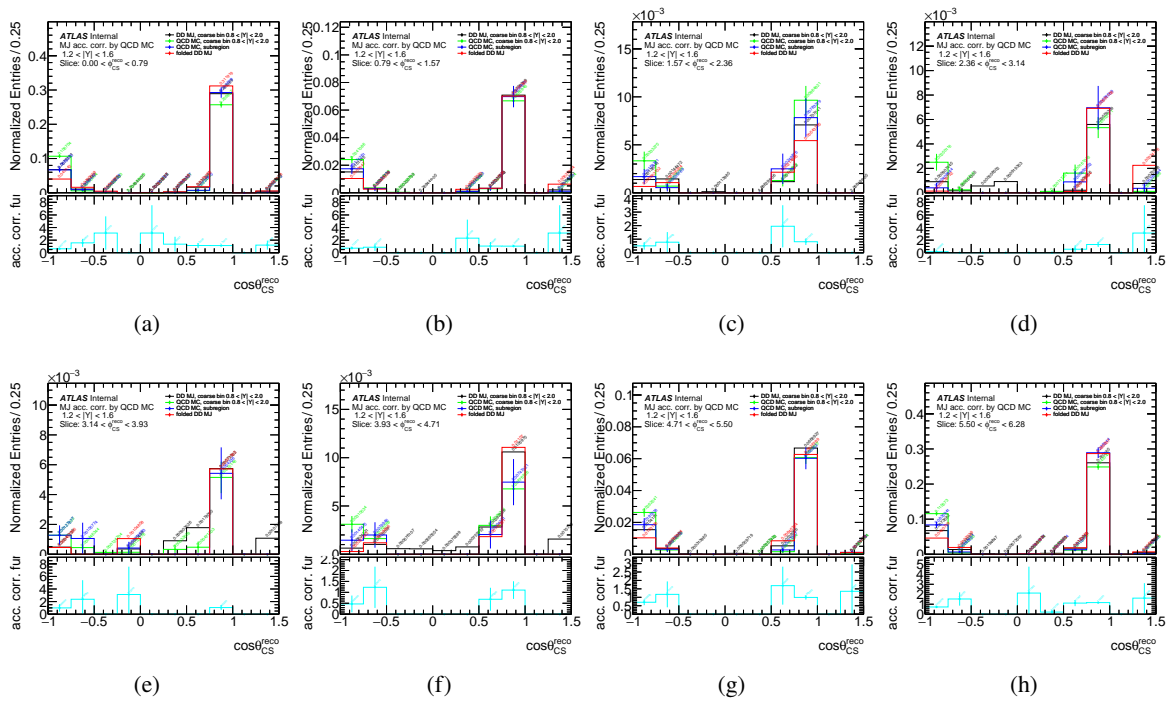


Figure 225: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

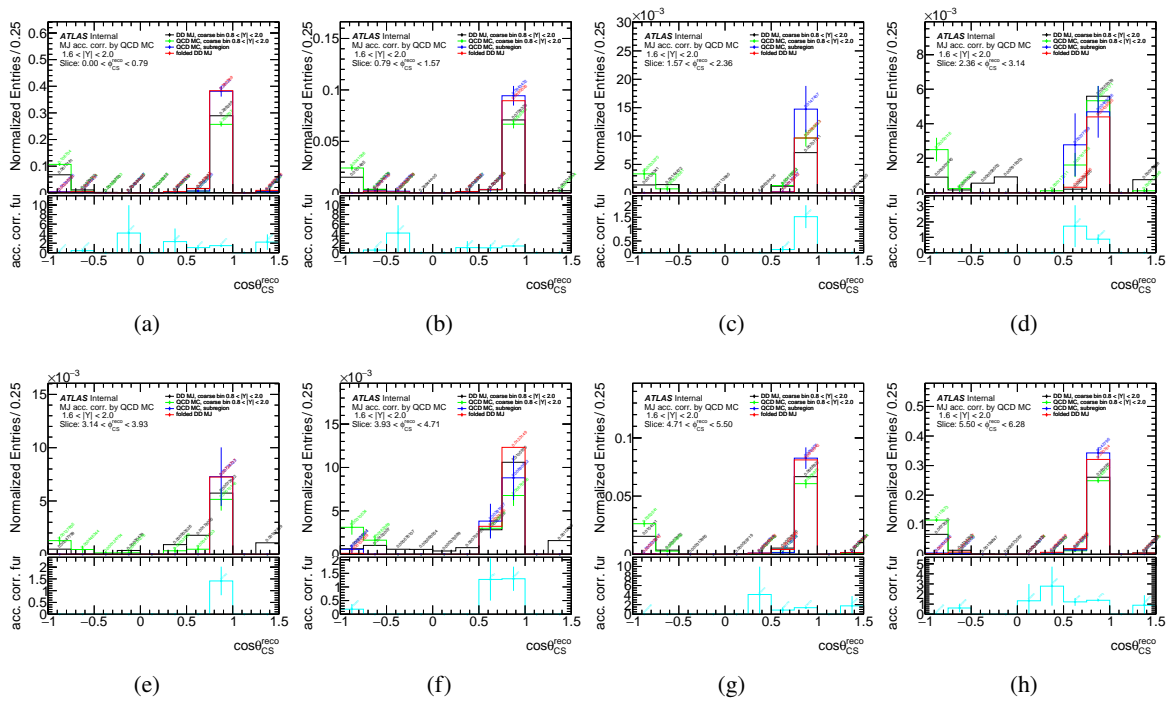


Figure 226: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

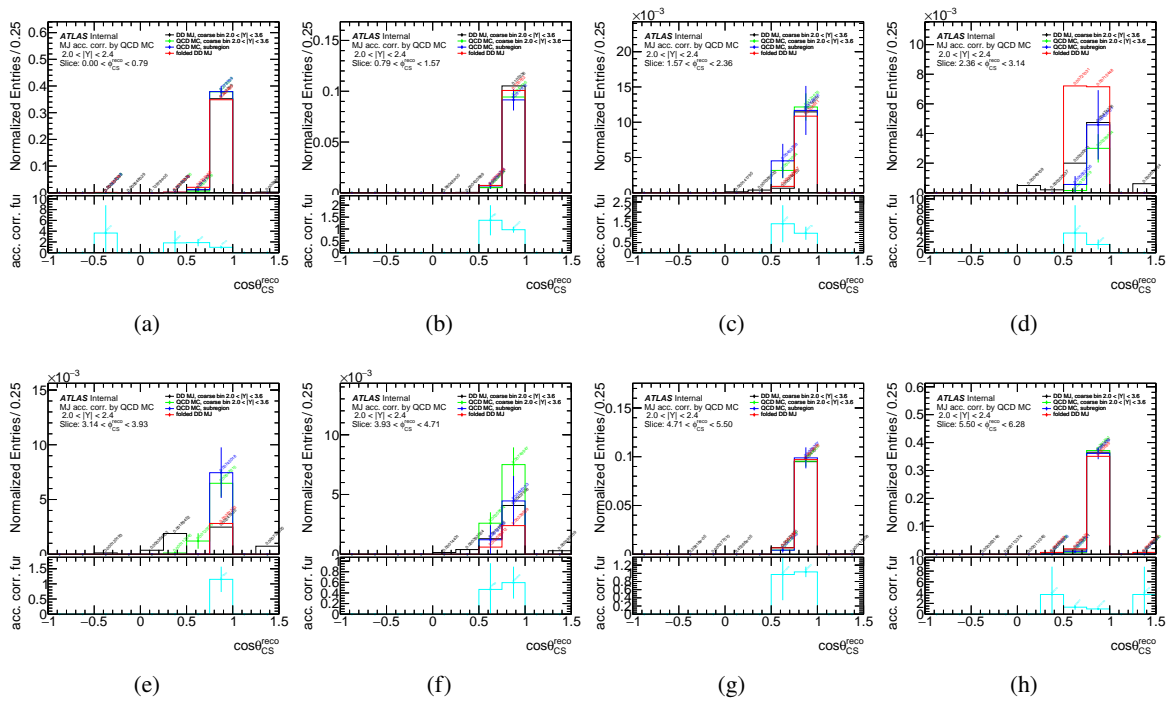


Figure 227: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

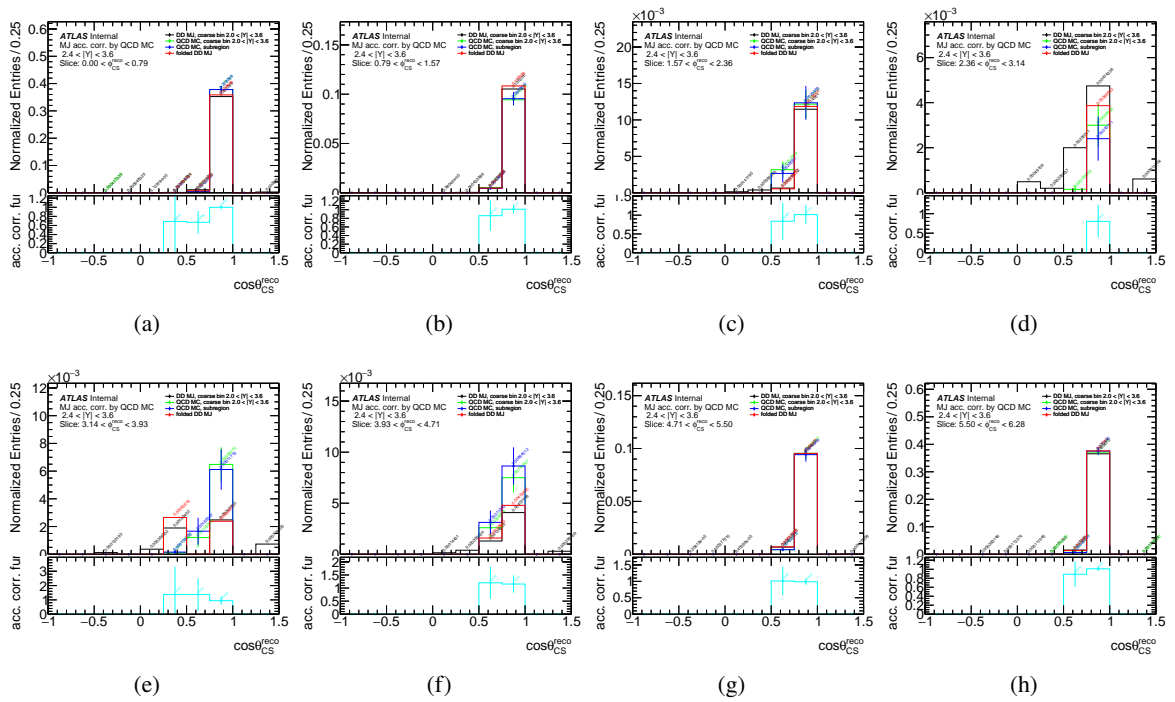


Figure 228: Multi-jet background template acceptance correction functions for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow \mu^+ \nu$ channel. Distributions are normalized over ϕ_{CS} slices. Error bands represents statistical uncertainties.

1287 E.6 MJ background 2D templates

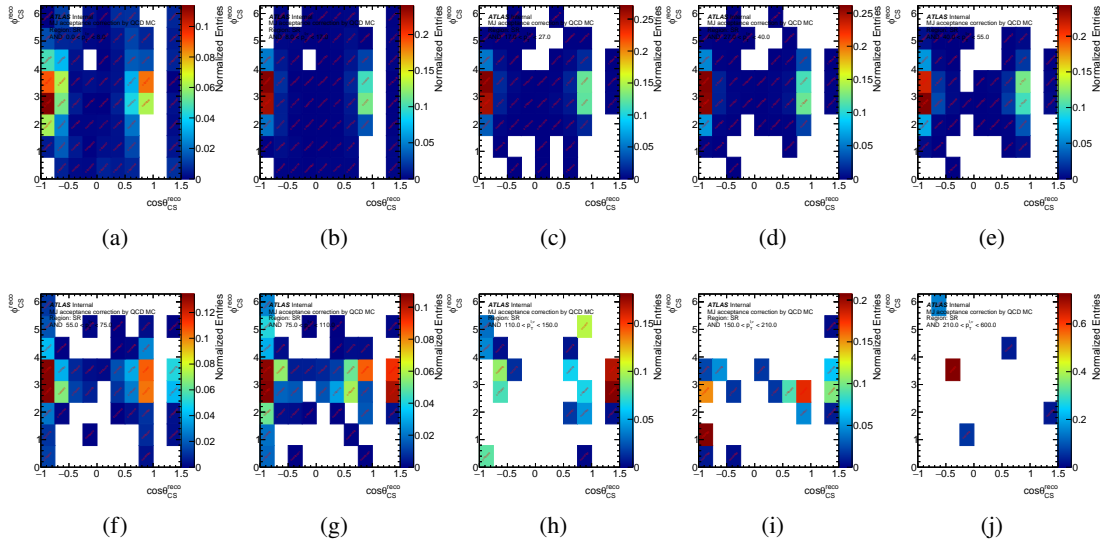


Figure 229: Multi-jet background 2D templates as a function of $p_T^{\ell, \nu}$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

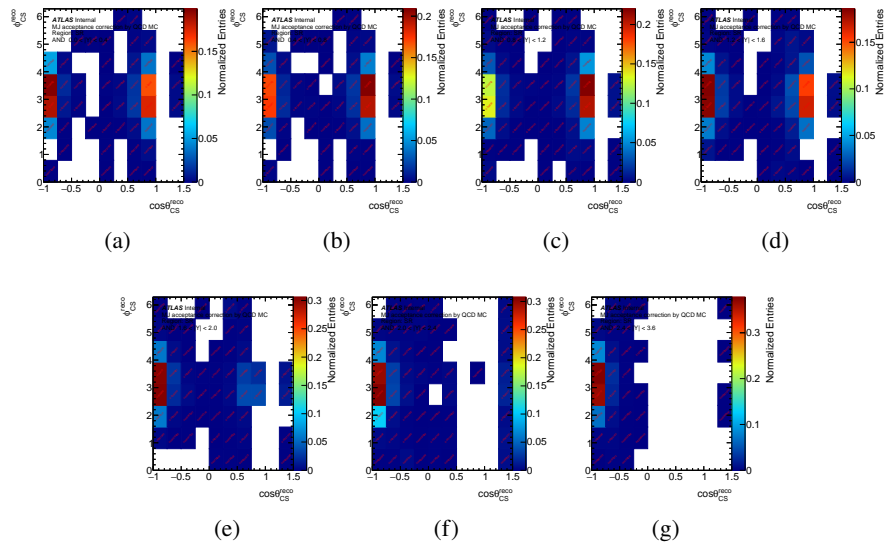


Figure 230: Multi-jet background 2D templates as a function of $|Y|$ bins for $W^- \rightarrow e^- \bar{\nu}$ channel.

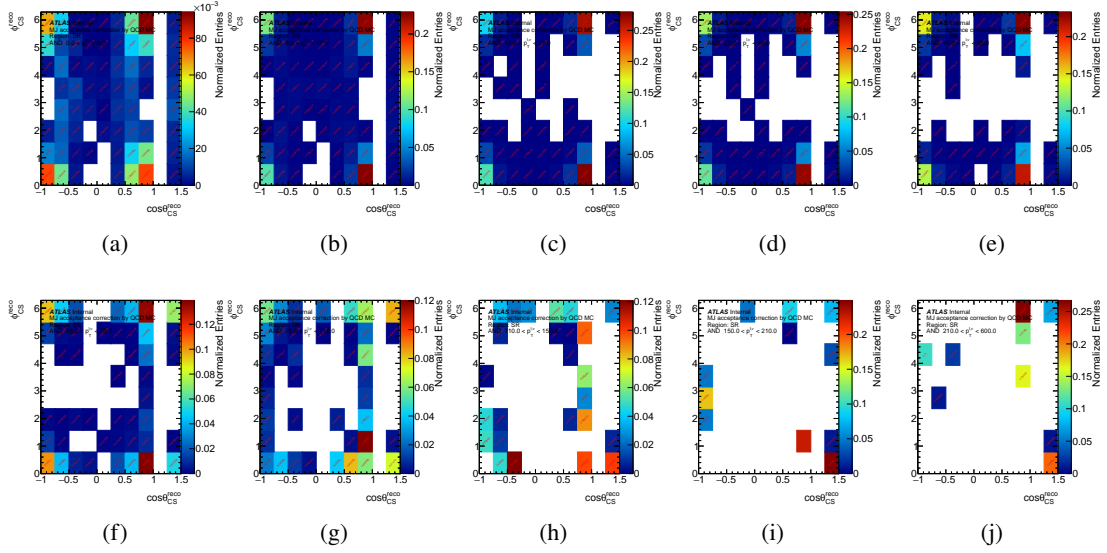


Figure 231: Multi-jet background 2D templates as a function of $p_T^{\ell,\nu}$ bins for $W^+ \rightarrow e^+\nu$ channel.

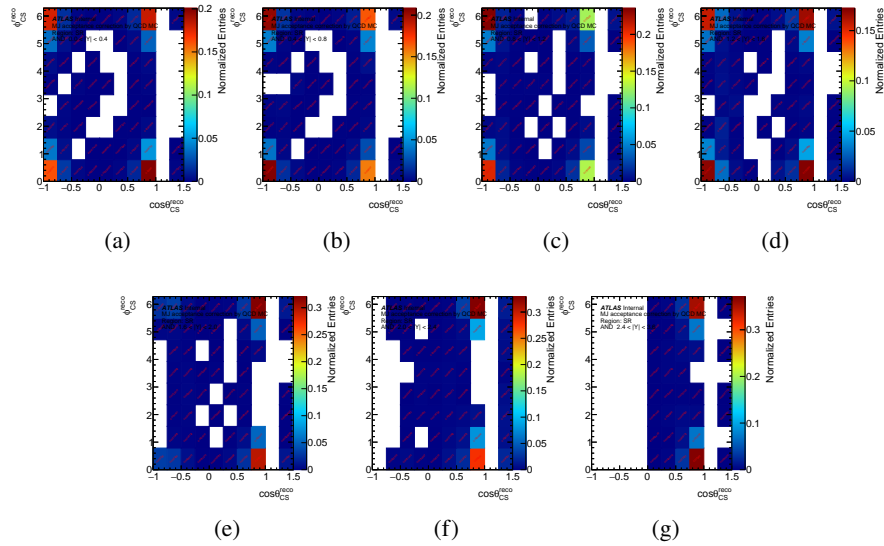


Figure 232: Multi-jet background 2D templates as a function of $|Y|$ bins for $W^+ \rightarrow e^+\nu$ channel.

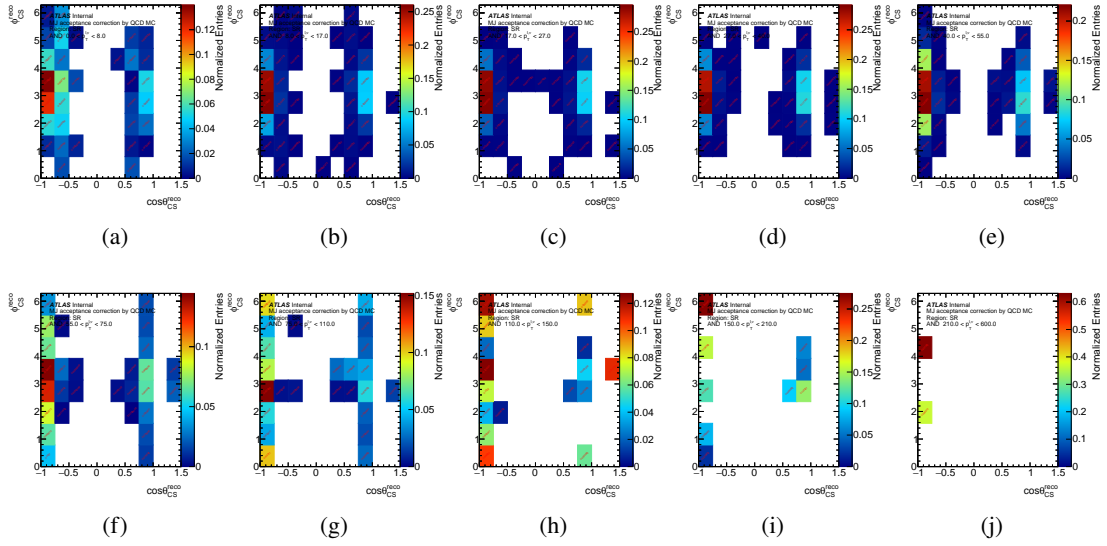


Figure 233: Multi-jet background 2D templates as a function of $p_T^{\ell,\nu}$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

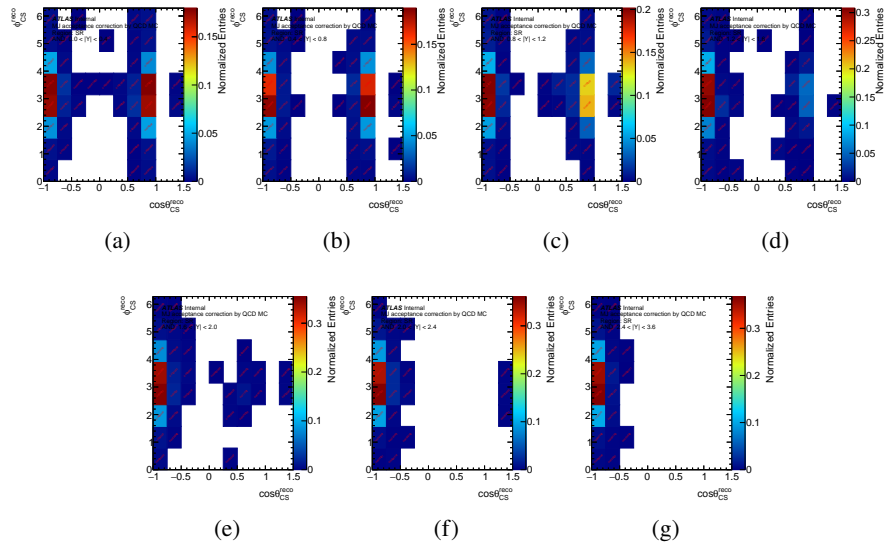


Figure 234: Multi-jet background 2D templates as a function of $|Y|$ bins for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

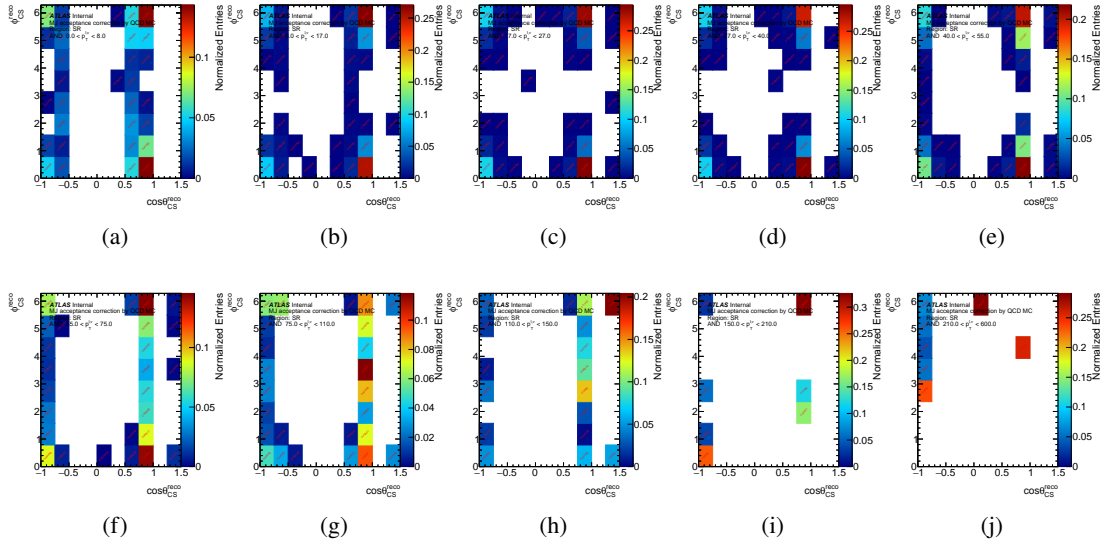


Figure 235: Multi-jet background 2D templates as a function of $p_T^{\ell,\nu}$ bins for $W^+ \rightarrow \mu^+\nu$ channel.

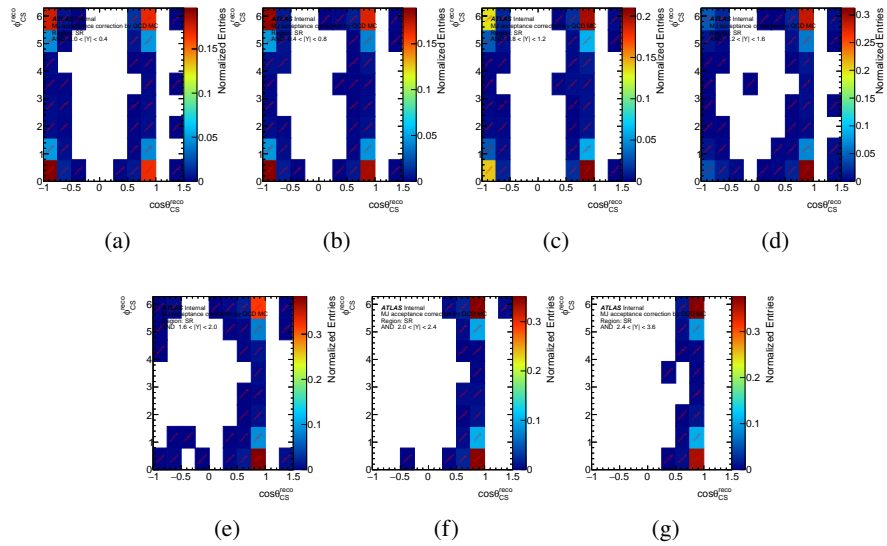


Figure 236: Multi-jet background 2D templates as a function of $|Y|$ bins for $W^+ \rightarrow \mu^+\nu$ channel.

1288 **E.7 Multi Jet shape systematics breakdown**

1289 **E.7.1 Electron channel**

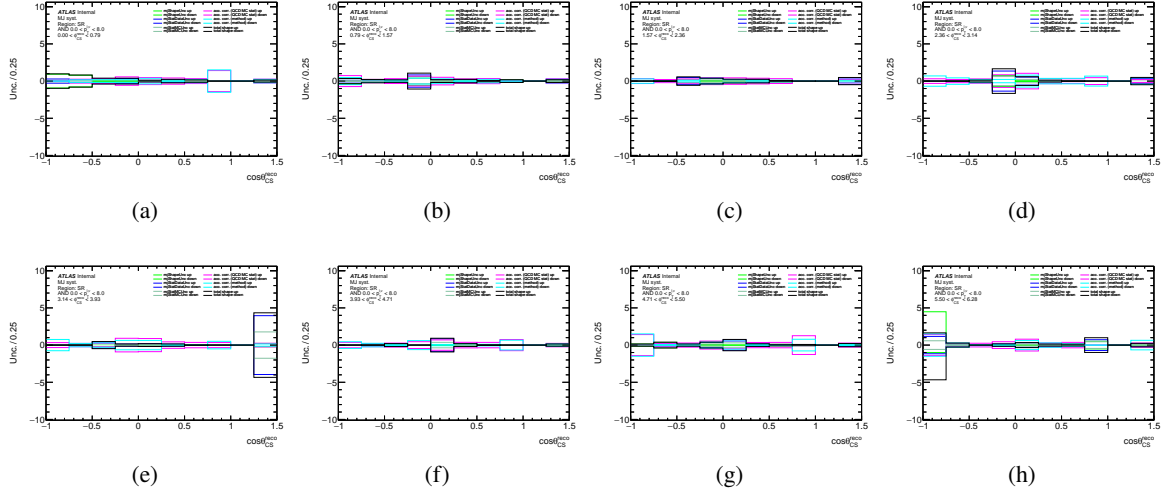


Figure 237: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

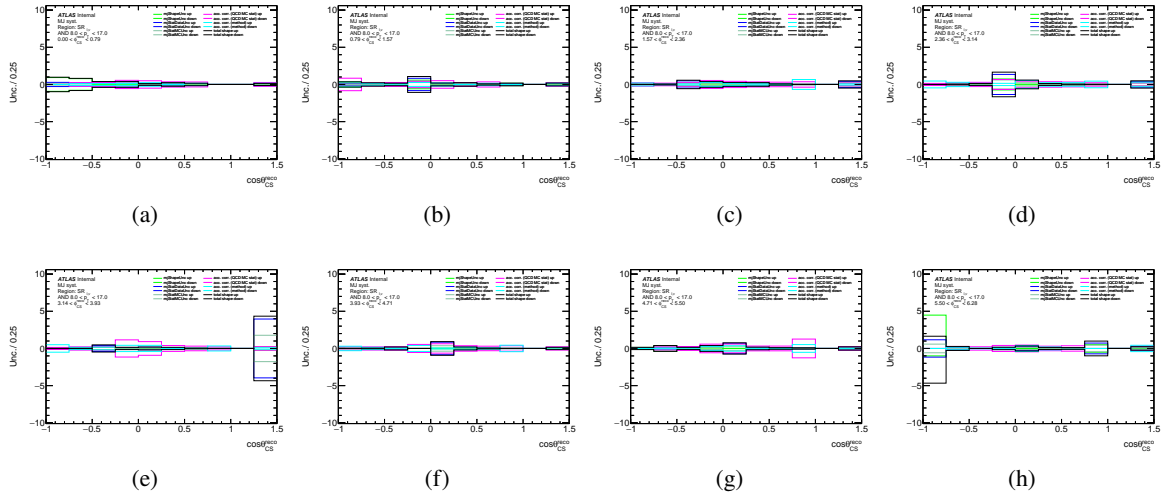


Figure 238: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

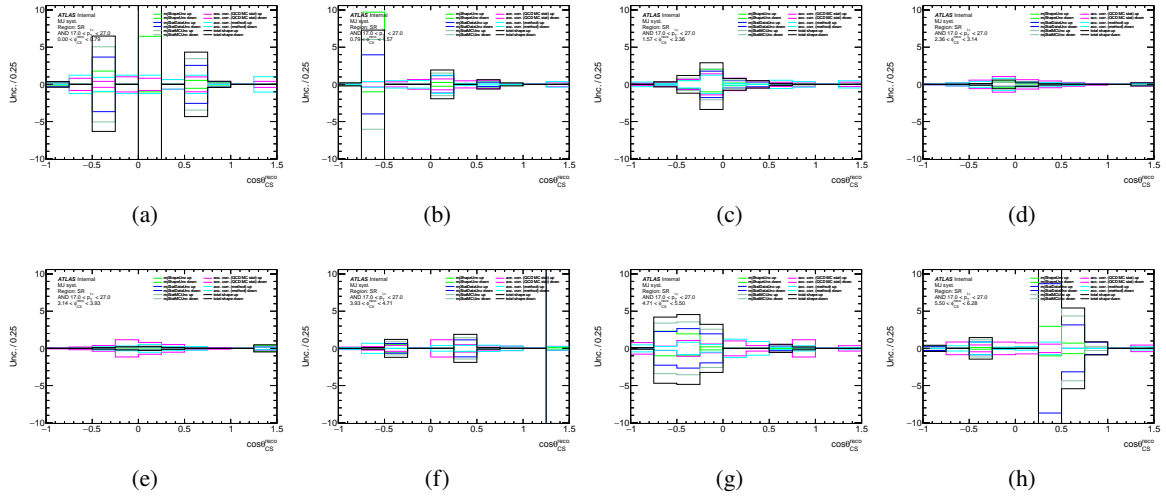


Figure 239: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

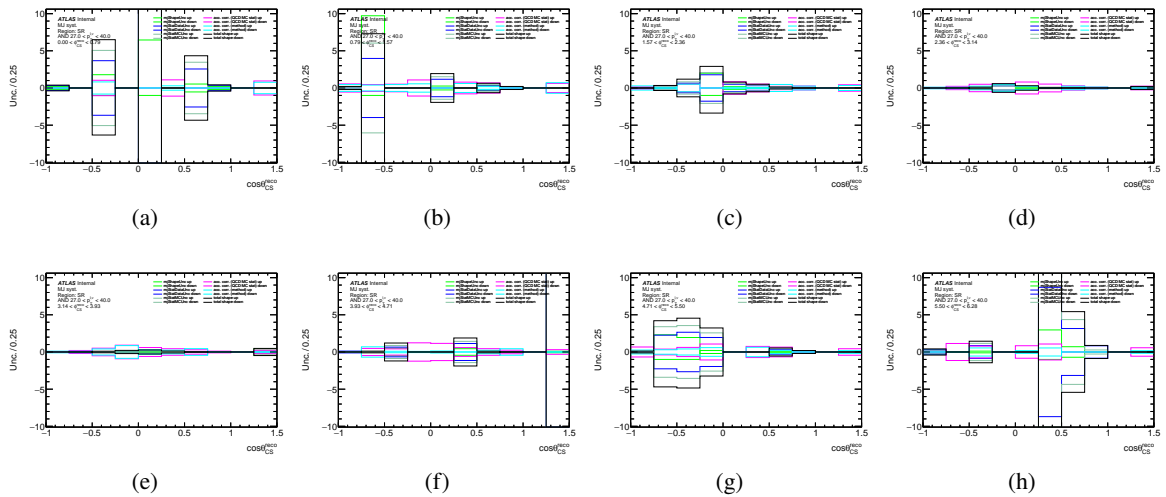


Figure 240: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

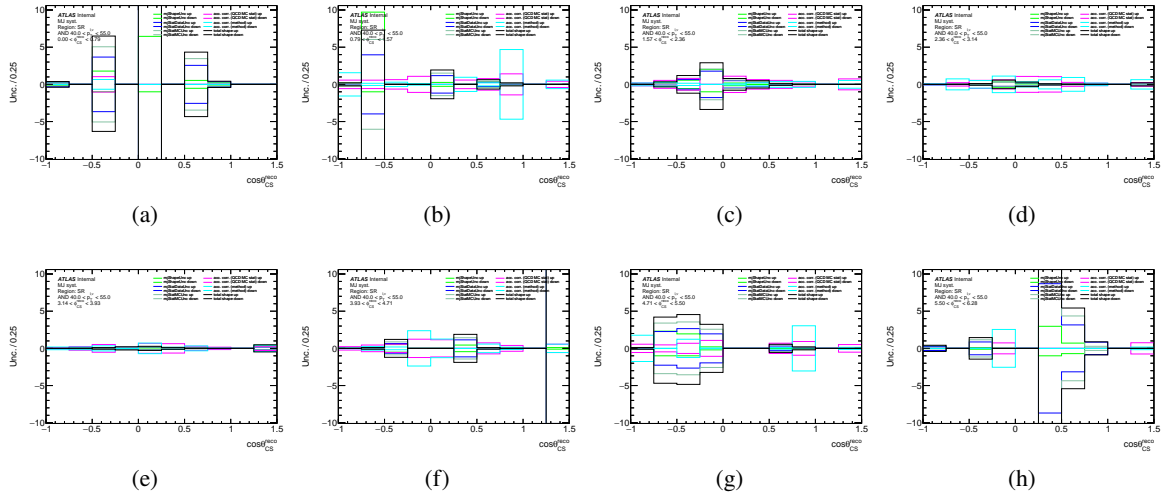


Figure 241: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

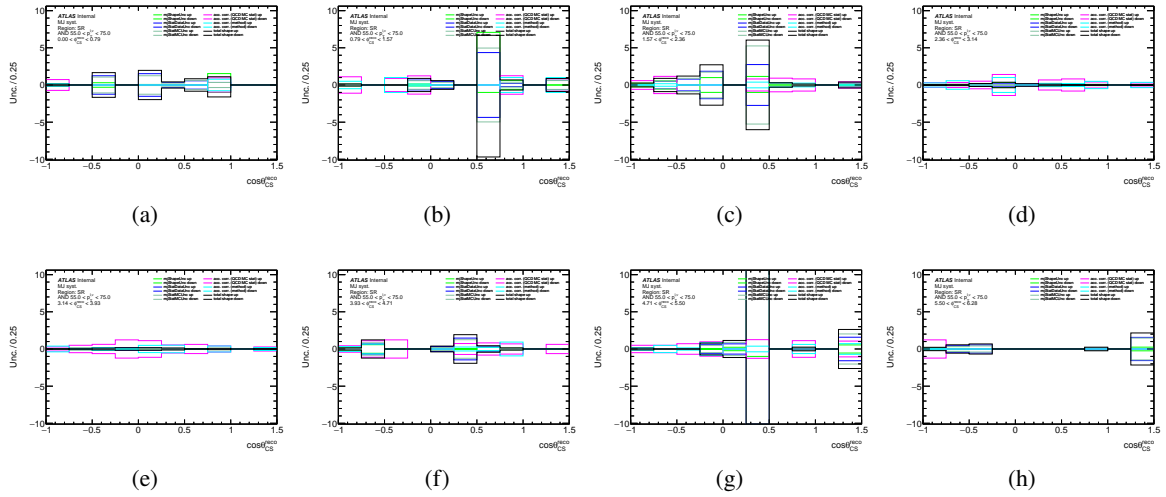


Figure 242: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

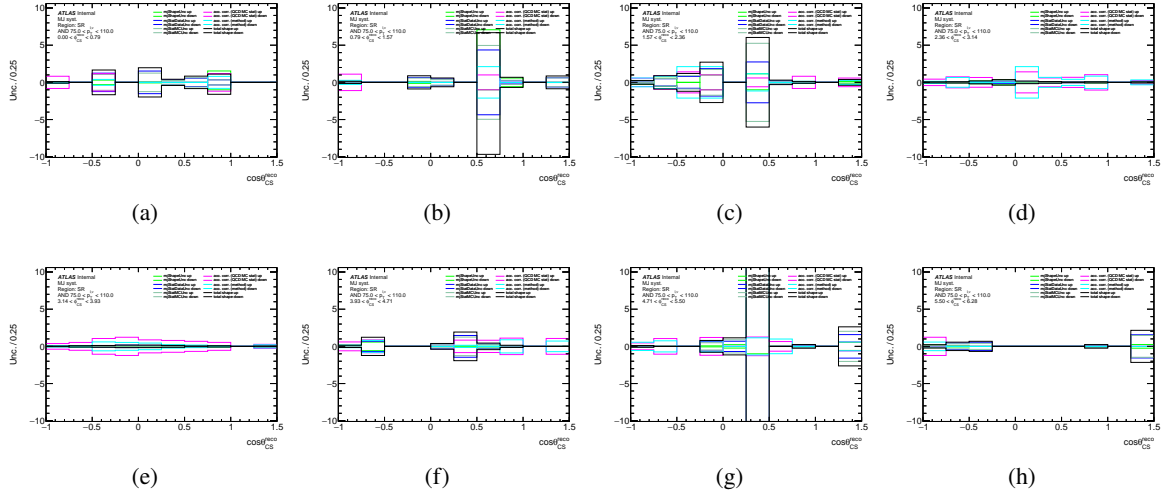


Figure 243: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

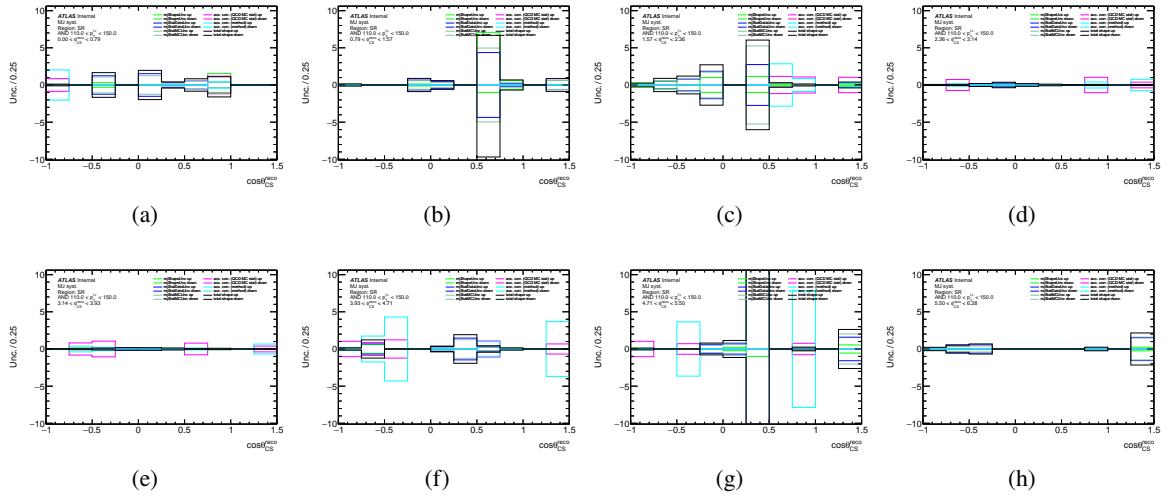


Figure 244: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

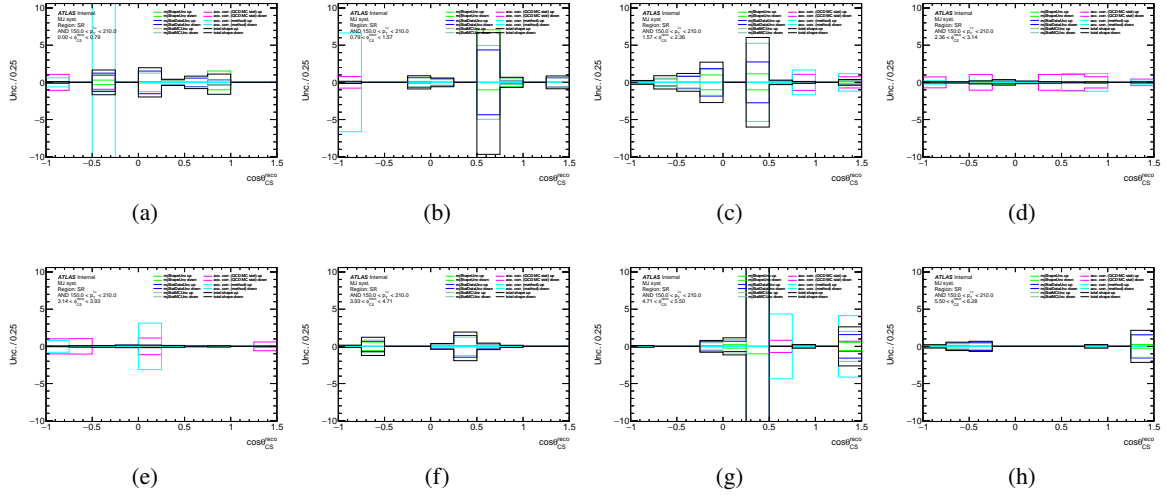


Figure 245: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

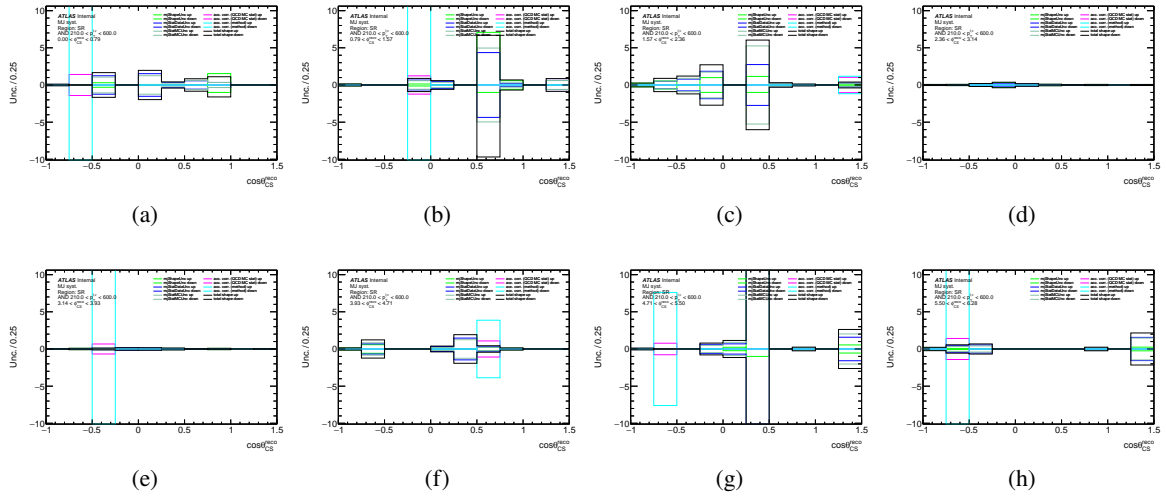


Figure 246: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

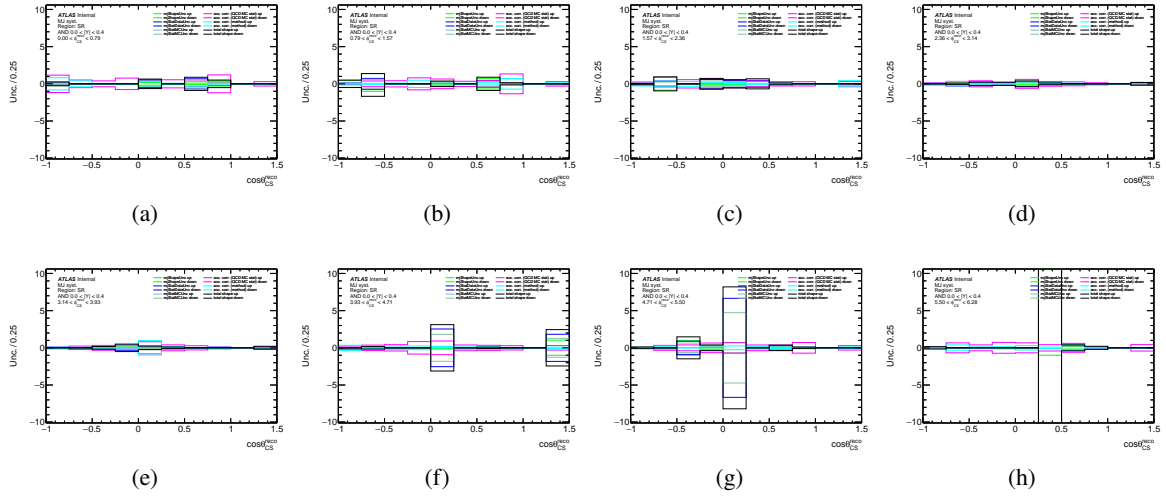


Figure 247: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

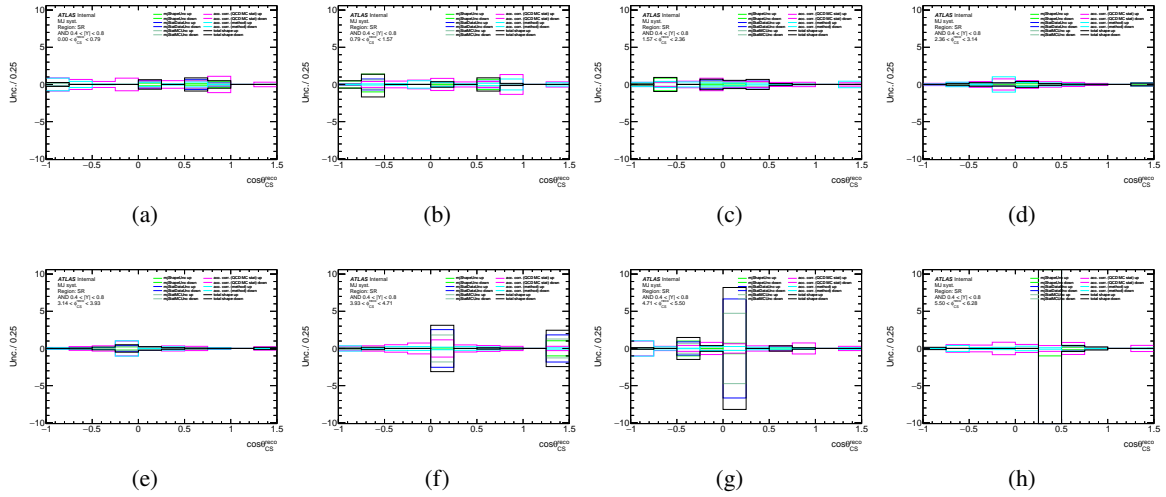


Figure 248: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

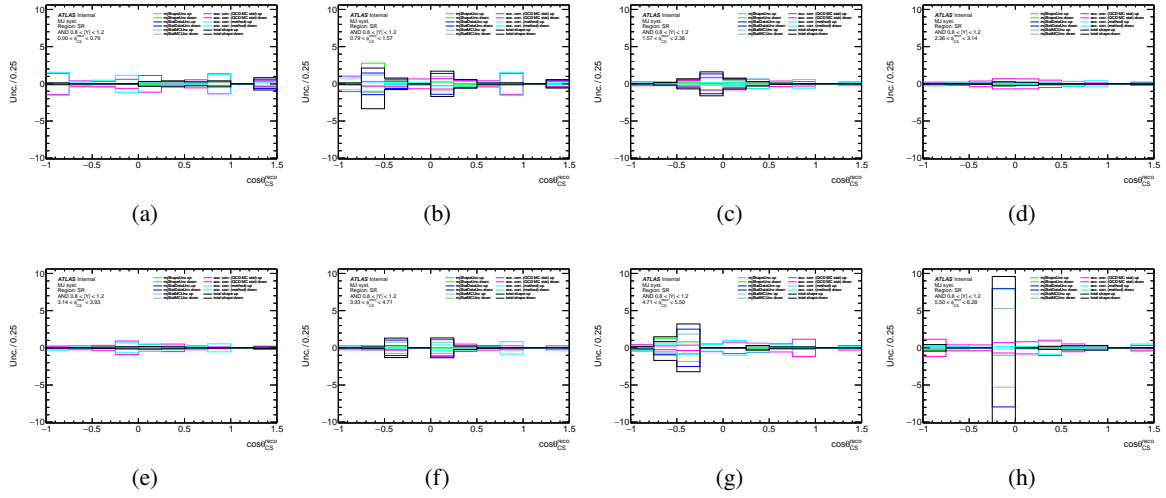


Figure 249: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

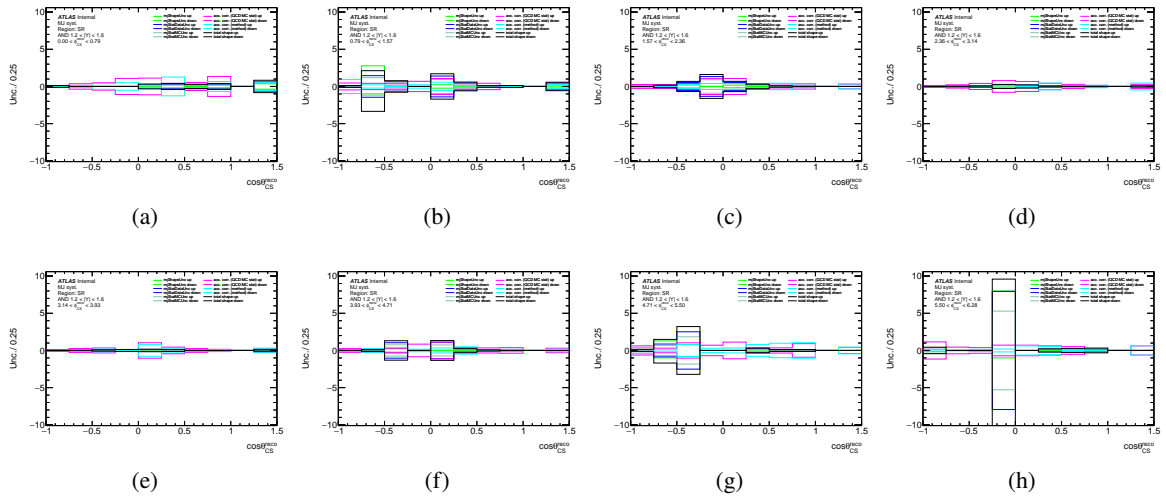


Figure 250: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

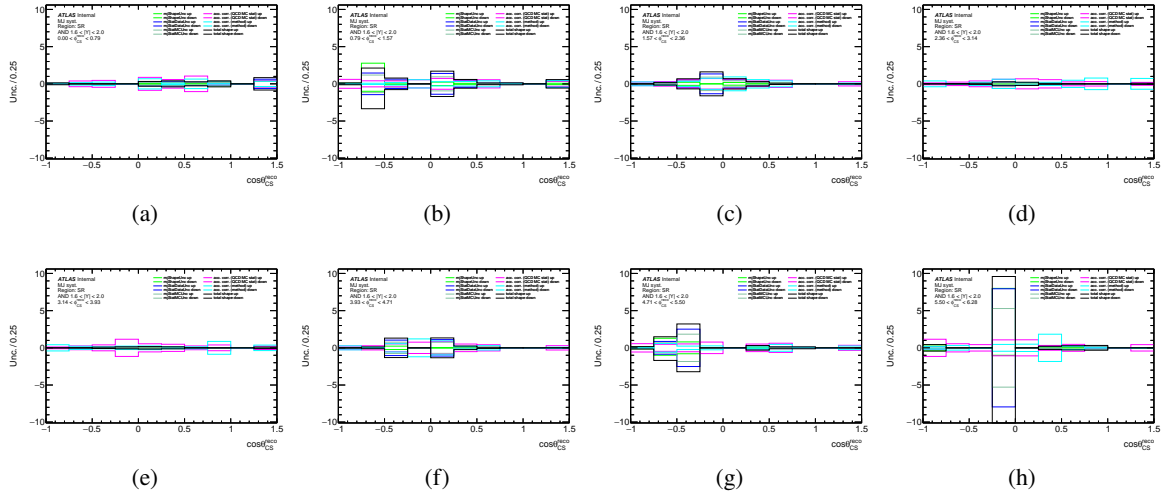


Figure 251: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

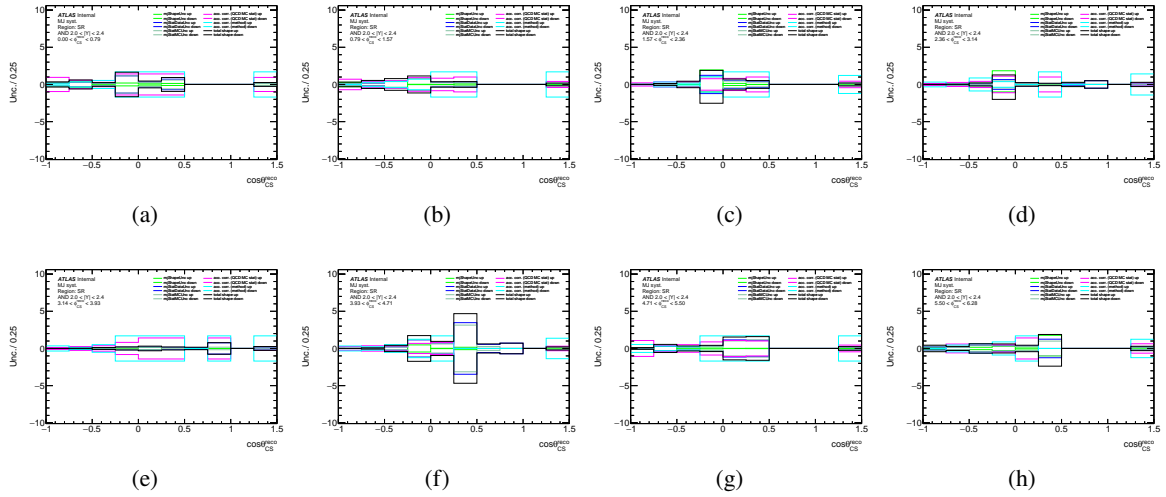


Figure 252: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

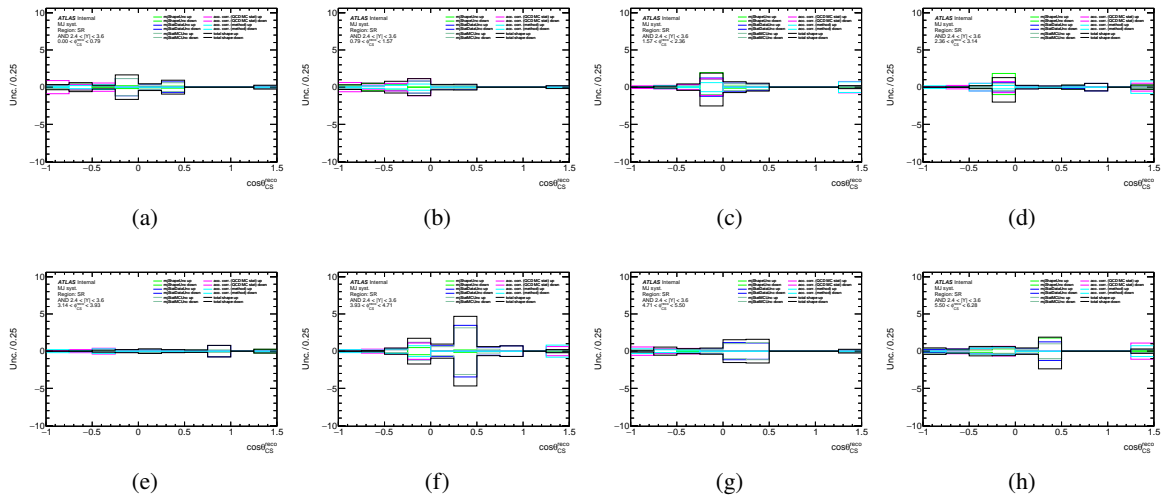


Figure 253: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

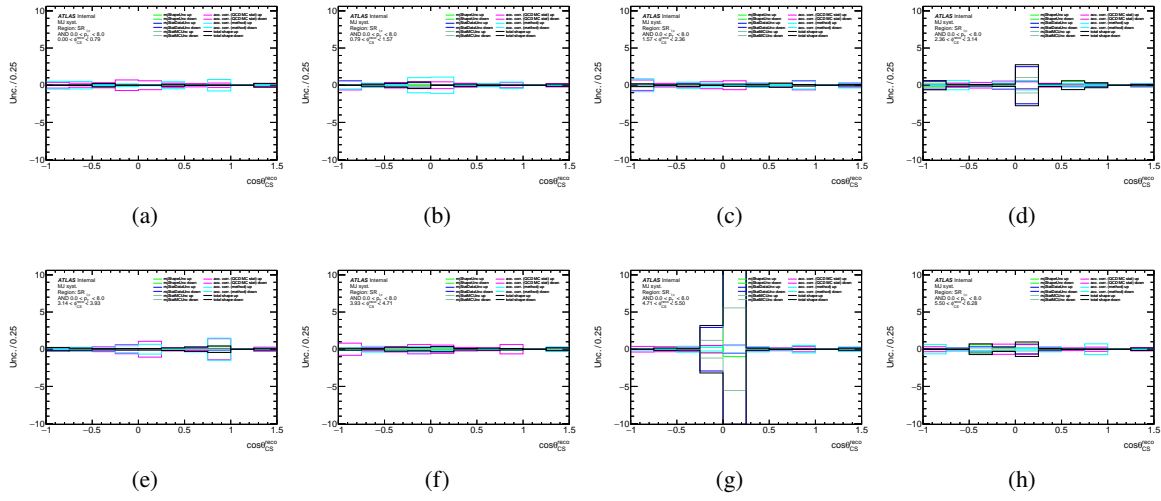


Figure 254: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

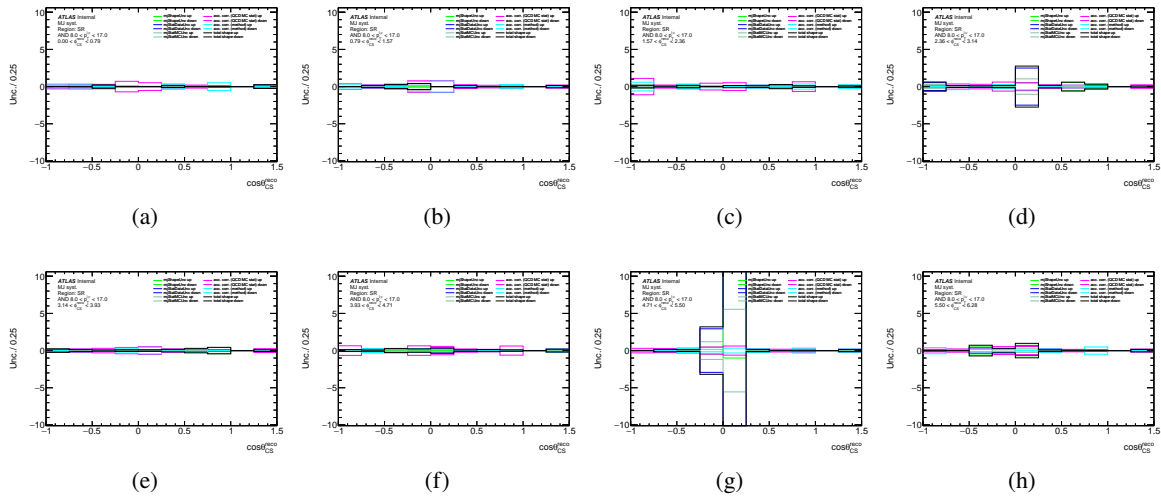


Figure 255: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

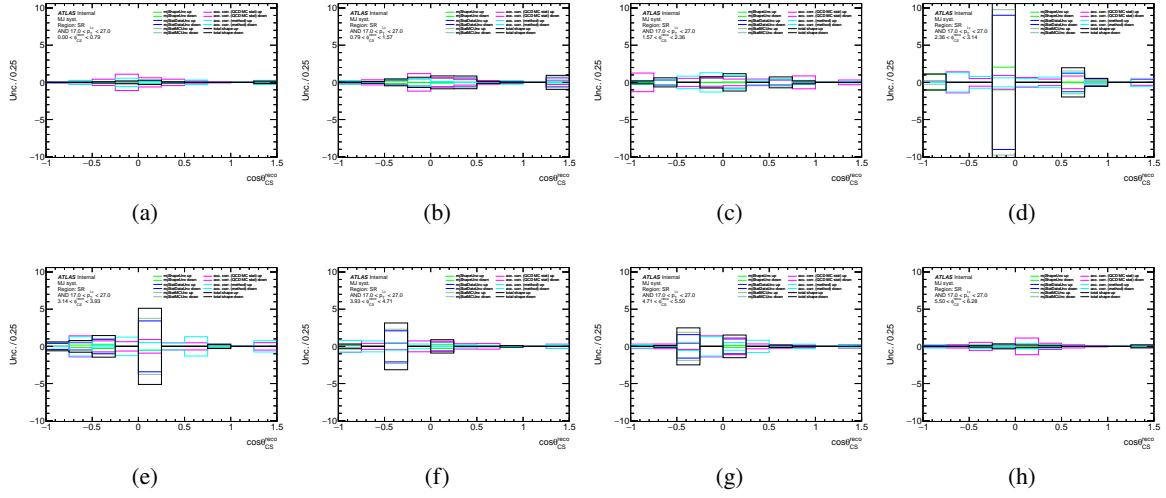


Figure 256: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

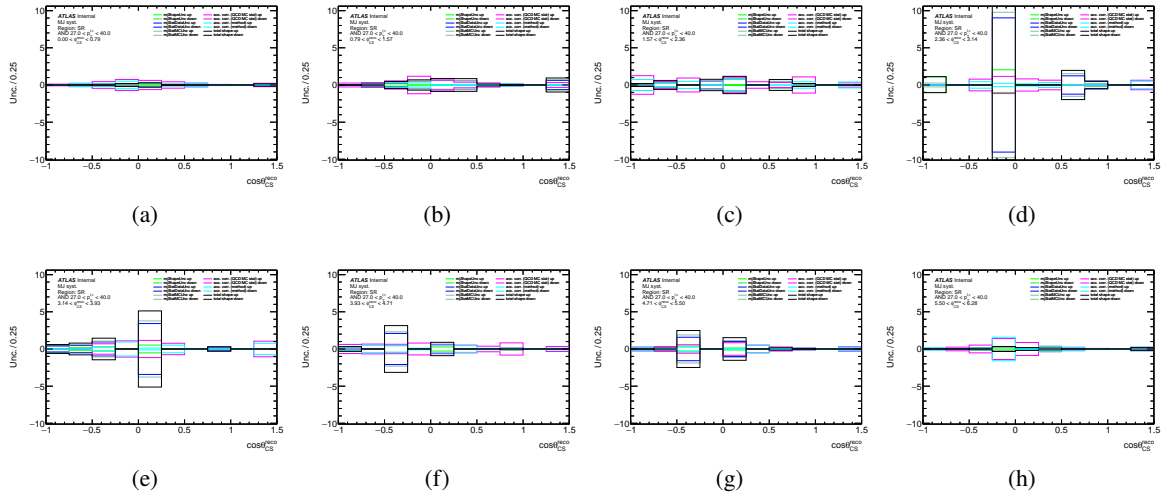


Figure 257: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

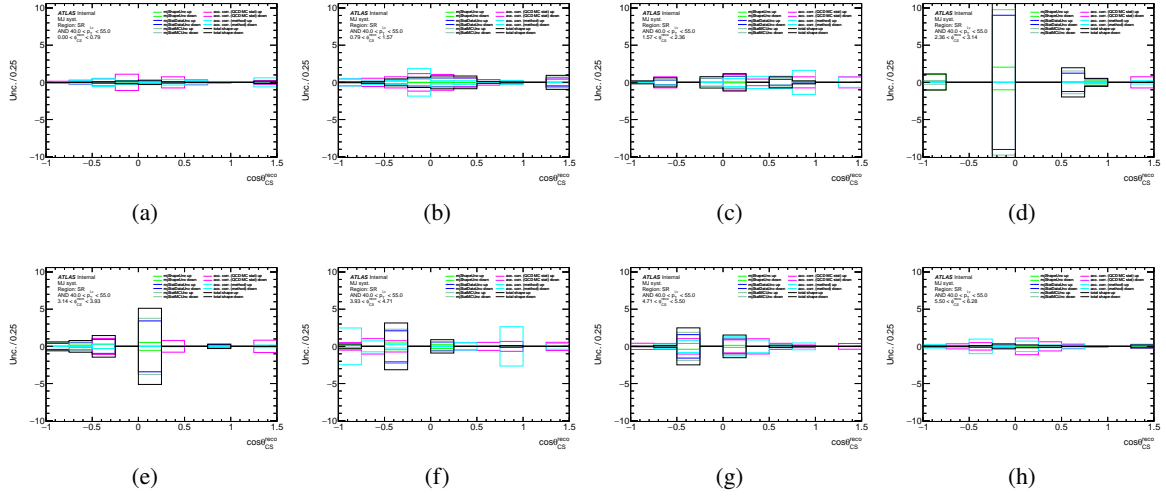


Figure 258: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell, \nu} < 55$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

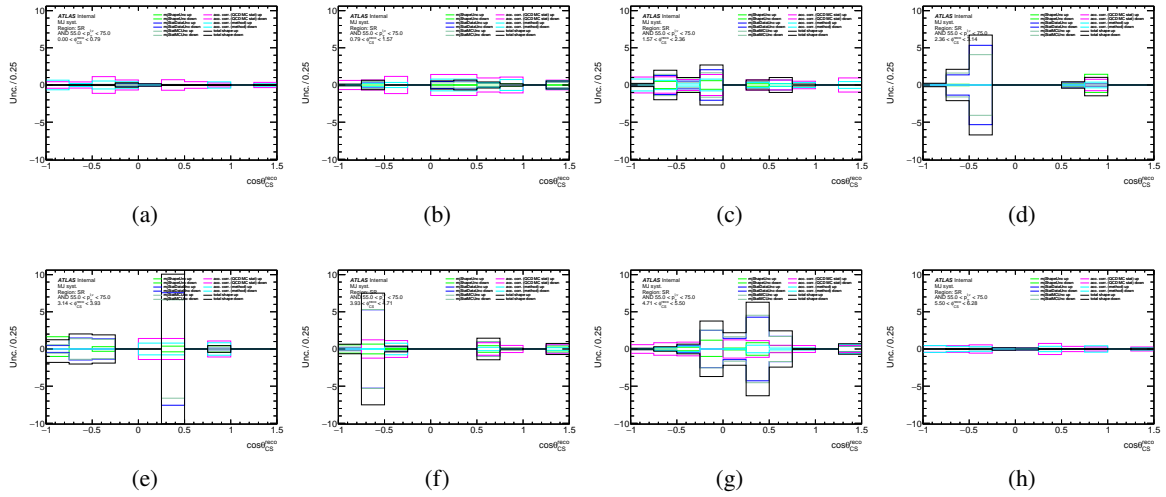


Figure 259: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell, \nu} < 75$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

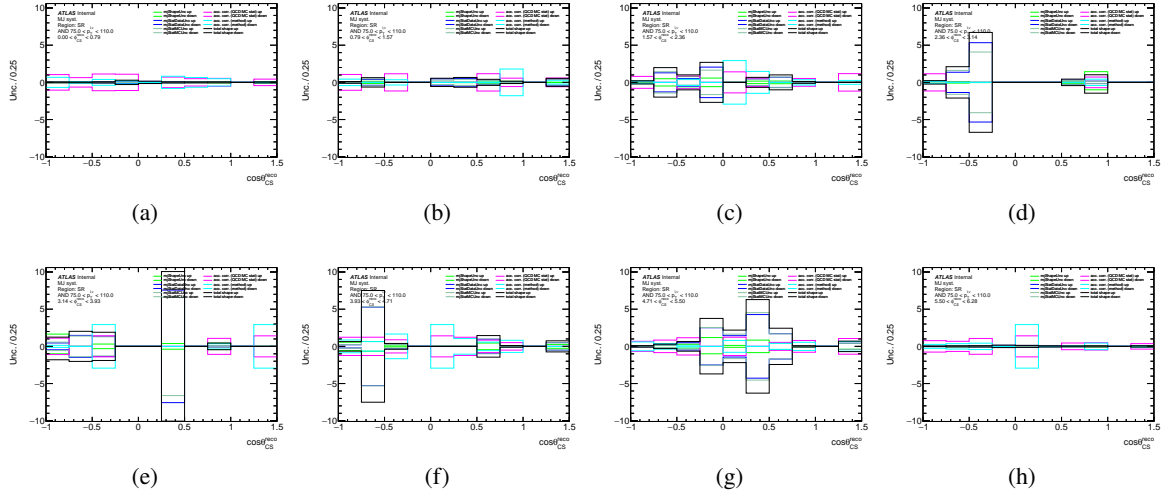


Figure 260: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

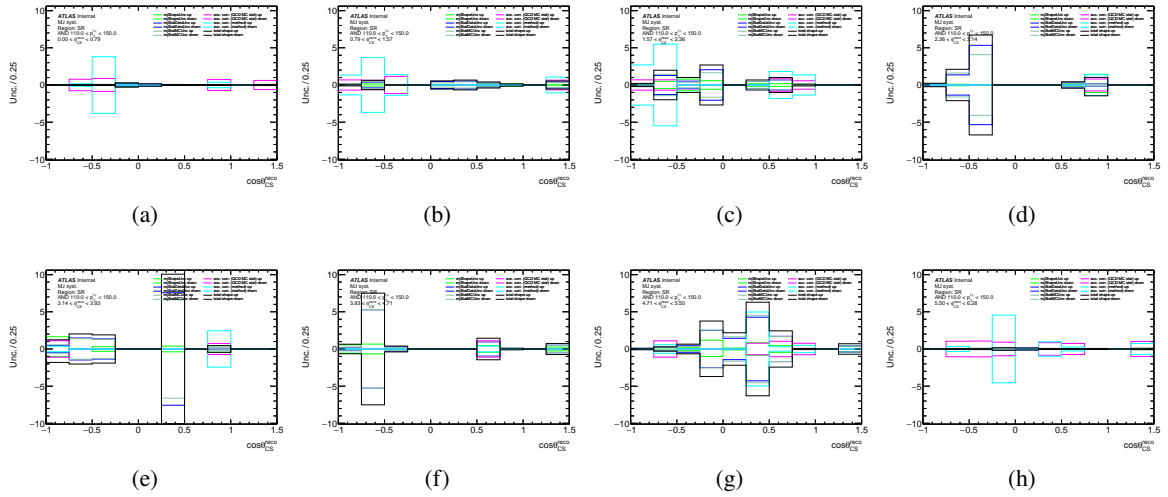


Figure 261: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

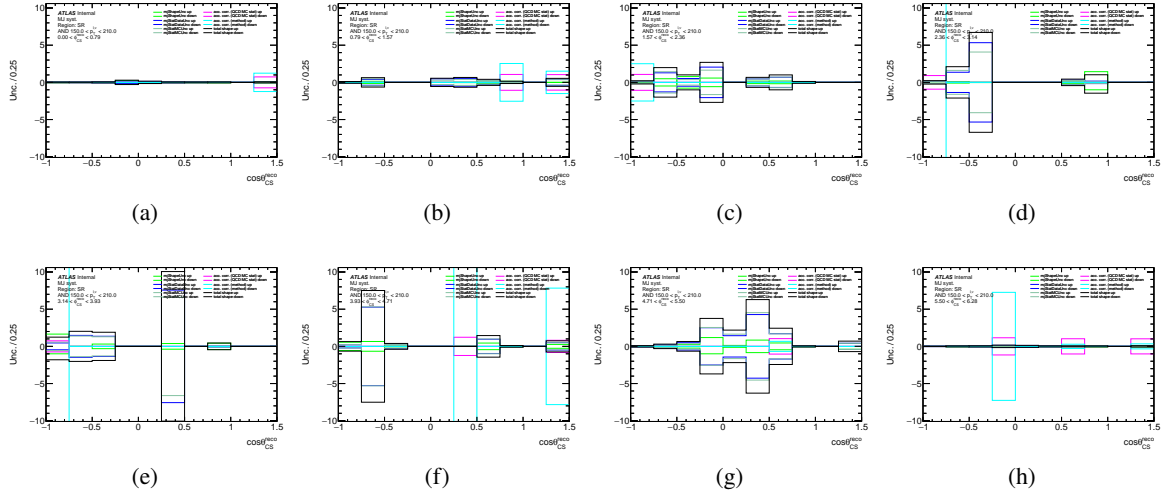


Figure 262: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

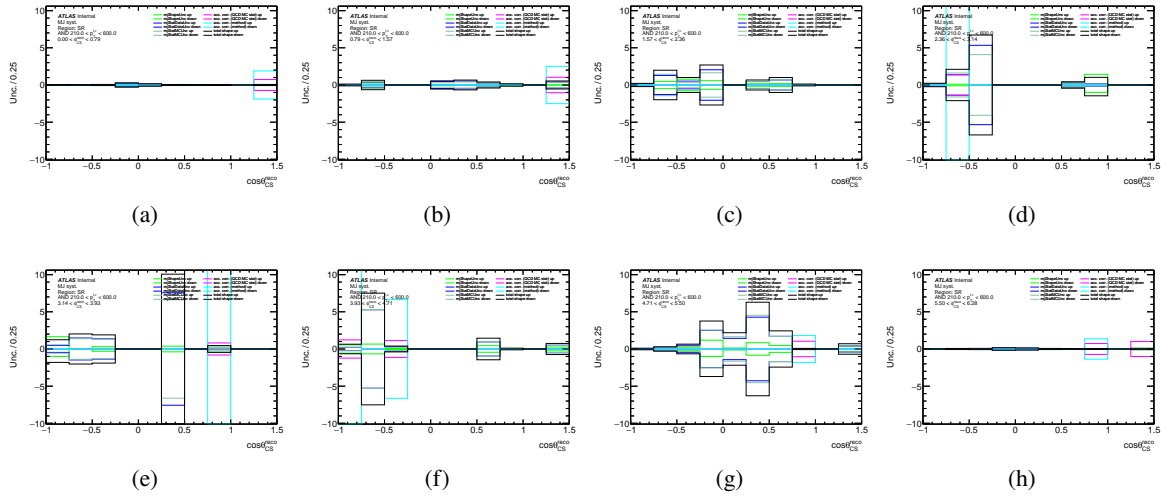


Figure 263: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

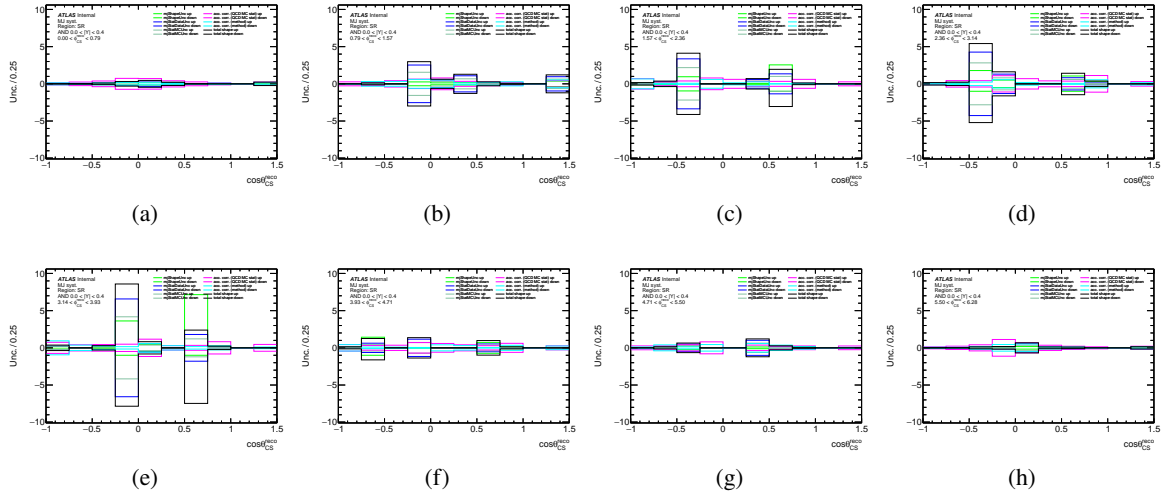


Figure 264: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^+ \rightarrow e^+ \nu$ channel.

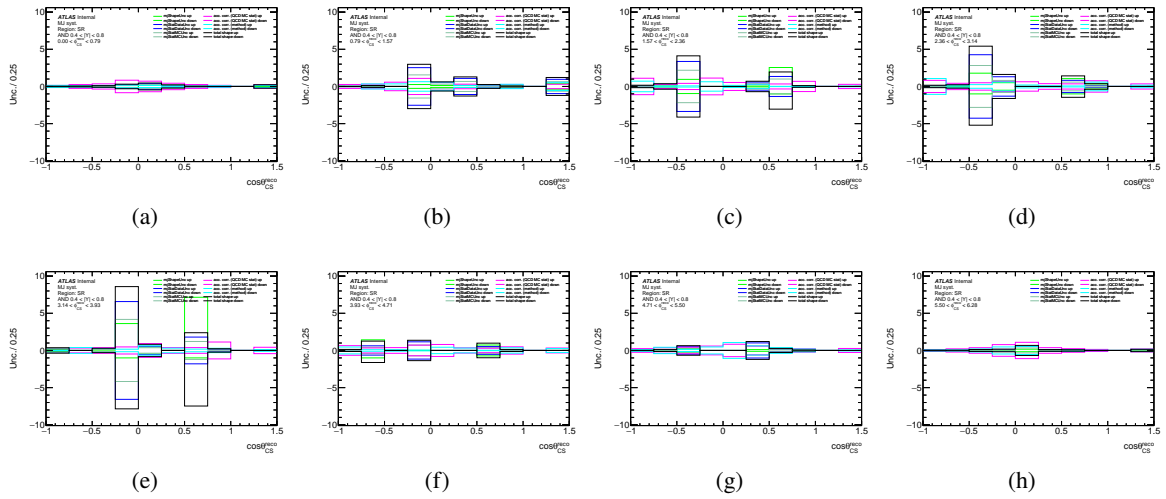


Figure 265: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow e^+ \nu$ channel.

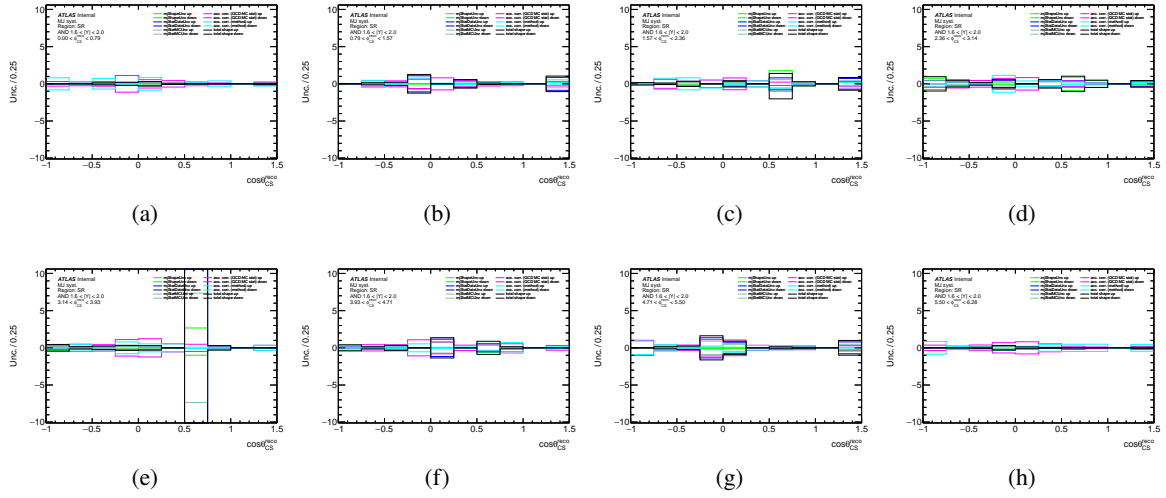


Figure 268: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow e^+\nu$ channel.

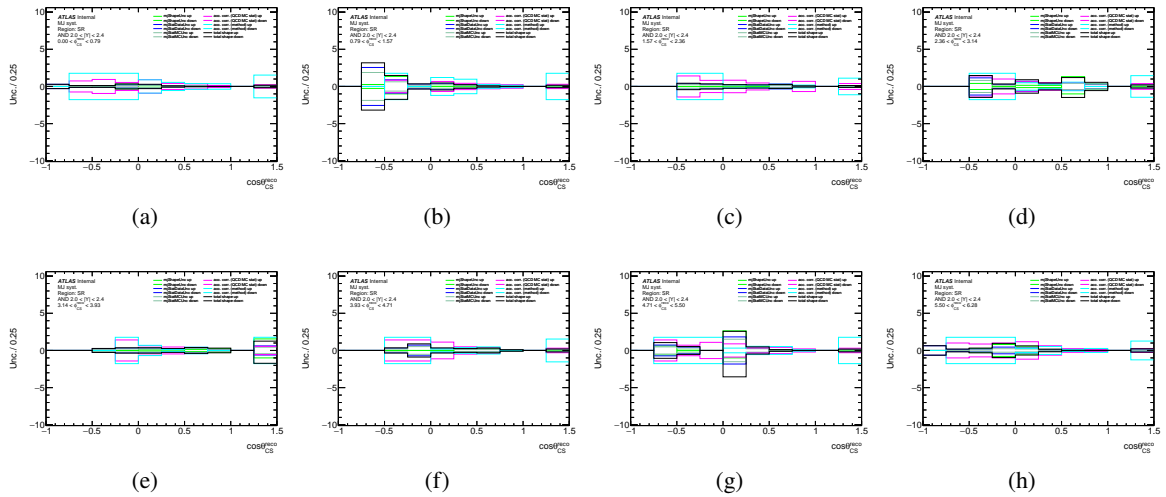


Figure 269: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow e^+\nu$ channel.

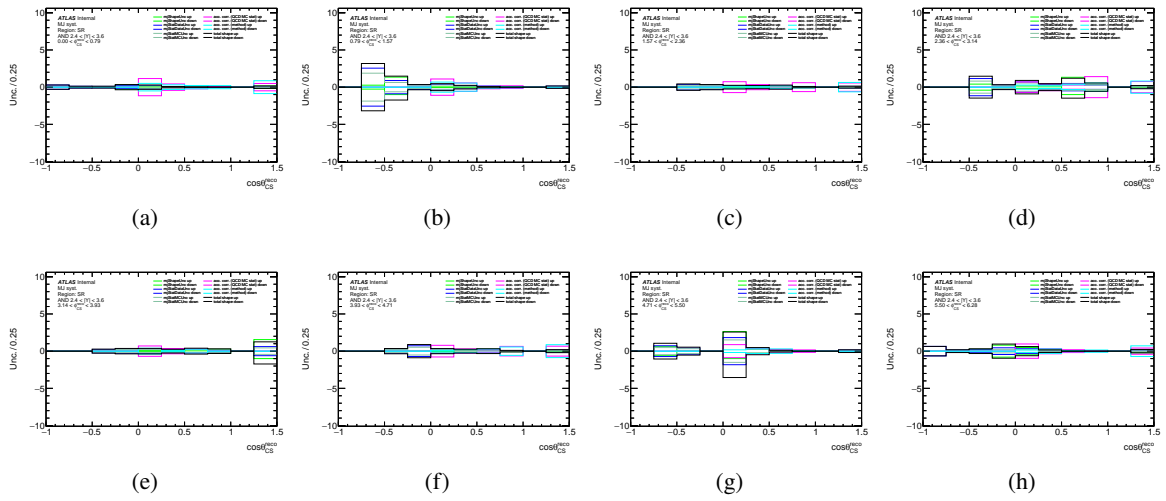


Figure 270: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow e^+\nu$ channel.

1291 **E.7.3 Muon channel**

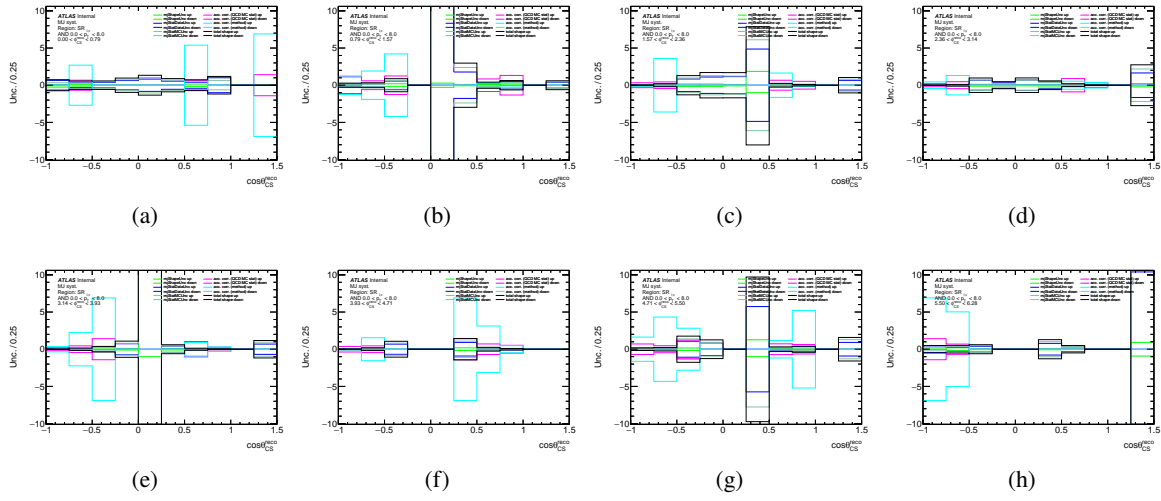


Figure 271: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

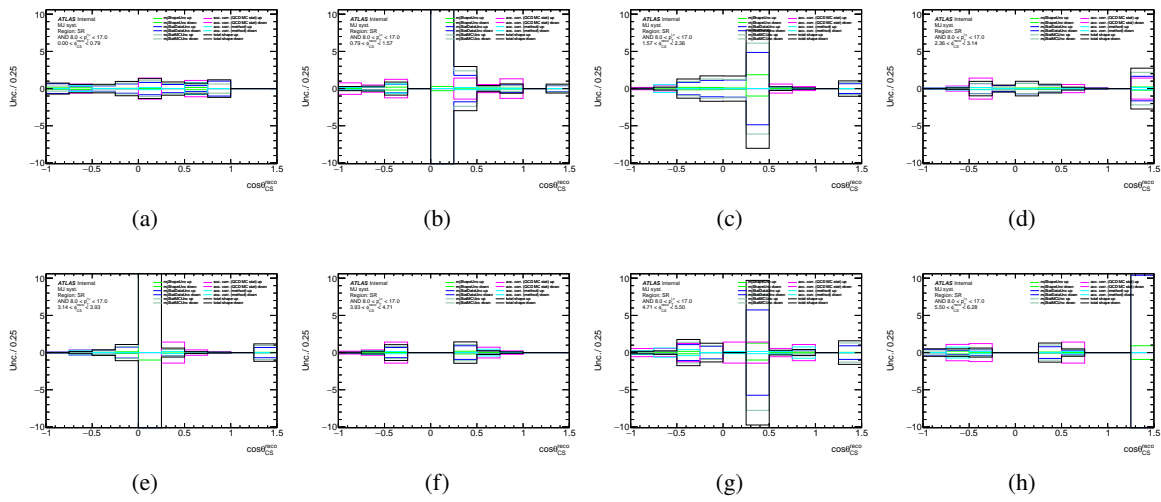


Figure 272: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

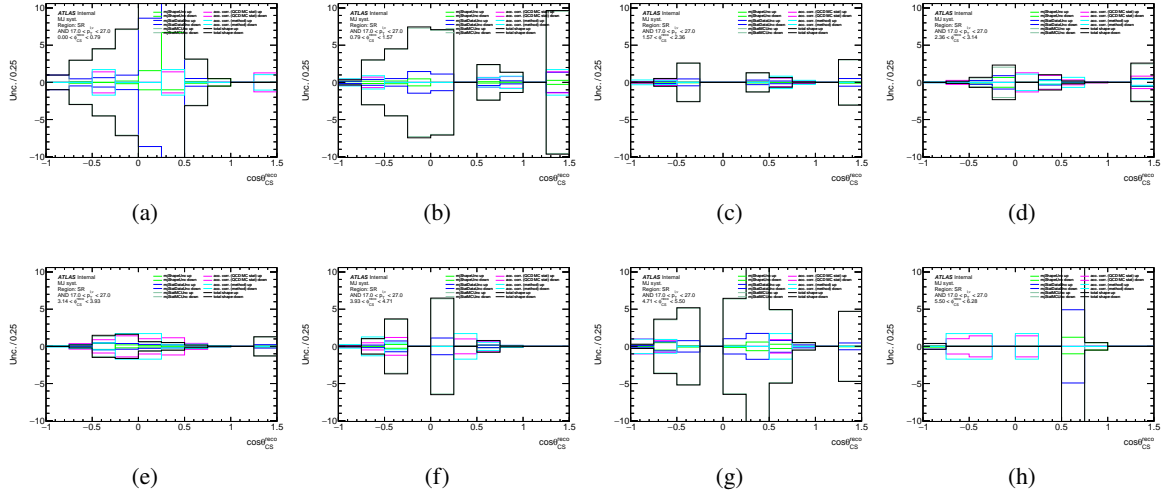


Figure 273: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

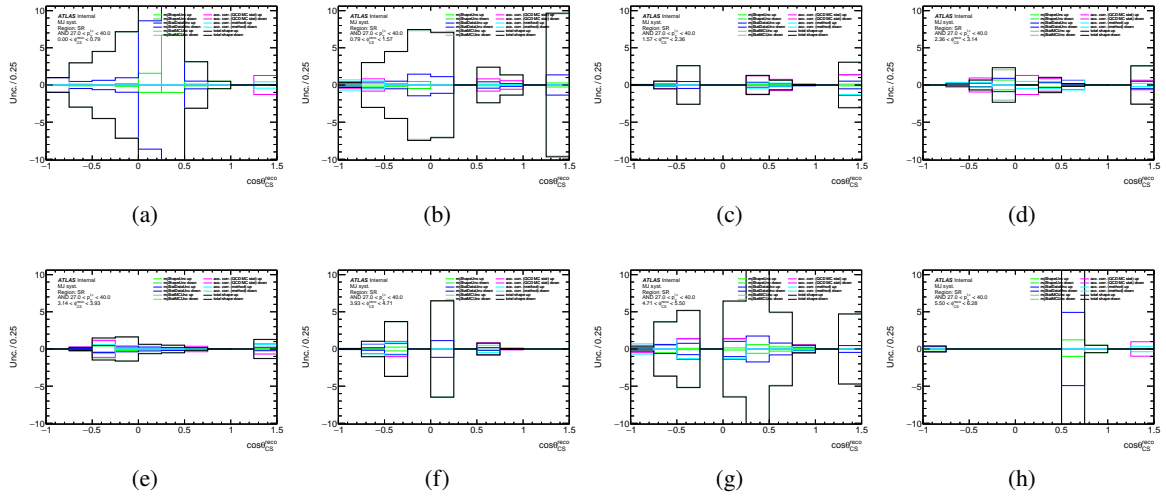


Figure 274: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

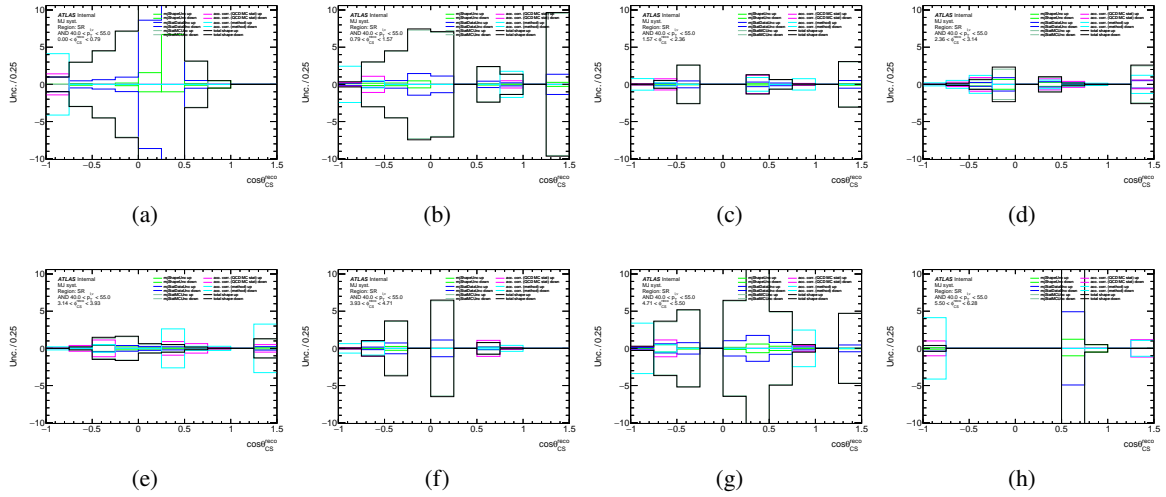


Figure 275: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

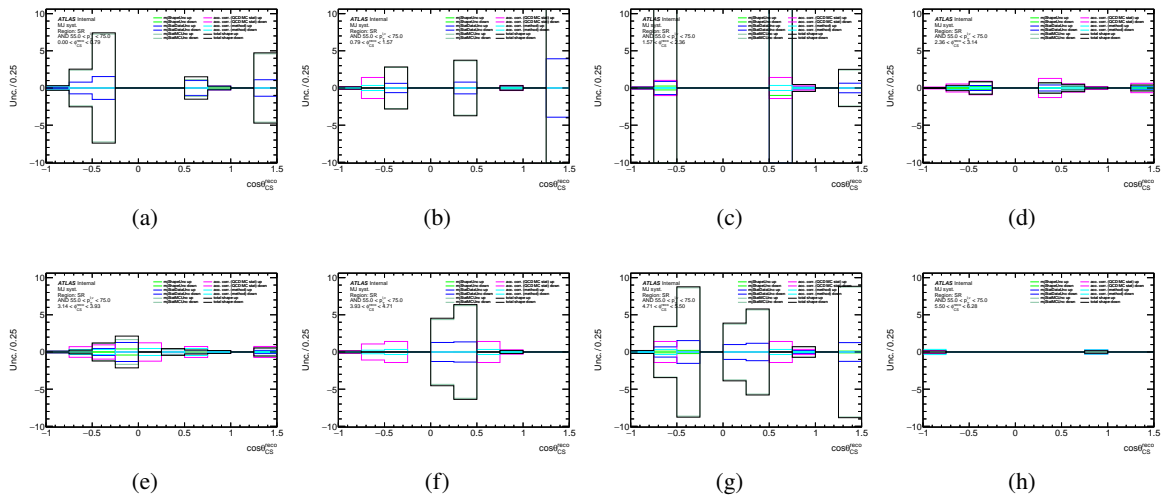


Figure 276: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

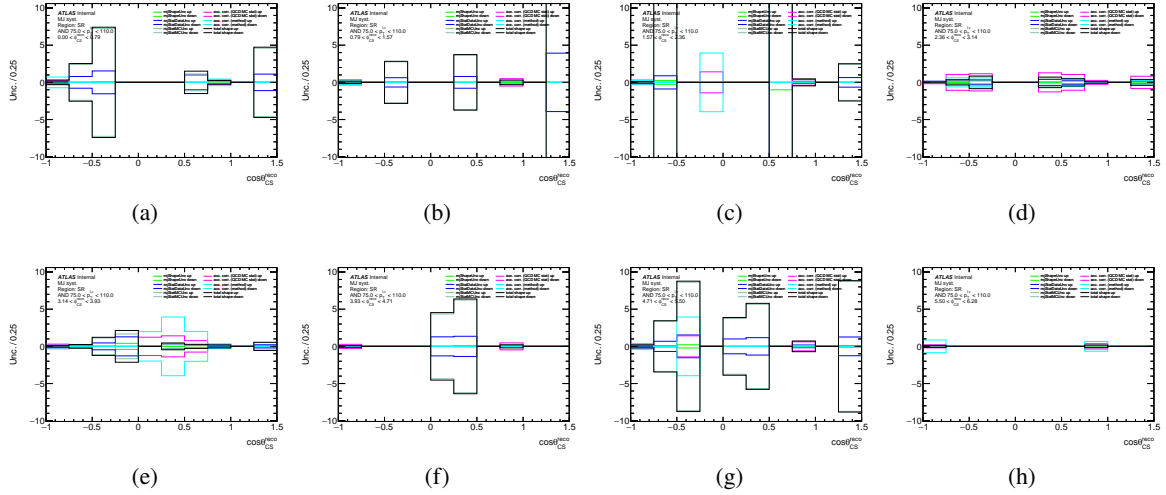


Figure 277: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

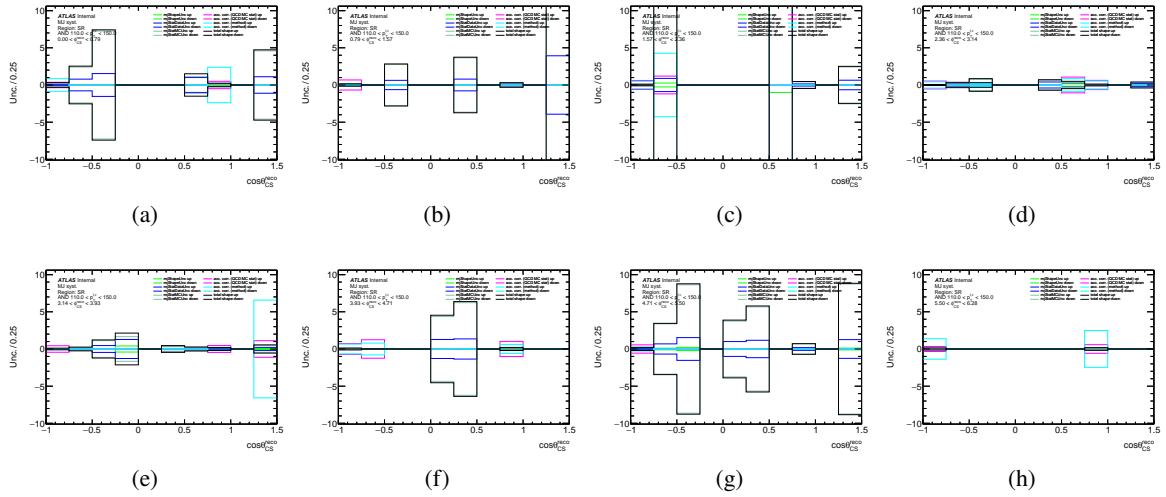


Figure 278: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

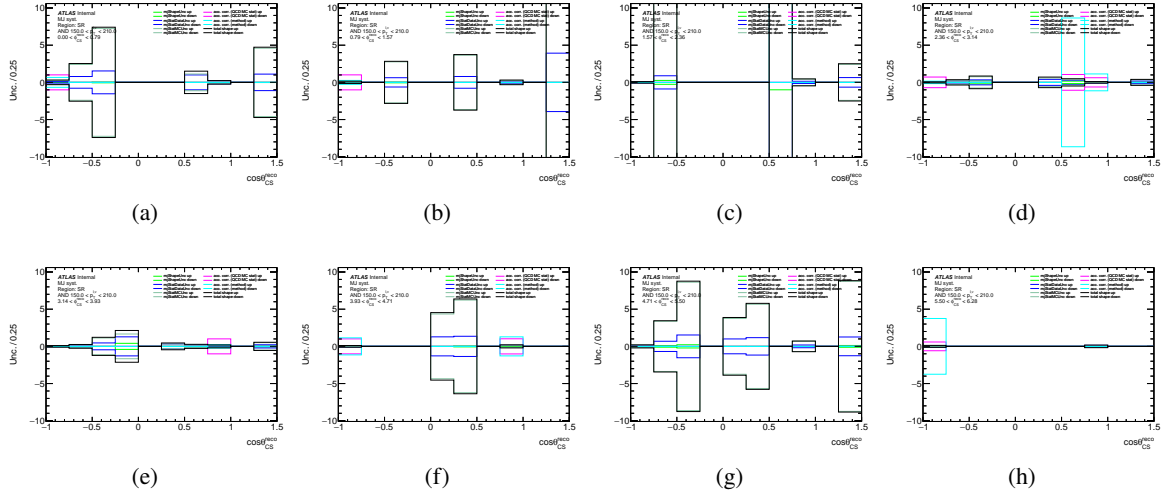


Figure 279: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

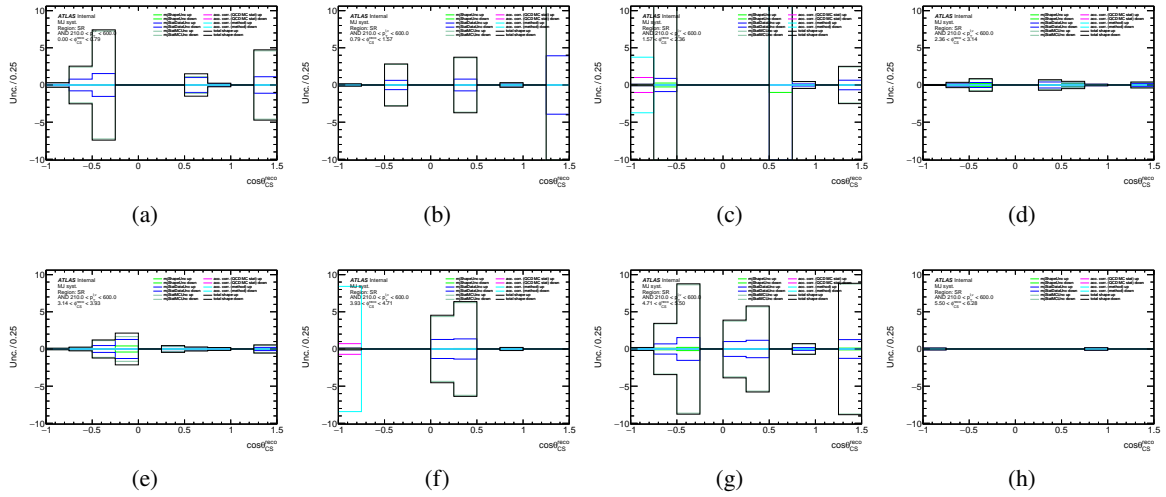


Figure 280: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

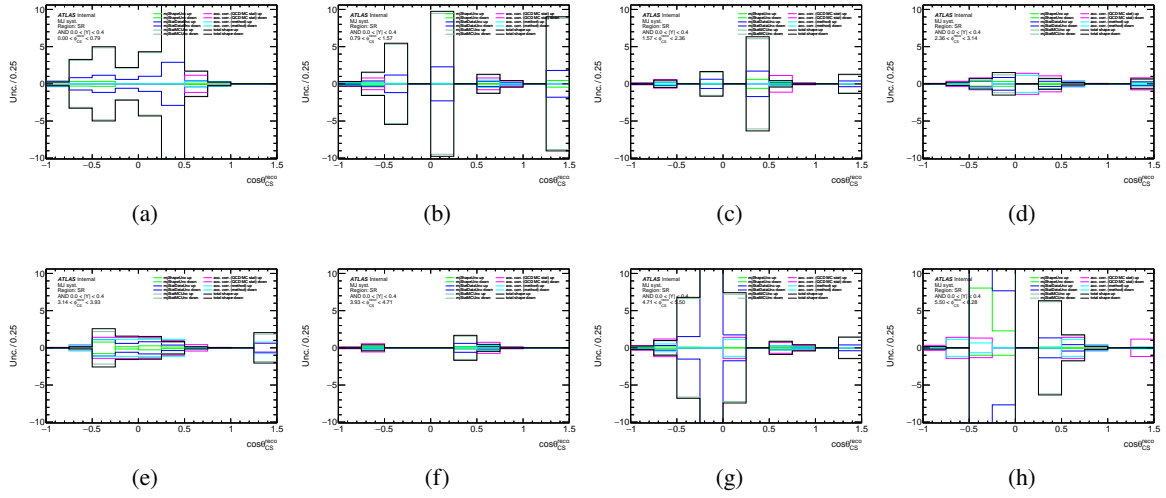


Figure 281: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

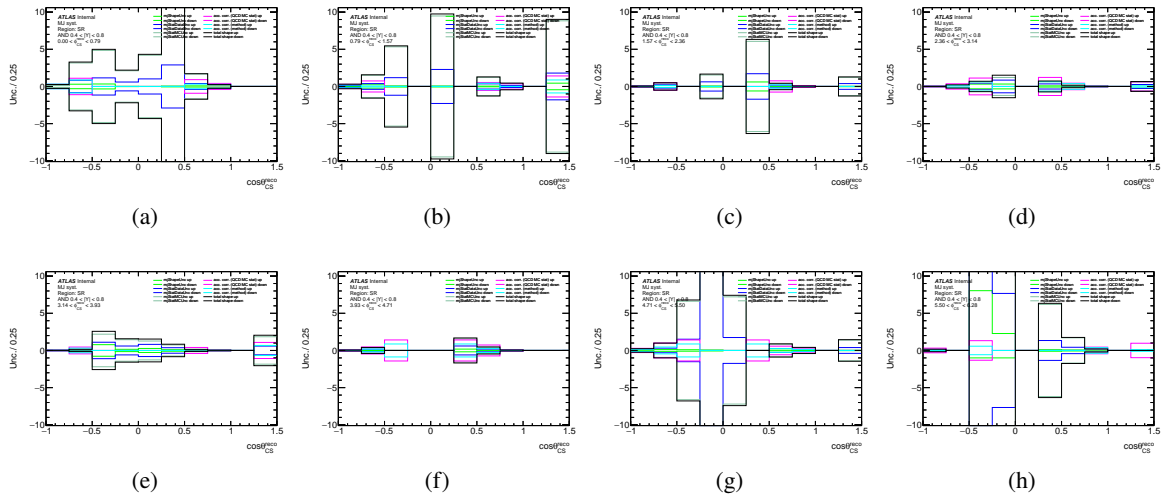


Figure 282: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

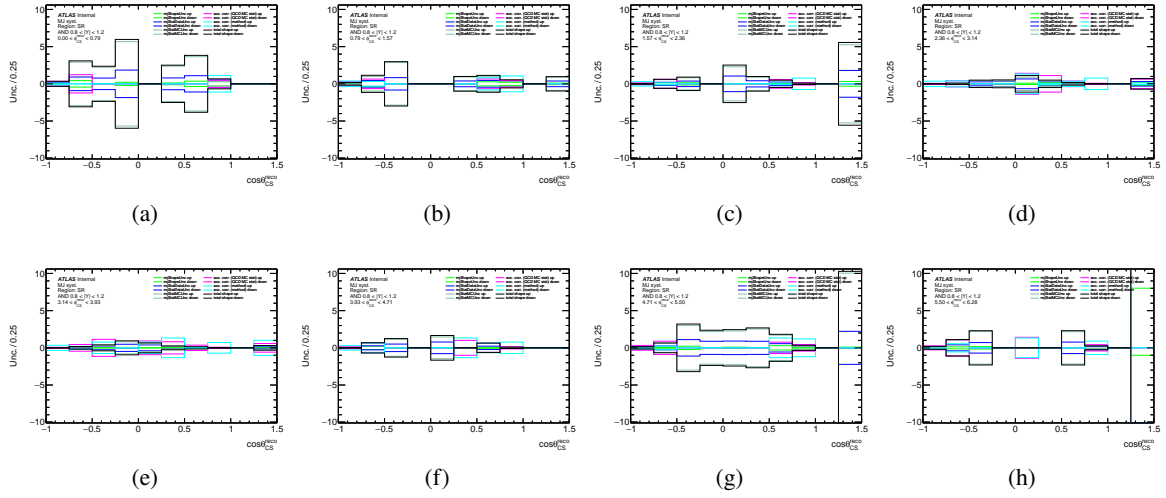


Figure 283: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

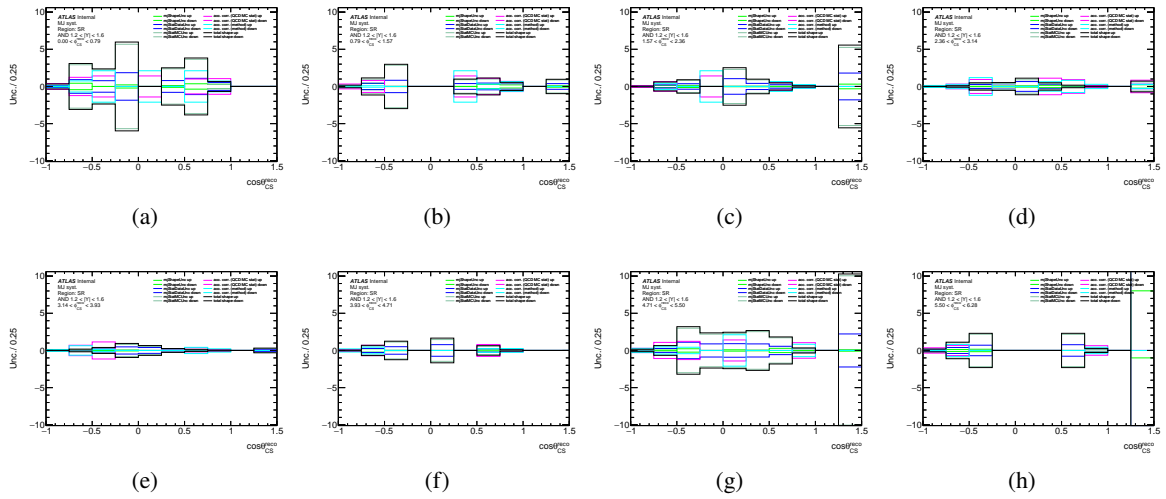


Figure 284: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

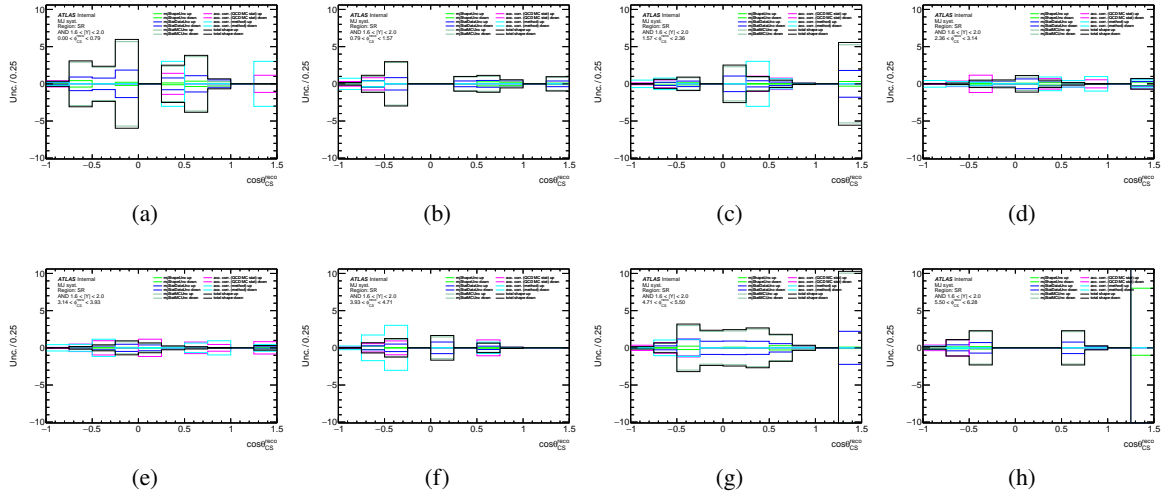


Figure 285: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

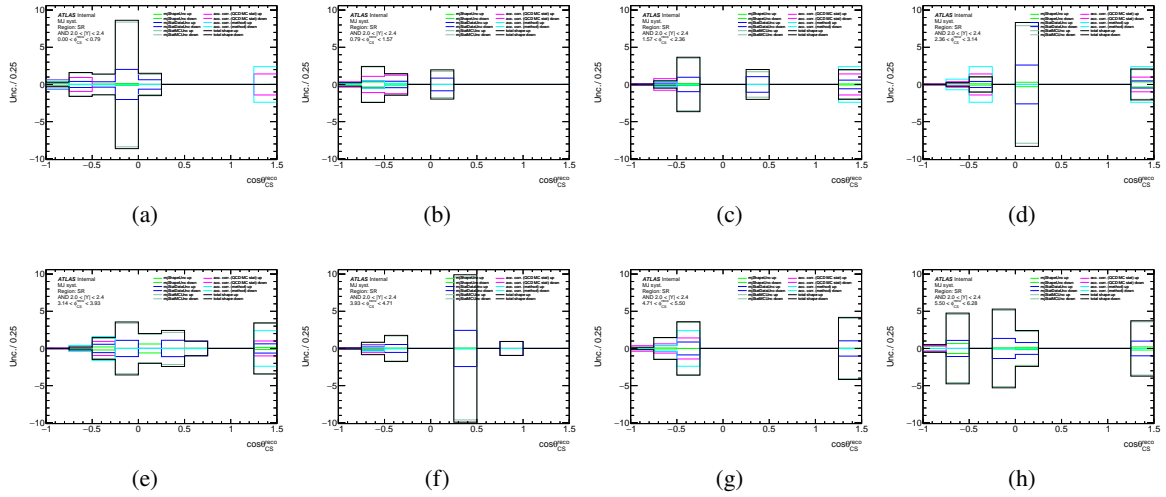


Figure 286: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

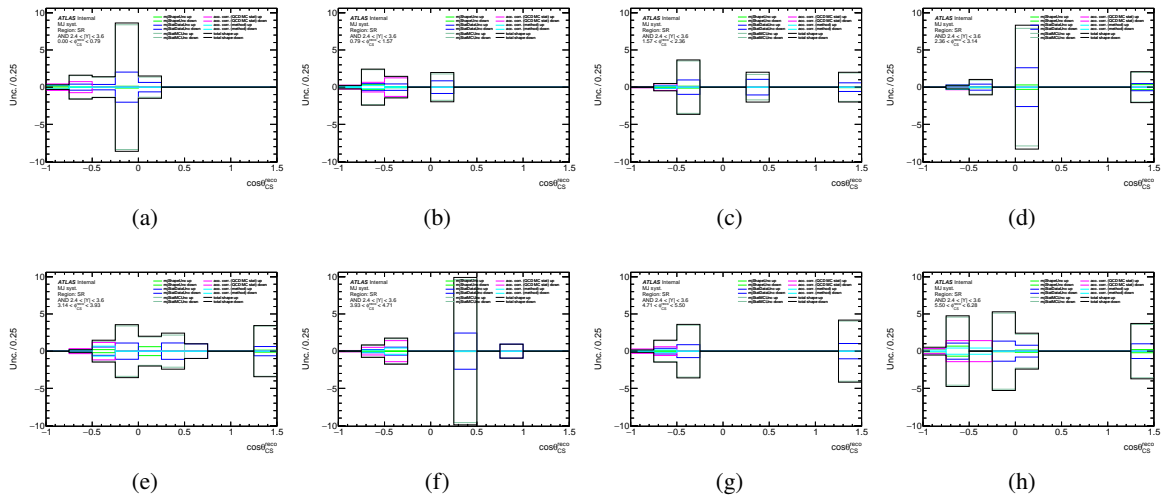


Figure 287: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

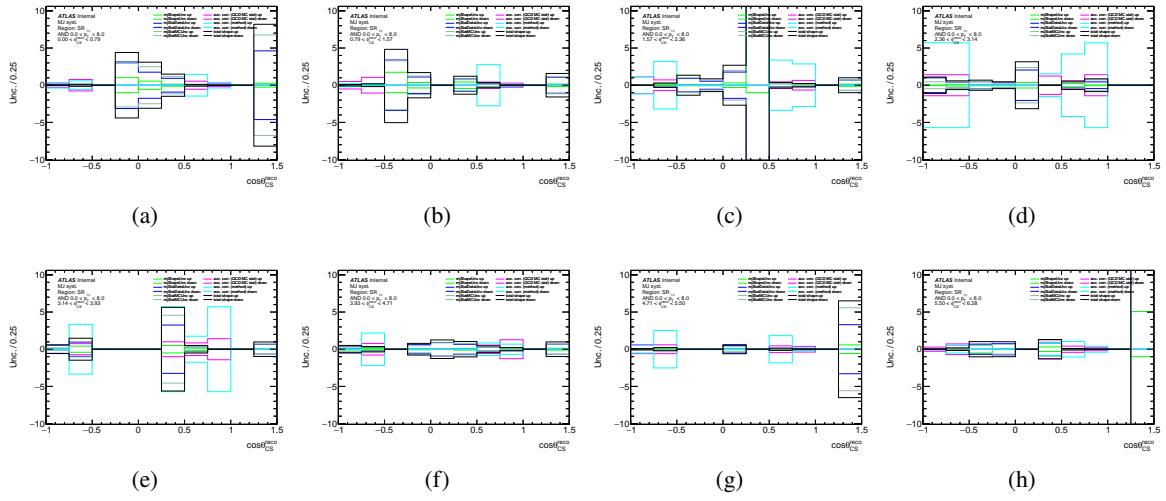


Figure 288: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

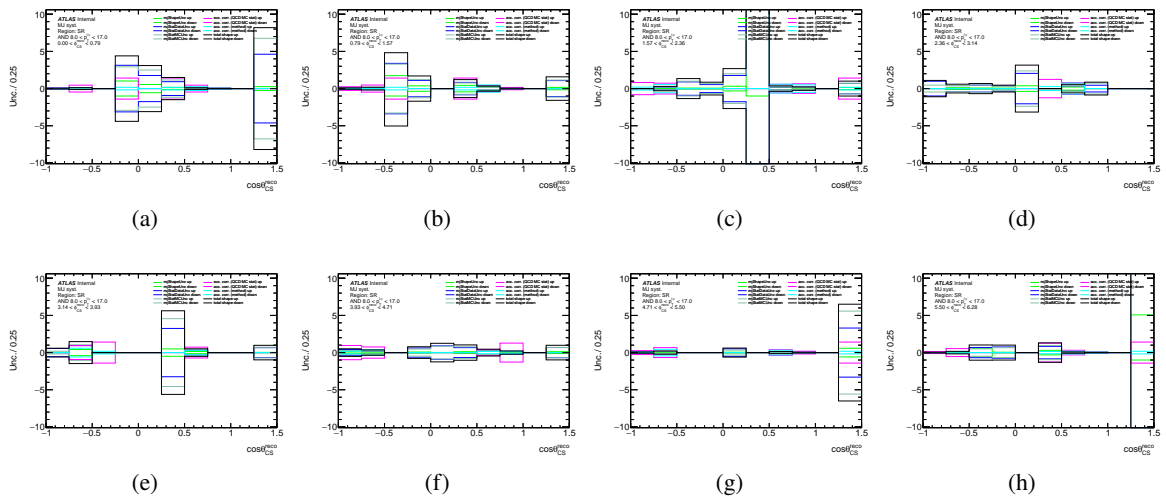


Figure 289: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $8 < p_T^{\ell,\nu} < 17$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

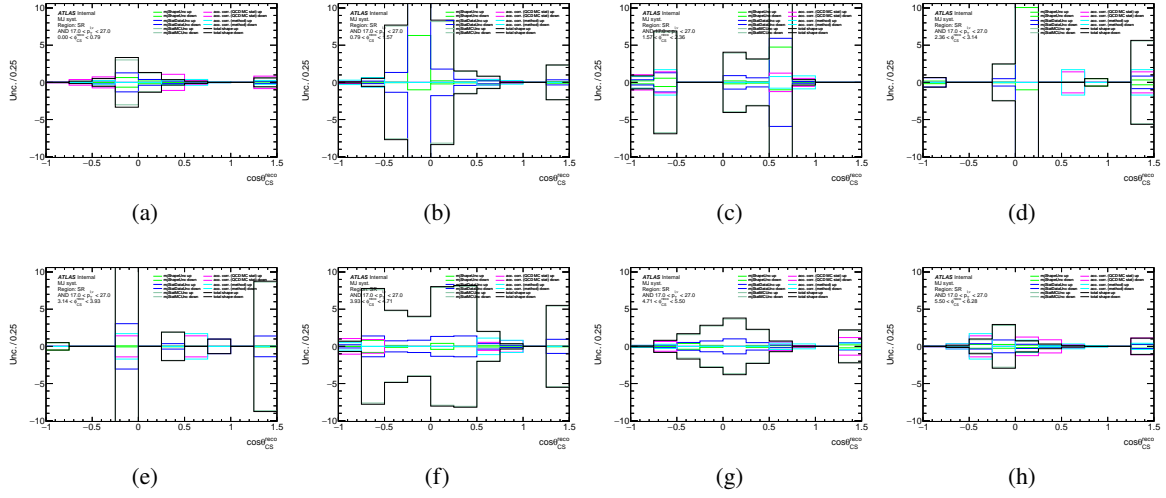


Figure 290: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $17 < p_T^{\ell,\nu} < 27$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

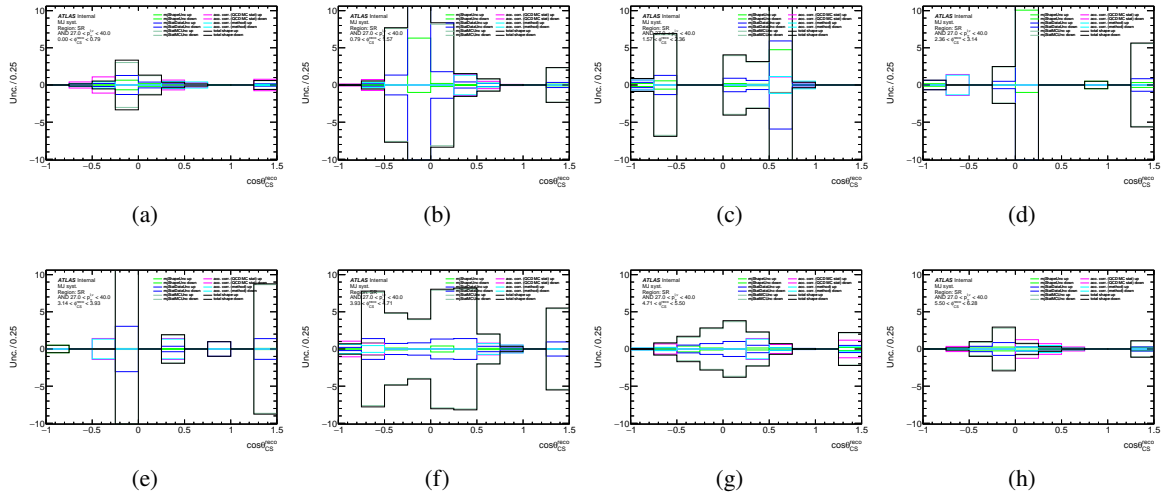


Figure 291: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

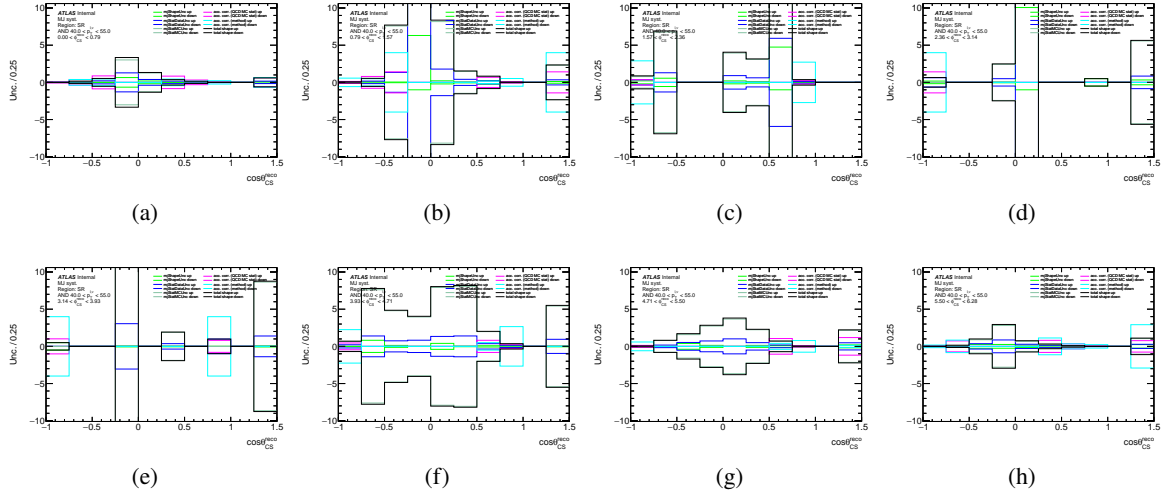


Figure 292: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

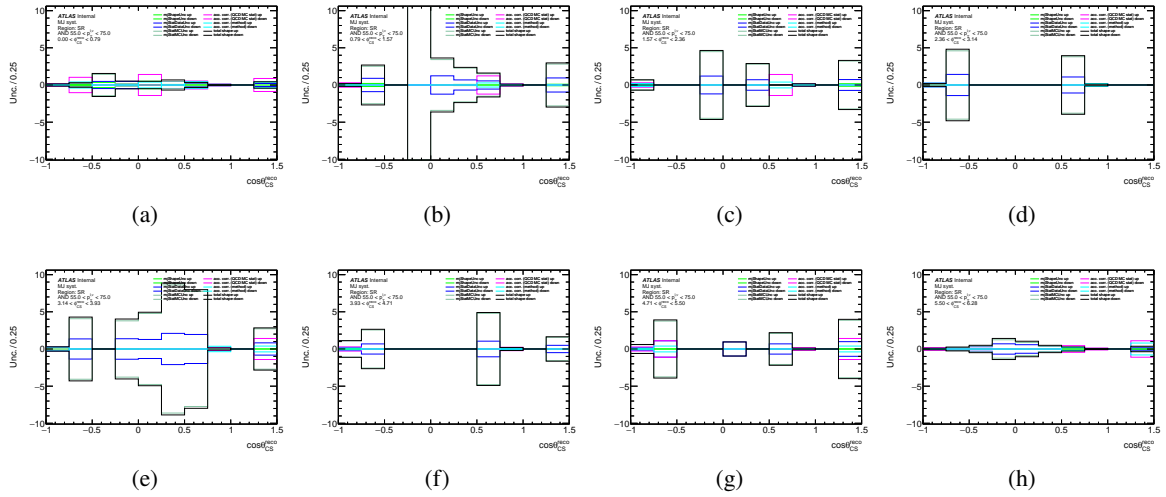


Figure 293: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

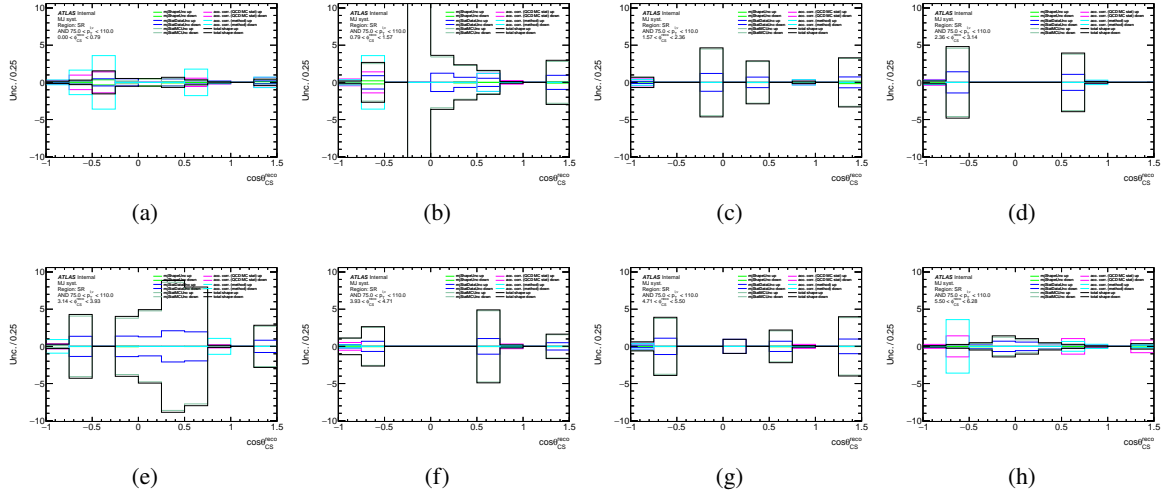


Figure 294: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

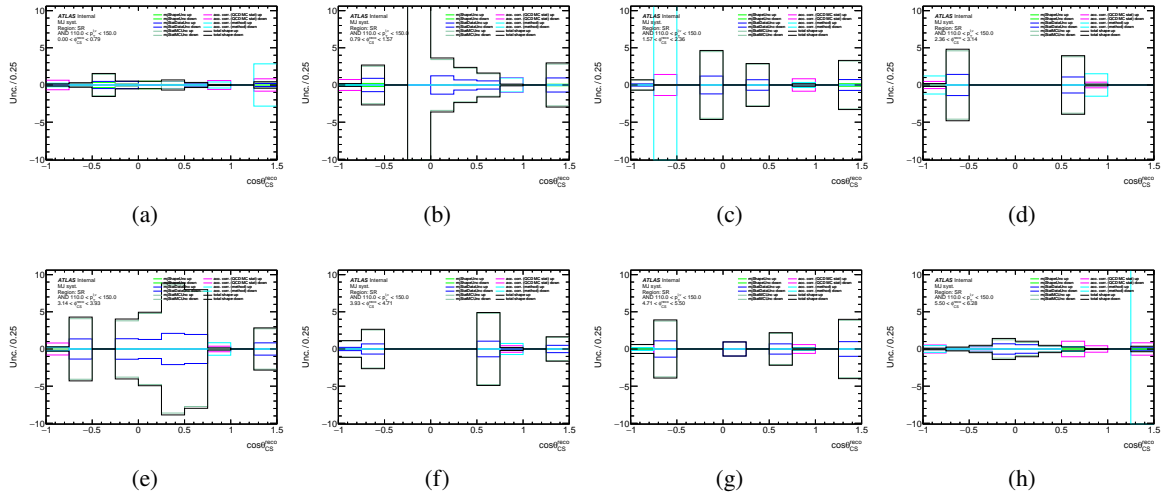


Figure 295: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

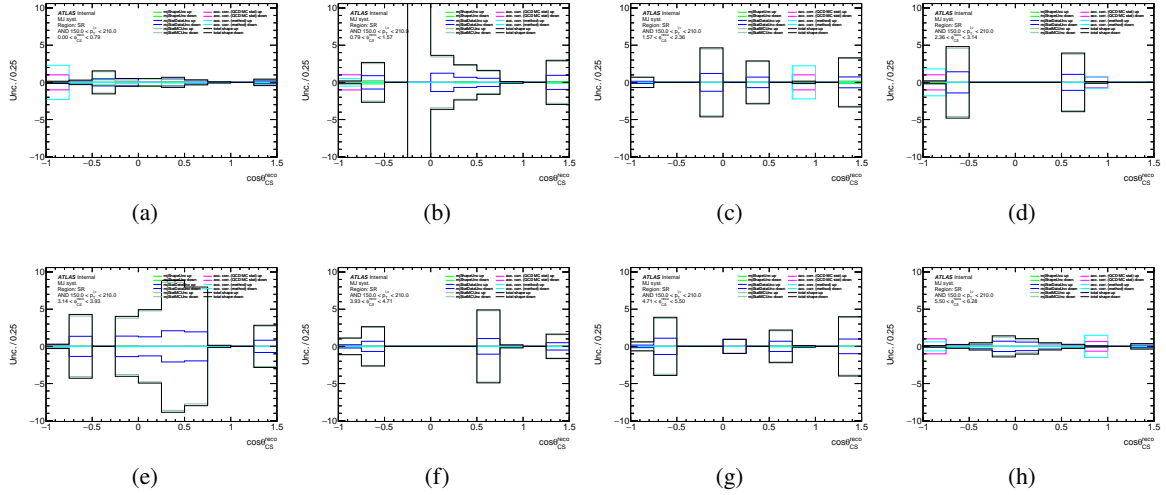


Figure 296: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

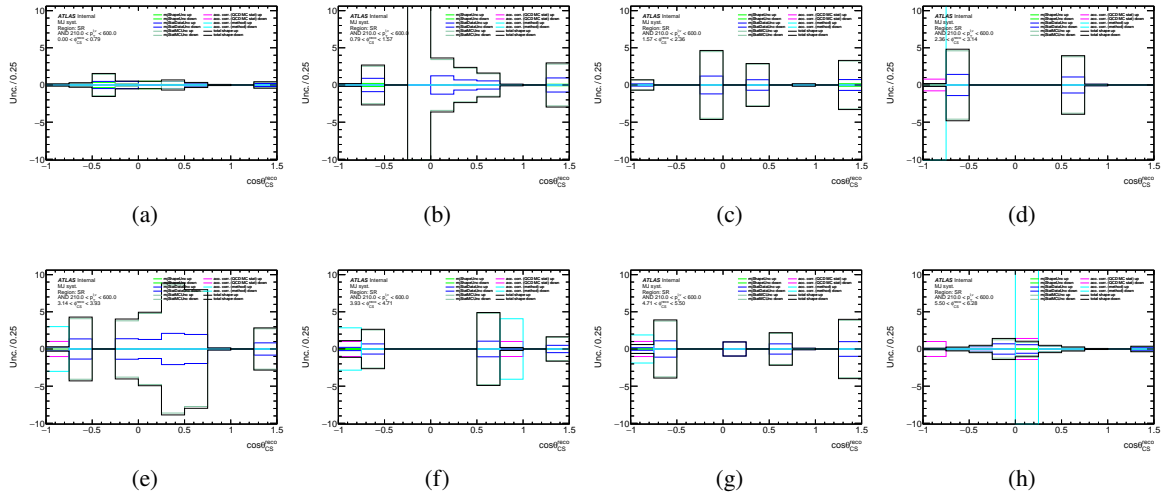


Figure 297: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $210 < p_T^{\ell,\nu} < 600$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

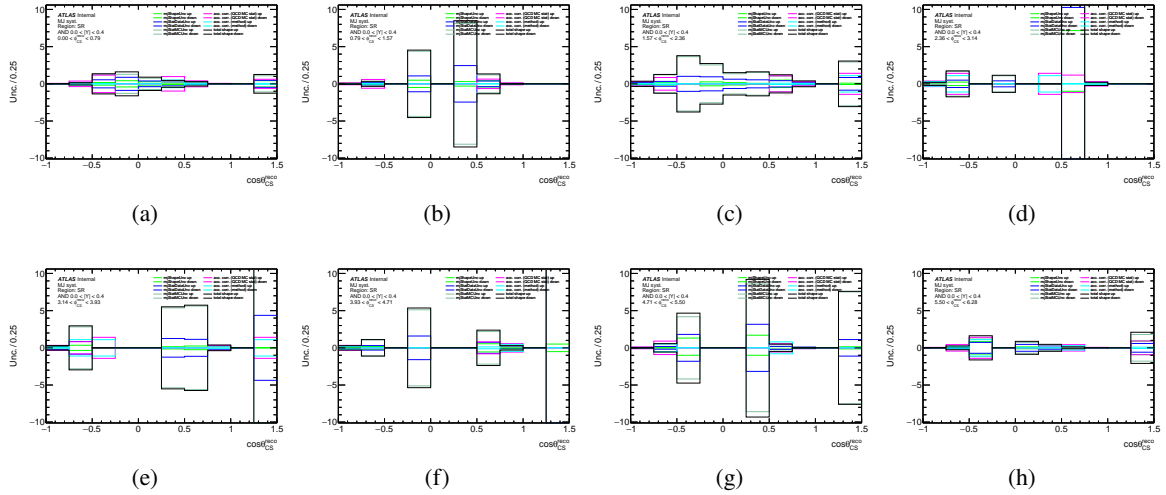


Figure 298: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0 < |Y| < 0.4$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

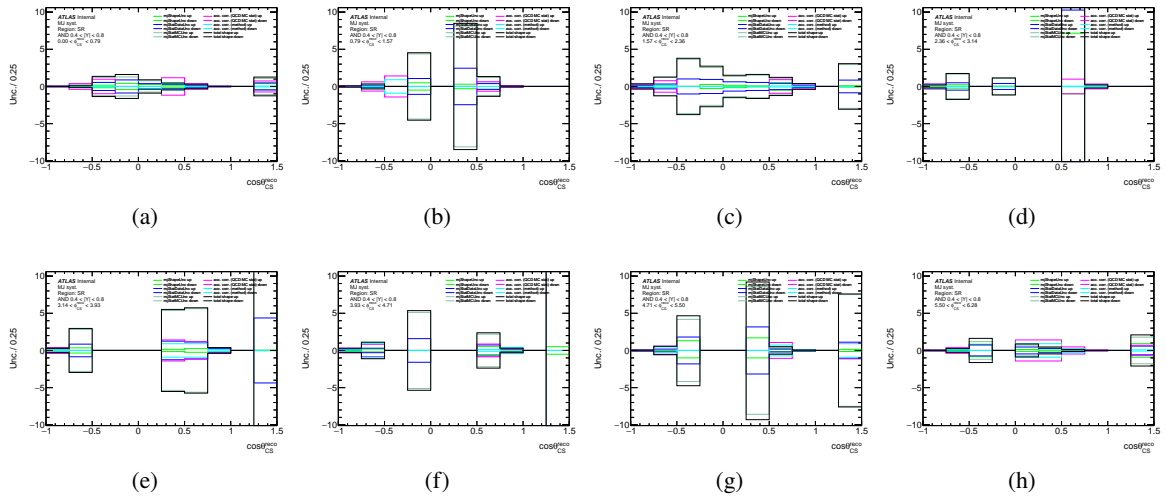


Figure 299: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

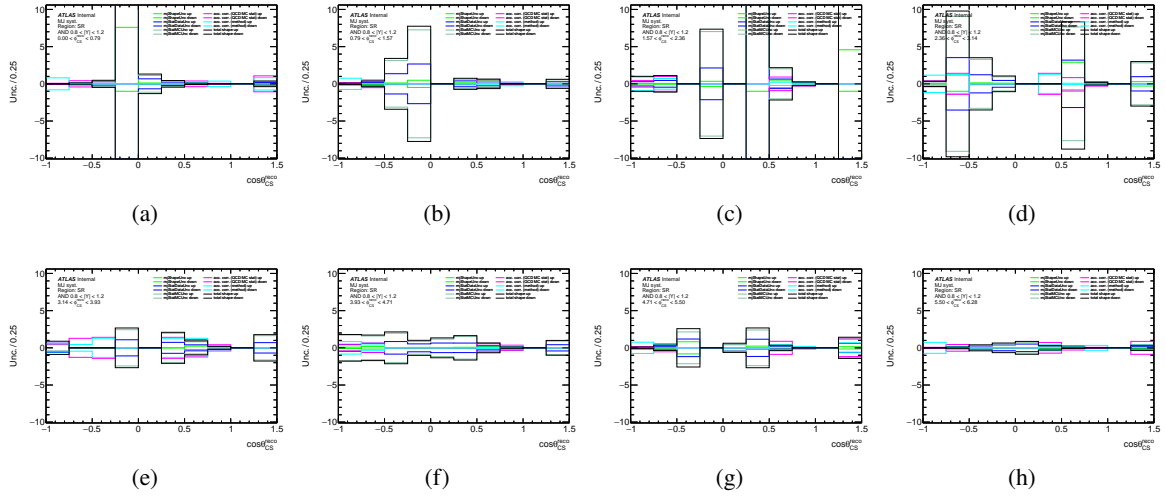


Figure 300: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $0.8 < |Y| < 1.2$ bin for $W^+ \rightarrow \mu^+\nu$ channel.

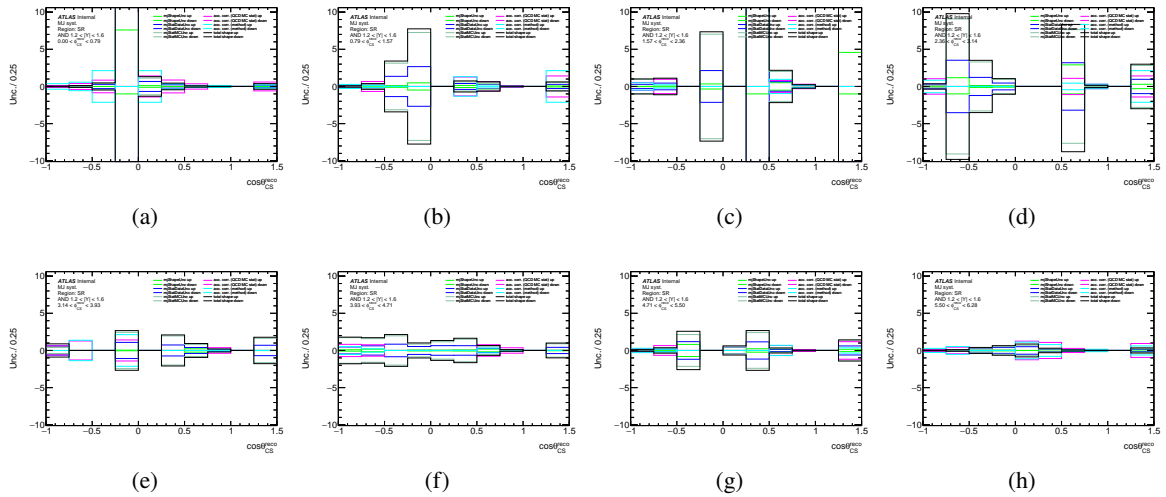


Figure 301: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.2 < |Y| < 1.6$ bin for $W^+ \rightarrow \mu^+\nu$ channel.

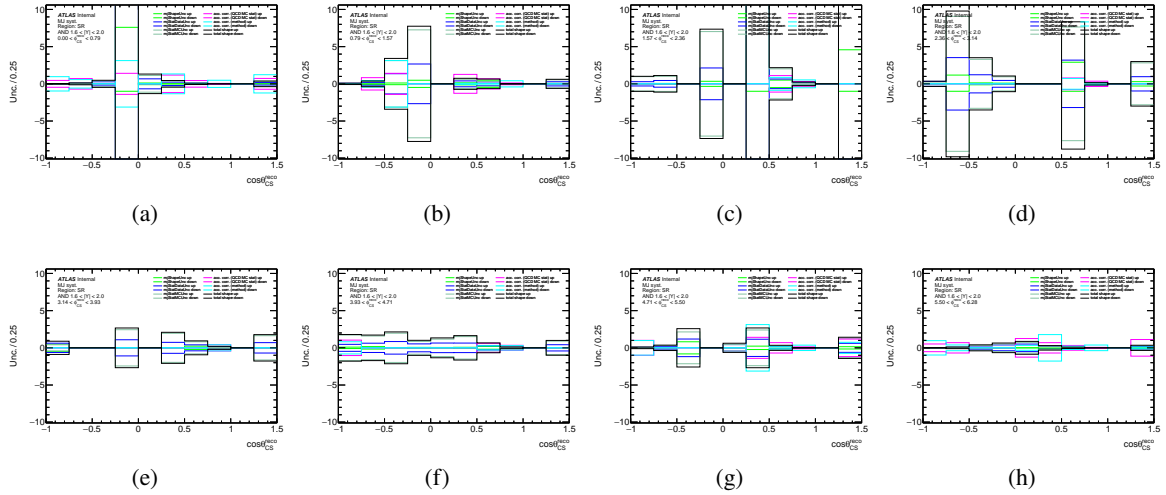


Figure 302: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow \mu^+\nu$ channel.

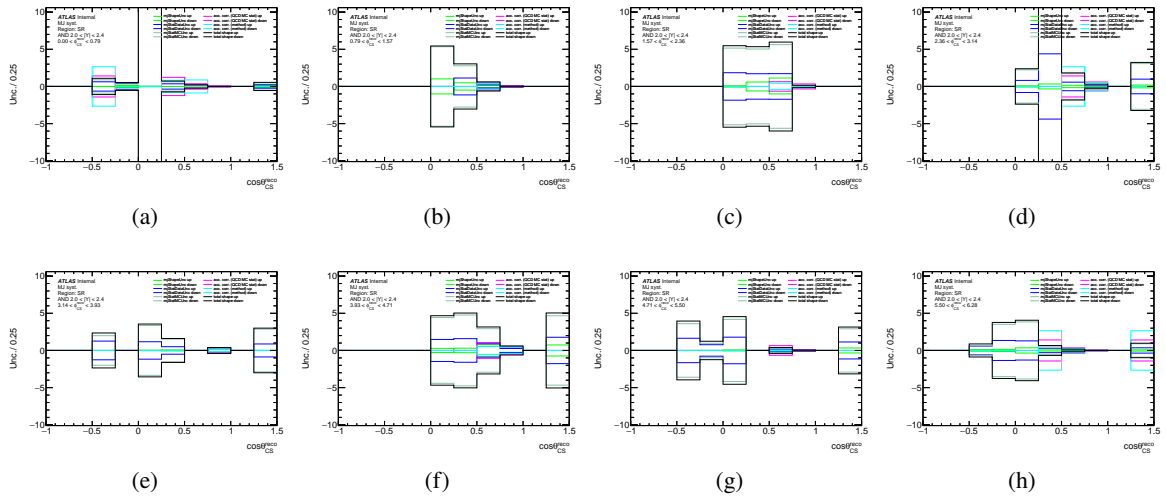


Figure 303: Multi-jet background shape systematics breakdown for $\cos\theta_{CS}$ as slices of ϕ_{CS} for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow \mu^+\nu$ channel.

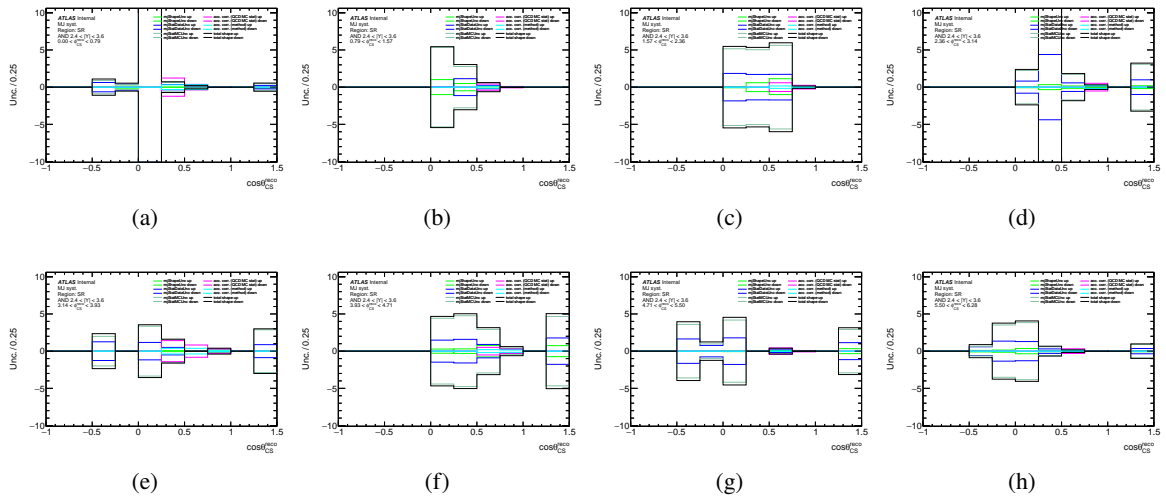


Figure 304: Multi-jet background shape systematics breakdown for $\cos \theta_{CS}$ as slices of ϕ_{CS} for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

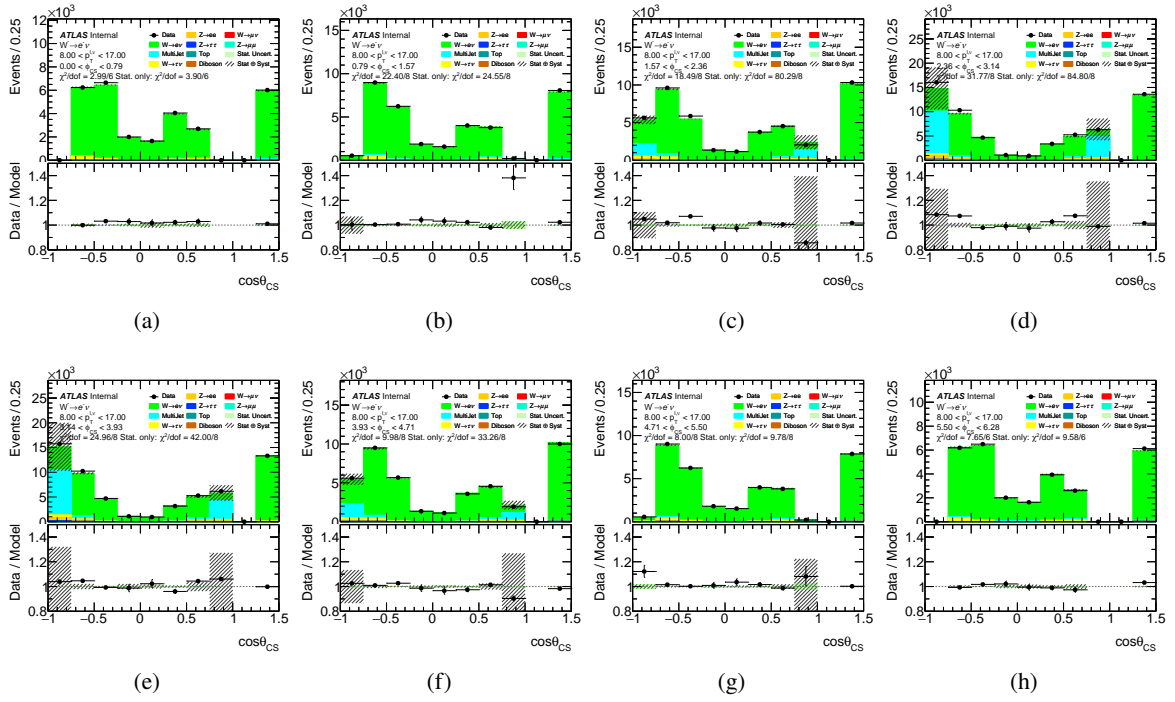


Figure 306: Control plots for $8 < p_T^{\ell, \nu} < 17$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

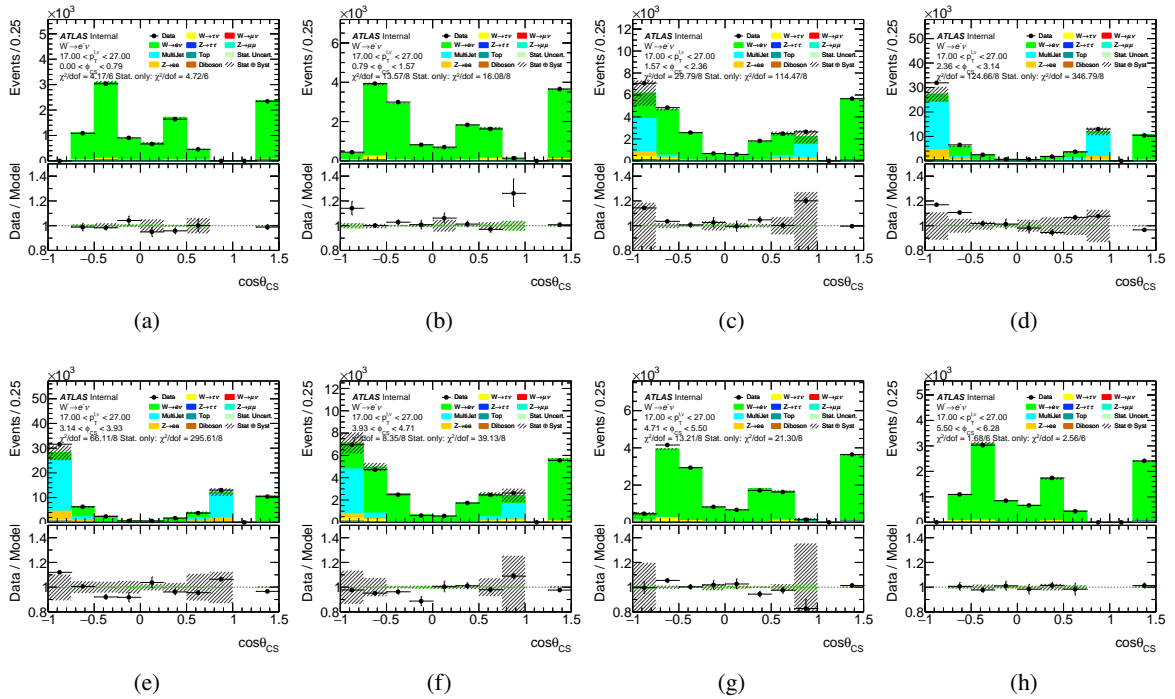


Figure 307: Control plots for $17 < p_T^{\ell, \nu} < 27$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

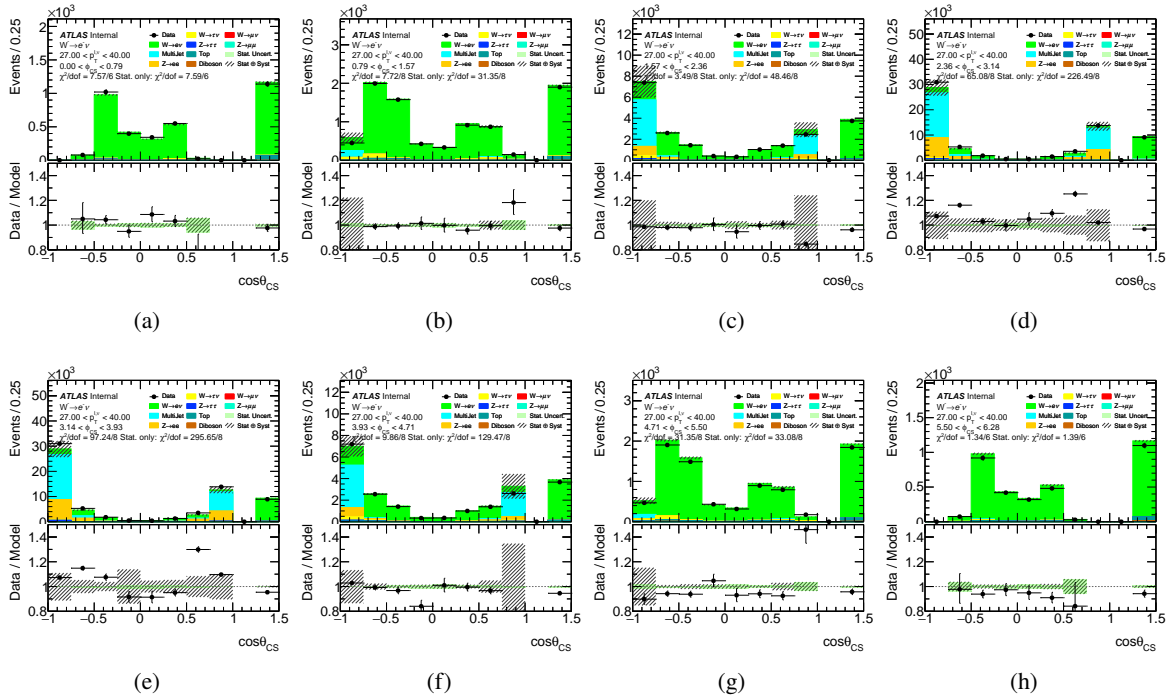


Figure 308: Control plots for $27 < p_T^{\ell,\nu} < 40$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

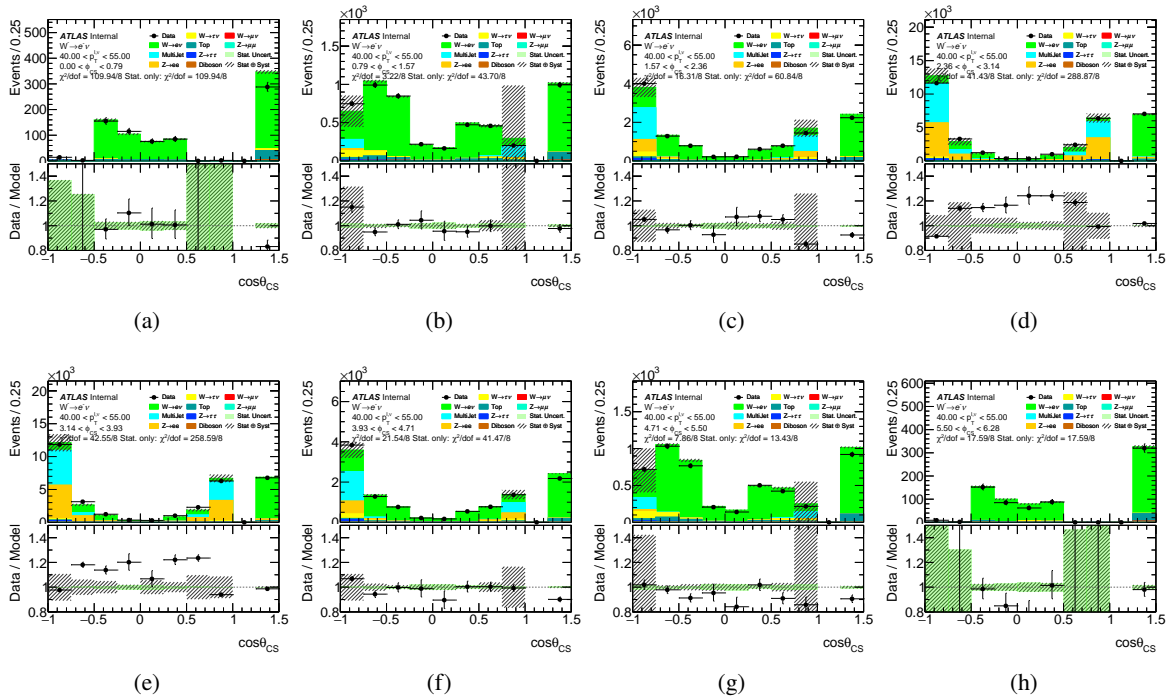


Figure 309: Control plots for $40 < p_T^{\ell,\nu} < 55$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

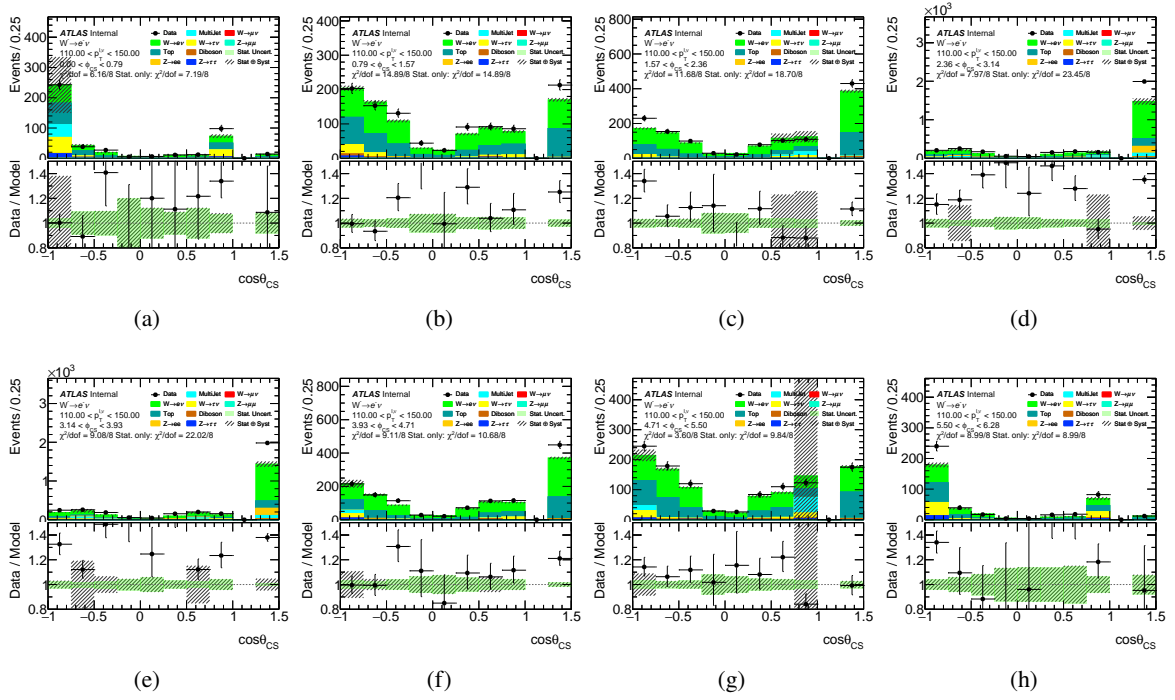


Figure 312: Control plots for $110 < p_T^{\ell, \nu} < 150$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

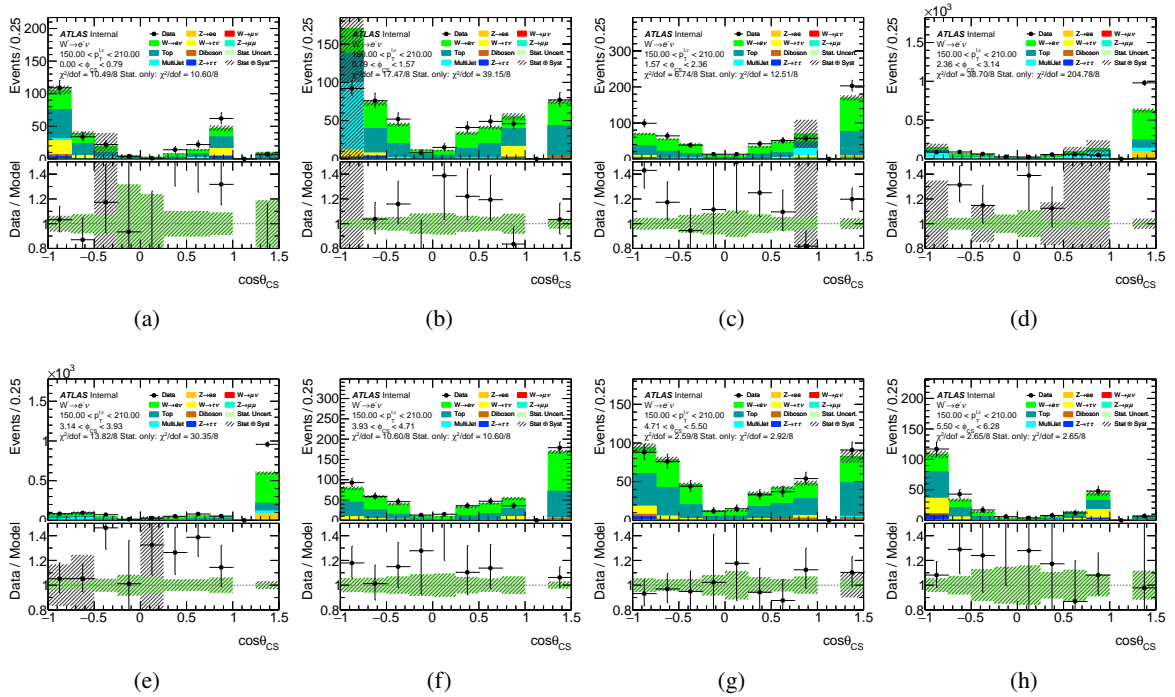


Figure 313: Control plots for $150 < p_T^{\ell, \nu} < 210$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

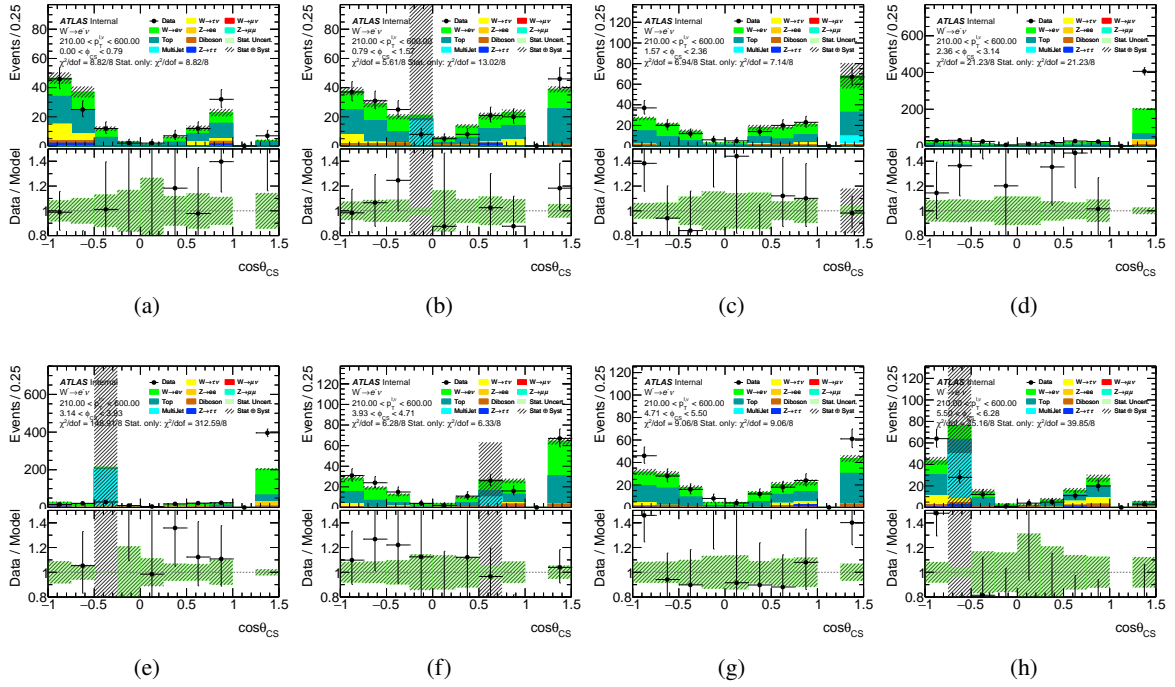


Figure 314: Control plots for $210 < p_T^{\ell, \nu} < 600$ GeV bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

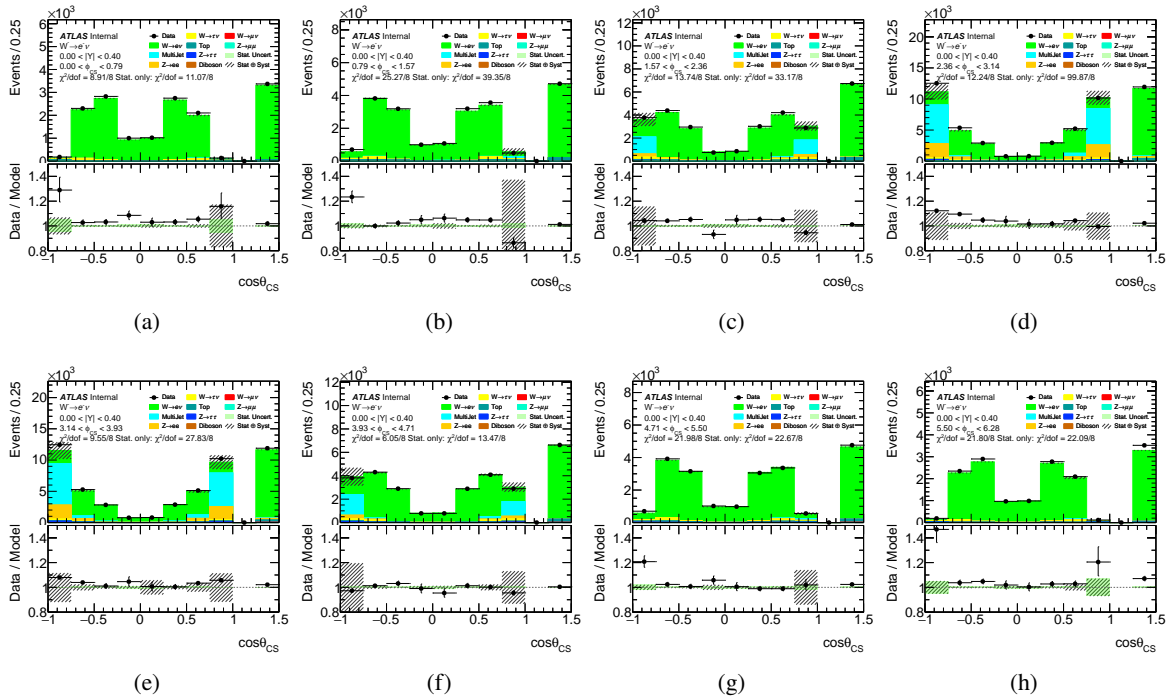


Figure 315: Control plots for $0 < |\eta| < 0.4$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

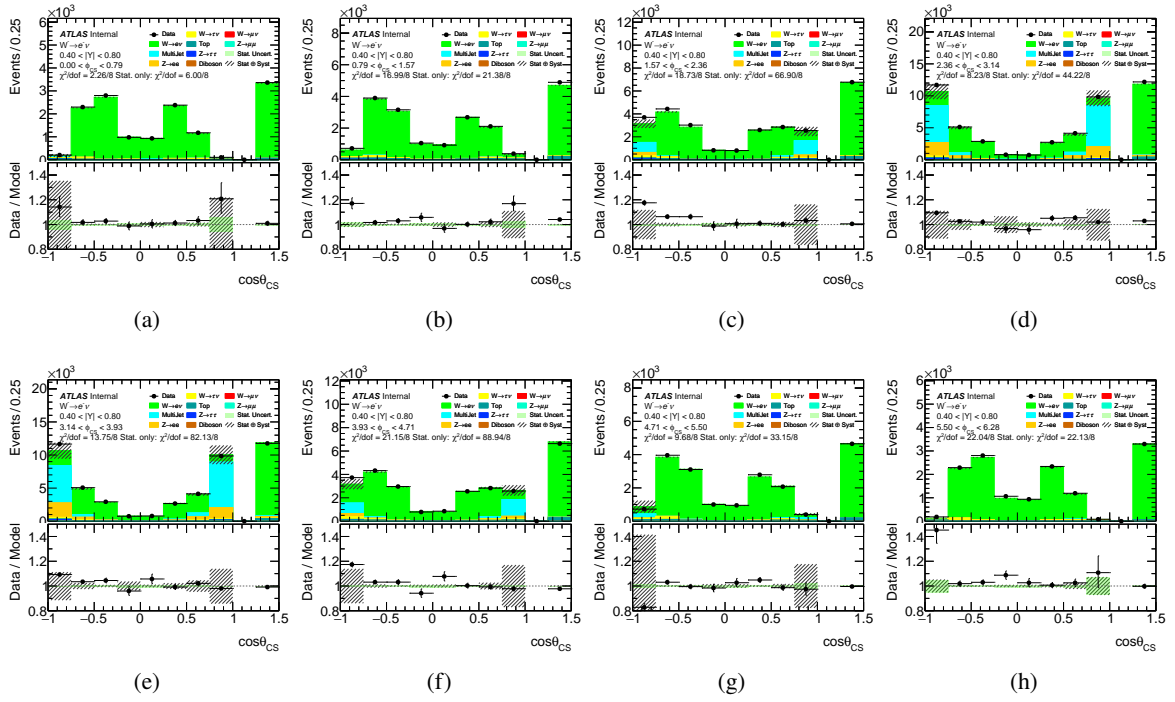


Figure 316: Control plots for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

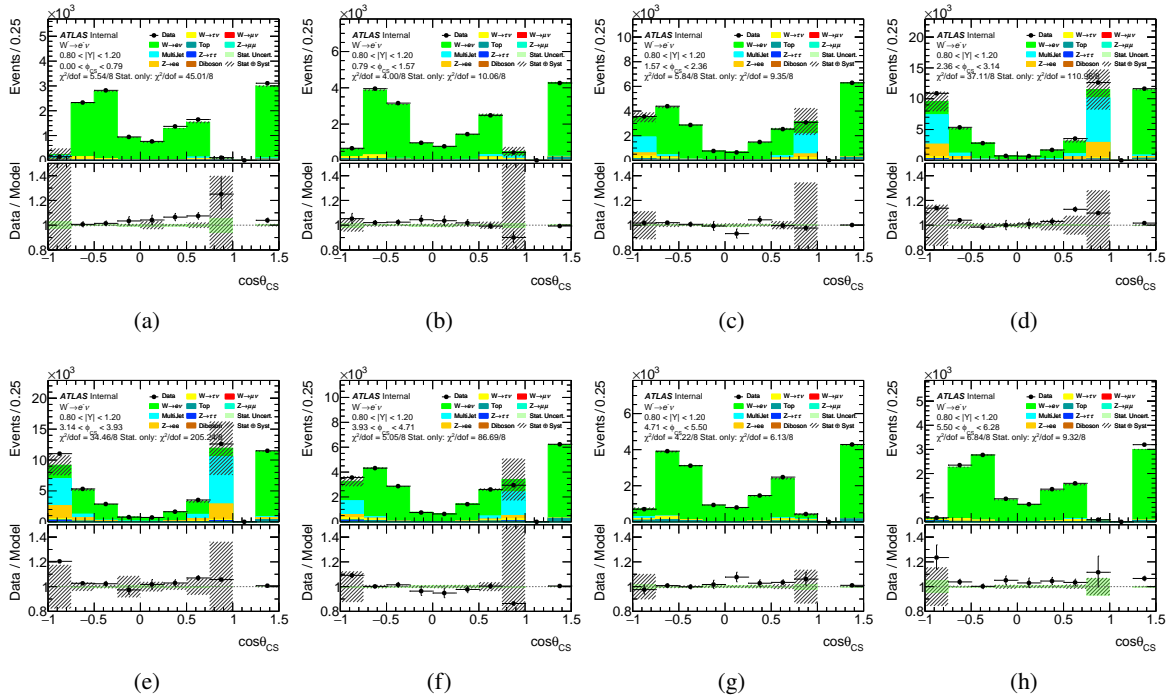


Figure 317: Control plots for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

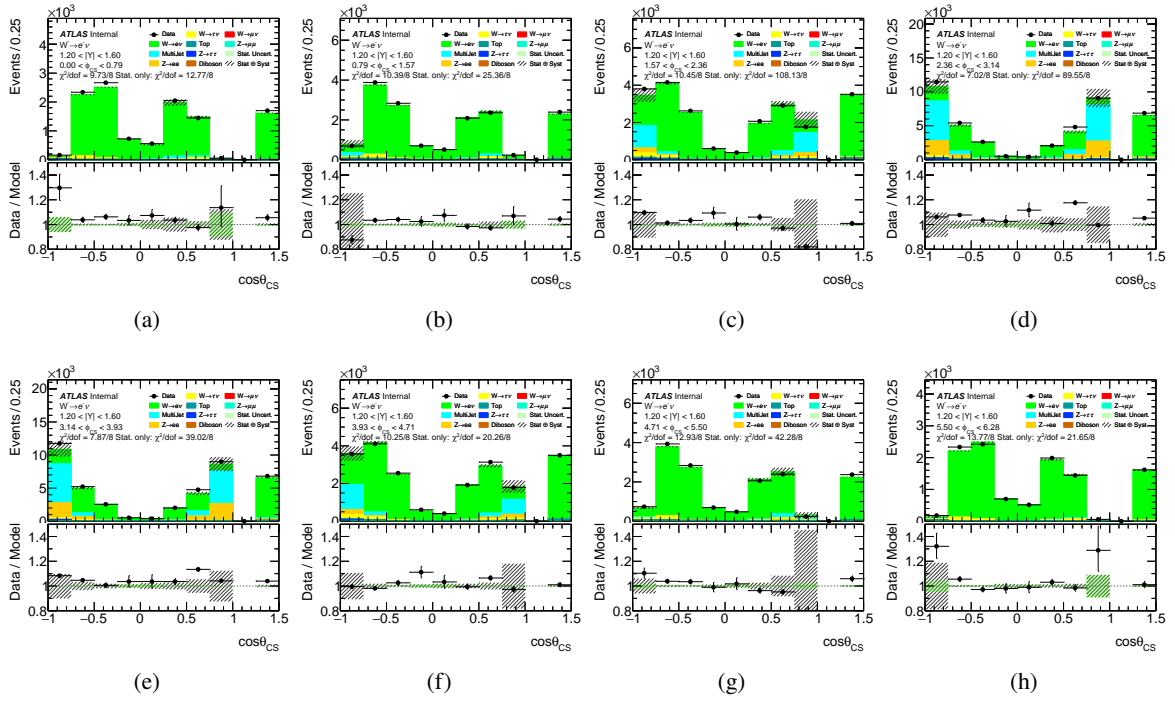


Figure 318: Control plots for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

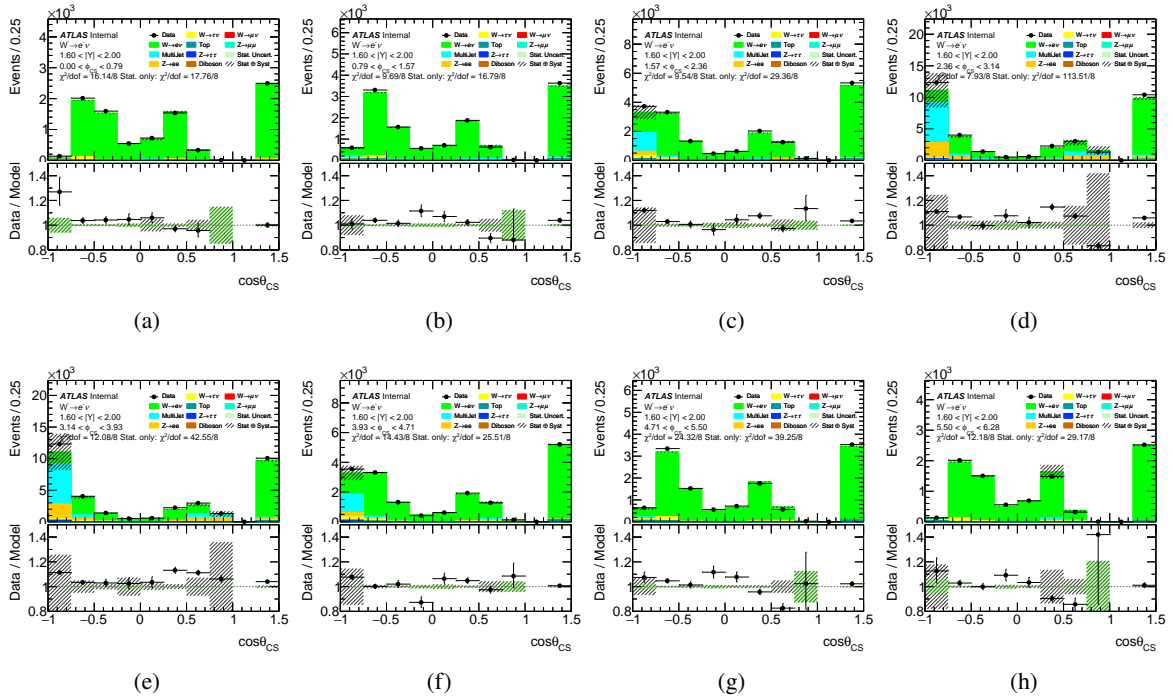


Figure 319: Control plots for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

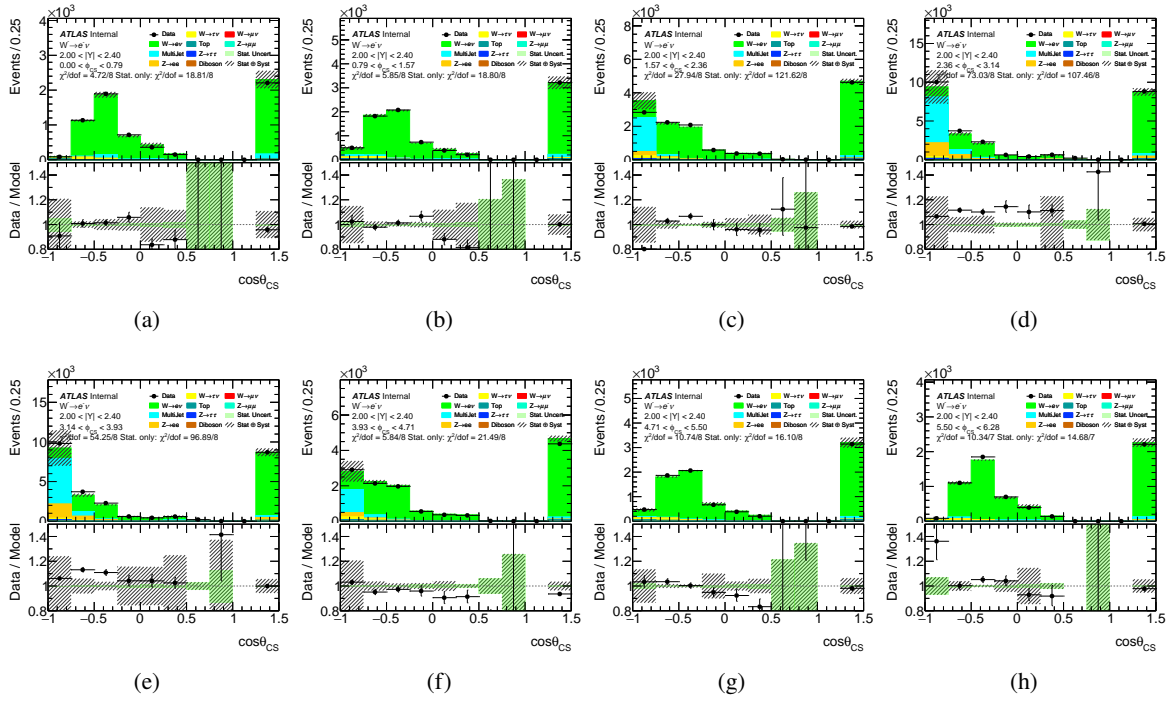


Figure 320: Control plots for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

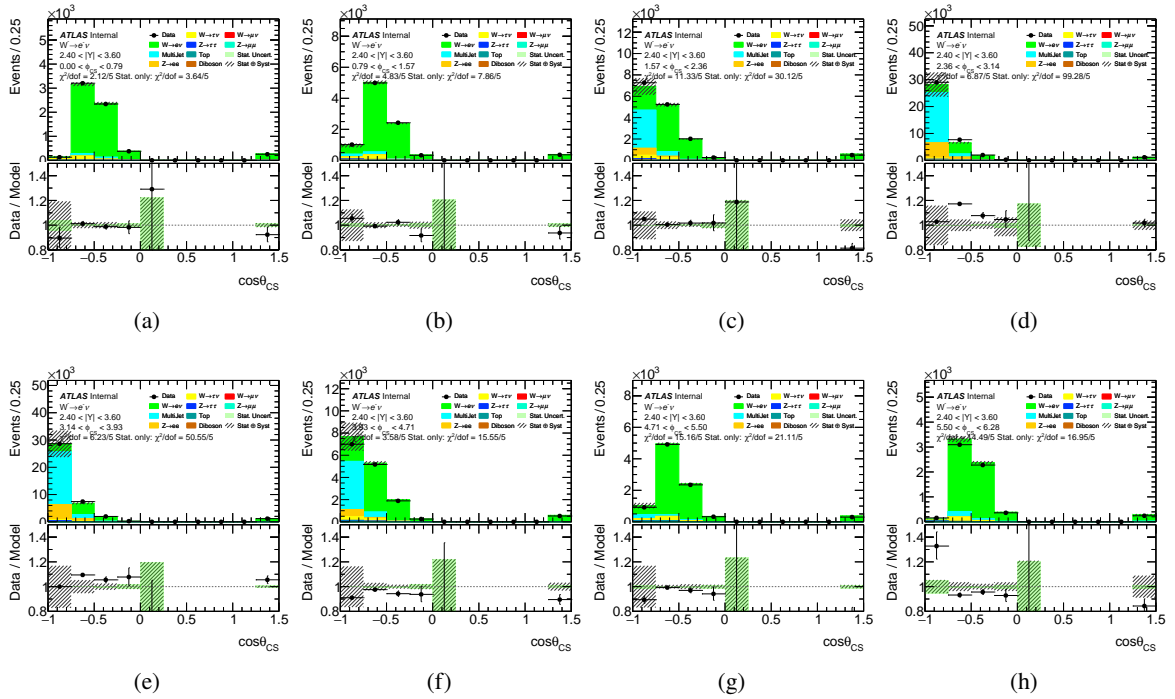


Figure 321: Control plots for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow e^- \bar{\nu}$ channel.

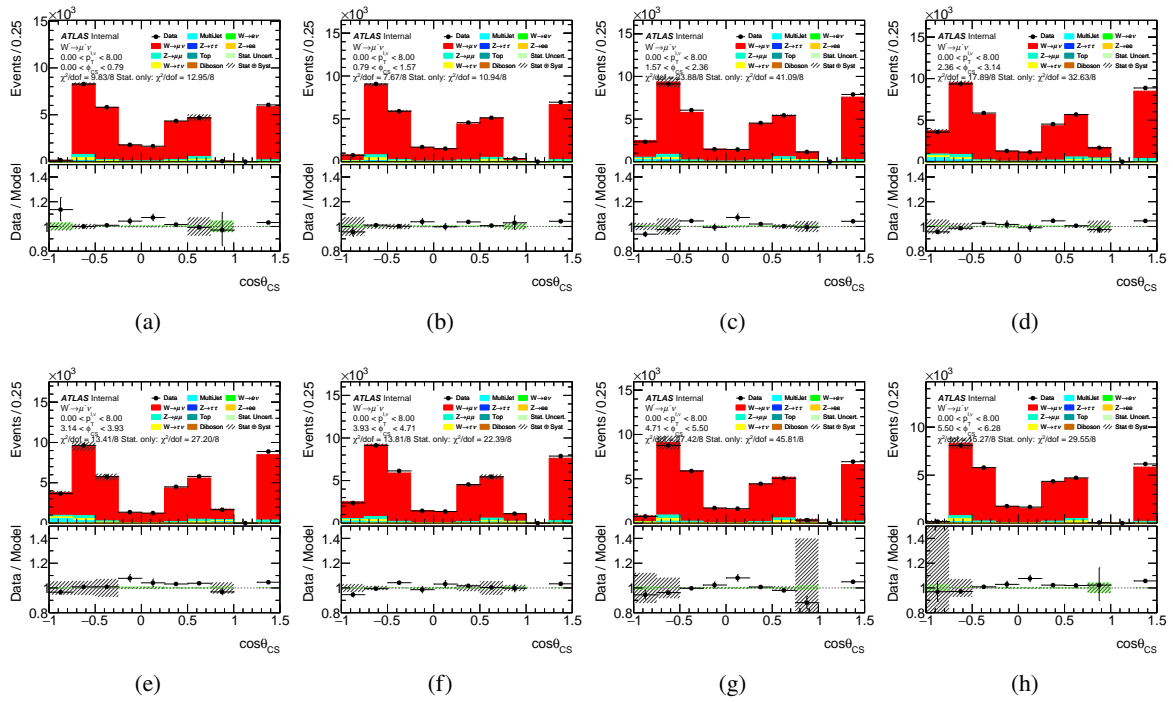


Figure 322: Control plots for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

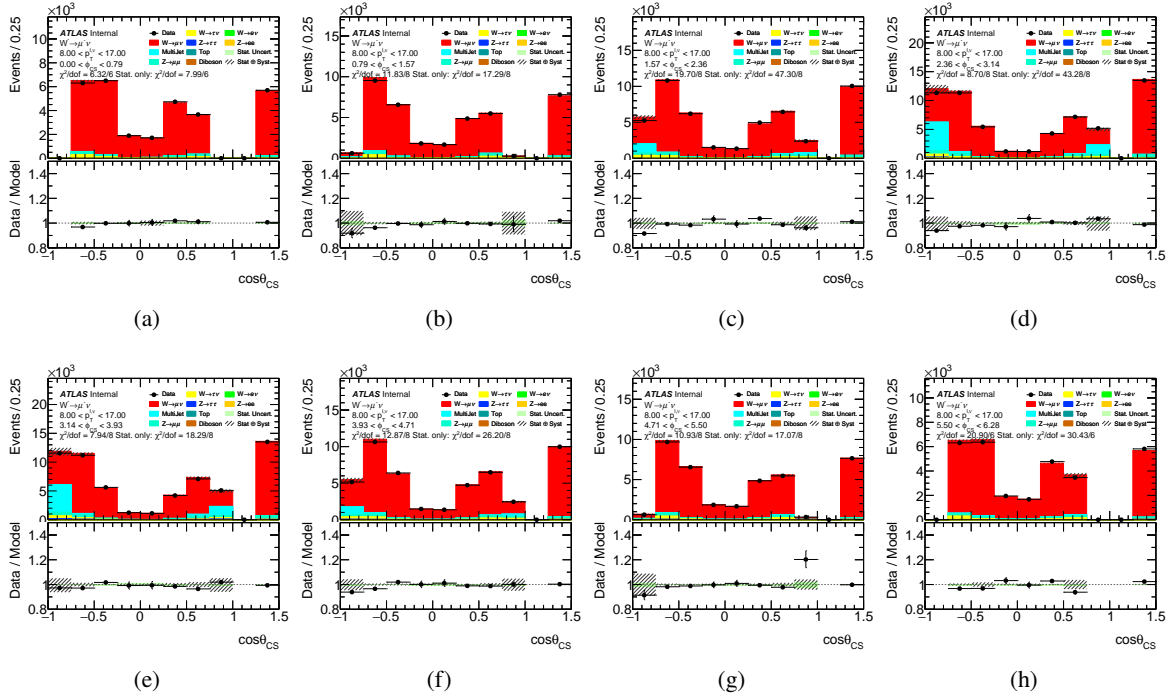


Figure 323: Control plots for $8 < p_T^{\ell, \nu} < 17$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

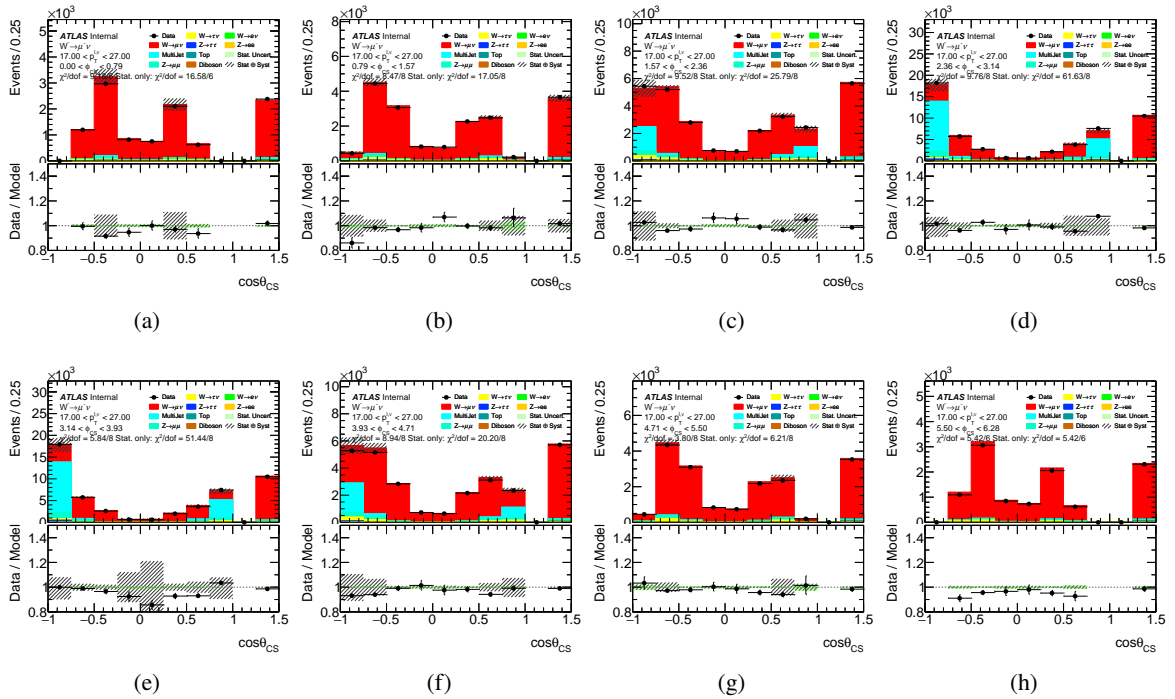


Figure 324: Control plots for $17 < p_T^{\ell, \nu} < 27$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

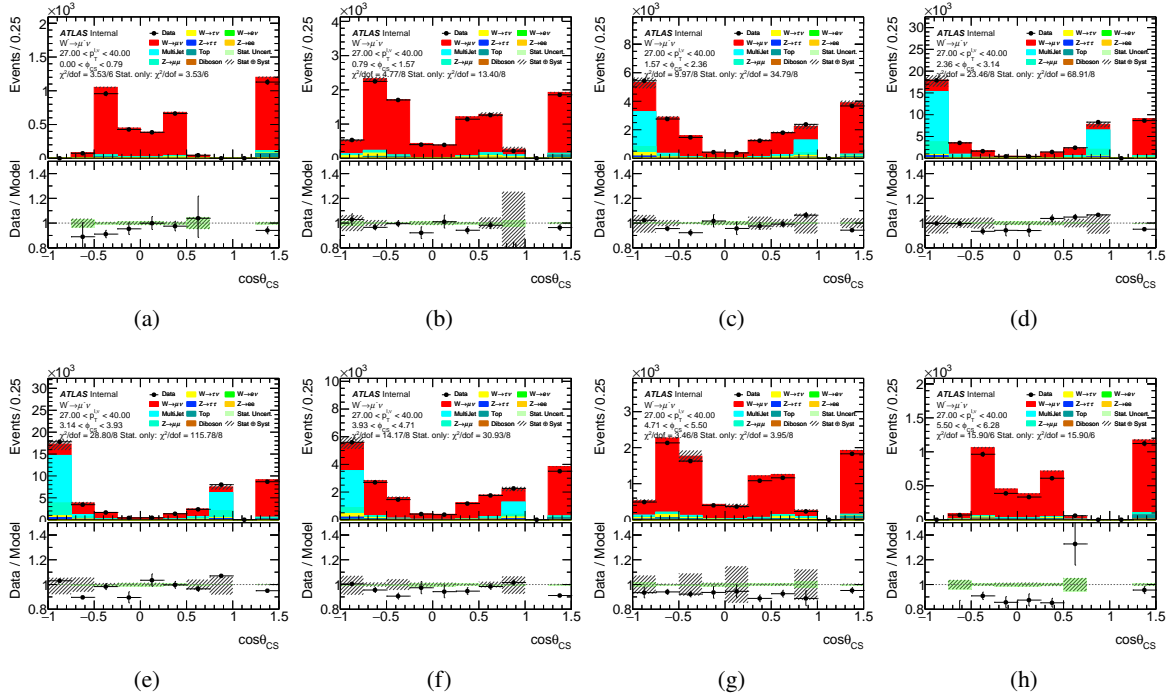


Figure 325: Control plots for $27 < p_T^{\ell, \nu} < 40$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

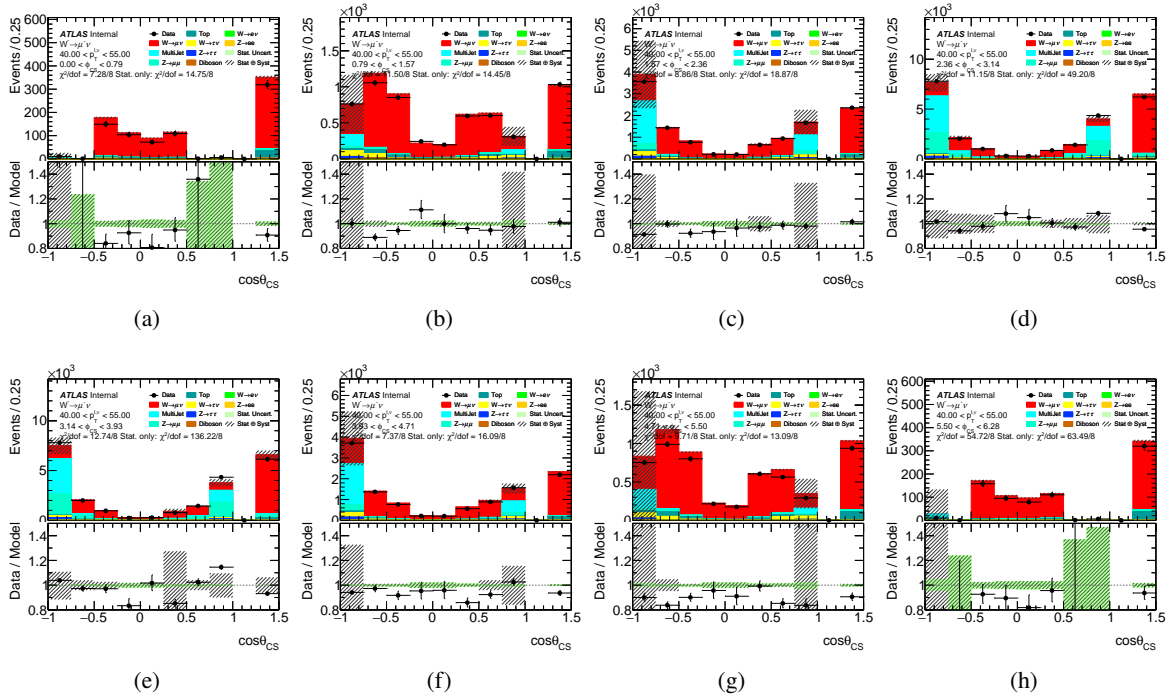


Figure 326: Control plots for $40 < p_T^{\ell, \nu} < 55$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

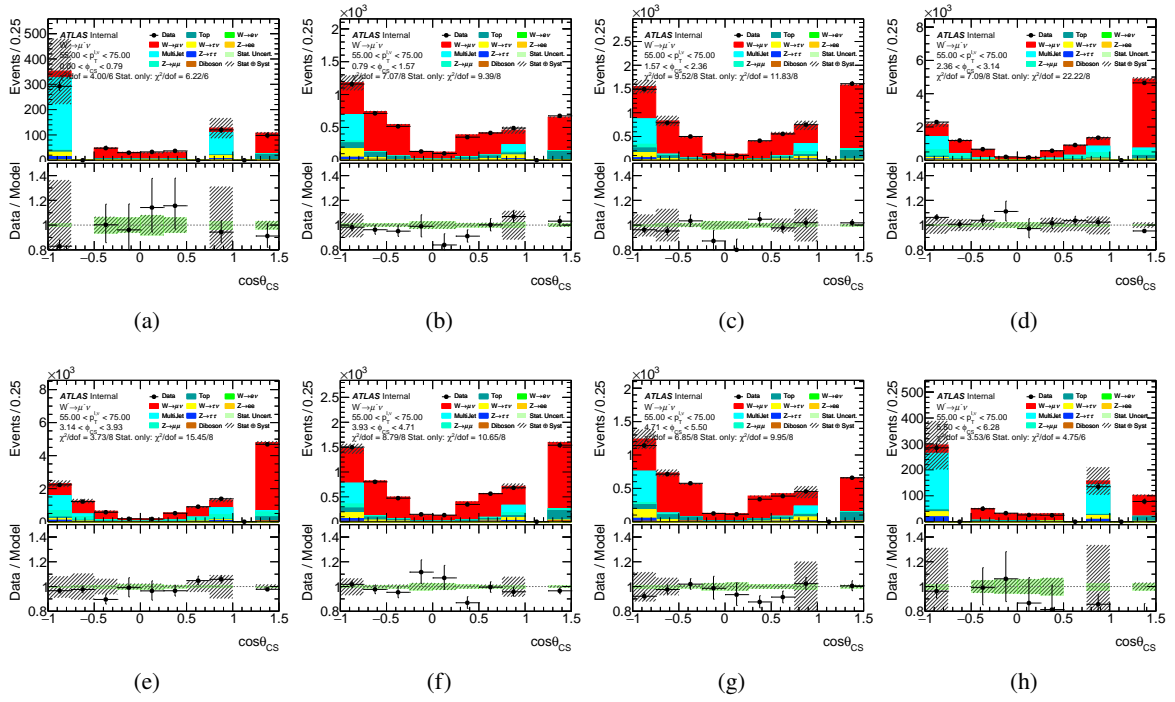


Figure 327: Control plots for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

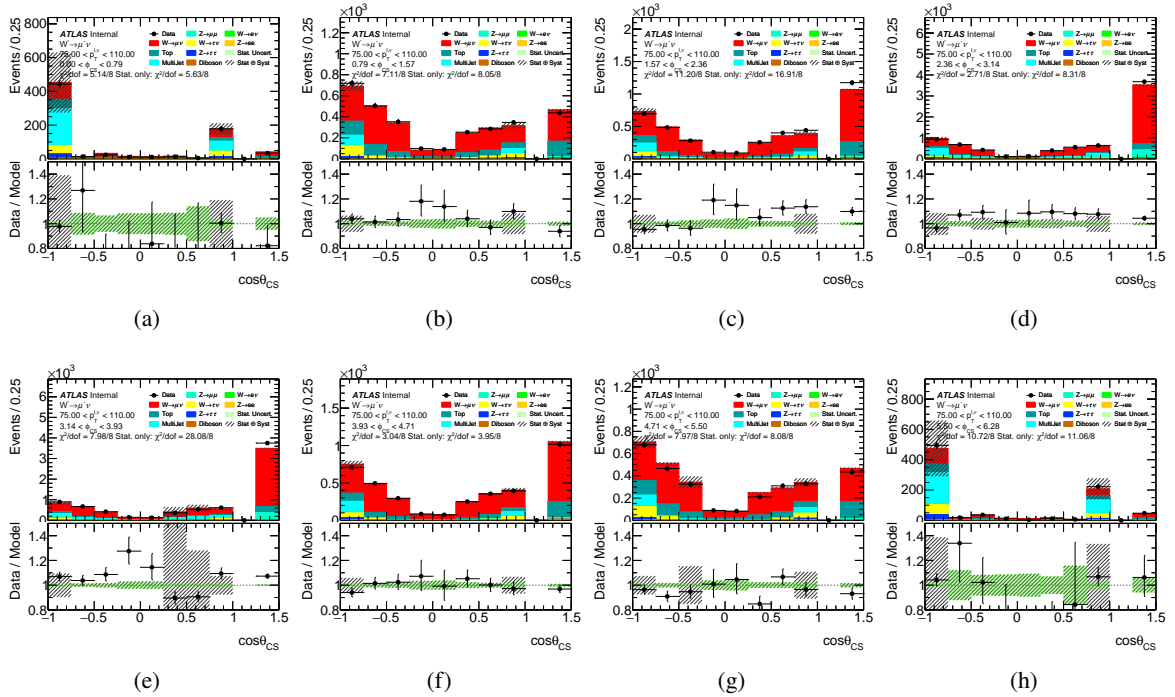


Figure 328: Control plots for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

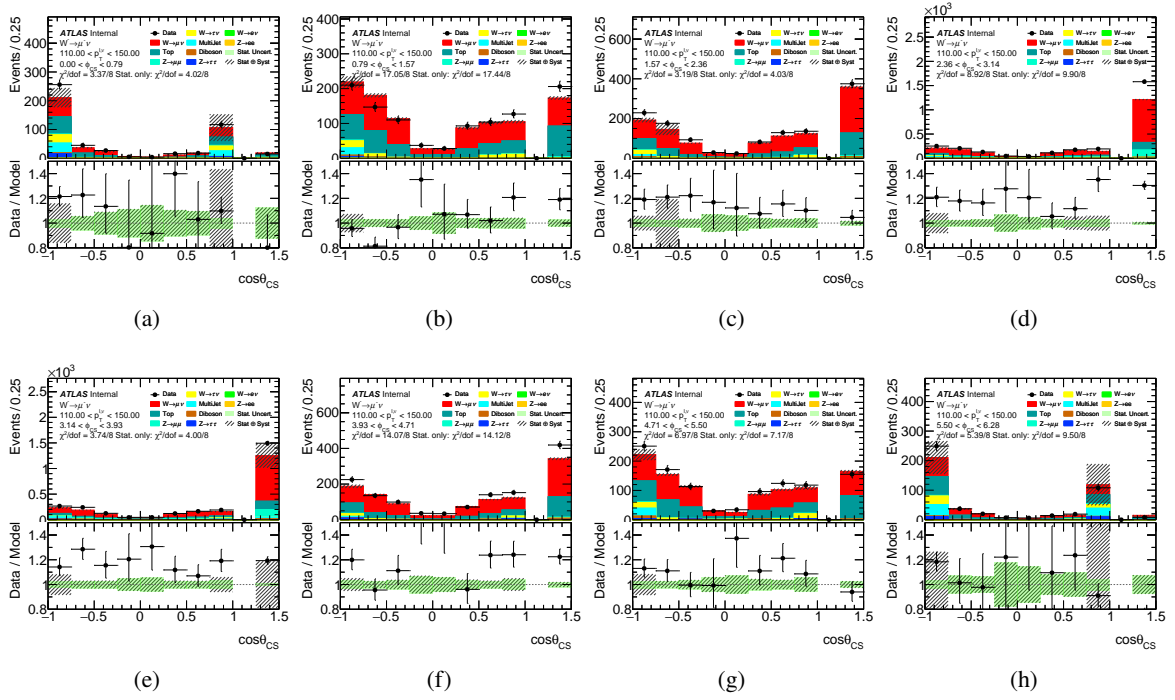


Figure 329: Control plots for $110 < p_T^{\ell,\nu} < 150$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

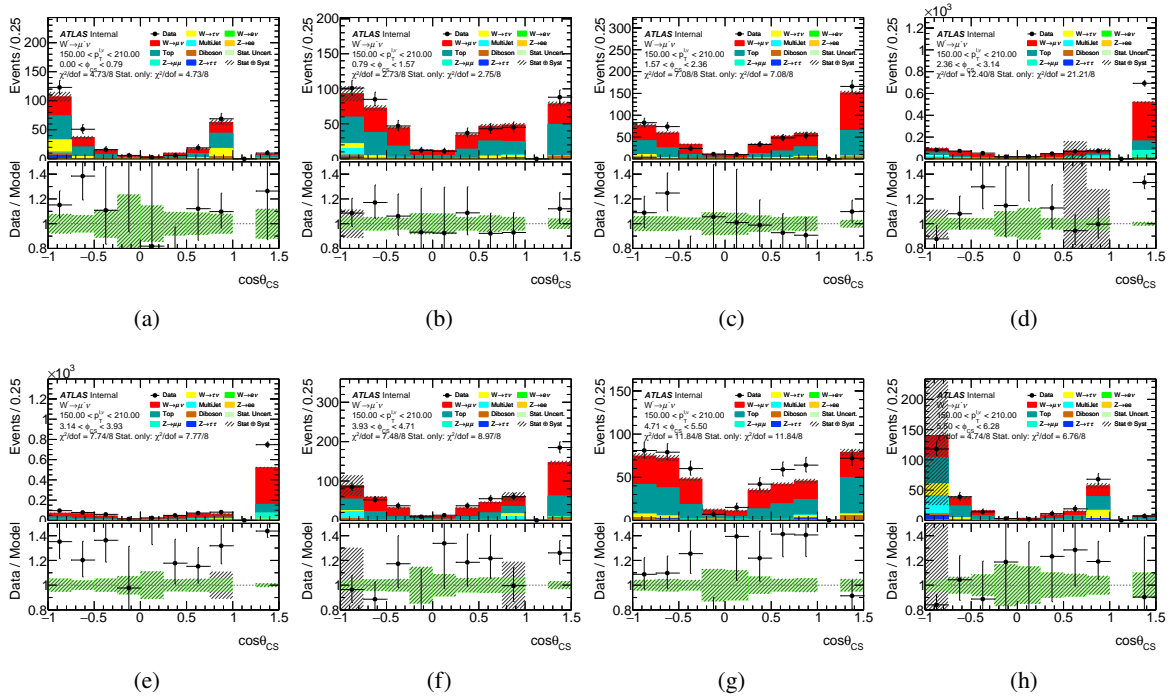


Figure 330: Control plots for $150 < p_T^{\ell,\nu} < 210$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

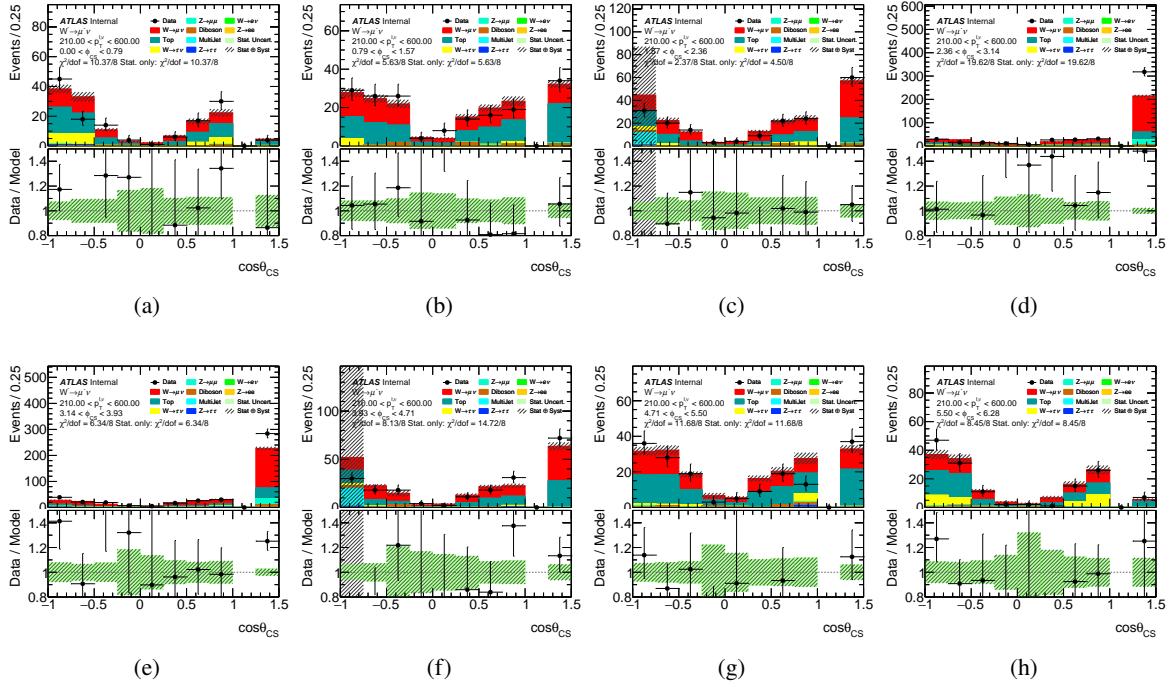


Figure 331: Control plots for $210 < p_T^{\ell, \nu} < 600$ GeV bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

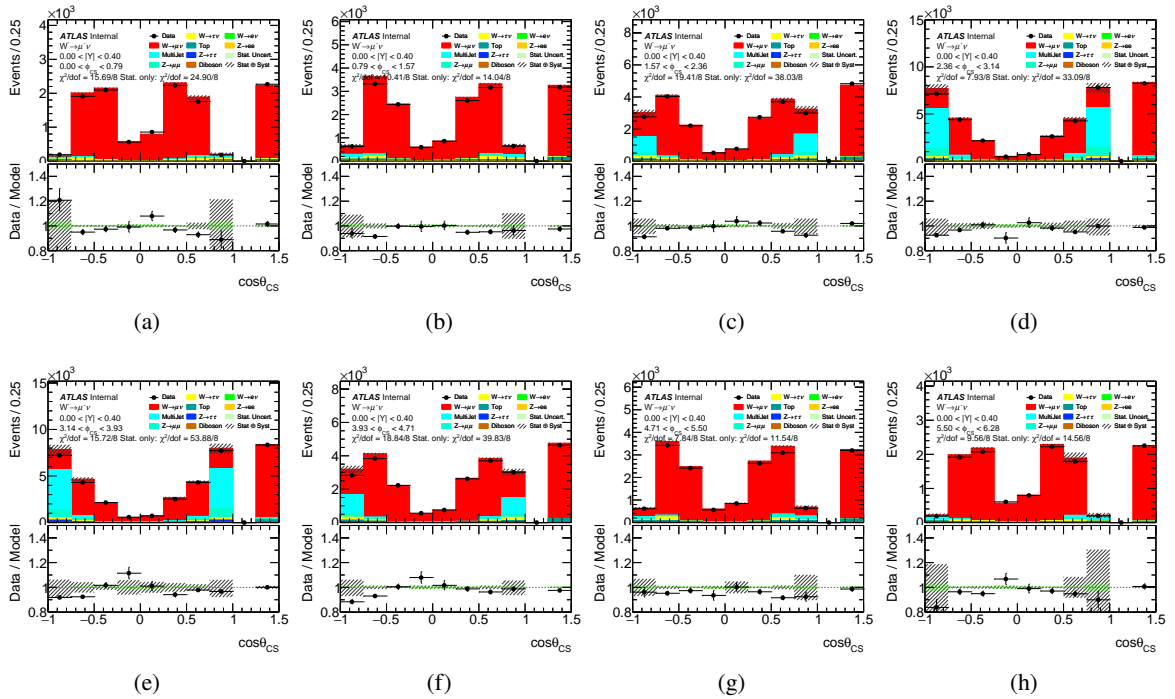


Figure 332: Control plots for $0 < |Y| < 0.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

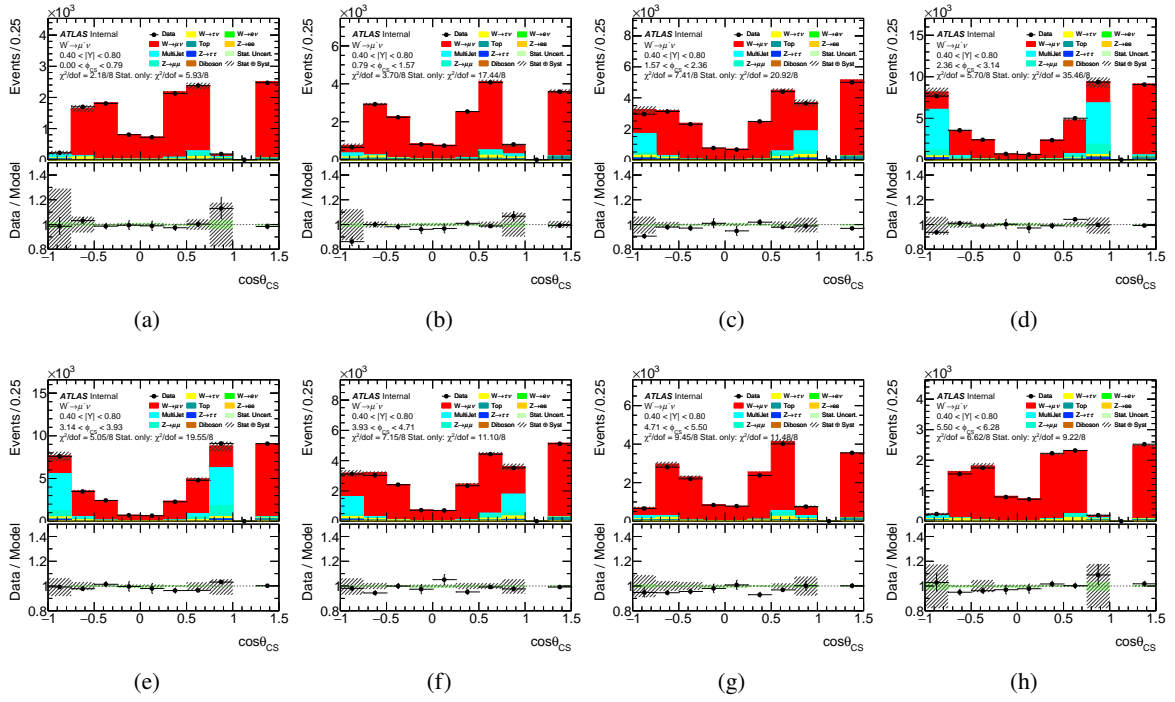


Figure 333: Control plots for $0.4 < |Y| < 0.8$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

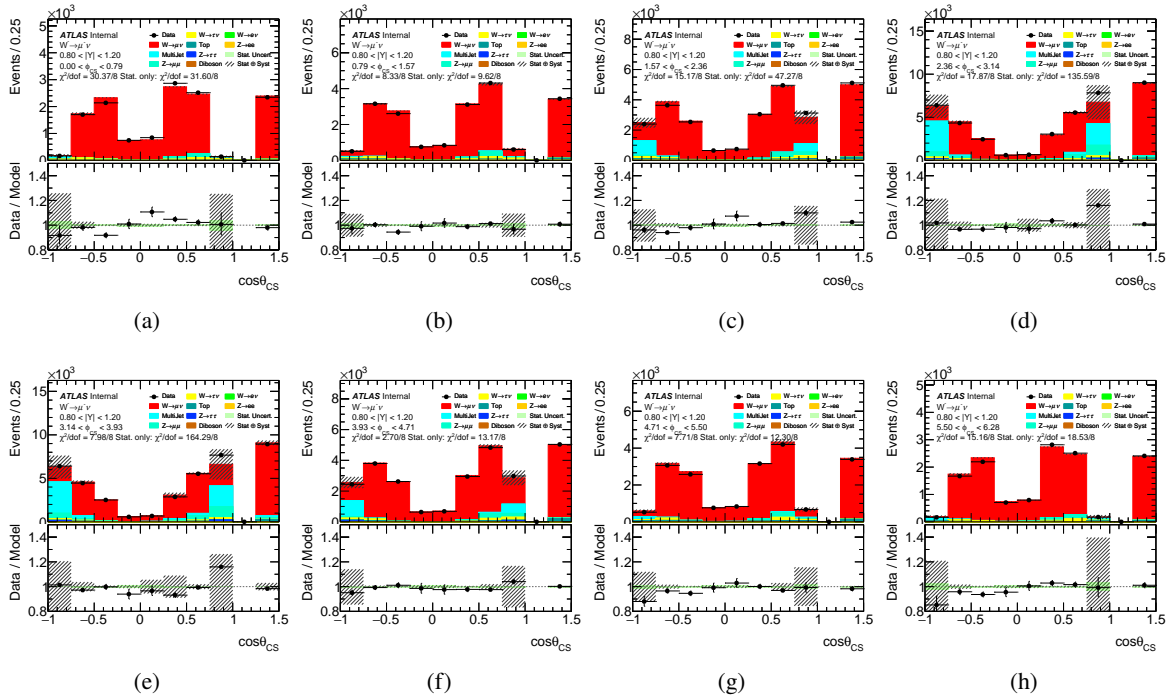


Figure 334: Control plots for $0.8 < |Y| < 1.2$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

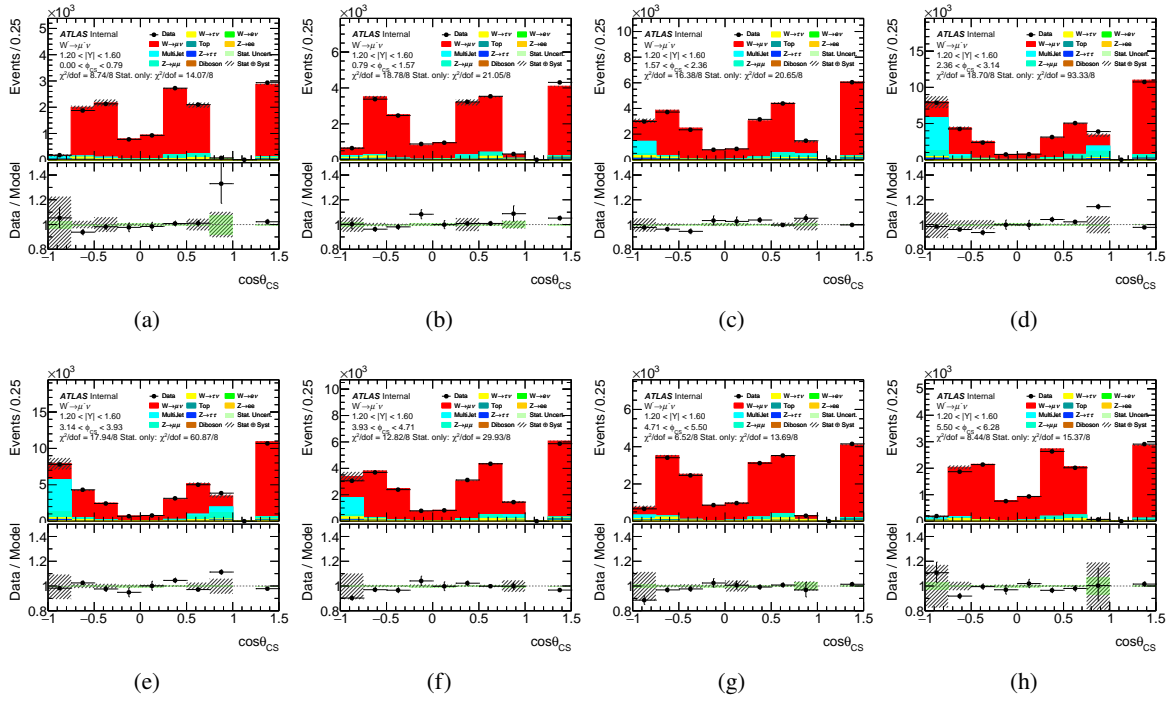


Figure 335: Control plots for $1.2 < |Y| < 1.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

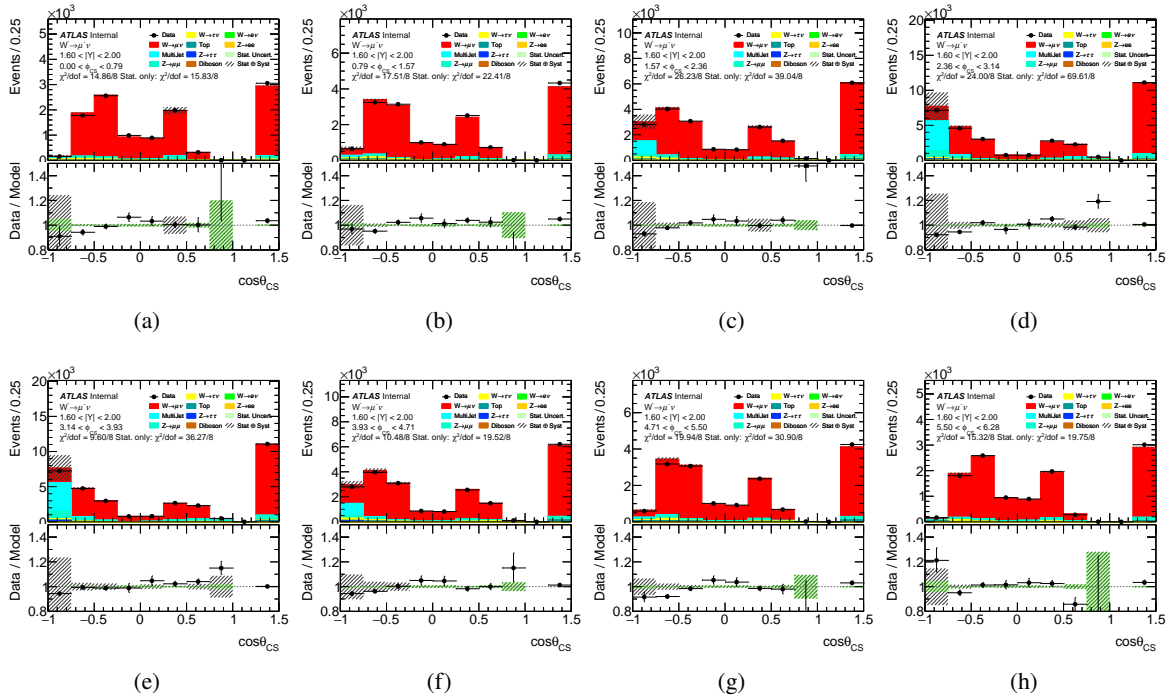


Figure 336: Control plots for $1.6 < |Y| < 2.0$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

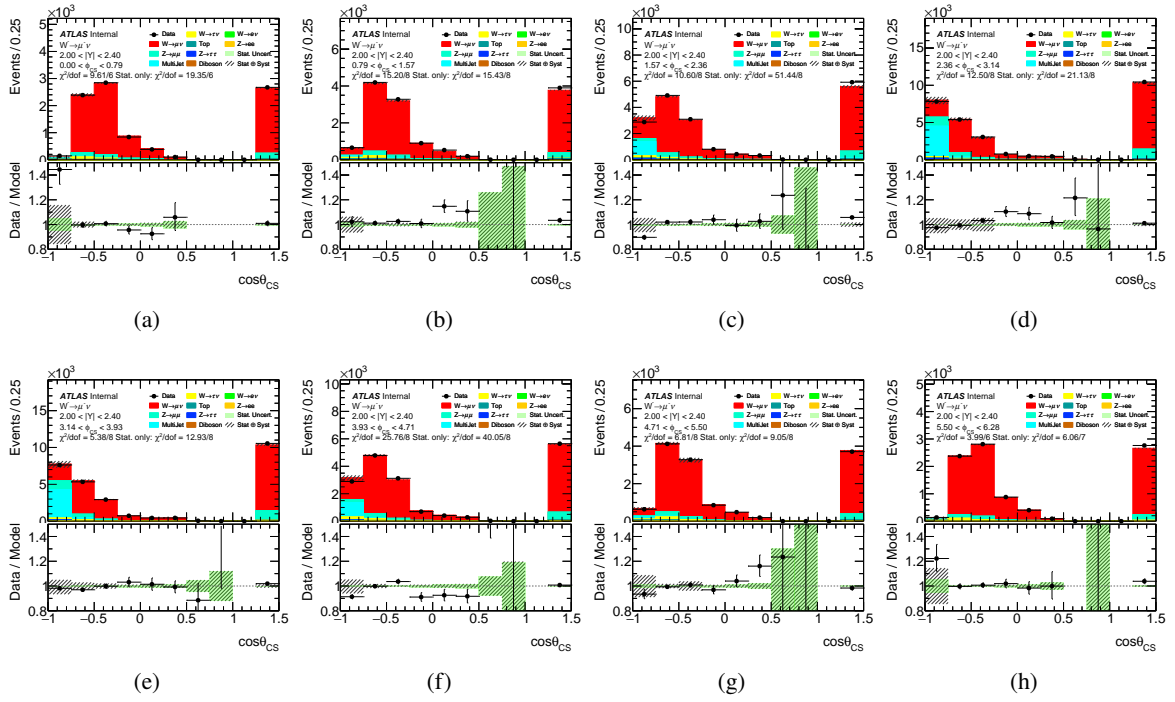


Figure 337: Control plots for $2.0 < |Y| < 2.4$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

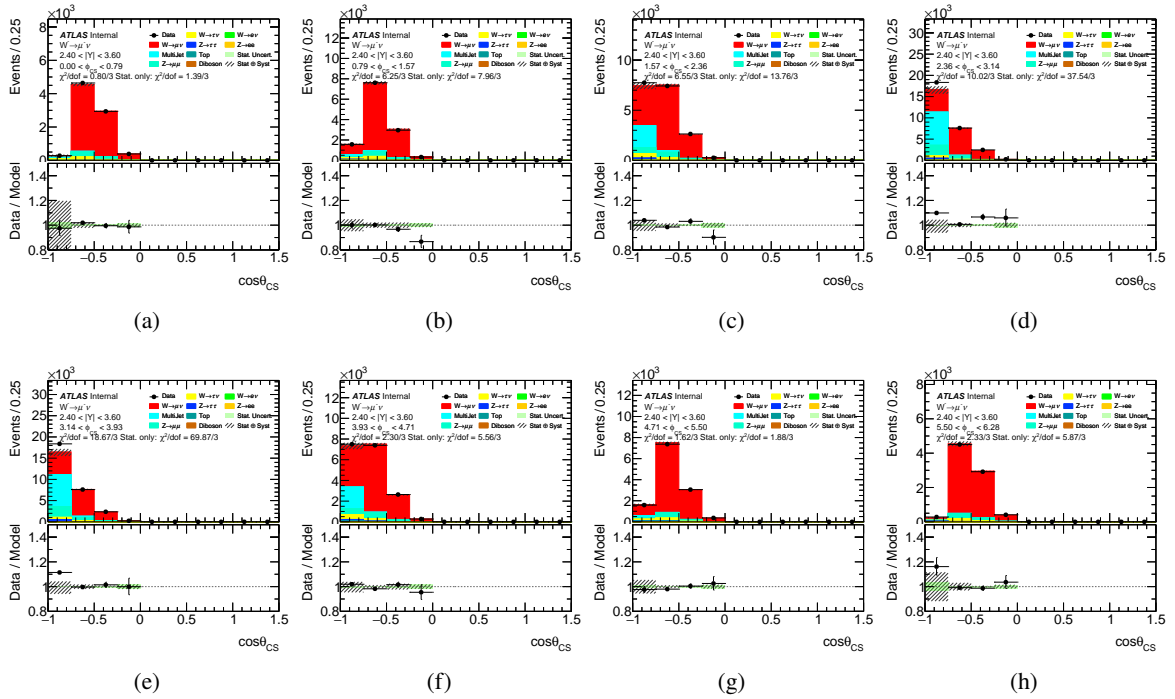


Figure 338: Control plots for $2.4 < |Y| < 3.6$ bin for $W^- \rightarrow \mu^- \bar{\nu}$ channel.

1302 **F.3 Double ratio $W^- \rightarrow e^- \bar{\nu} / W^- \rightarrow \mu^- \bar{\nu}$**

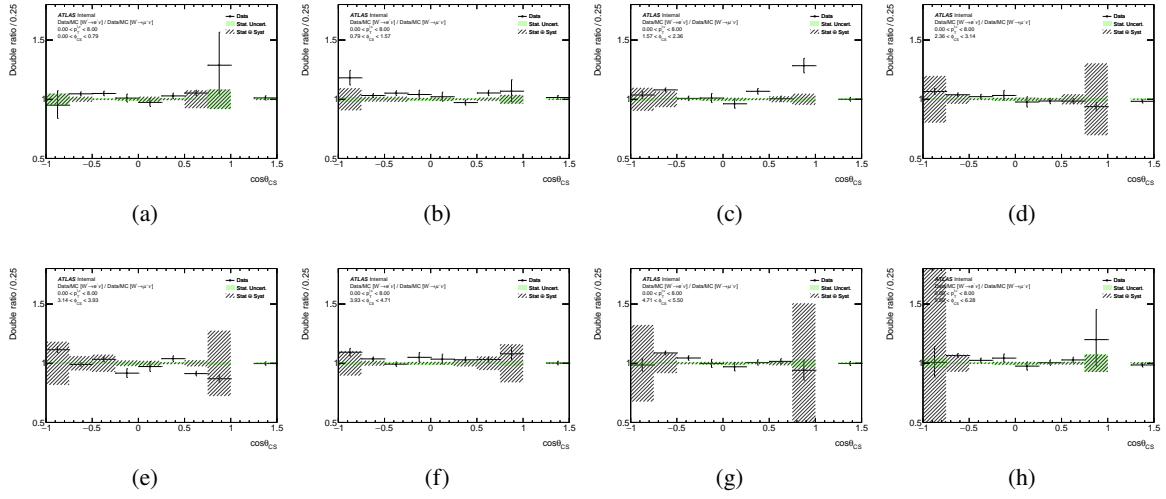


Figure 339: Double ratio plots for e/μ for $0 < p_T^{\ell,\nu} < 8$ GeV bin.

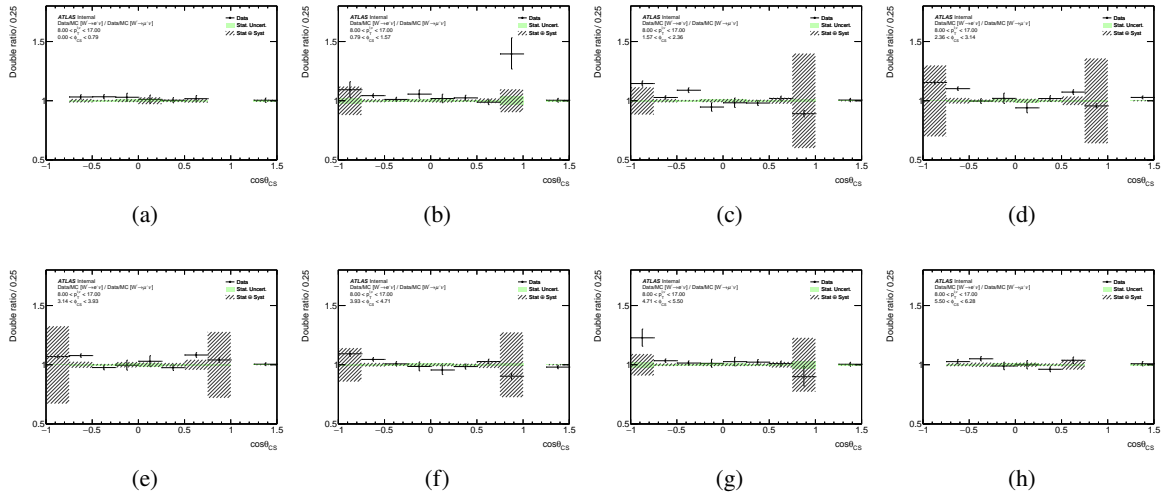


Figure 340: Double ratio plots for e/μ for $8 < p_T^{\ell,\nu} < 17$ GeV bin.

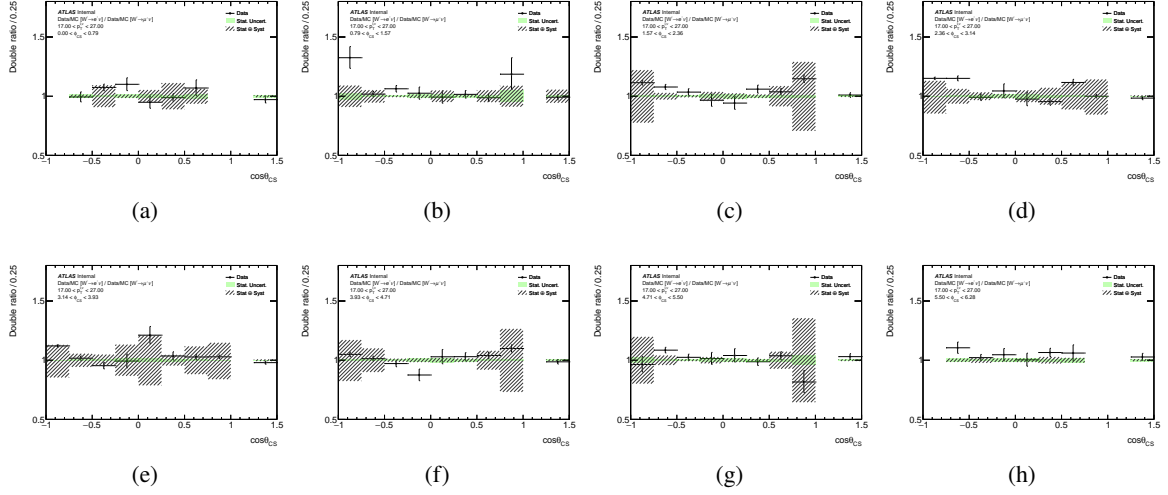


Figure 341: Double ratio plots for e/μ for $17 < p_T^{\ell,\nu} < 27$ GeV bin.

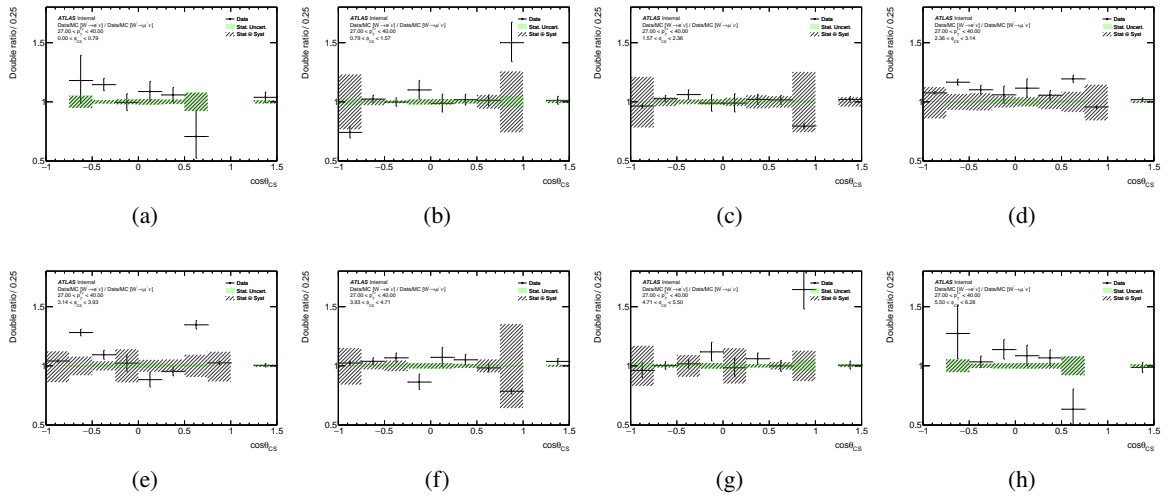


Figure 342: Double ratio plots for e/μ for $27 < p_T^{\ell,\nu} < 40$ GeV bin.

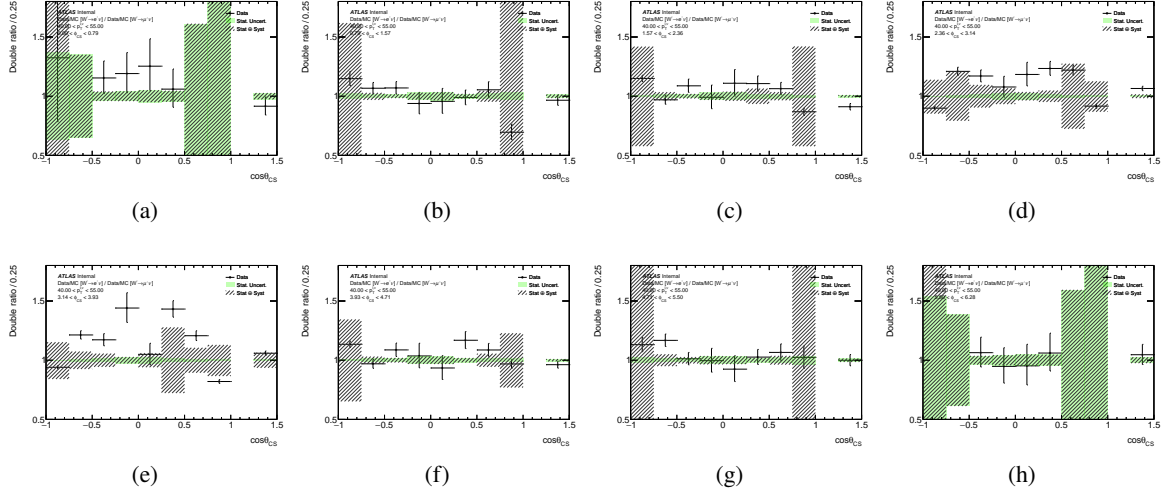


Figure 343: Double ratio plots for e/μ for $40 < p_T^{\ell, \nu} < 55$ GeV bin.

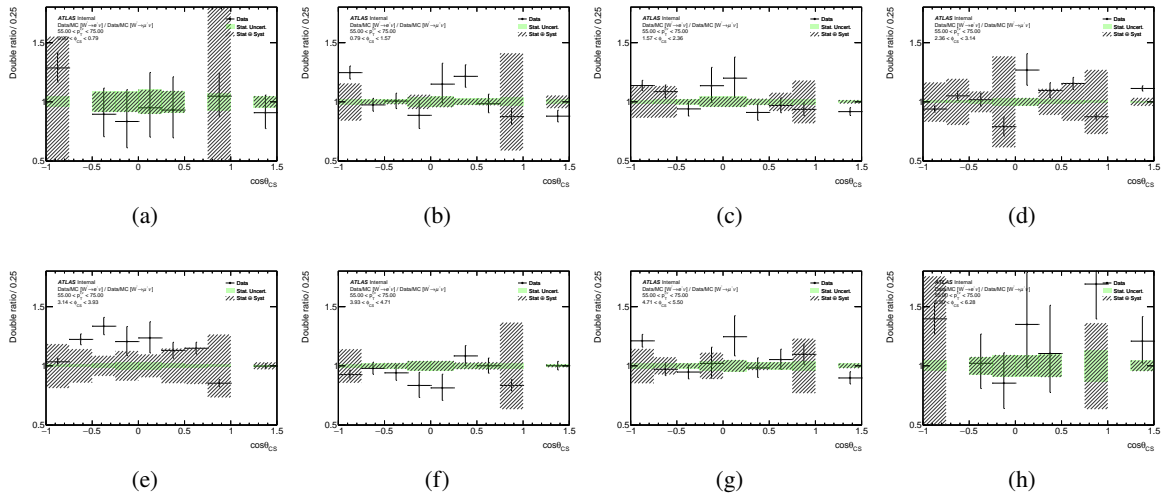


Figure 344: Double ratio plots for e/μ for $55 < p_T^{\ell, \nu} < 75$ GeV bin.

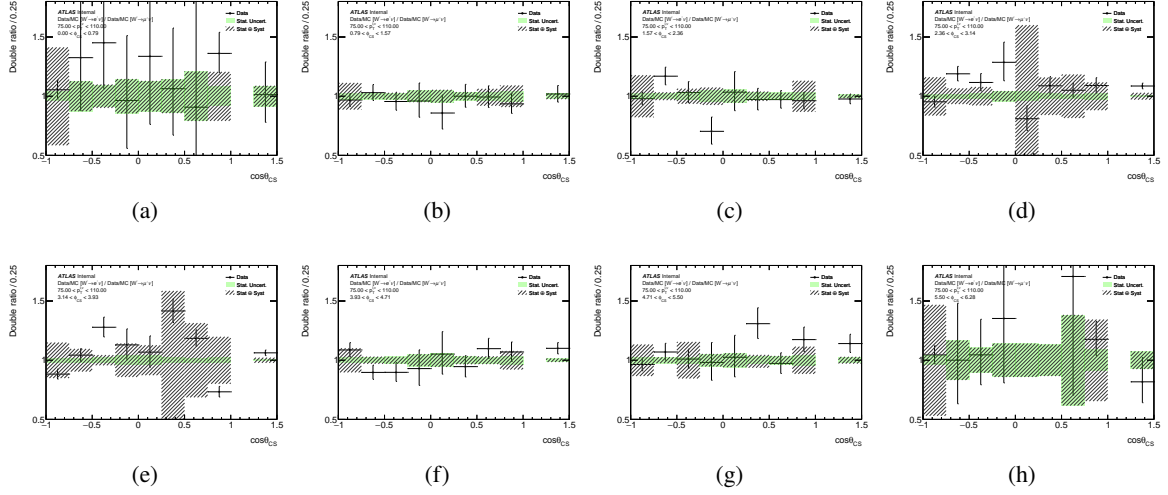


Figure 345: Double ratio plots for e/μ for $75 < p_T^{\ell,\nu} < 110$ GeV bin.

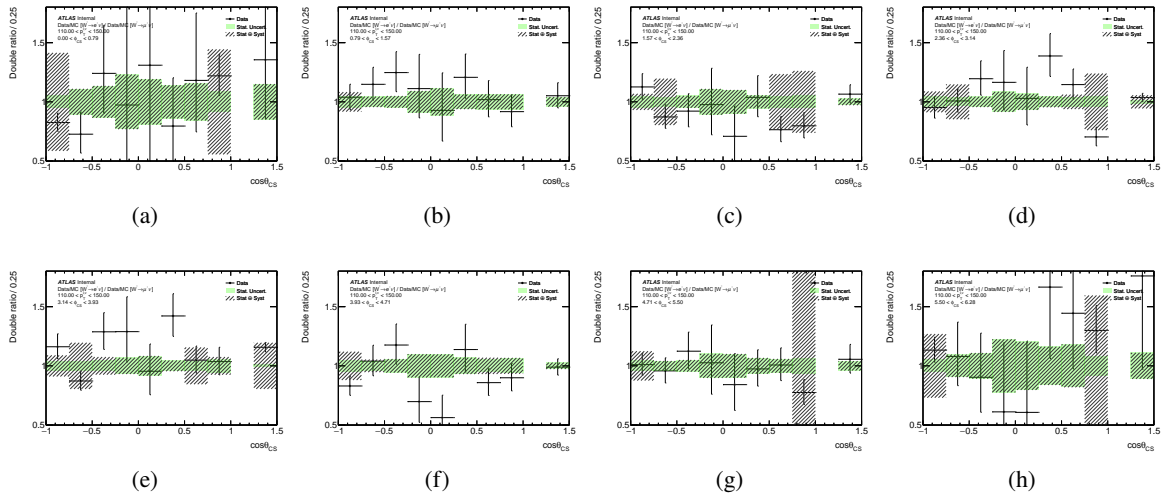


Figure 346: Double ratio plots for e/μ for $110 < p_T^{\ell,\nu} < 150$ GeV bin.

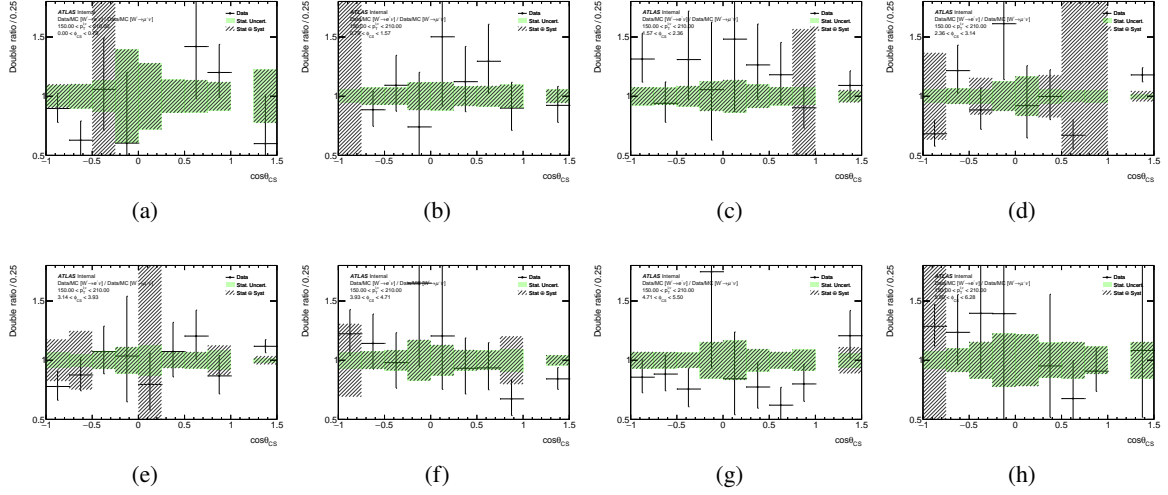


Figure 347: Double ratio plots for e/μ for $150 < p_T^{\ell,\nu} < 210$ GeV bin.

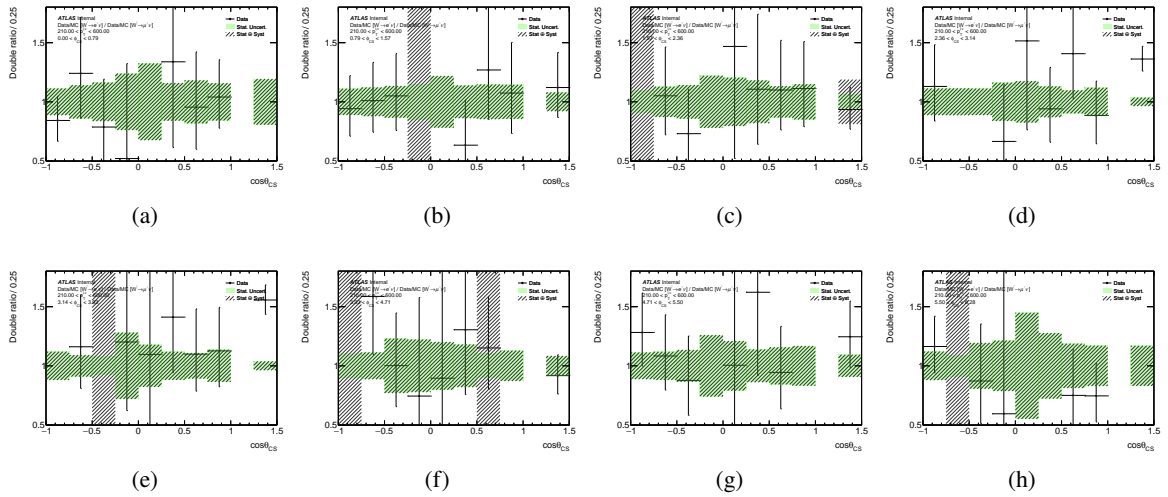


Figure 348: Double ratio plots for e/μ for $210 < p_T^{\ell,\nu} < 600$ GeV bin.

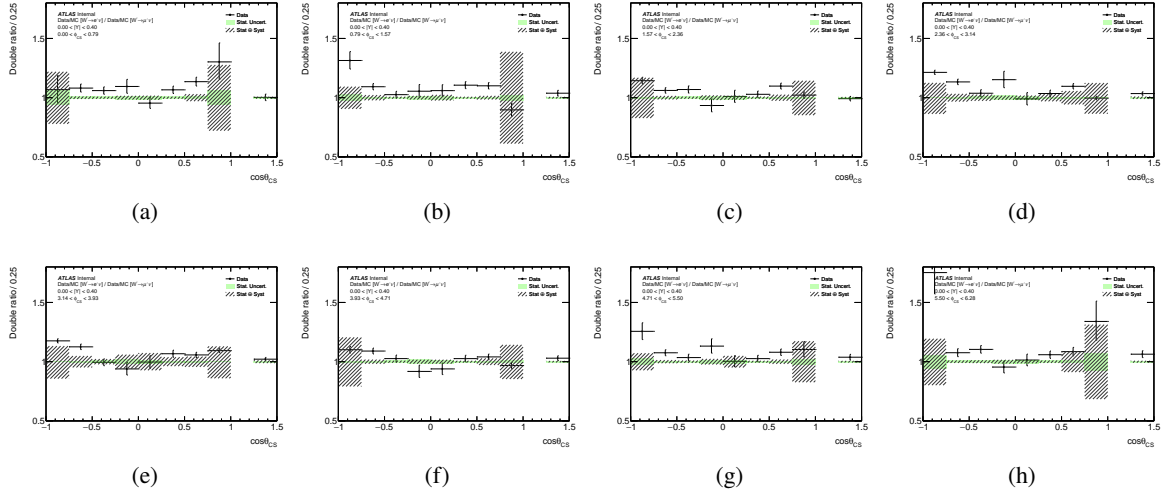


Figure 349: Double ratio plots for e/μ for $0 < |Y| < 0.4$ bin.

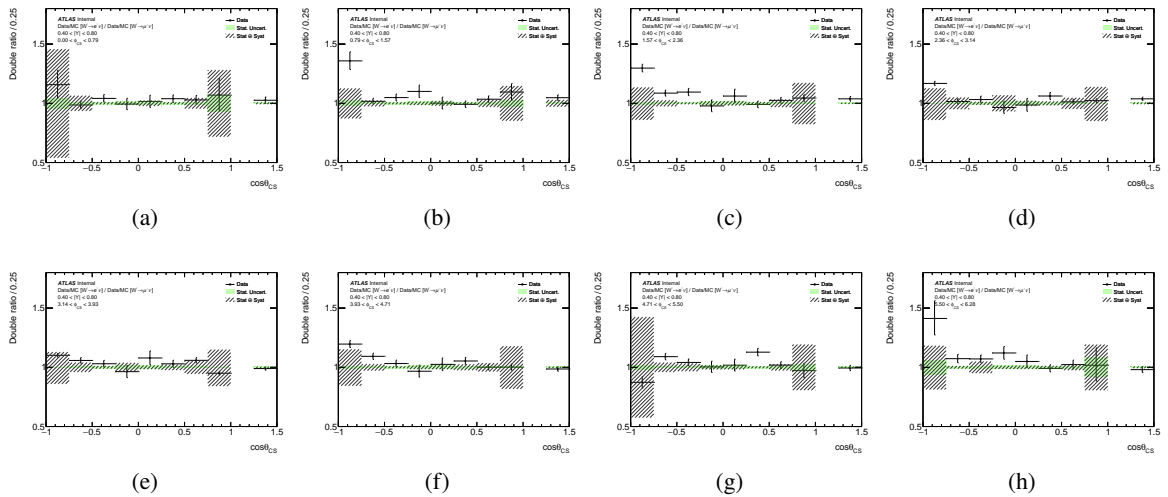


Figure 350: Double ratio plots for e/μ for $0.4 < |Y| < 0.8$ bin.

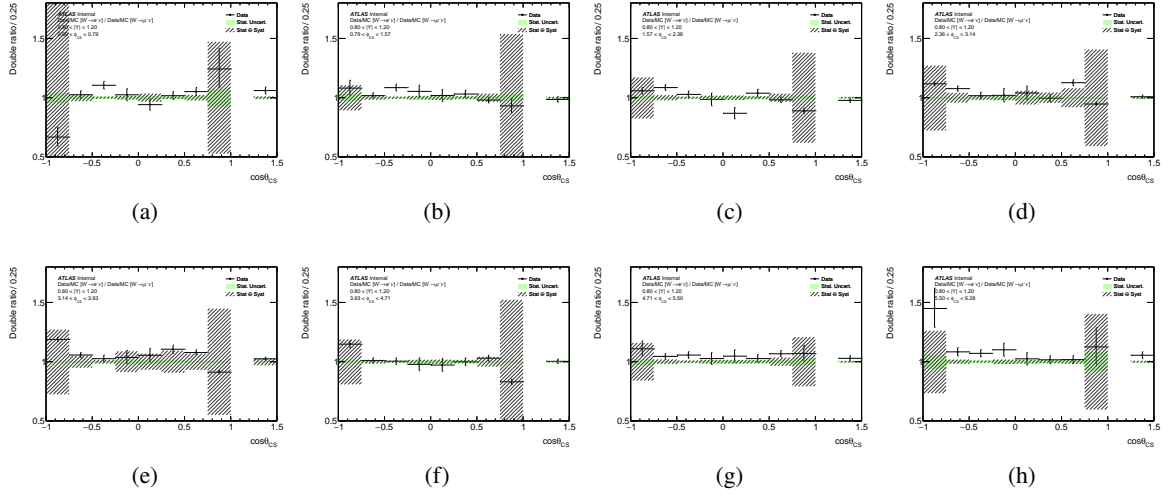


Figure 351: Double ratio plots for e/μ for $0.8 < |Y| < 1.2$ bin.

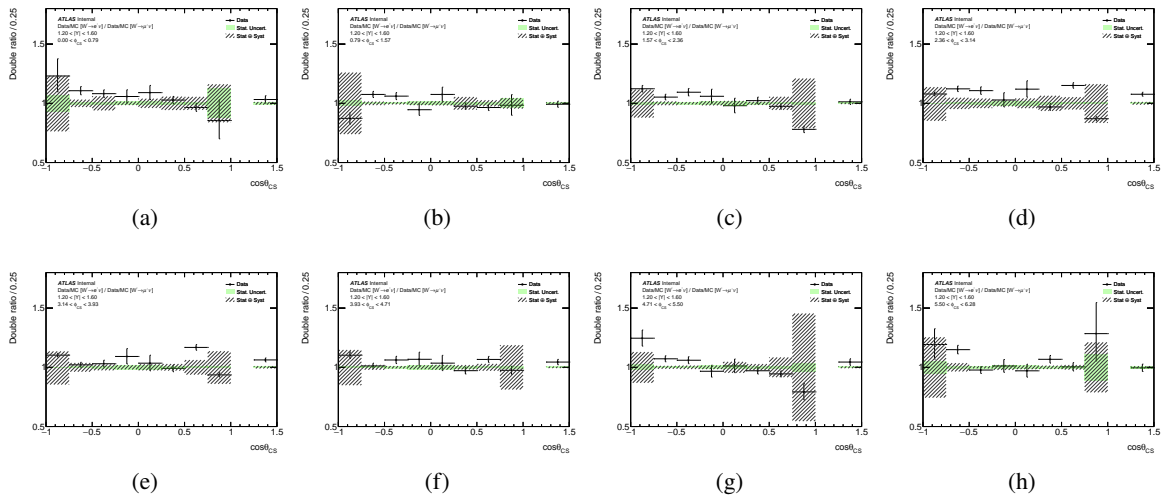


Figure 352: Double ratio plots for e/μ for $1.2 < |Y| < 1.6$ bin.

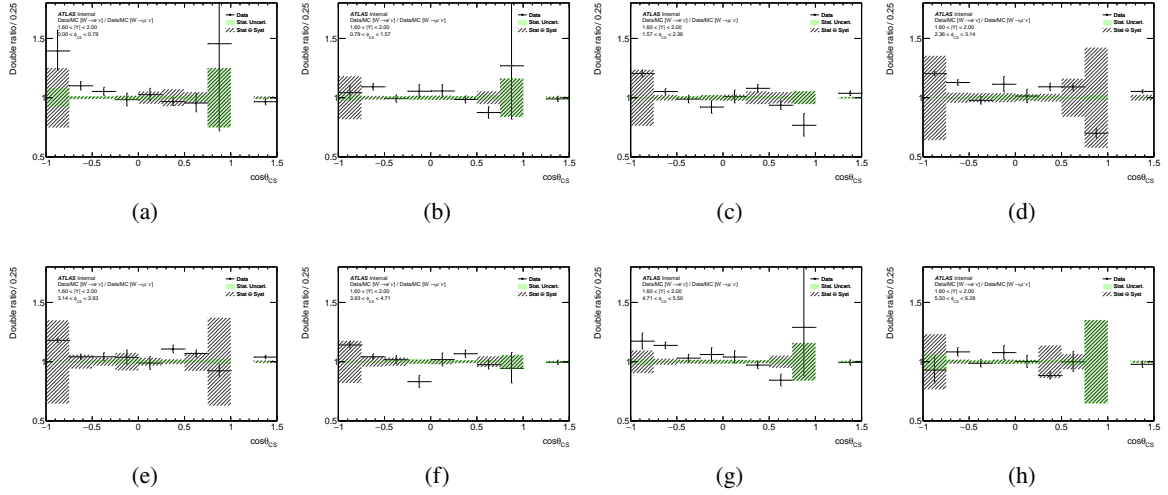


Figure 353: Double ratio plots for e/μ for $1.6 < |Y| < 2.0$ bin.

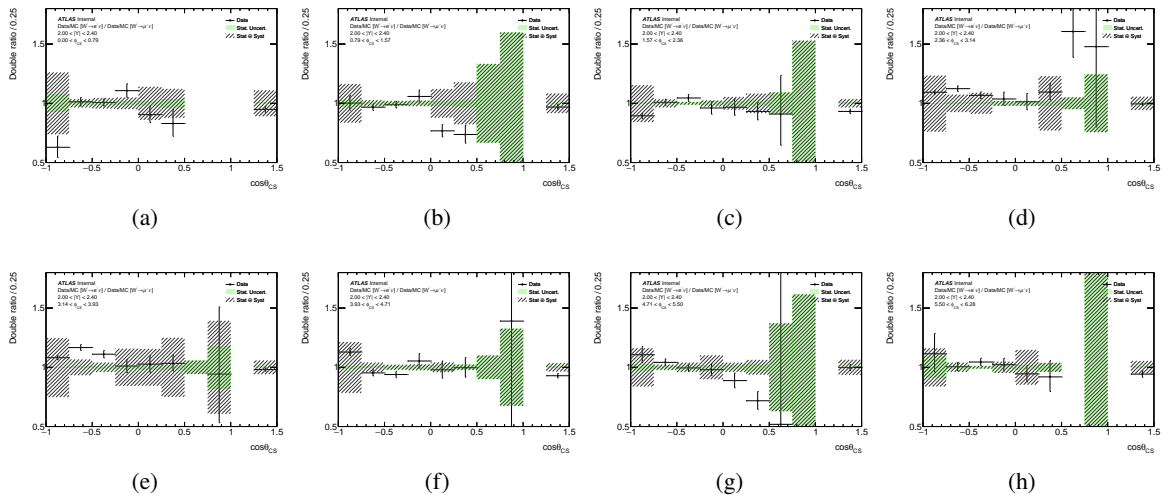


Figure 354: Double ratio plots for e/μ for $2.0 < |Y| < 2.4$ bin.

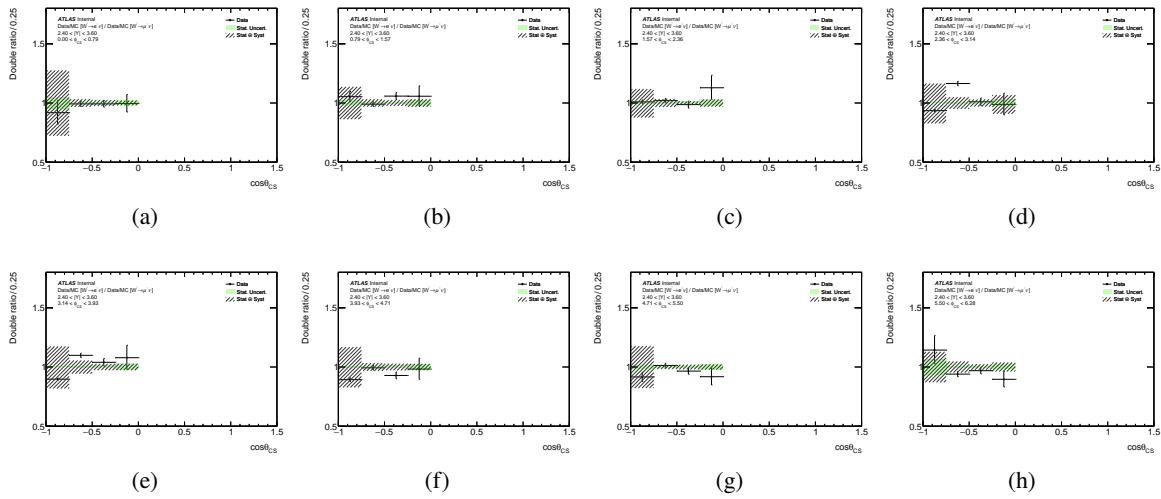


Figure 355: Double ratio plots for e/μ for $2.4 < |Y| < 3.6$ bin.

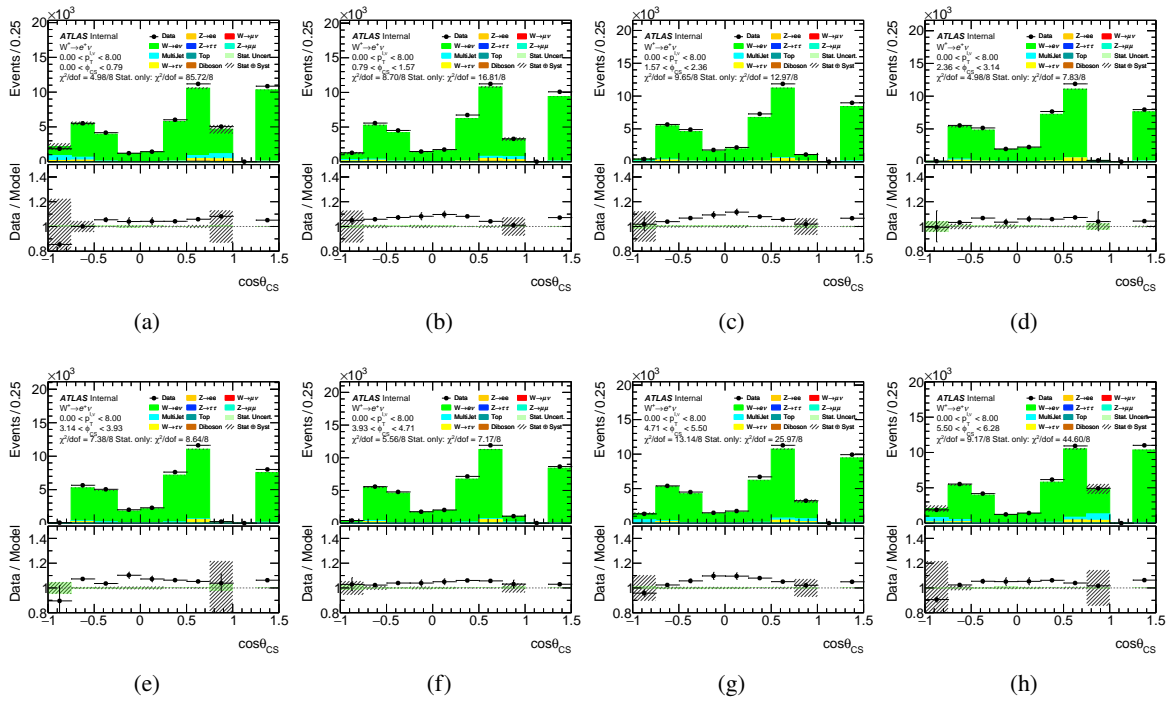


Figure 356: Control plots for $0 < p_T^{e^+, \nu} < 8$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

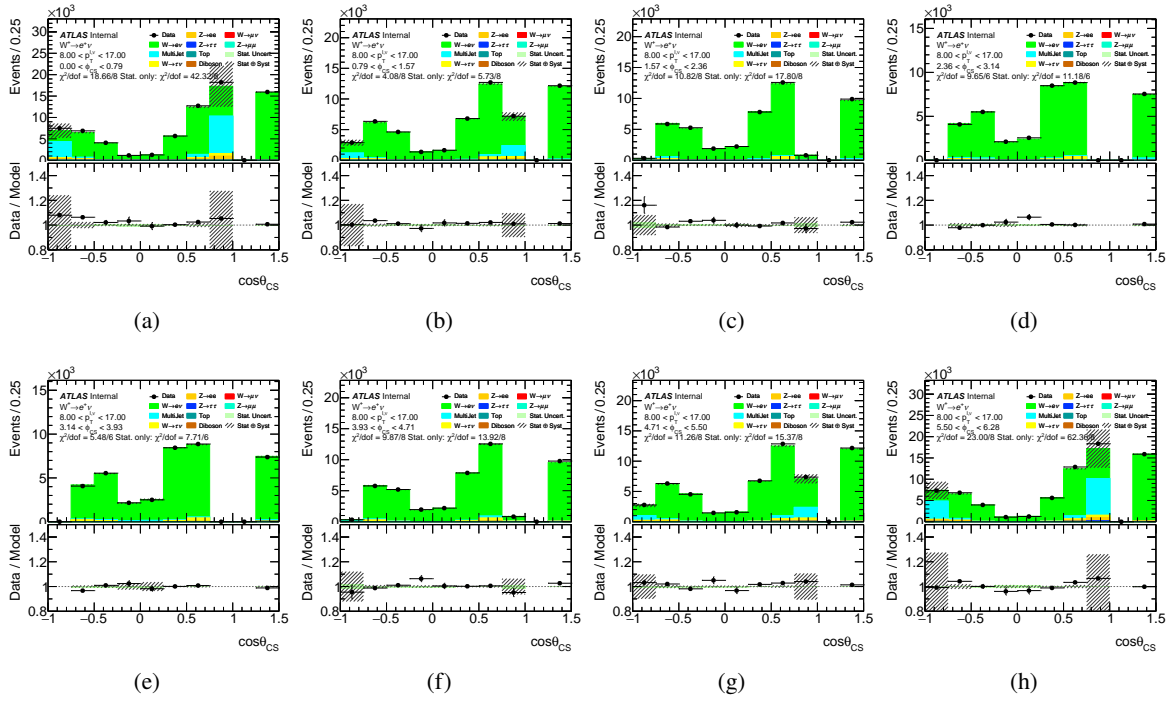


Figure 357: Control plots for $8 < p_T^{\ell, \nu} < 17$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

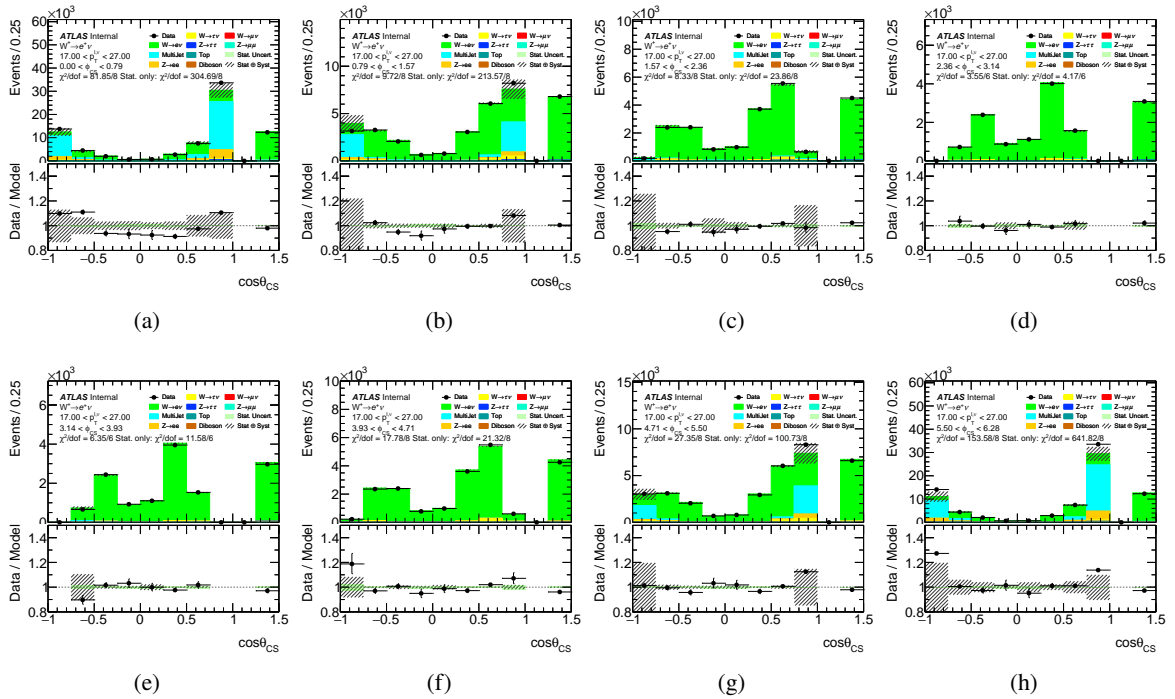


Figure 358: Control plots for $17 < p_T^{\ell, \nu} < 27$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

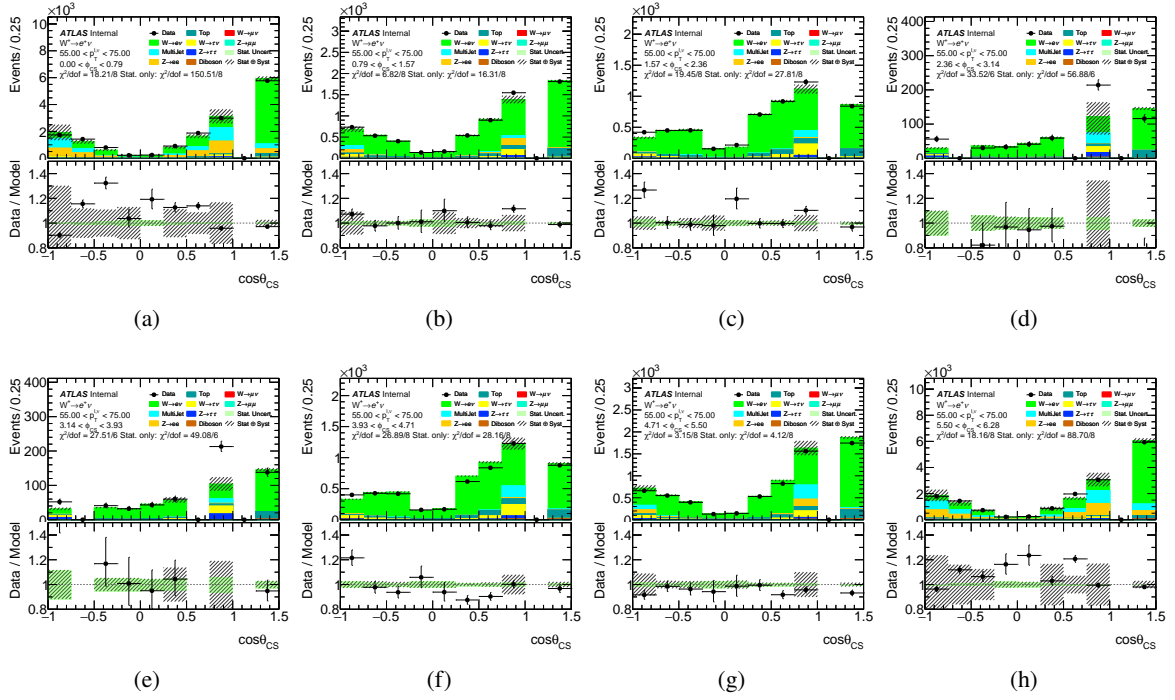


Figure 361: Control plots for $55 < p_T^{\ell,\nu} < 75$ GeV bin for $W^+ \rightarrow e^+\nu$ channel.

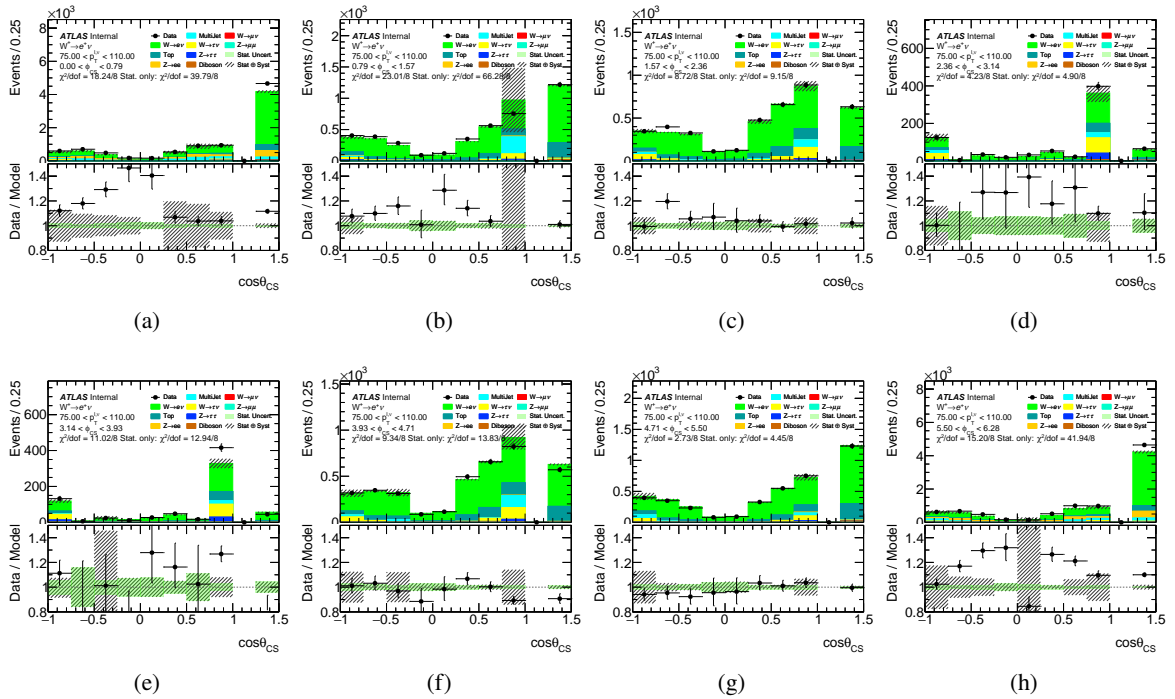


Figure 362: Control plots for $75 < p_T^{\ell,\nu} < 110$ GeV bin for $W^+ \rightarrow e^+\nu$ channel.

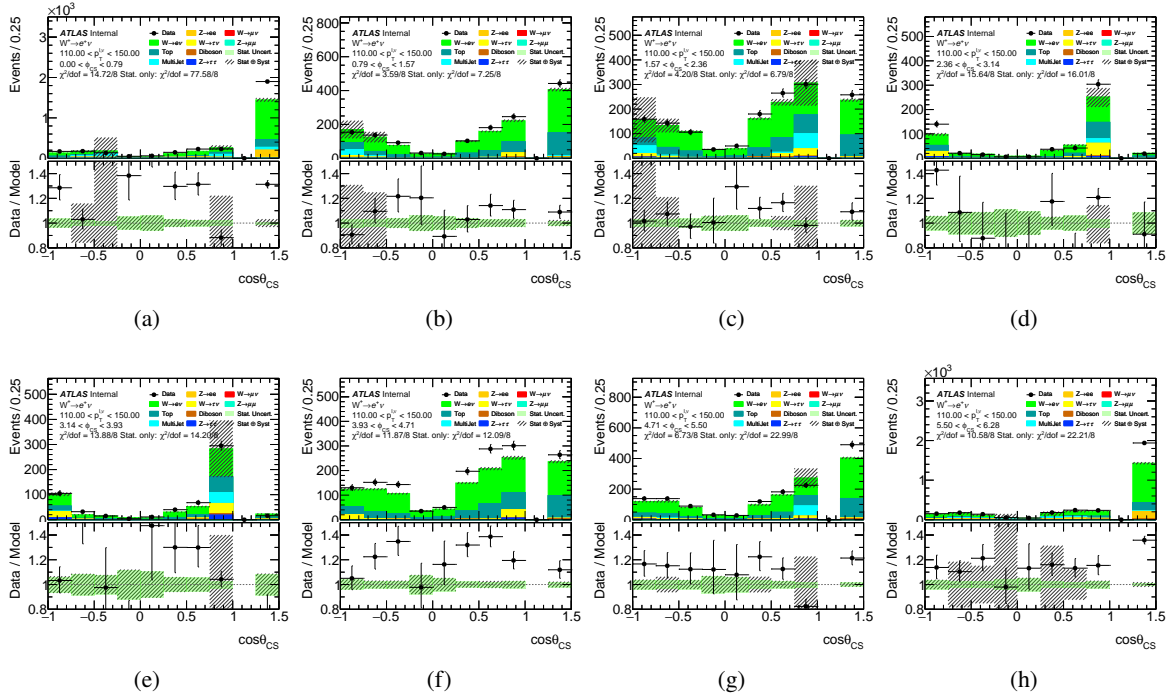


Figure 363: Control plots for $110 < p_T^{\ell, \nu} < 150$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

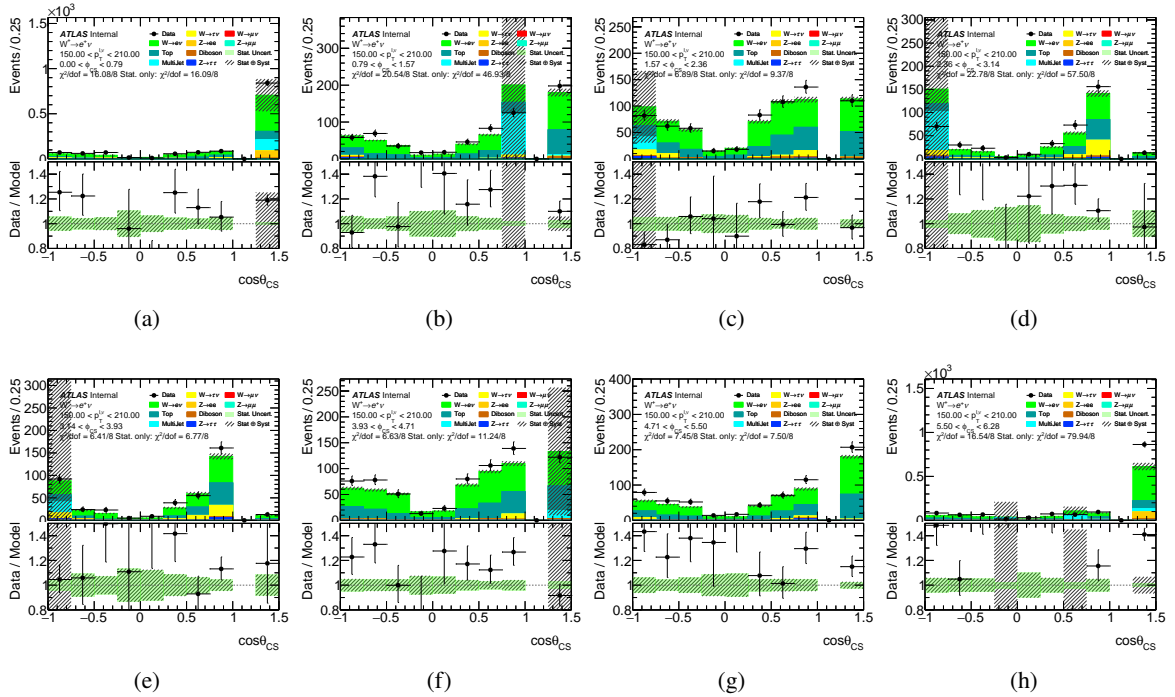


Figure 364: Control plots for $150 < p_T^{\ell, \nu} < 210$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

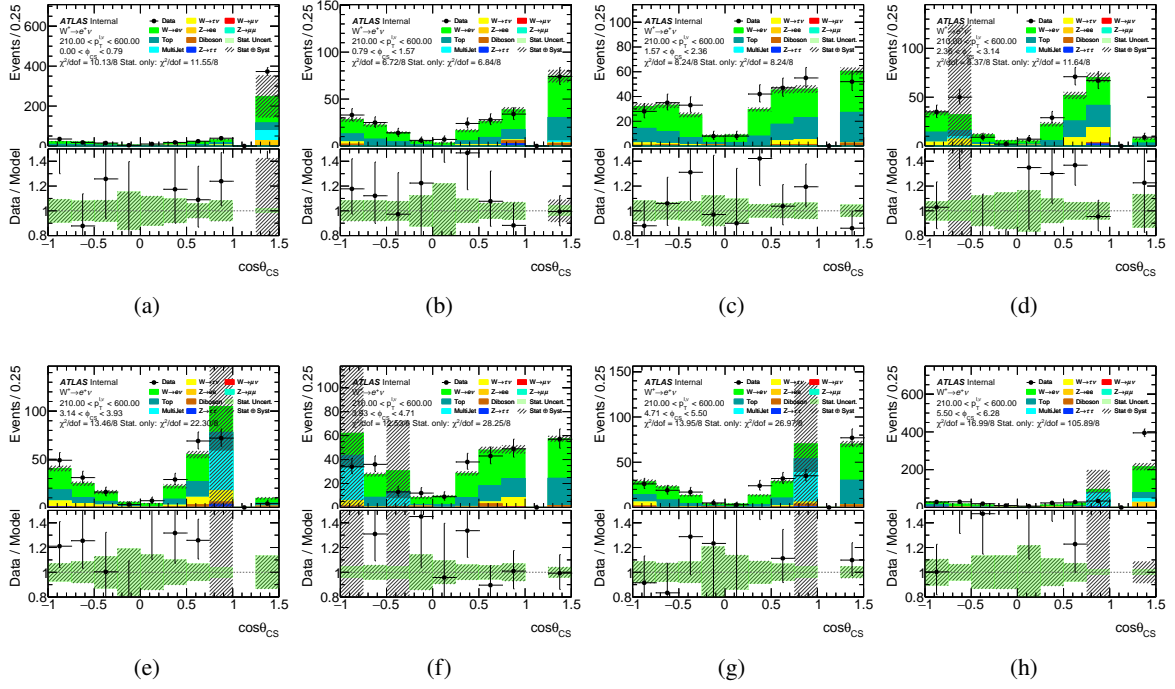


Figure 365: Control plots for $210 < p_T^{\ell, \nu} < 600$ GeV bin for $W^+ \rightarrow e^+ \nu$ channel.

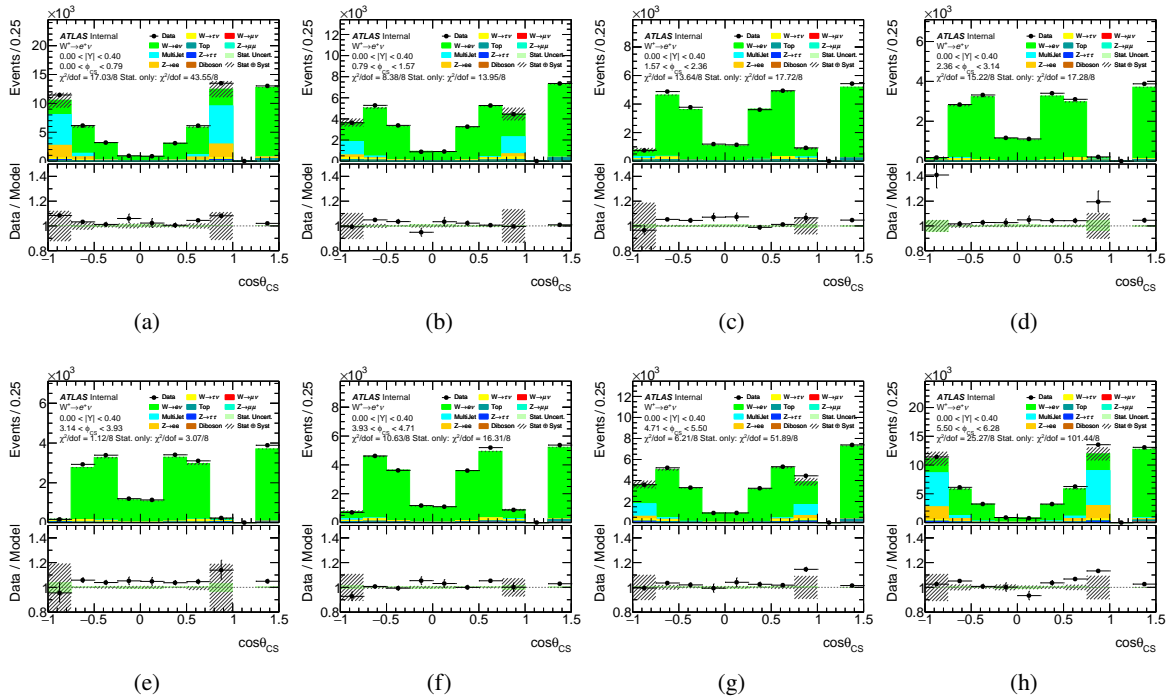


Figure 366: Control plots for $0 < |\eta| < 0.4$ bin for $W^+ \rightarrow e^+ \nu$ channel.

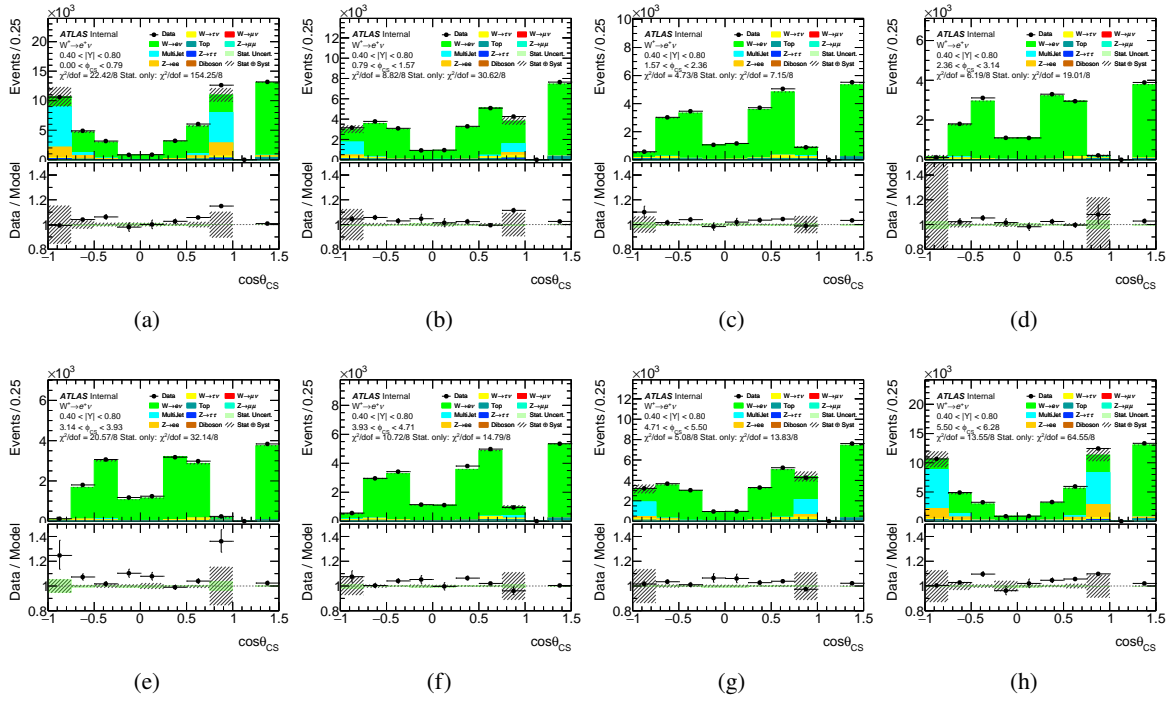


Figure 367: Control plots for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow e^+\nu$ channel.

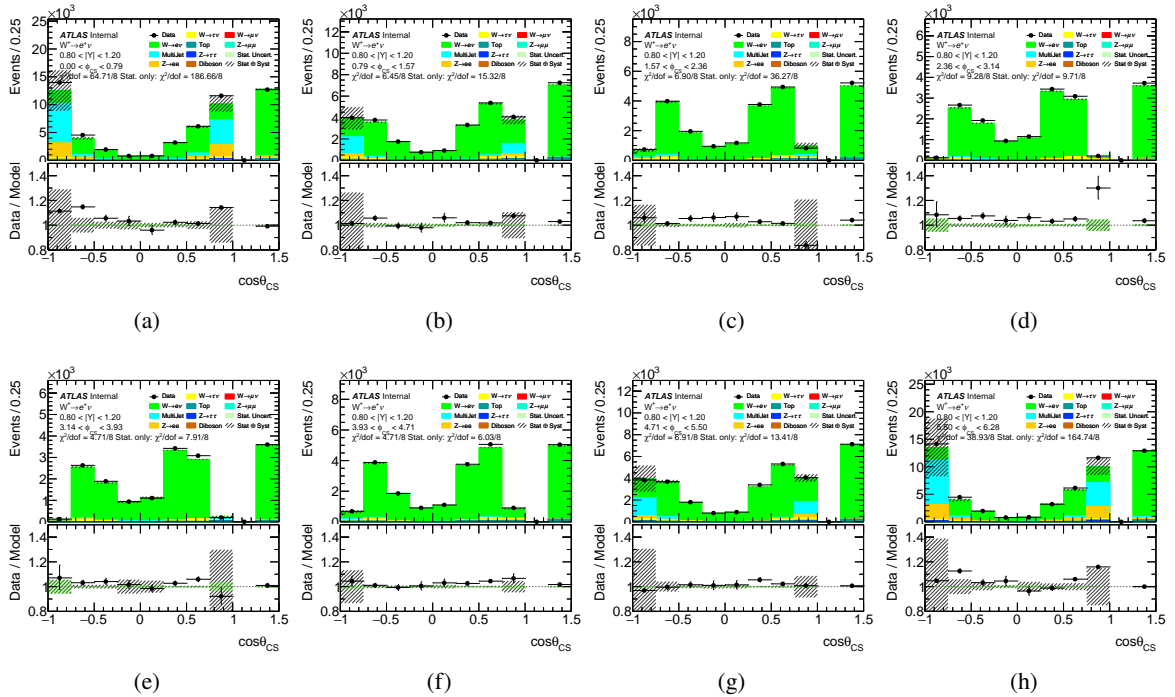


Figure 368: Control plots for $0.8 < |Y| < 1.2$ bin for $W^+ \rightarrow e^+\nu$ channel.

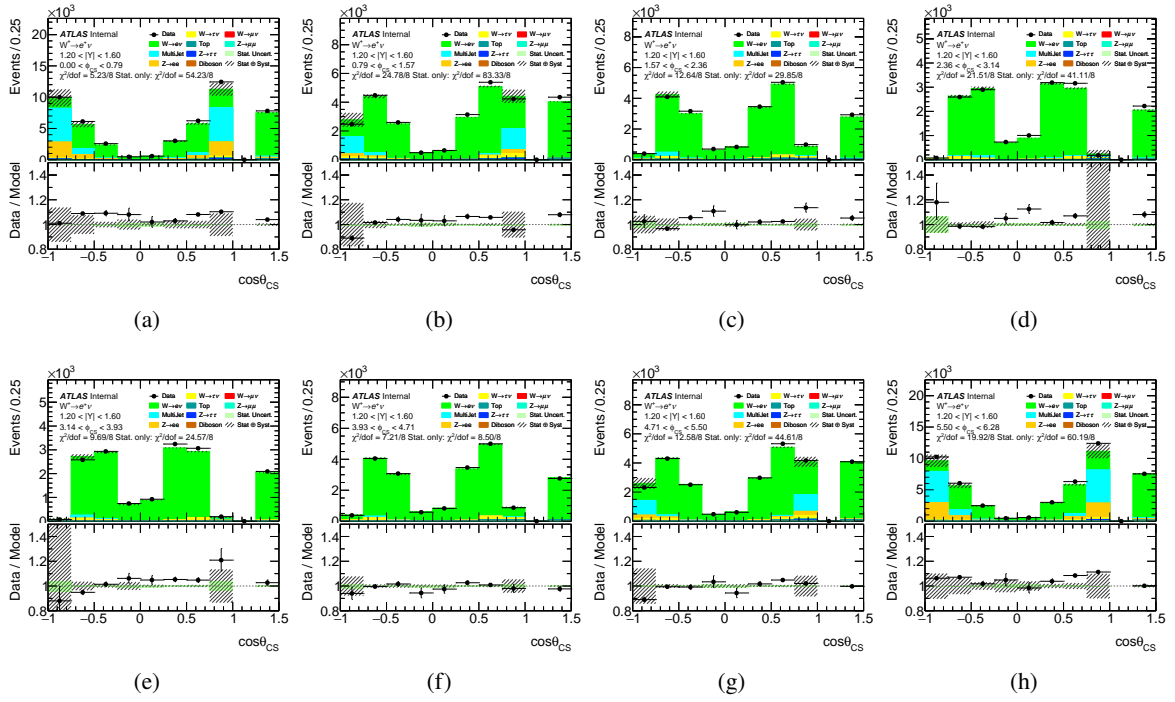


Figure 369: Control plots for $1.2 < |Y| < 1.6$ bin for $W^+ \rightarrow e^+ \nu$ channel.

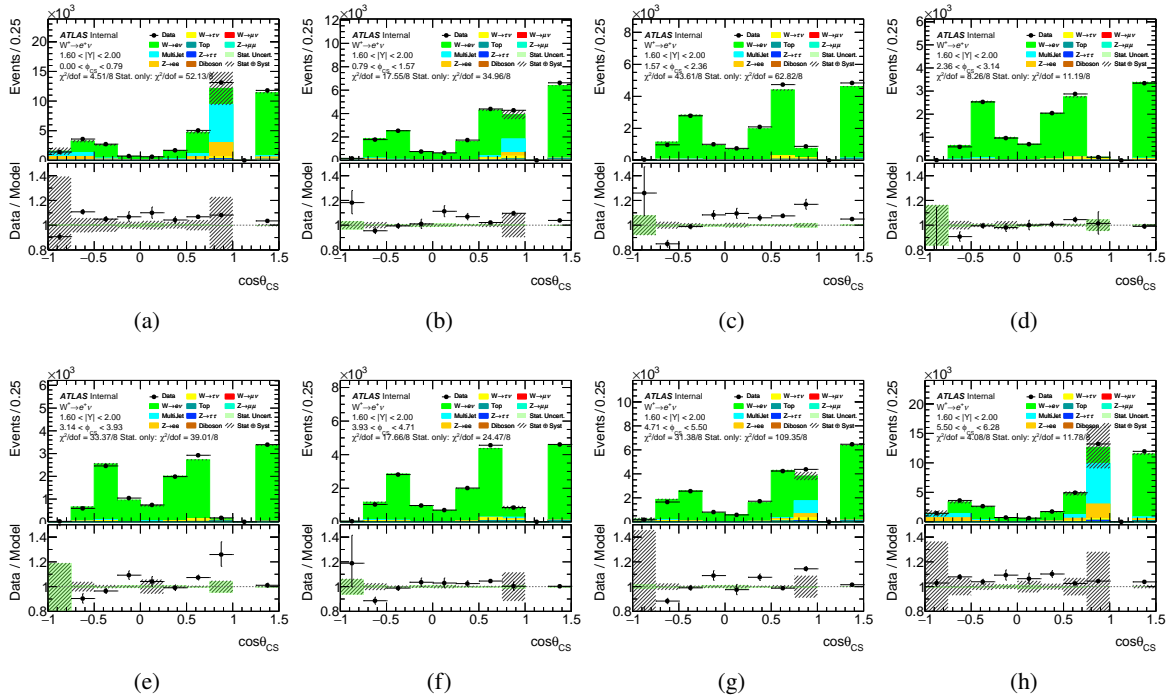


Figure 370: Control plots for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow e^+ \nu$ channel.

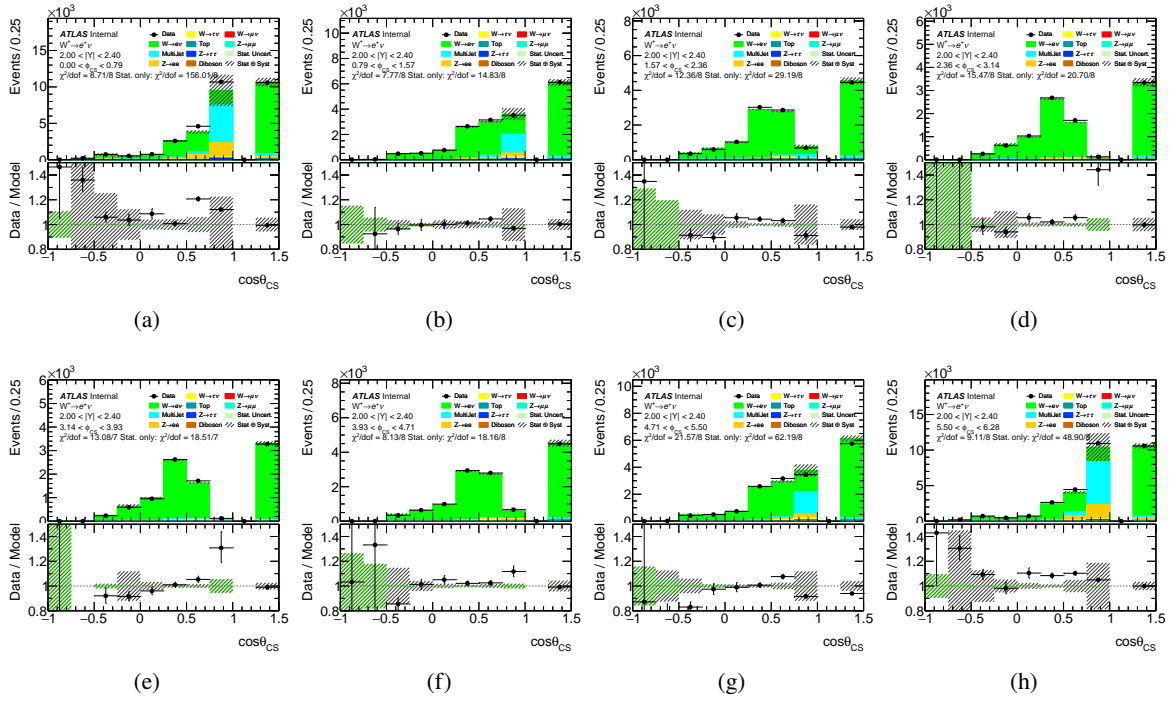


Figure 371: Control plots for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow e^+ \nu$ channel.

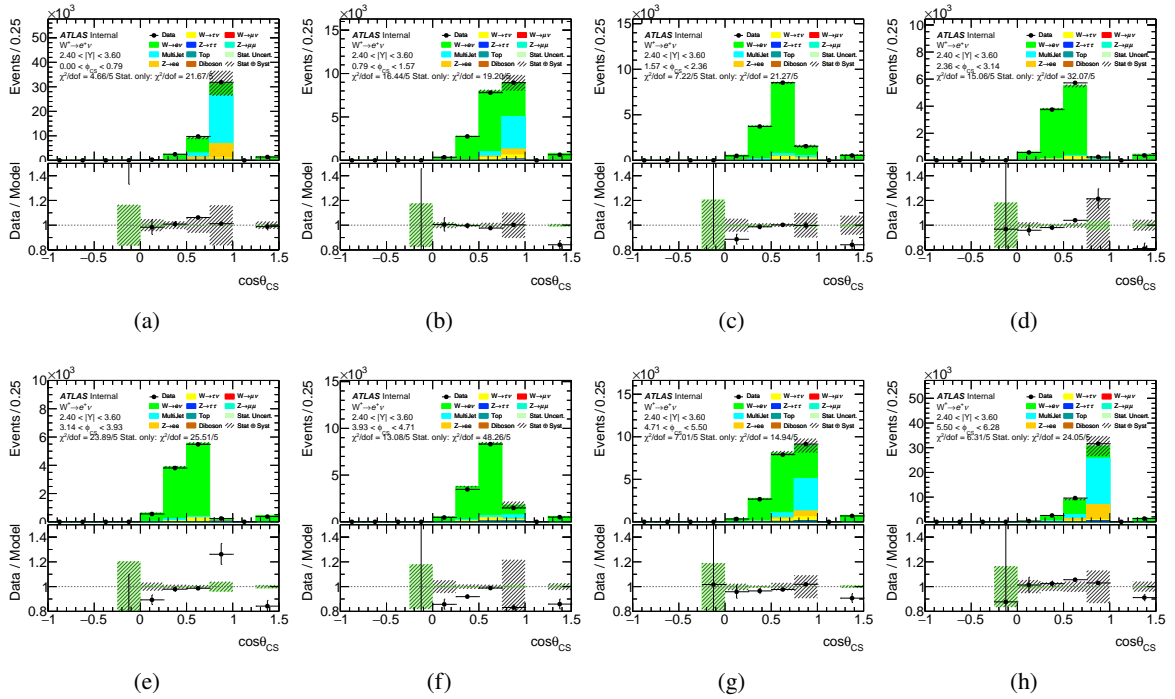


Figure 372: Control plots for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow e^+ \nu$ channel.

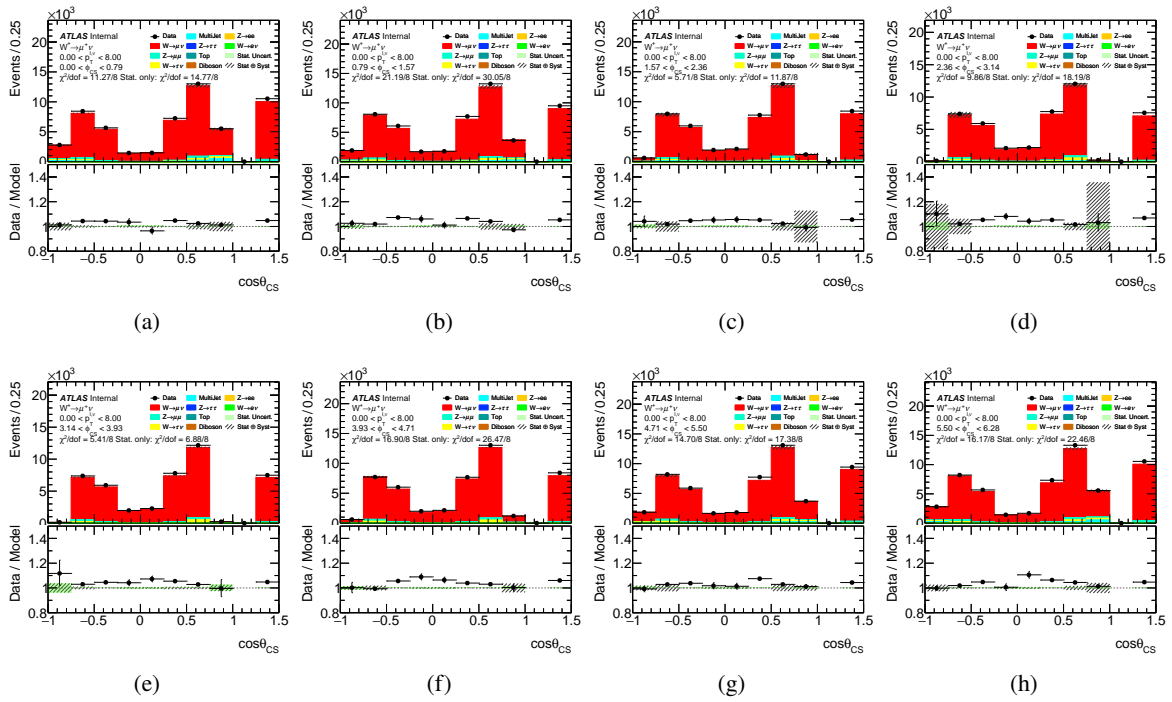


Figure 373: Control plots for $0 < p_T^{\ell,\nu} < 8$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

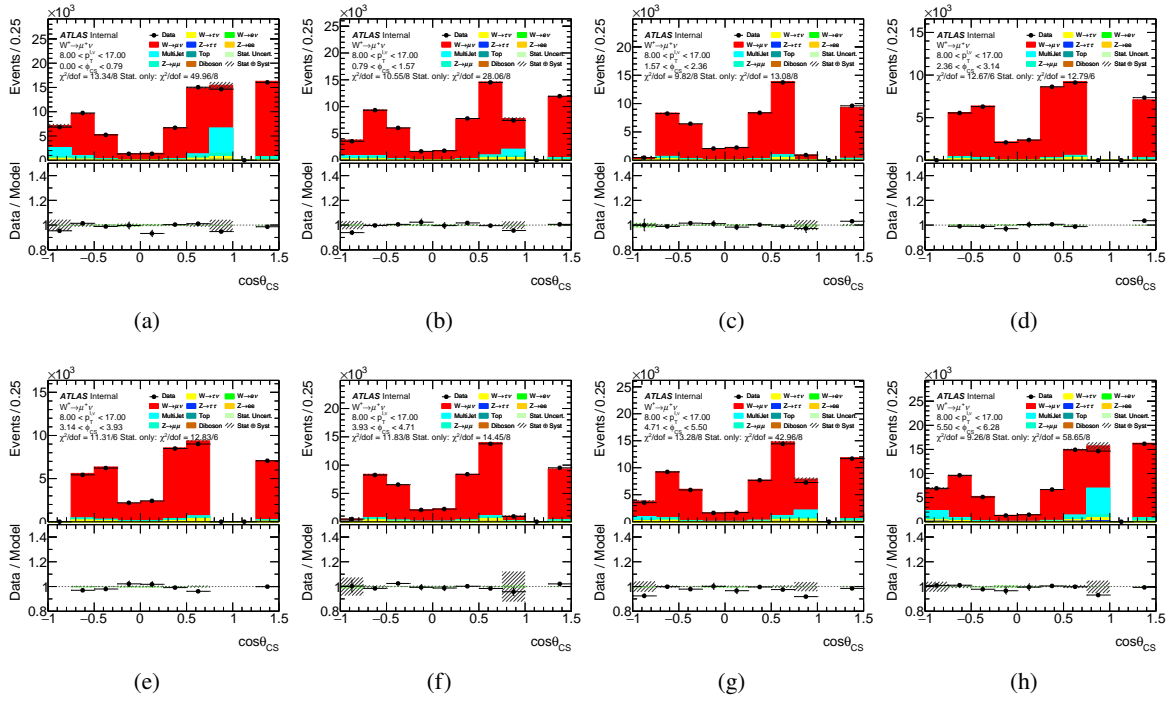


Figure 374: Control plots for $8 < p_T^{\ell, \nu} < 17$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

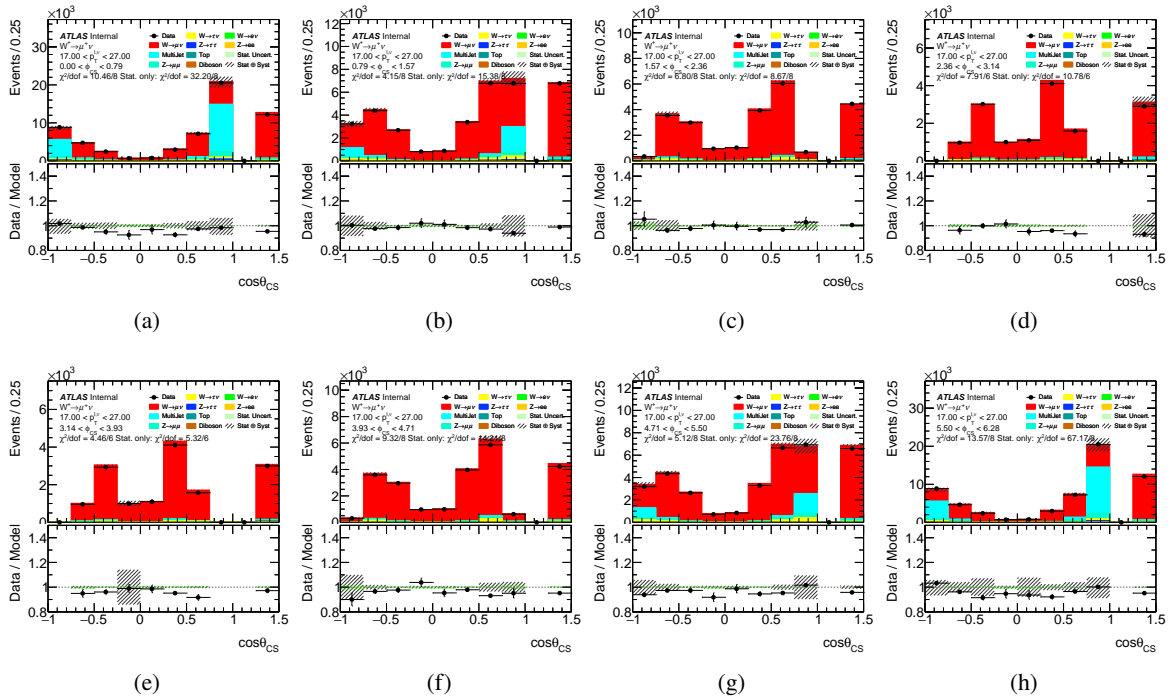


Figure 375: Control plots for $17 < p_T^{\ell, \nu} < 27$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

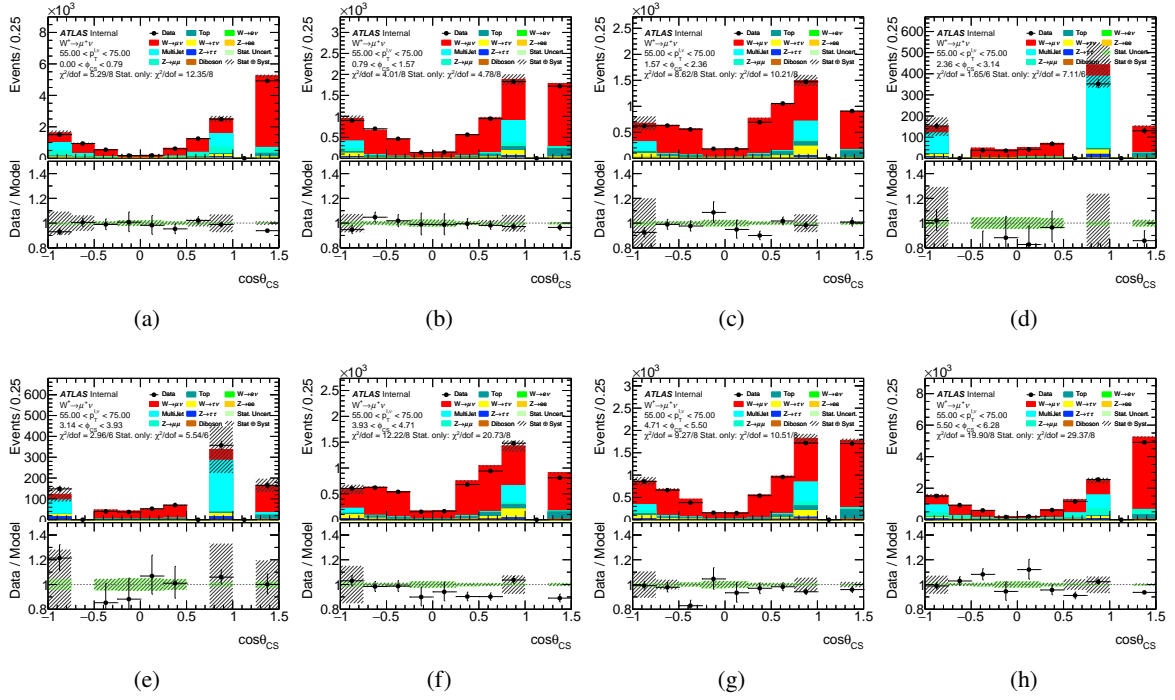


Figure 378: Control plots for $55 < p_T^{\ell, \nu} < 75$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

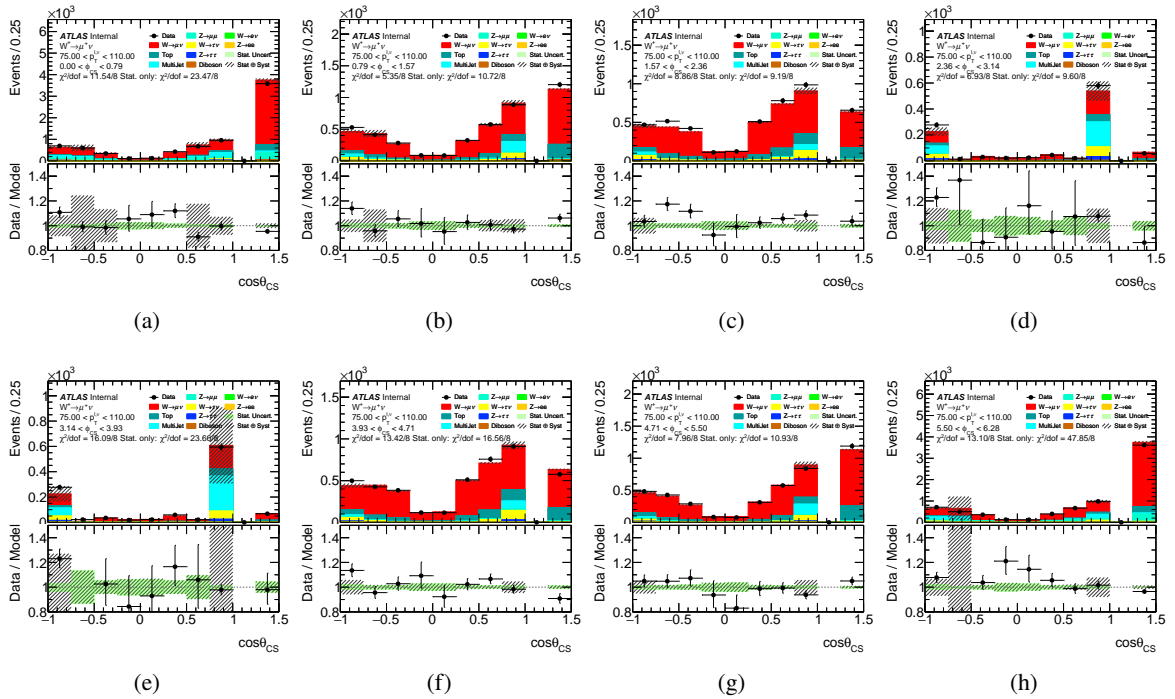


Figure 379: Control plots for $75 < p_T^{\ell, \nu} < 110$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

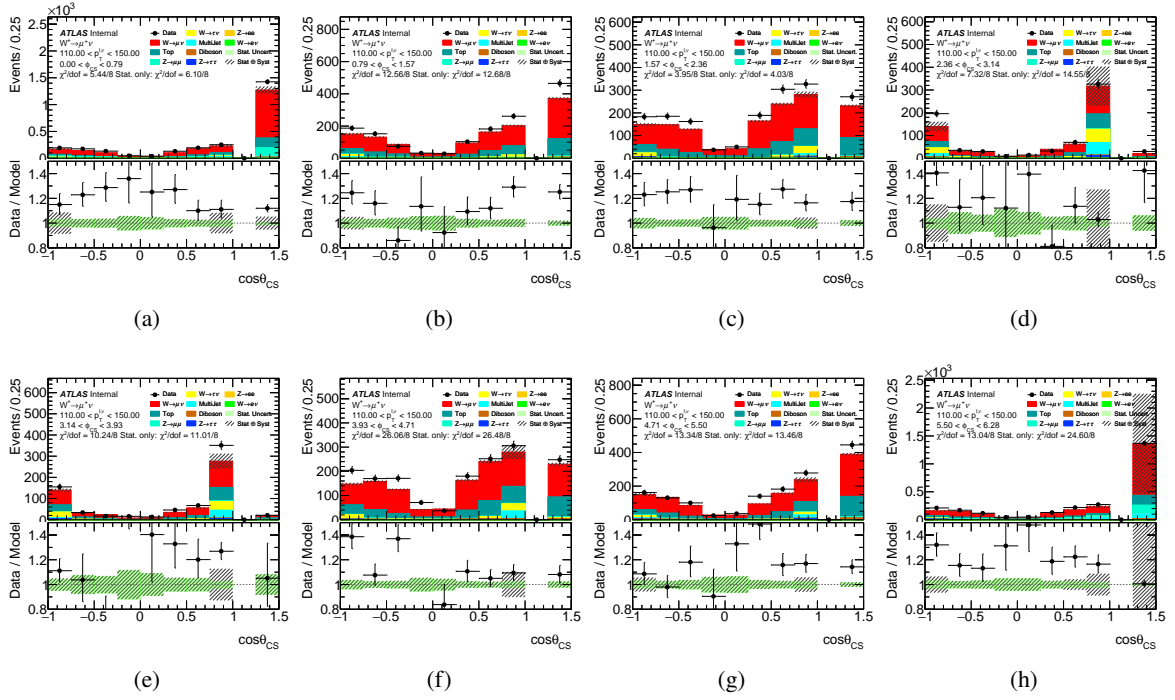


Figure 380: Control plots for $110 < p_T^{\ell, \nu} < 150$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

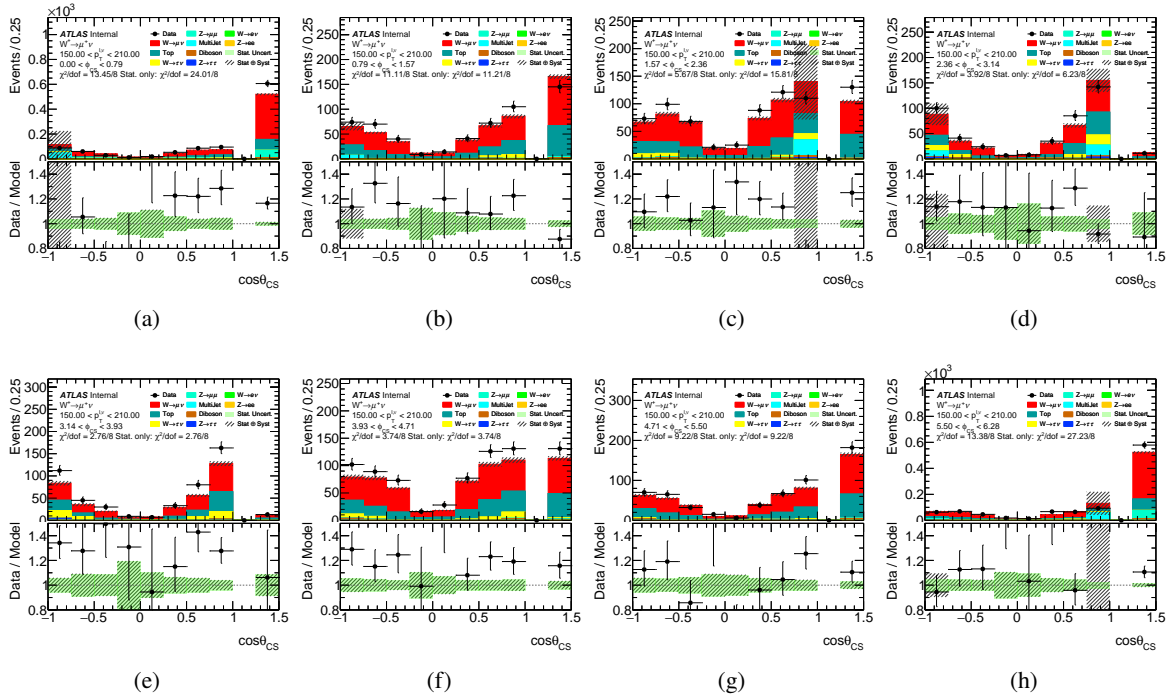


Figure 381: Control plots for $150 < p_T^{\ell, \nu} < 210$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

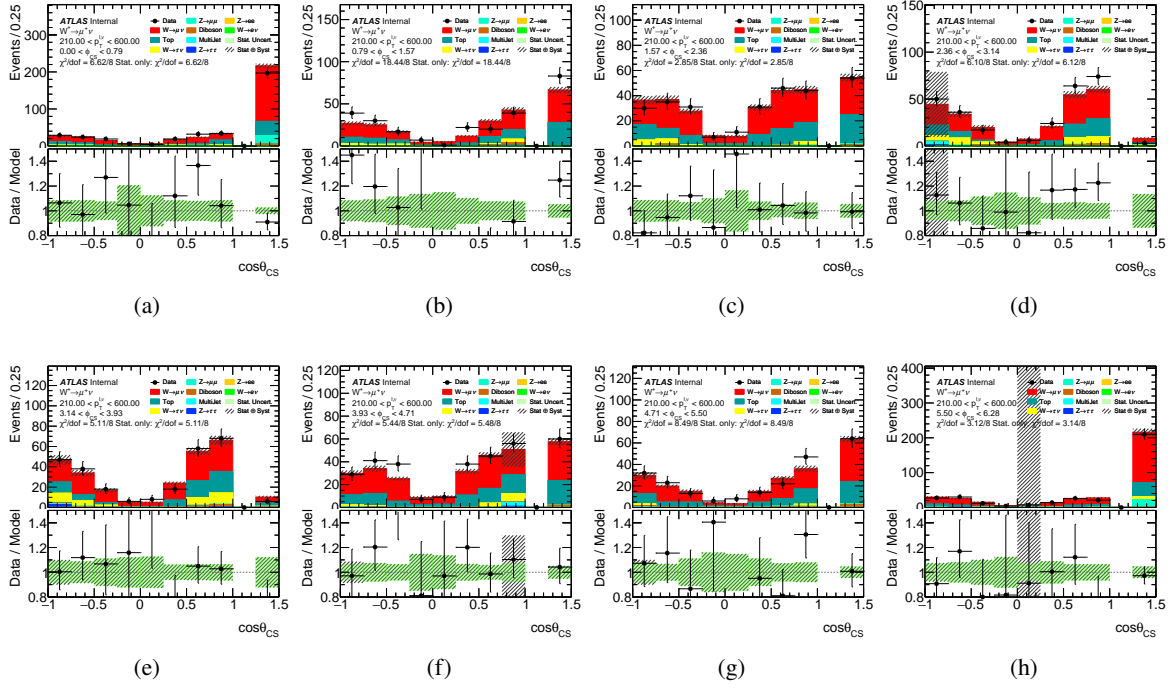


Figure 382: Control plots for $210 < p_T^{\ell, \nu} < 600$ GeV bin for $W^+ \rightarrow \mu^+ \nu$ channel.

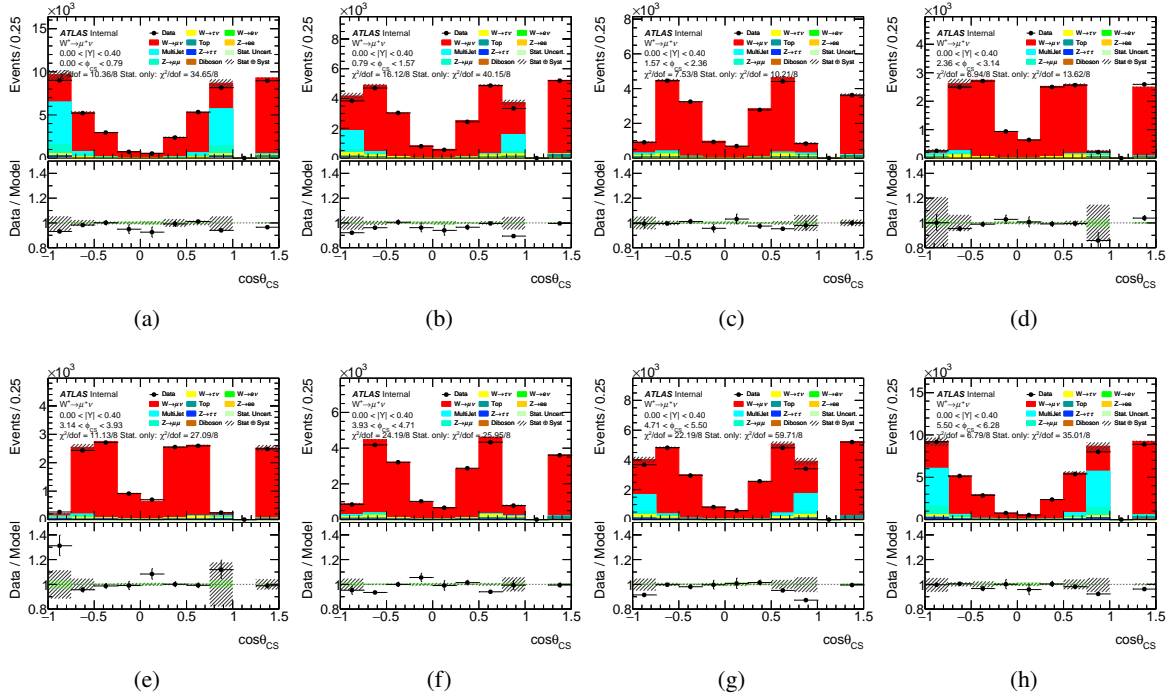


Figure 383: Control plots for $0 < |Y| < 0.4$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

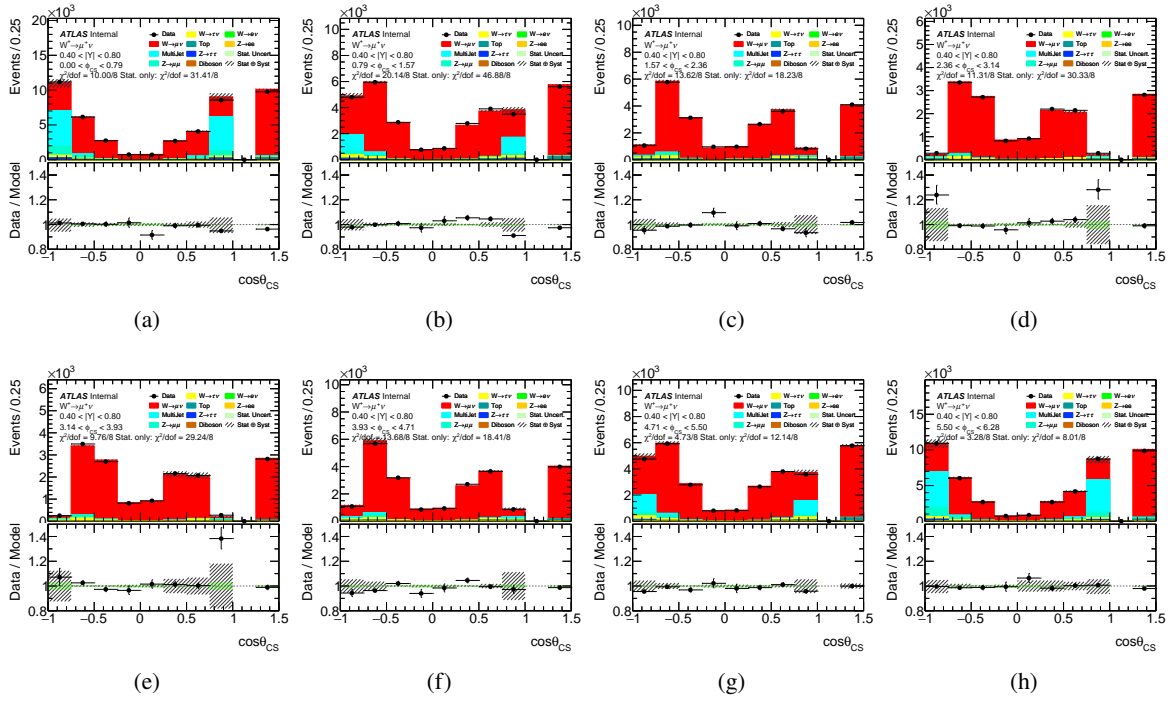


Figure 384: Control plots for $0.4 < |Y| < 0.8$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

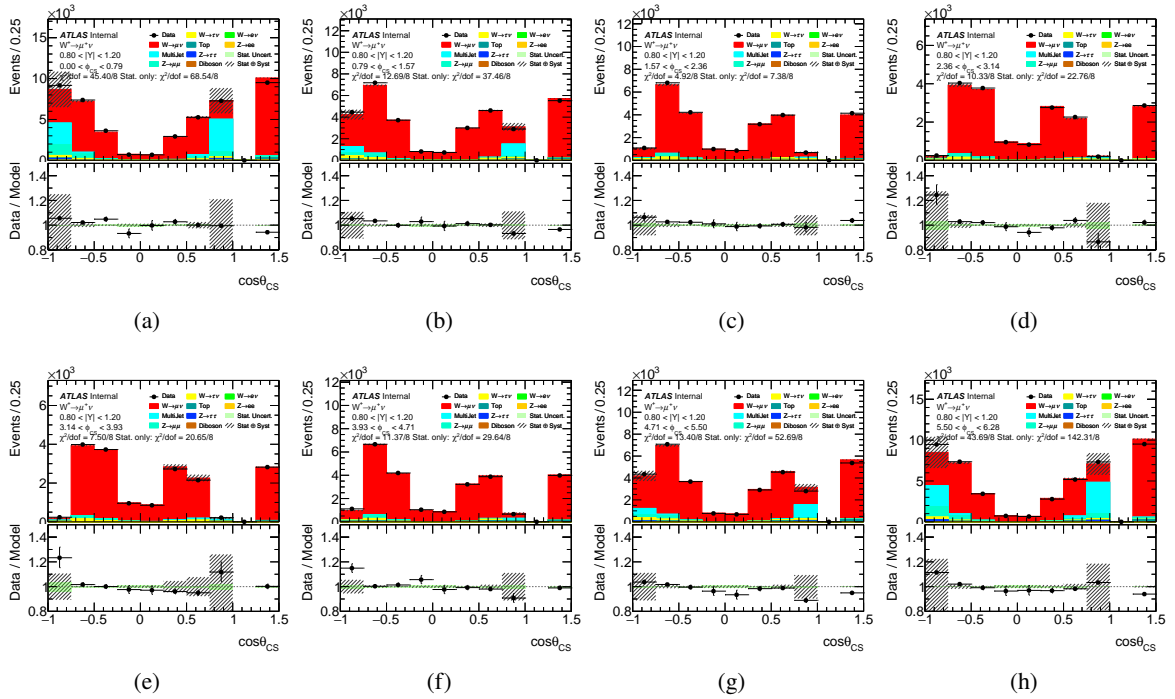


Figure 385: Control plots for $0.8 < |Y| < 1.2$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

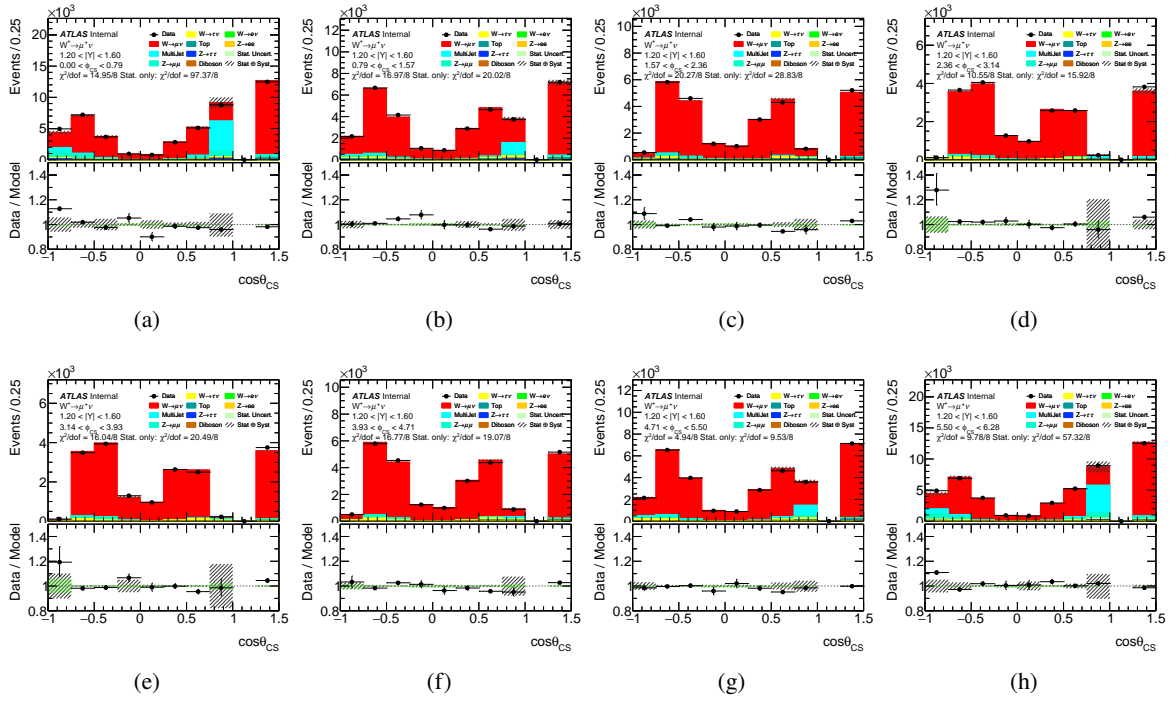


Figure 386: Control plots for $1.2 < |Y| < 1.6$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

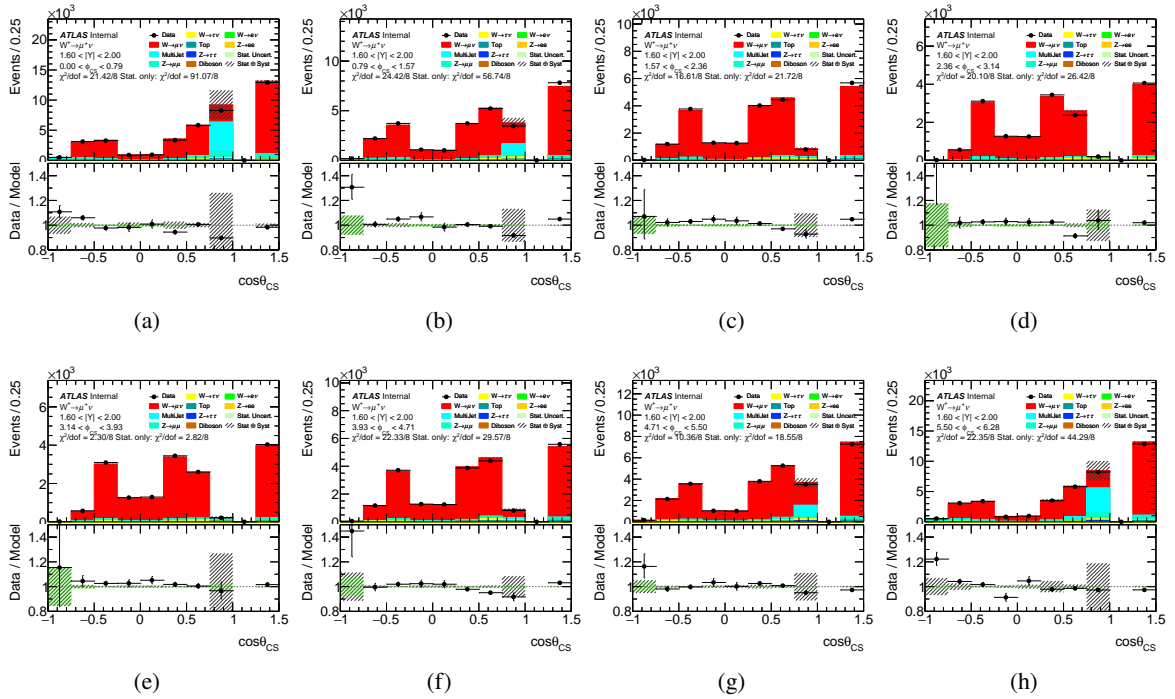


Figure 387: Control plots for $1.6 < |Y| < 2.0$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

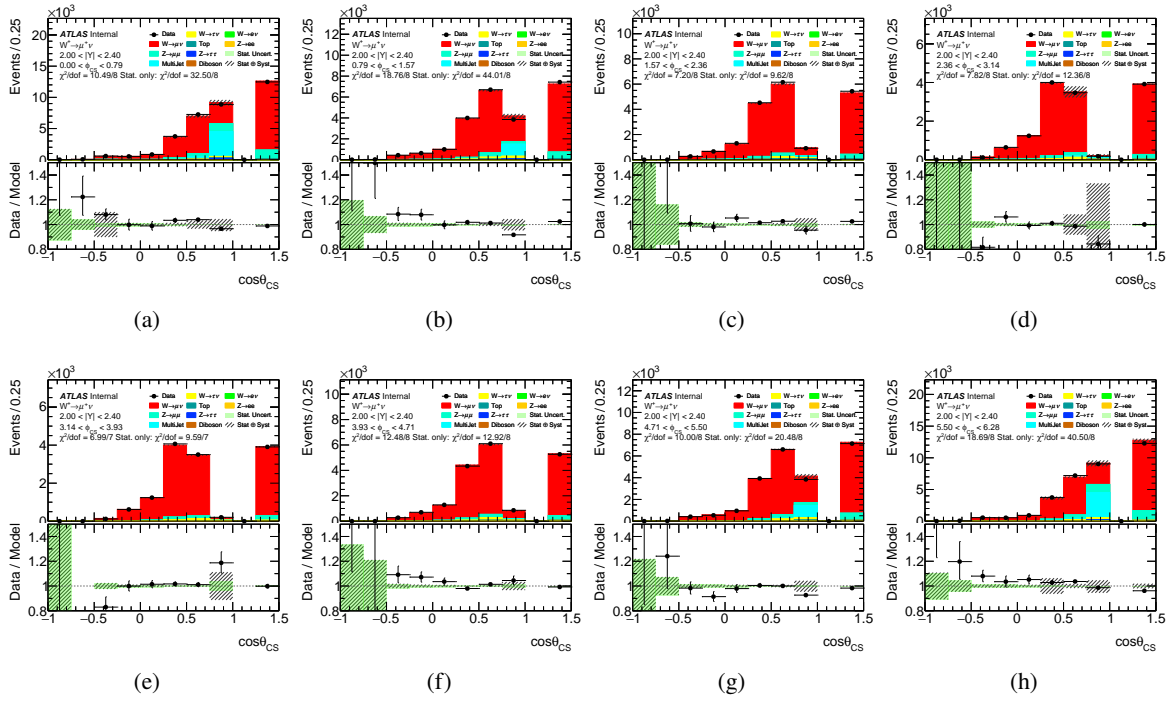


Figure 388: Control plots for $2.0 < |Y| < 2.4$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

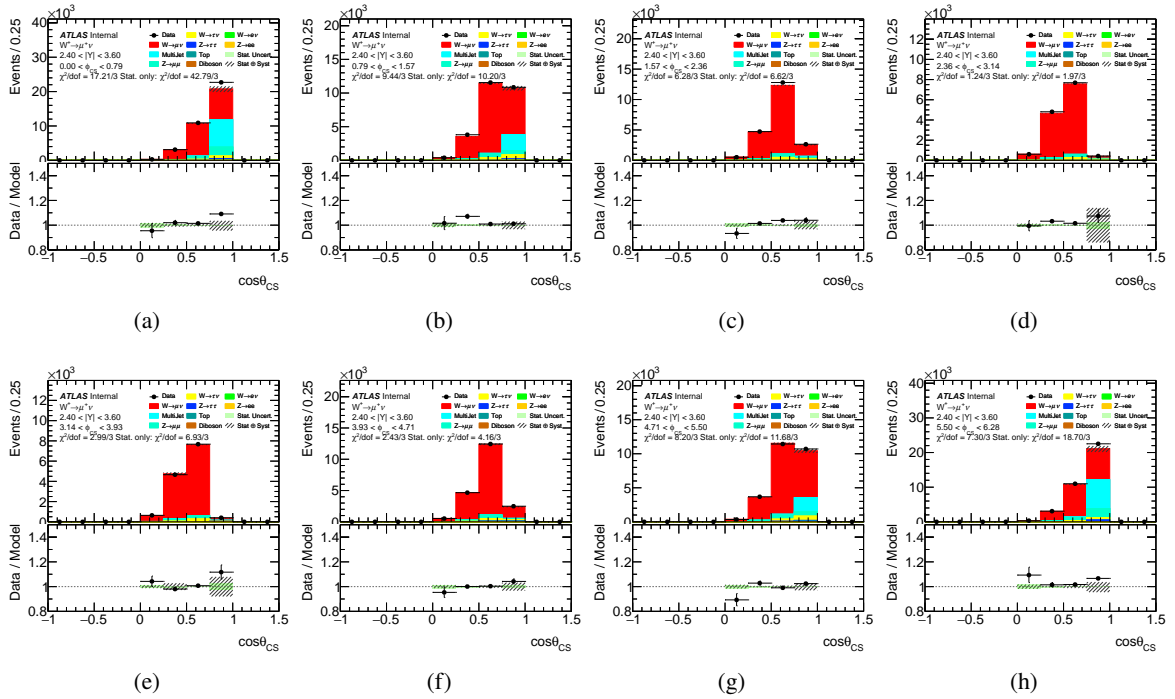


Figure 389: Control plots for $2.4 < |Y| < 3.6$ bin for $W^+ \rightarrow \mu^+ \nu$ channel.

1305 **F.6 Double ratio $W^+ \rightarrow e^+\nu / W^+ \rightarrow \mu^+\nu$**

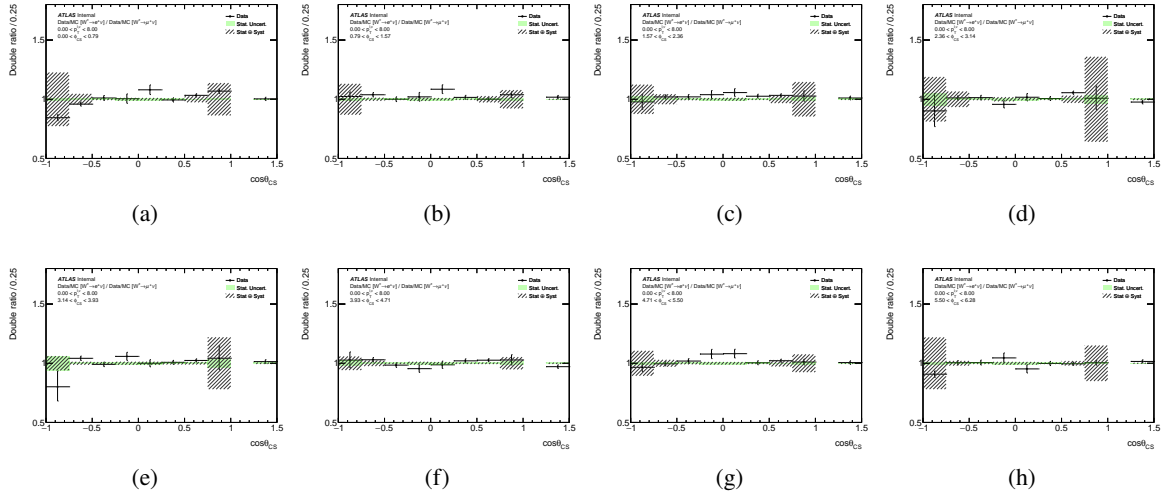


Figure 390: Double ratio plots for e/μ for $0 < p_T^{\ell,\nu} < 8$ GeV bin.

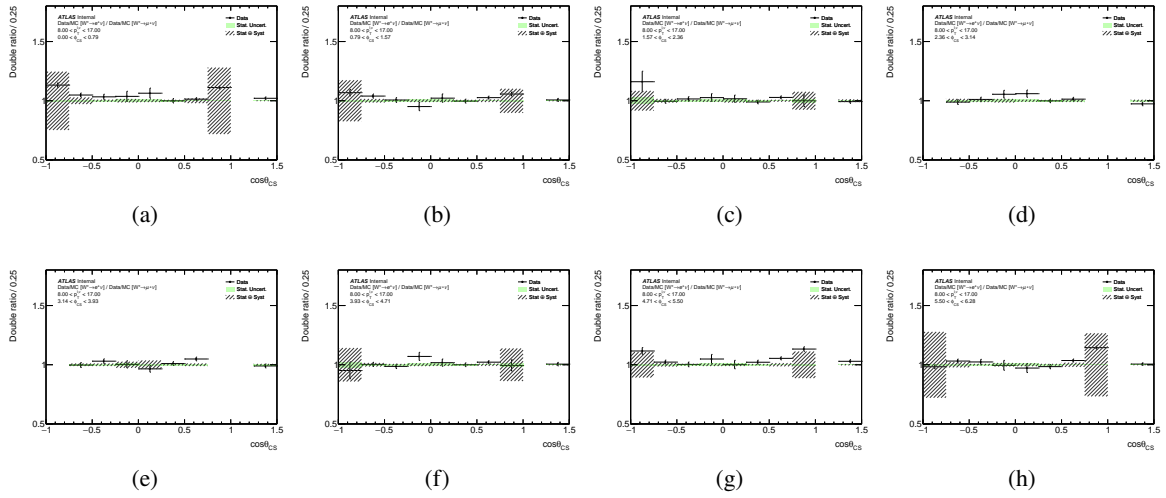


Figure 391: Double ratio plots for e/μ for $8 < p_T^{\ell,\nu} < 17$ GeV bin.

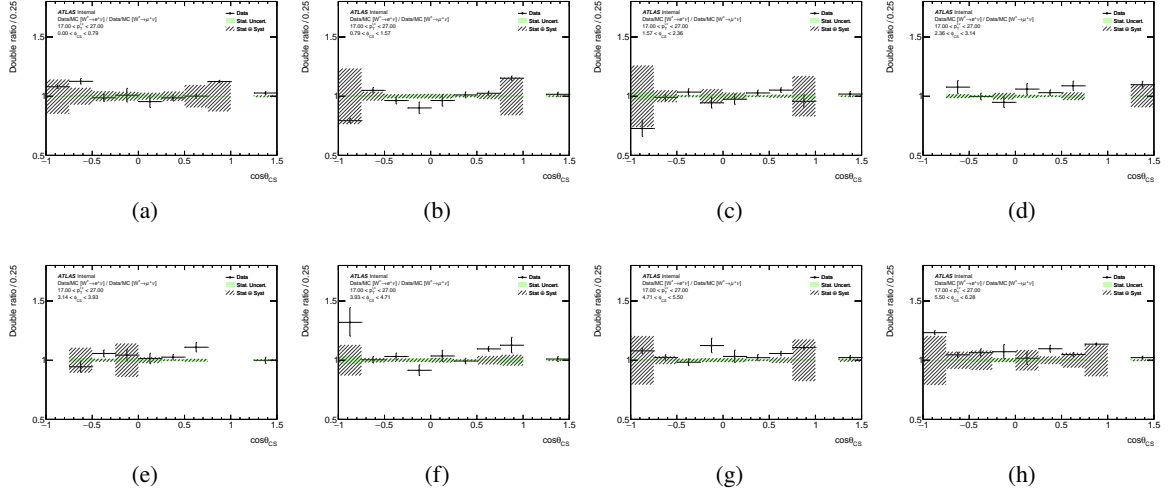


Figure 392: Double ratio plots for e/μ for $17 < p_T^{\ell,\nu} < 27$ GeV bin.

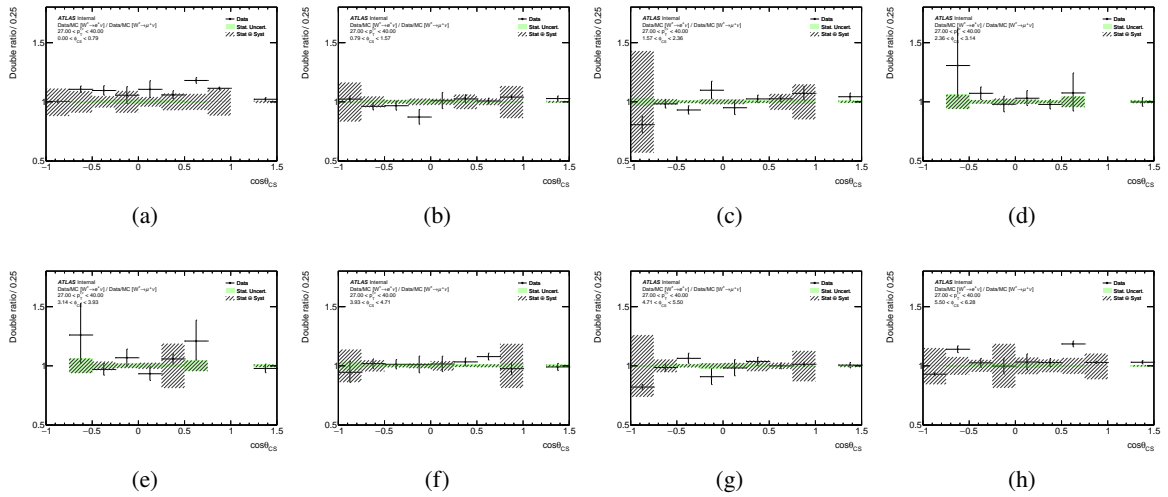


Figure 393: Double ratio plots for e/μ for $27 < p_T^{\ell,\nu} < 40$ GeV bin.

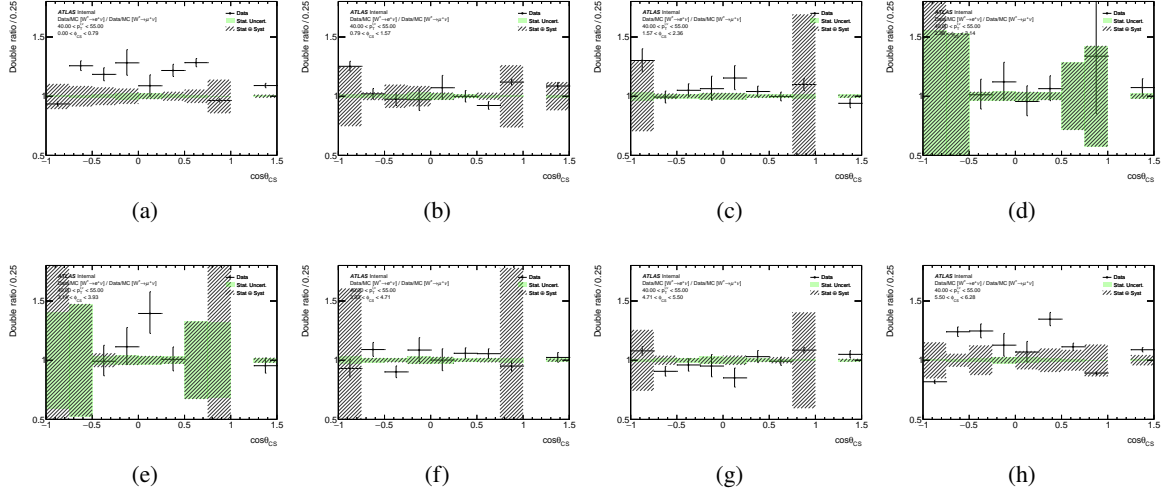


Figure 394: Double ratio plots for e/μ for $40 < p_T^{\ell,\nu} < 55$ GeV bin.

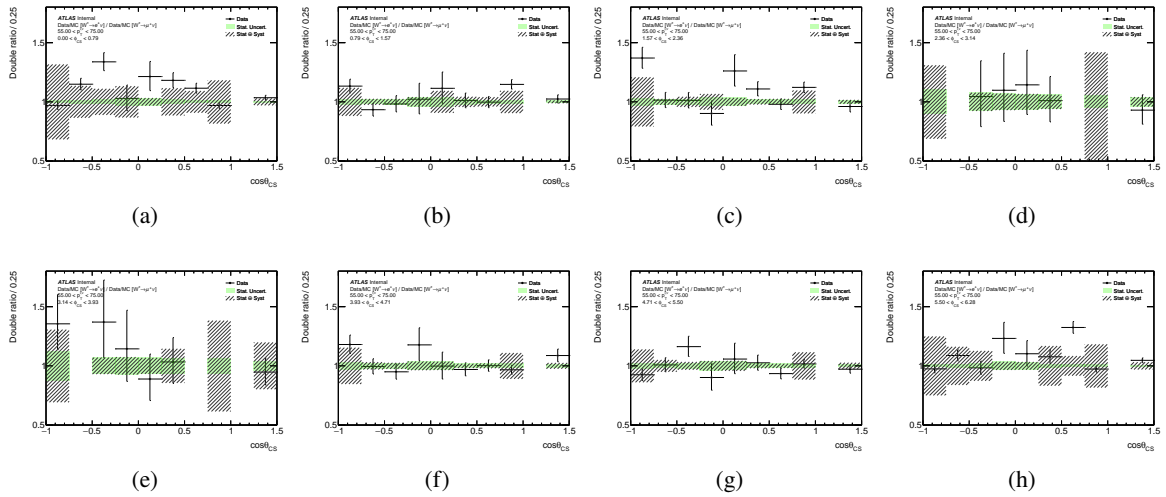


Figure 395: Double ratio plots for e/μ for $55 < p_T^{\ell,\nu} < 75$ GeV bin.

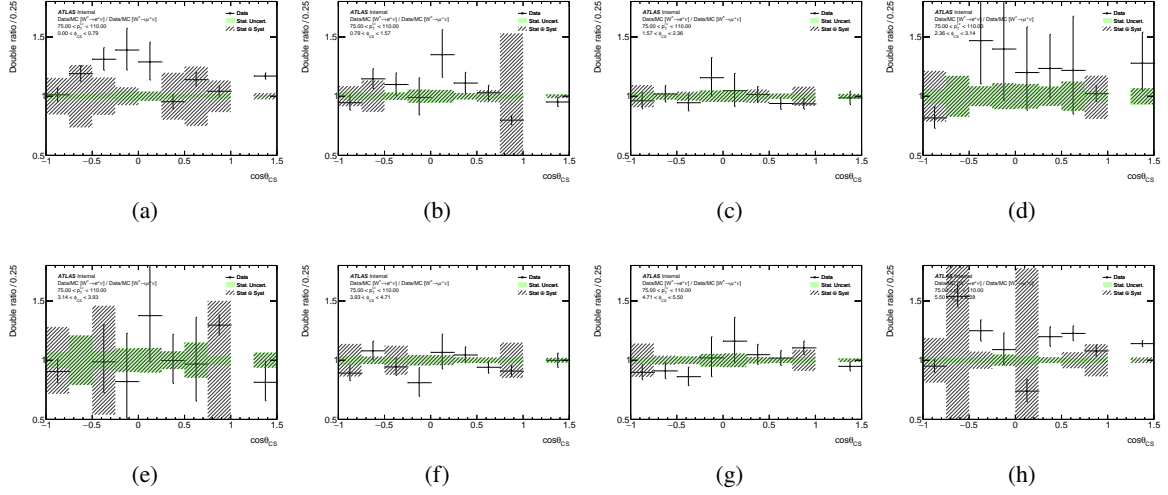


Figure 396: Double ratio plots for e/μ for $75 < p_T^{\ell,\nu} < 110$ GeV bin.

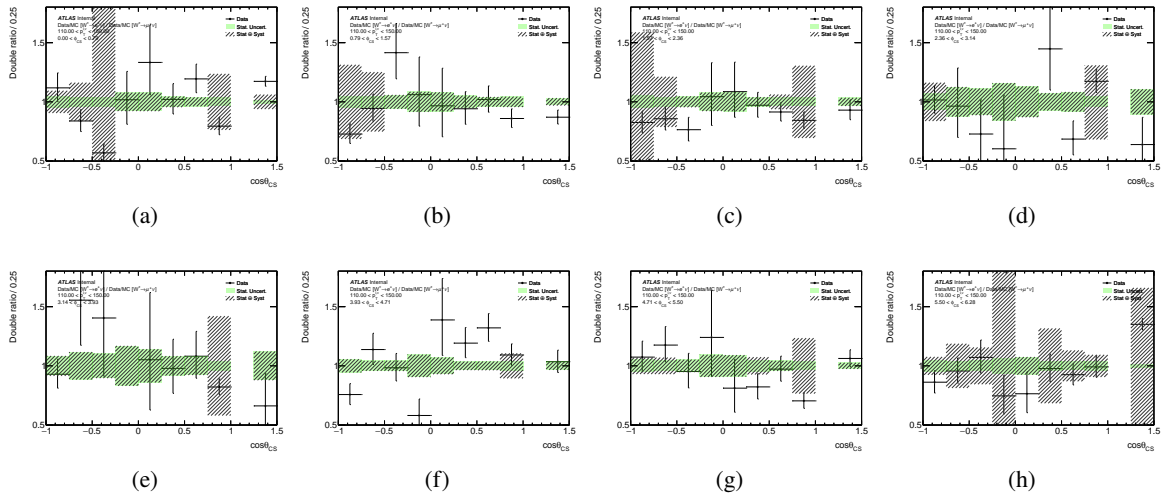


Figure 397: Double ratio plots for e/μ for $110 < p_T^{\ell,\nu} < 150$ GeV bin.

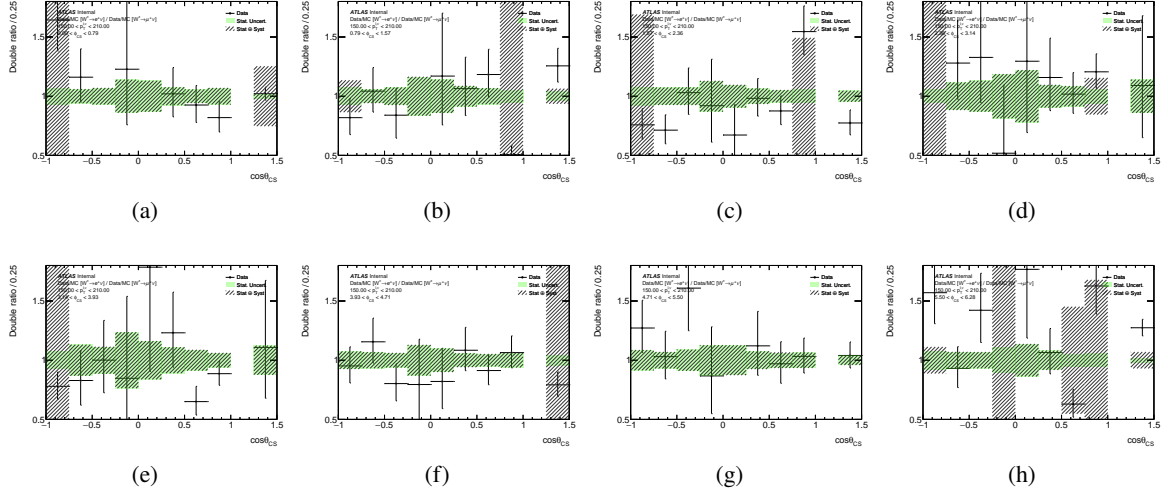


Figure 398: Double ratio plots for e/μ for $150 < p_T^{\ell,\nu} < 210$ GeV bin.

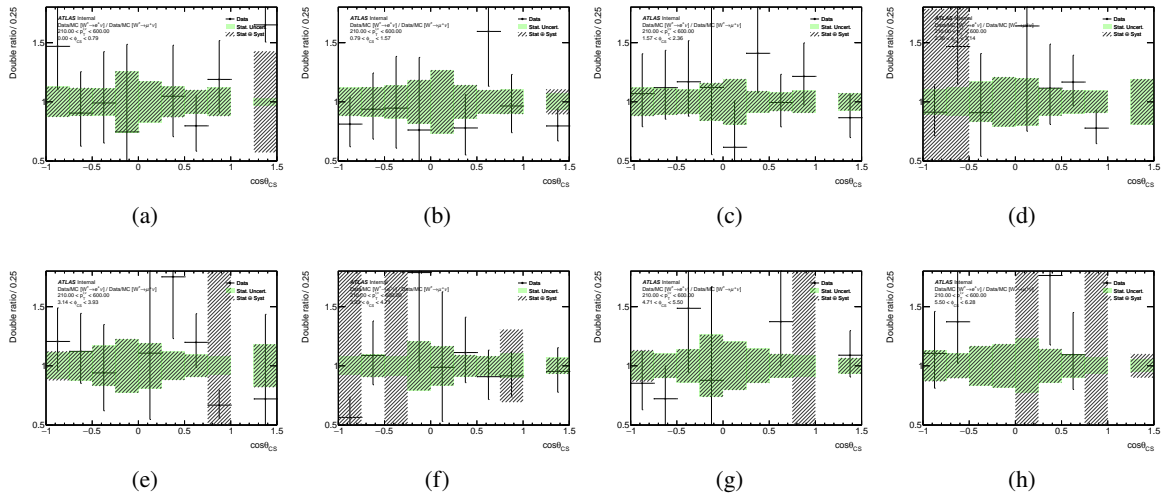


Figure 399: Double ratio plots for e/μ for $210 < p_T^{\ell,\nu} < 600$ GeV bin.

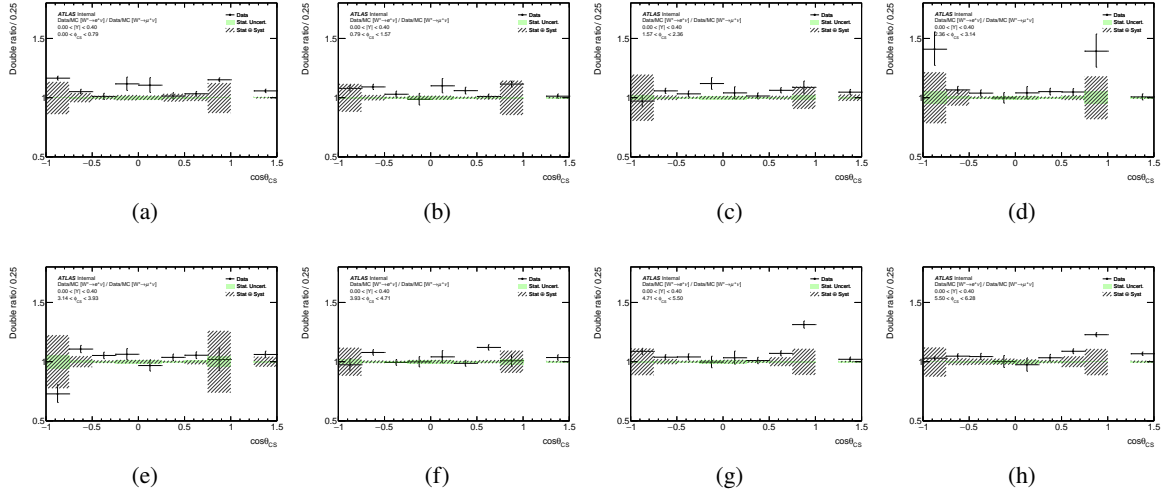


Figure 400: Double ratio plots for e/μ for $0 < |Y| < 0.4$ bin.

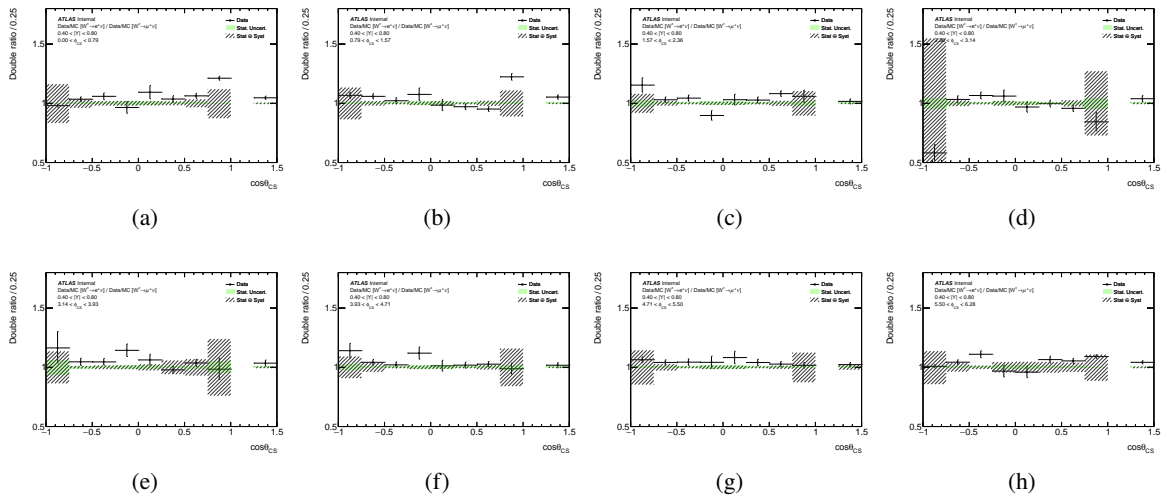


Figure 401: Double ratio plots for e/μ for $0.4 < |Y| < 0.8$ bin.

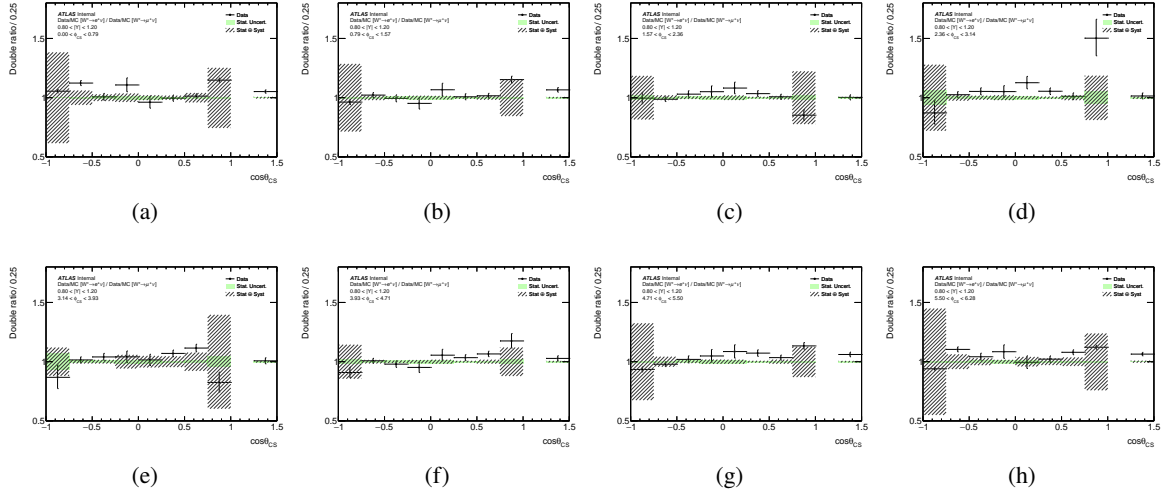


Figure 402: Double ratio plots for e/μ for $0.8 < |Y| < 1.2$ bin.

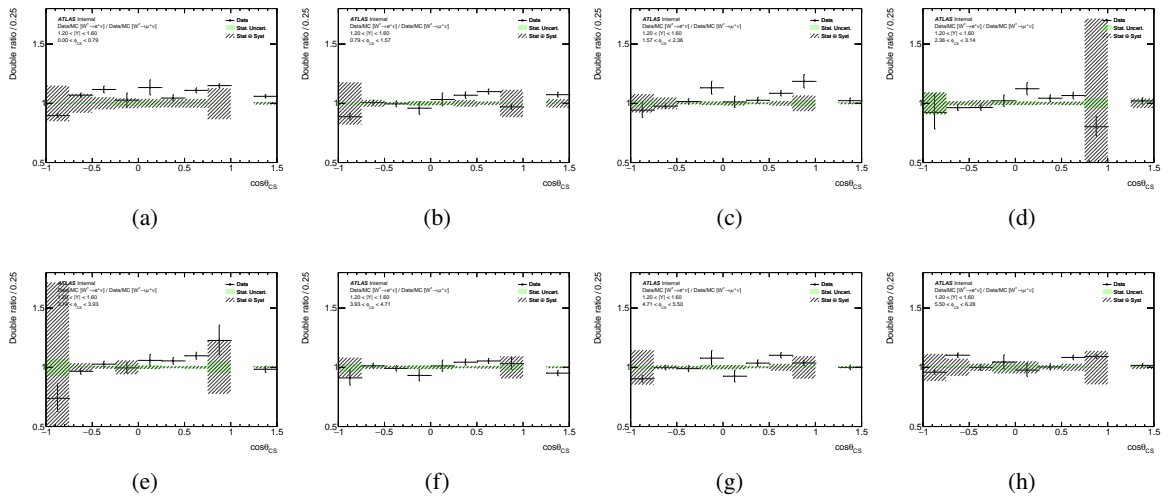


Figure 403: Double ratio plots for e/μ for $1.2 < |Y| < 1.6$ bin.

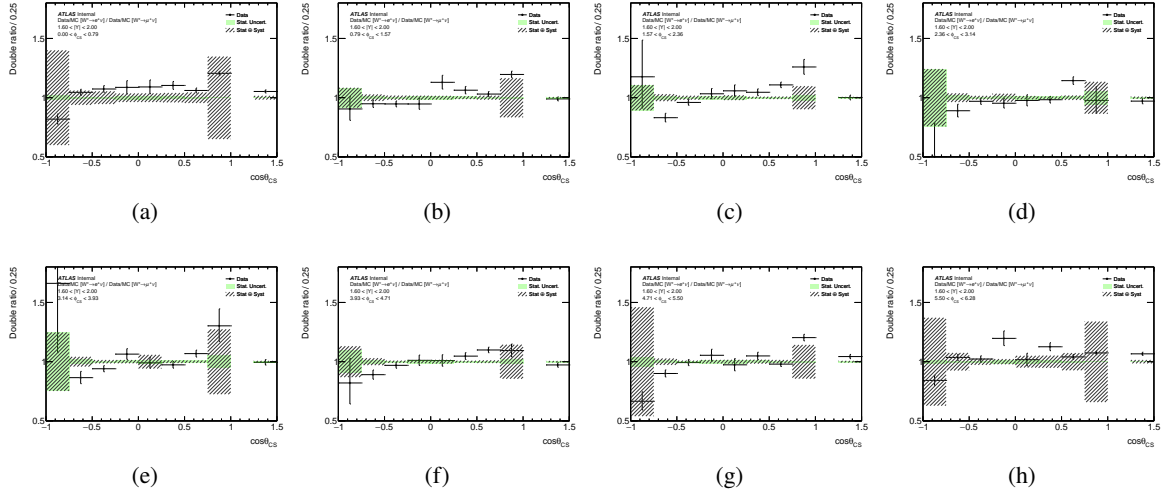


Figure 404: Double ratio plots for e/μ for $1.6 < |Y| < 2.0$ bin.

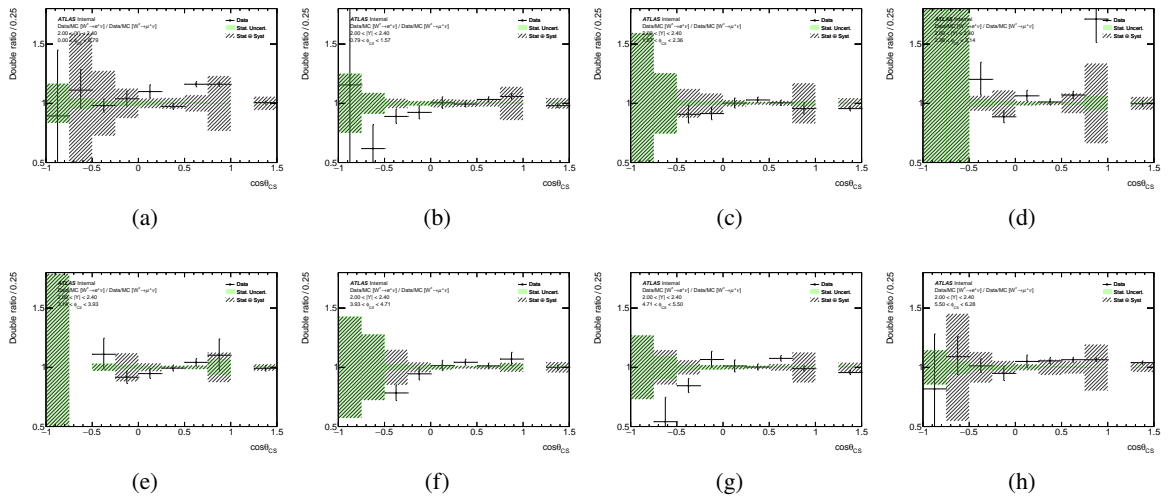


Figure 405: Double ratio plots for e/μ for $2.0 < |Y| < 2.4$ bin.

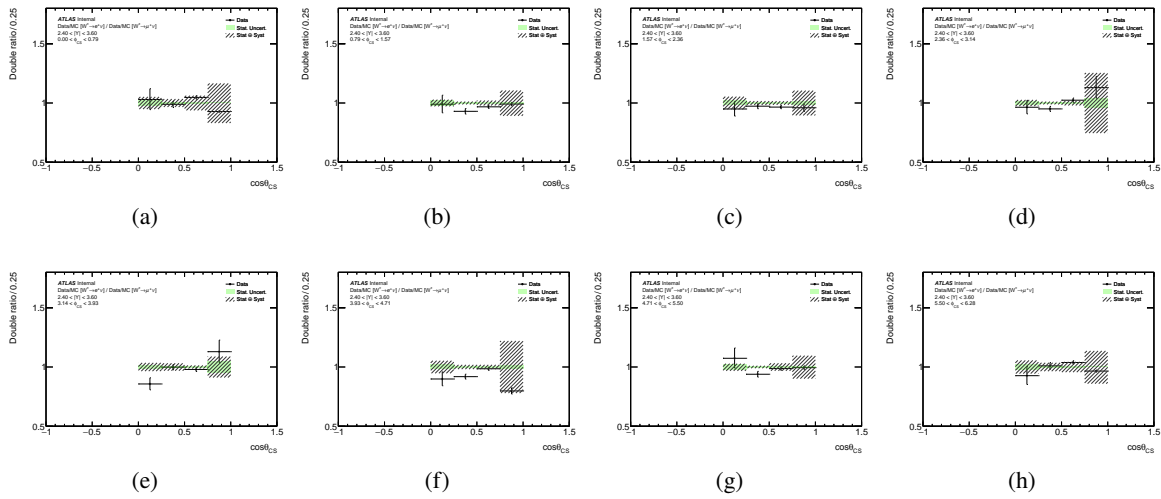


Figure 406: Double ratio plots for e/μ for $2.4 < |Y| < 3.6$ bin.

1306 G FTL

1307 G.1 Newton's Method

1308 Based on Newton's method which is an iterative root-finding method based on Taylor expansion of a
1309 function. In 1D:

$$y \approx f(x_n) + \delta f(x_n) / \delta x \times (x_{n+1} - x_n) \approx 0 \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{[\delta f(x_n) / \delta x]}. \quad (65)$$

1310 To minimize a function $g(x)$, in the case of this analysis $-\ln \mathcal{L}$, we set $f(x) = \delta g(x) / \delta x$ to find
1311 $\delta g(x_n) / \delta x = 0$ so x_{n+1} becomes

$$x_{n+1} = x_n - \frac{\delta g(x_n) / \delta x}{[\delta^2 g(x_n) / \delta^2 x]}. \quad (66)$$

1312 Extrapolating this to N dimensions

$$\begin{aligned} x &\rightarrow \vec{x} \\ \delta g(x_n) / \delta x &\rightarrow \nabla g(\vec{x}) \\ 1 / [\delta^2 g(x_n) / \delta^2 x] &\rightarrow \mathbf{H}^{-1} = [\nabla^2 g(\vec{x})]^{-1} \\ \Rightarrow \vec{x}_{n+1} &= \vec{x}_n - \mathbf{H}^{-1} \nabla g(\vec{x}) \end{aligned} \quad (67)$$

1313 Can introduce μ parameter as $\vec{x}_{n+1} = \vec{x}_n - \mu \mathbf{H}^{-1} \nabla g(\vec{x})$ and perform line search to find minimum $-\ln \mathcal{L}$
1314 for each iteration n. The bulk of the computation time is spent on computing \mathbf{H} and solving the system of
1315 equations for $\mathbf{H} \vec{x}_{\text{corr}} = \nabla g(\vec{x})$.

1316 G.2 Uncertainty Decomposition

1317 The total uncertainty on a given parameter is the square root of the diagonal of the covariance matrix \mathbf{H}^{-1} .
1318 For a group of parameters 'm' we want to find the component uncertainty of we need to block decompose
1319 covariance matrix M:

$$M = \begin{pmatrix} A & B \\ B^\top & C \end{pmatrix} \quad (68)$$

1320 If M is of size $n \times n$ then A is $(n - m) \times (n - m)$, C is $m \times m$, and B is $(n - m) \times m$. The Schur complement
1321 of M, $M/C = A - BC^{-1}B^\top$, is the conditional covariance matrix of M given C, which is the covariance
1322 matrix with parameters in C fixed to constant. The diagonals of $BC^{-1}B^\top$ gives us the square of the
1323 component uncertainty for the parameters A due to the parameters in C. This process is then repeated for
1324 each group of parameters.

1325 A multivariate Gaussian likelihood can be built directly from the Hessian \mathbf{H} , parameters \vec{x} , and their best fit
1326 values $\vec{\mu}$ as $-\ln \mathcal{L}(\vec{x}) = 0.5 (\vec{x} - \vec{\mu}) \mathbf{H} (\vec{x} - \vec{\mu})^\top$ which could be used to find the uncertainty decomposition
1327 for alternative parameterizations.

1328 H ID and Isolation Scale Factors

1329 Relaxing the kinematic selection requirements to not include one for E_T^{miss} or m_T was chosen to allow for
1330 more signal at high p_T^W but allowed for more MJ background. In order to suppress the MJ an additional
1331 TightLLH_d0z0 ID selection was added to electron channels and a calorimeter isolation requirement of
1332 $\text{topoetcone20}/p_T^\ell < 0.05$ for all channels. These selection requirements need associated scale factors (SFs)
1333 to account for the discrepancy between MC and data efficiencies so this appendix covers the process of
1334 deriving the SFs.

1335 To produce these SFs events are pre-selected to have leptons that pass all of the other selection requirements
1336 used in the analysis so that these SFs are applied in addition the SFs used already. During the selection
1337 process the events were weighted with the SFs for existing selection requirements like ID, isolation, and
1338 calibration. From there the tag and probe method was used with events broken up into two categories,

- 1339 • Leading lepton - tag — subleading lepton - probe
- 1340 • Subleading lepton - tag — leading lepton - probe

1341 These were then combined into one sample before calculating the efficiency, $\epsilon = N_{\text{probes}}^{\text{passed}}/N_{\text{probes}}$ for data
1342 and MC using Bayesian statistics. The SF is then calculated by taking the ratio of the data efficiency over
1343 the MC efficiency. This was done as a function of p_T^ℓ and η^ℓ as seen in Figures 407, 408, and 409 for
1344 electron TightLLH_d0z0 ID, electron calorimeter isolation, and muon isolation respectively.

1345 To estimate a systematic uncertainty instead of combining the previously stated tag and probe categories
1346 they were processed separately to produce SFs denoted by the red and blue points in Figures 410, 411, and
1347 412. With the separate SFs a weighted average is taken between them denoted by the magenta points in the
1348 figures. The systematic uncertainty is then the difference between the SF from combining the samples and
1349 the weighted average between them.

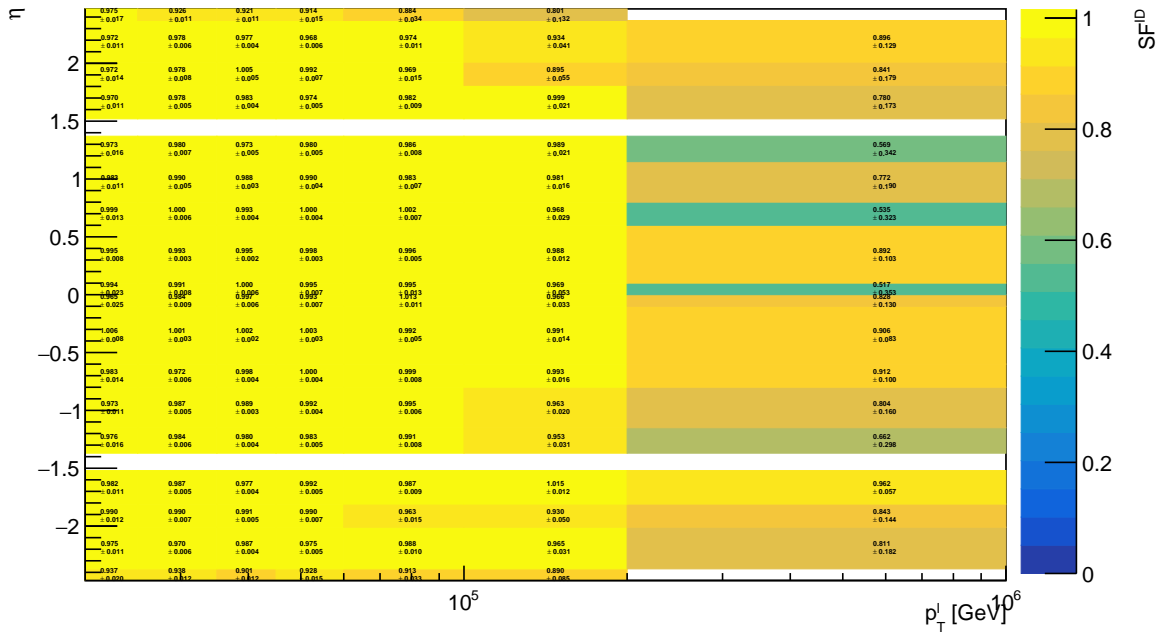


Figure 407: Scale factors derived for TightLLH_d0z0 ID selection requirements for electrons as a function of p_T^e and η^e .

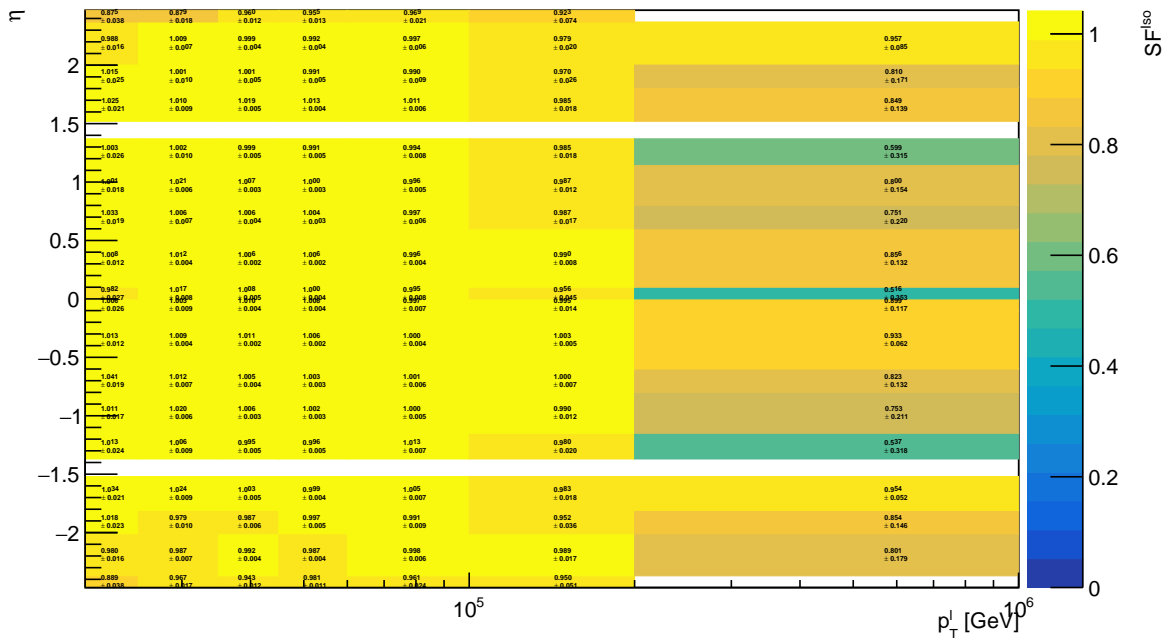


Figure 408: Scale factors derived for $\text{topoetcone20}/p_T^e < 0.05$ isolation selection requirements for electrons as a function of p_T^e and η^e .

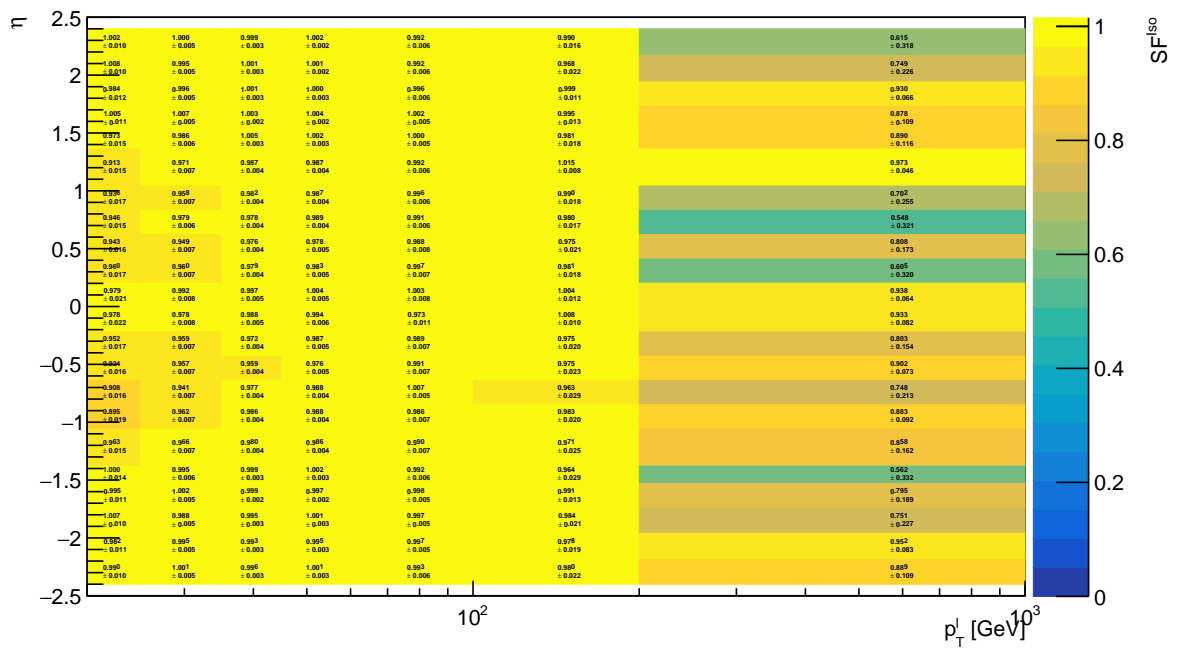


Figure 409: Scale factors derived for $\text{topoetcone20}/p_T^\mu < 0.05$ isolation selection requirements for electrons as a function of p_T^μ and η^μ .

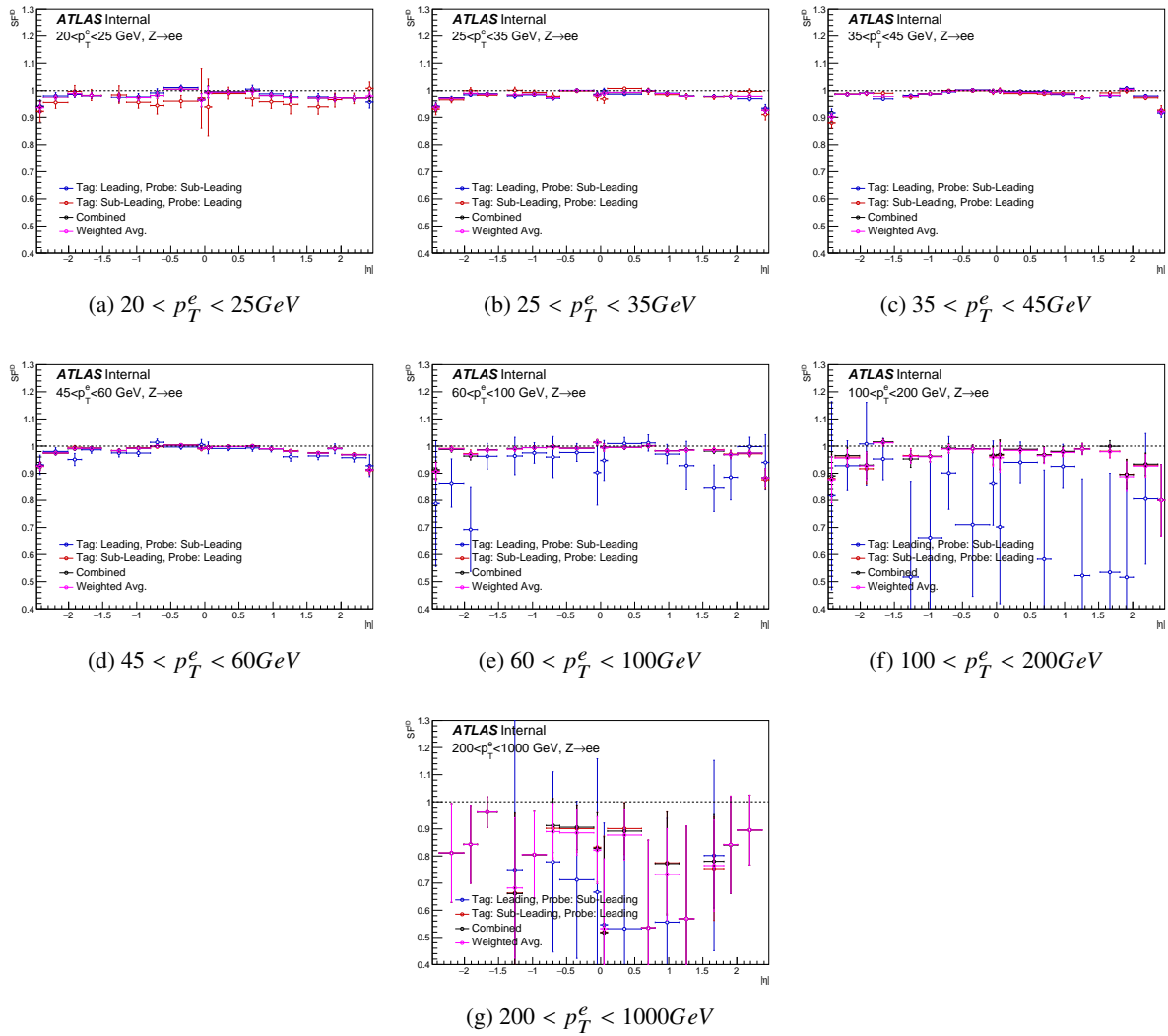


Figure 410: Scale factors derived for TightLLH_d0z0 ID selection requirements for electrons as a function for individual p_T^e slice. The final SFs used are denoted by combined in black and are compared to deriving using each category individually in red and blue and the weighted average of these categories in magenta.

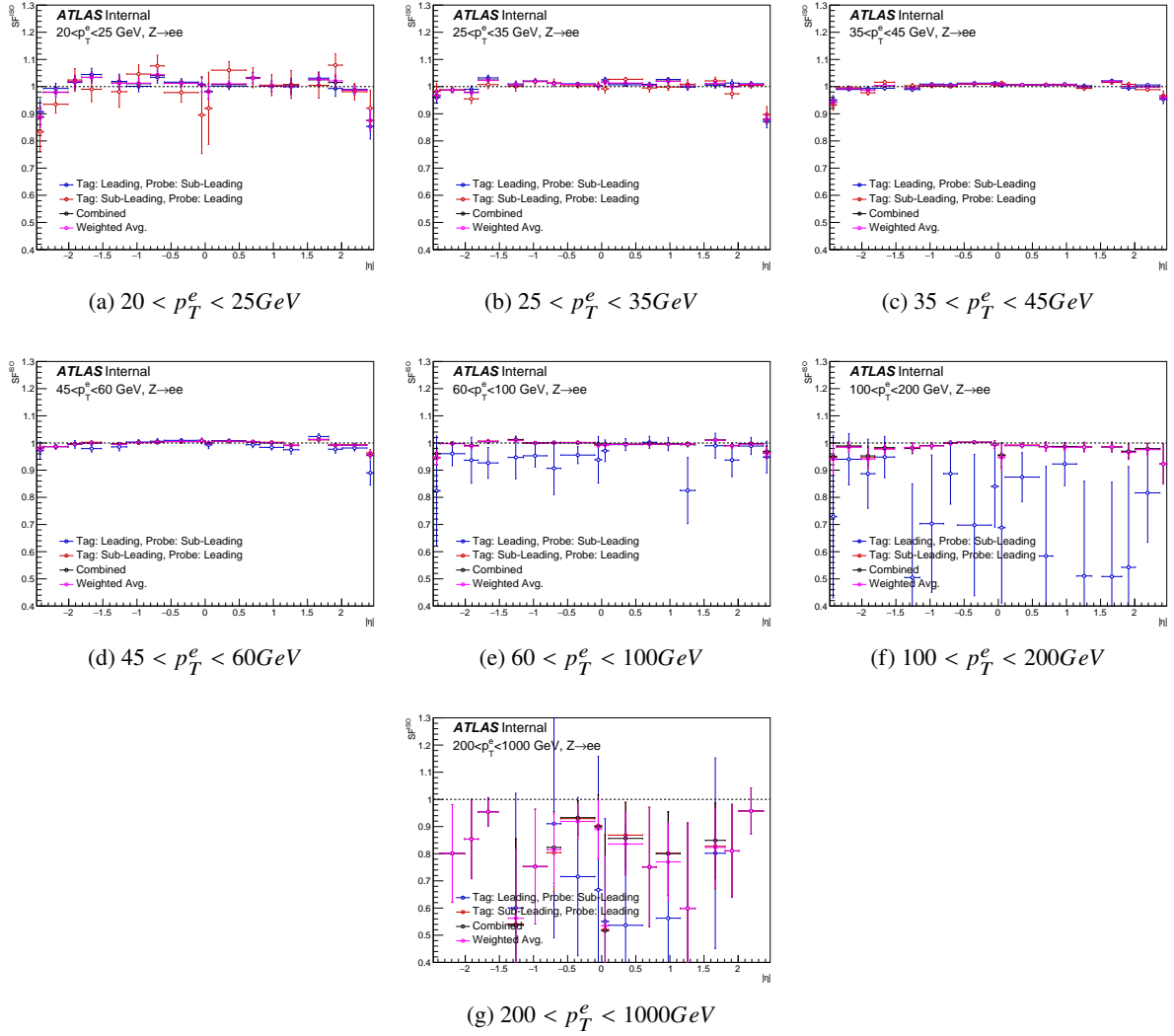


Figure 411: Scale factors derived for topoetcone20/ $p_T^e < 0.05$ isolation selection requirements for electrons as a function for individual p_T^e slice. The final SFs used are denoted by combined in black and are compared to deriving using each category individually in red and blue and the weighted average of these categories in magenta.

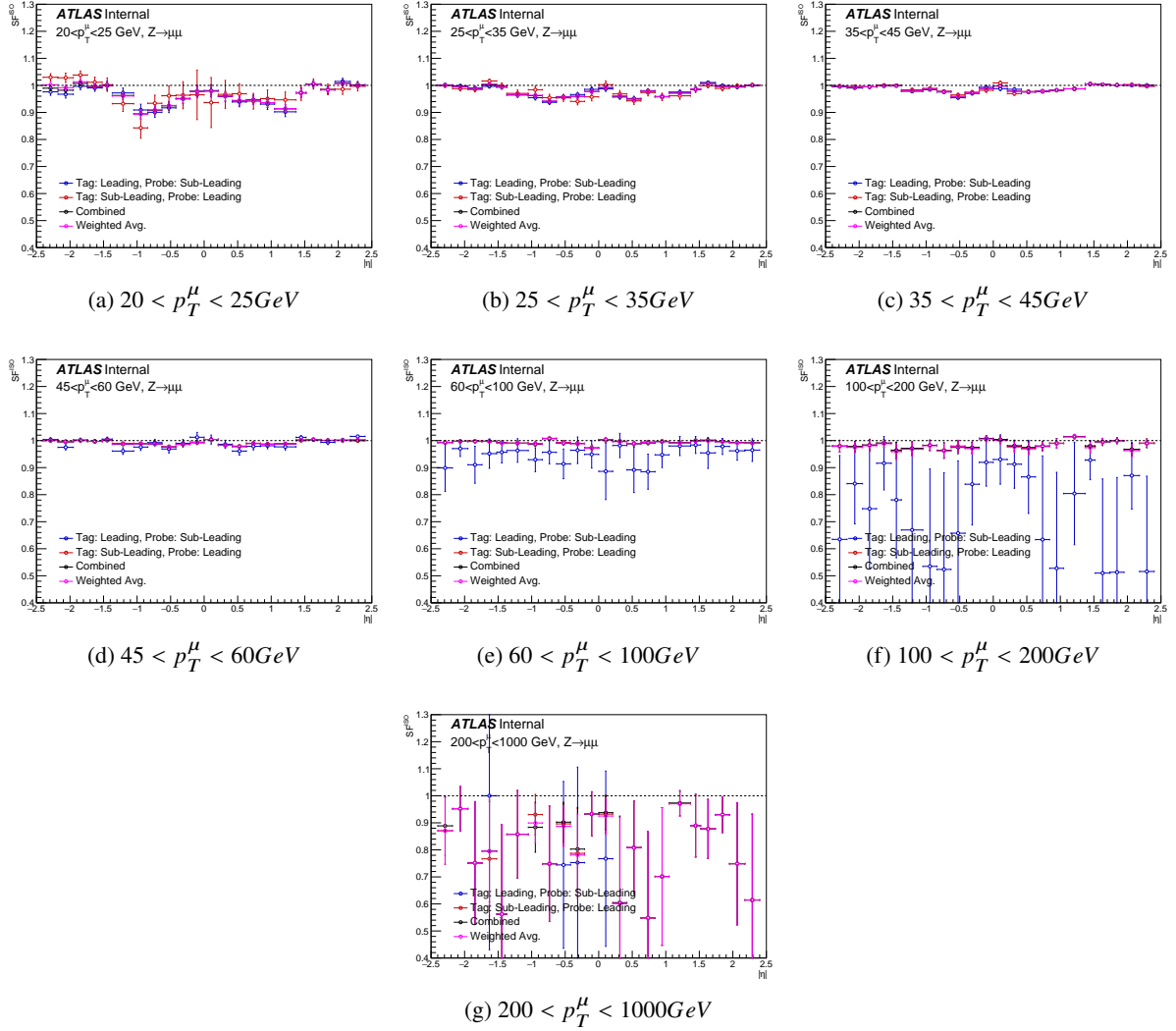


Figure 412: Scale factors derived for $\text{topoetcone}20/p_T^\mu < 0.05$ isolation selection requirements for electrons as a function for individual p_T^μ slice. The final SFs used are denoted by combined in black and are compared to deriving using each category individually in red and blue and the weighted average of these categories in magenta.