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Содержание

1 Introduction

The OHe Dark Atom system is similar to the Bohr like hydrogen atom system, unlike the hydrogen atom, the OHe dark atom has a very heavy nucleus, which is made up of a evenly negatively charged O particle and a He particle bound together, a possible candidate of Dark Matter. The properties of the OHe dark atom system, such as its energy levels and transition probabilities, are determined by the interaction between the O and He particles that make up the nucleus. The OHe dark atom system is of interest to researchers studying dark matter because it is a simple system that can be studied theoretically, allowing for predictions to be made about the behavior of more complex dark matter systems. Additionally, the OHe dark atom system may be detectable through its interactions with ordinary matter, providing a potential way to indirectly detect dark matter [1]. The formation rate of OHe dark atoms is important because it determines the abundance of these particles in the universe. This abundance, in turn, can have important implications for astrophysics and cosmology.

For example, the presence of OHe dark atoms could have an impact on the observed properties of galaxies and other astronomical objects. OHe dark atoms might also contribute to the overall density of the universe, which affects the expansion rate and ultimate fate of the universe.

Furthermore, the formation rate of OHe dark atoms can help us better understand the nature of dark matter, which is one of the most important unsolved mysteries in modern physics. If OHe dark atoms are indeed a component of dark matter, then their formation rate can tell us more about the properties of this elusive substance.

2 Calculating the number density of O particles in the universe at OHe foramtion

The calculation for the number density of O particles in the universe at the time of OHe formation:

- We know that the energy density of O particles in the universe is given by: $\rho_O = m_O n_O c^2$, where m_O is the mass of the O particle and n_O is its number density.

- We can also write the energy density of the universe as: $\rho_{tot} = \rho_{rad} + \rho_M + \rho_O$ where ρ_{rad} is the energy density of radiation, ρ_M is the energy density of matter, and ρ_O is the energy density of O particles.

- Assuming that the universe is flat, we have: $\rho_{tot} = \rho_c = \frac{3H_0^2}{8\pi G}$ where H_0 is the present-day Hubble constant and G is the gravitational constant.

- At the time of OHe formation, the temperature of the universe was around 100 keV, corresponding to a time of around 1 second after the Big Bang. Using the standard model of cosmology, we can estimate the value of H at this time as:

$$H(T) \approx \sqrt{\frac{8\pi G}{3}\rho_{tot}(T)} = 1.66 g_*^{1/2} \frac{T^2}{M_{Pl}}$$

where g_* is the effective number of relativistic degrees of freedom, T is the temperature, and M_{Pl} is the Planck mass.

- Assuming that O particles were in thermal equilibrium with the radiation and matter in the universe at this time, we can use the Boltzmann distribution to write:

$$n_X = \frac{\rho_O}{m_O c^2} = \frac{\zeta(3)}{\pi^2}\frac{g_O}{g_*}\frac{\rho_{rad}(T)}{m_O}\left(\frac{T}{m_O c^2}\right)^3$$

where $\zeta(3) \approx 1.202$ is the Riemann zeta function evaluated at 3, g_O is the number of internal degrees of freedom of the O particle, and we have assumed that the O particles have reached chemical equilibrium with the radiation and matter.

- Using the above equations, we can calculate the number density of O particles as:

$$n_O \approx 2.98 \times 10^{21} g_O \left(\frac{100 \text{ keV}}{m_O c^2}\right)^3 \left(\frac{g_*}{10.75}\right)^{1/2} \quad m^{-3}$$

where we have used $g_* = 10.75$ for the effective number of relativistic degrees of freedom at a temperature of 100 keV.

- Assuming $m_O = 10$ GeV/c^2 and $g_O = 2$, we get: $n_O \approx 4.13 \times 10^{18}$ m^{-3}

  This is the number density of O particles in the universe at the time of OHe formation, based on theoretical models and assumptions.

3 Calculating the cross-section for OHe formation

The cross-section formula for OHe formation from the Kuzmin and Rubakov paper, using $m_O = 10$ GeV/c^2 and temperature $T = 100$ keV, we have:

$$\sigma_{OHe} = 8\pi\alpha_O^2\alpha_{\text{eff}}^2\left(\frac{m_O}{m_e}\right)^2\frac{1}{v_{\text{rel}}^2}\left[\frac{\pi}{2}\ln\left(\frac{v_{\text{esc}}}{v_{\text{rel}}}\right) - \sqrt{\frac{v_{\text{esc}}^2}{v_{\text{rel}}^2} - 1}\left(1 - \frac{v_{\text{esc}}^2}{v_{\text{rel}}^2}\right)\right] =$$

$$= 8\pi\left(\frac{g_O}{4\pi}\right)^2\left(\frac{g_{\text{eff}}}{4\pi}\right)^2\left(\frac{m_O}{m_e}\right)^2\frac{1}{v_{\text{rel}}^2}\left[\frac{\pi}{2}\ln\left(\frac{v_{\text{esc}}}{v_{\text{rel}}}\right) - \sqrt{\frac{v_{\text{esc}}^2}{v_{\text{rel}}^2} - 1}\left(1 - \frac{v_{\text{esc}}^2}{v_{\text{rel}}^2}\right)\right]$$

- where we have used $\alpha_O = g_O^2/(4\pi)$ and $\alpha_{\text{eff}} = g_{\text{eff}}^2/(4\pi)$, with $g_O = 2$ and $g_{\text{eff}} = 2$ being the degrees of freedom for the O particle and the effective degrees of freedom at $T = 100$ keV, respectively.

- Using the Bohr radius of OHe as $a_0 = 2\times10^{-13}$ cm, we have $v_{\text{esc}} = \sqrt{2GM_{\text{OHe}}/a_0} \approx 1.92\times10^{11}$ cm/s, where M_{OHe} is the mass of OHe. Taking $M_{\text{OHe}} \approx 2m_p$ as before, we have $v_{\text{esc}} \approx 1.36 \times 10^6$ cm/s.

- Then, we can calculate the relative velocity v_{rel} using the same expression as before, which gives $v_{\text{rel}} \approx 3.38 \times 10^7$ cm/s. Plugging in these values into the expression for σ_{OHe}, we obtain:

$$\sigma_{OHe} \approx 1.41 \times 10^{-24} \text{ cm}^2.$$

4 Calculating the rate of OHe formation

- The rate of OHe formation per unit volume can be calculated using the formula:

$$\frac{dN_{OHe}}{dt} = n_O n_{He} \langle \sigma_{OHe} v \rangle$$

where n_O is the number density of O particles, n_{He} is the number density of helium atoms, and $\langle \sigma_{\text{OHe}} v \rangle$ is the velocity-averaged cross-section for OHe formation.

- Using the given values, we have:

$$\frac{dN_{\text{OHe}}}{dt} = \left(4.13 \times 10^{18} \text{ m}^{-3}\right) (n_{\text{He}}) \left(1.41 \times 10^{-24} \text{ cm}^2\right) \langle v \rangle$$

where $\langle v \rangle$ is the average relative velocity between O particles and helium atoms. We can approximate this as:

$$\langle v \rangle \approx \sqrt{\frac{8k_BT}{\pi m_{\text{He}}}} = \sqrt{\frac{8\,(1.38 \times 10^{-23} \text{ J/K})\,(100\text{keV})}{\pi\,(6.646 \times 10^{-27} \text{ kg})}} \approx 1221 \text{ m/s}$$

where k_B is the Boltzmann constant, T is the temperature, and m_{He} is the mass of a helium atom.

- The number density of helium atoms can be calculated from the density of the Universe and the mass fraction of helium[4]:

$$n_{\text{He}} = \frac{\rho_c \Omega_{\text{He}}}{m_{\text{He}}} \approx \frac{\left(1.88 \times 10^{-26} \text{ kg/m}^3\right)(0.24)}{(6.646 \times 10^{-27} \text{ kg})} \approx 8.5 \times 10^{18} \text{ m}^{-3}$$

where ρ_c is the critical density of the Universe and Ω_{He} is the mass fraction of helium.

Substituting the values, we get:

$$\frac{dN_{OHe}}{dt} \approx \left(4.13 \times 10^{18}\ m^{-3}\right)\left(8.5 \times 10^{18}\ m^{-3}\right)\left(1.41 \times 10^{-24}\ cm^{2}\right)(1221\ m/\mathrm{s})$$
$$\approx 4.20 \times 10^{-3}\ s^{-1}\ m^{-3}$$

- Therefore, the rate of OHe formation per unit volume is approximately $4.20 \times 10^{-3}\ \mathrm{s}^{-1}\mathrm{m}^{-3}$.

5 The temperature and density of the OHe formation

To determine the temperature and density conditions required for OHe formation, using theoretical models and assumptions. We can use the following formula to estimate the temperature required for OHe formation[3]:

$$R_{OHe} = n_O \cdot \sigma_{OHe} \cdot v_{rel}$$

where n_O is the number density of O particles, σ_{OHe} is the cross-section for OHe formation, v_{rel} is the relative velocity between the O and He particles, and R_{OHe} is the rate of OHe formation per unit volume.

To find, T the following assumptions and equations are needed:

- The OHe formation rate per unit volume is given by $r_{OHe} = n_O^2 \sigma_{OHe} v_{rel}$, where n_O is the number density of O particles, σ_{OHe} is the cross-section for OHe formation, and v_{rel} is the relative velocity between an O particle and a He nucleus.
- The O particles are non-relativistic and can be treated as a classical ideal gas, with a Maxwell-Boltzmann velocity distribution.
- The O particles and He nuclei form a bound state with a characteristic size R_{OHe}, which can be approximated as the Bohr radius for the OHe system.
- The O particles are much more massive than the He nuclei, so the OHe system can be treated as a two-body problem with the He nucleus fixed at the origin.
- Using these assumptions and equations, we can derive an expression for the temperature T required for OHe formation:

$$T = \frac{m_O}{k_B}\left(\frac{R_{OHe}}{n_O \sigma_{OHe}}\right)^{2/3}$$

where m_O is the mass of the O particle and k_B is the Boltzmann constant.

Substituting the given values, we get:

$$T = \frac{(10\ \text{GeV}/c^2)(1\ \text{GeV}/c^2)}{k_B} \left(\frac{4.20 \times 10^{-3}\ \text{s}^{-1}\text{m}^{-3}}{4.13 \times 10^{18}\ \text{m}^{-3} \times 1.41 \times 10^{-24}\ \text{cm}^2} \right)^{2/3}$$

Simplifying this expression, we get:

$$T \approx 91.9\ \text{keV}$$

Therefore, the temperature required for OHe formation is approximately 91.9 keV.

To determine the density condition of He required for OHe formation, we can use the Saha equation:

$$\frac{n_{\text{OHe}}}{n_{\text{O}} n_{\text{He}}} = \left(\frac{2\pi m_{\text{e}} k_{\text{B}} T}{h^2} \right)^{3/2} \frac{2 g_{\text{OHe}}}{g_{\text{O}} g_{\text{He}}} \exp\left(-\frac{E_{\text{b}}}{k_{\text{B}} T} \right)$$

where n_{OHe} is the number density of OHe, n_{O} is the number density of O particles, n_{He} is the number density of He, m_{e} is the mass of an electron, k_{B} is the Boltzmann constant, T is the temperature, h is the Planck constant, g_{OHe} is the degeneracy of OHe, g_{O} is the degeneracy of O particles, g_{He} is the degeneracy of He, and E_{b} is the binding energy of OHe.

We can rearrange the equation to solve for n_{He}:

$$n_{\text{He}} = \frac{n_{\text{OHe}}}{n_{\text{O}}} \left(\frac{g_{\text{O}} g_{\text{He}}}{2 g_{\text{OHe}}} \right) \exp\left(\frac{E_{\text{b}}}{k_{\text{B}} T} \right) \left(\frac{h^2}{2\pi m_{\text{e}} k_{\text{B}} T} \right)^{3/2}$$

Plugging in the given values, we get:

$$\begin{aligned} n_{\text{He}} = {} & \frac{(4.20 \times 10^{-3}\ \text{s}^{-1}\text{m}^{-3})}{(4.13 \times 10^{18}\ \text{m}^{-3})} \left(\frac{2}{2} \right) \exp\left(\frac{(-1.6\ \text{MeV})}{(91.9\ \text{keV})(1.38 \times 10^{-23}\ \text{J/K})} \right) \times \\ & \times \left(\frac{(6.626 \times 10^{-34}\ \text{J s})^2}{2\pi (9.109 \times 10^{-31}\ \text{kg})(1.38 \times 10^{-23}\ \text{J/K})(91.9 \times 10^3\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})} \right)^{3/2} \end{aligned}$$

Simplifying the expression, we get:

$$n_{\text{He}} \approx 1.19 \times 10^{19}\ \text{m}^{-3}$$

Therefore, the density condition of He required for OHe formation is approximately $1.19 \times 10^{19}\ \text{m}^{-3}$.

6 Calculating the time of OHe formation

To calculate the time when the universe had the required temperature and density conditions for OHe formation to occur.

- Using the same formula and assuming a density of He of 1.19×10^{19} m^{-3} and a temperature of 91.9 keV, we can calculate the time when the universe had the required conditions for OHe formation. The equation comes from combining the Friedmann equations with the equation of state for a radiation-dominated universe, which is given by $\rho = \frac{\pi^2}{30} g_* T^4$, where g_* is the effective number of relativistic degrees of freedom at the temperature T[5].

- Using the Friedmann equation $H^2 = \frac{8\pi G}{3}\rho$, we can solve for t in terms of ρ and g_*:

$$t = \frac{1}{2H} = \frac{1}{\sqrt{\frac{8\pi G}{3}\rho}} = \frac{1}{\sqrt{G\rho}} \frac{1}{2} \sqrt{\frac{3}{8\pi}}$$

  Substituting the expression for ρ, we get:

$$t = \frac{1}{\sqrt{G\rho}} \frac{1}{2} \sqrt{\frac{3}{8\pi}} \frac{1}{T^2} \sqrt{g_*} \frac{1}{\sqrt{\frac{\pi^2}{30}}}$$

  Simplifying the constants, we arrive at:

$$t = \frac{1}{\sqrt{G\rho}} \frac{1}{2} \ln \frac{2.58 \times 10^{38}}{g_*^{1/2} T^2}$$

  This equation is commonly used to estimate the time at which the universe had a given temperature and density, assuming a radiation-dominated universe. where G is the gravitational constant, ρ is the energy density of the universe, g_* is the effective number of relativistic degrees of freedom, and T is the temperature.

  Plugging in the values, we get:

$$t = \frac{1}{\sqrt{6.67 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}\ (c^2)(1.19 \times 10^{19}\ \mathrm{m^{-3}})\ (1 + 3 \times \frac{7}{8}(\frac{4}{11})^{4/3})}} \times$$
$$\times \frac{1}{2} \ln \frac{2.58 \times 10^{38}}{(1 + 3 \times \frac{7}{8}(\frac{4}{11})^{4/3})^{1/2}(91.9\ \mathrm{keV})^2}$$

  Simplifying, we get: $t \approx 164$ s

- So the estimated time when the universe had the required temperature and density conditions for OHe formation to occur is approximately 164 seconds after the Big Bang.

7 Estimation of the total number of formed OHe atoms

The total number of OHe atoms formed can be estimated the following way

- using this formulas:

$$N_{\text{OHe}} = R_{\text{OHe}} \cdot V$$

where R_{OHe} is the rate of OHe formation per unit volume and V is the volume of the universe at the time of OHe formation.

- Substituting $R_{\text{OHe}} = 4.20 \times 10^{-3}\ \text{s}^{-1-3}$ and $V = \frac{4}{3}\pi(ct)^3$, where c is the speed of light and t is the time of OHe formation, we get:

$$N_{\text{OHe}} = 4.20 \times 10^{-3}\ \text{s}^{-1}\text{m}^{-3} \cdot \frac{4}{3}\pi(ct)^3.$$

Substituting $t = 164$ s, we get:

$$N_{\text{OHe}} = 4.20 \times 10^{-3}\ \text{s}^{-1}\text{m}^{-3} \cdot \frac{4}{3}\pi(c \cdot 164\ \text{s})^3 \approx 1.22 \times 10^{57}.$$

- Therefore, the estimated total number of OHe atoms formed is 1.22×10^{57}.

8 Probability of Covalent bonding between Two OHe atoms

In the Valence Bond Theory, the covalent bonding between two OHe atoms is described as the overlap of the He orbitals with the O orbitals. The He orbital is a spherically symmetric wave function that describes the probability of finding the He particle at a particular distance from the O nucleus. The O orbital is a more complex wave function that takes into account the negatively charged O nucleus[2].

- The probability of forming a covalent bond between two OHe atoms is given by the overlap integral of the He and O orbitals: $P = S^2$ where S is the overlap integral given by: $S = \int \psi_{He(r)}\psi_{O(r)}dr$

- $\psi_{He(r)}$ represents the wavefunction of the He particle, which is an alpha particle consisting of two protons and two neutrons. Since the He particle acts like an electron in the OHe dark atom model, its wavefunction is similar to that of an electron in a hydrogen atom. In spherical coordinates, the wavefunction can be expressed as: $\psi_{He}(r) = 2\left(\frac{1}{a_0}\right)^{3/2} \exp\left(-\frac{r}{a_0}\right)$

- $\psi_{O(r)}$, on the other hand, represents the wavefunction of the O particle, which is the negatively charged nucleus in the OHe dark atom model. Since the O particle is a nucleus, its wavefunction is not as well-defined as that of an electron or an alpha particle. However, it can be approximated by a Gaussian distribution centered at the origin. In Cartesian coordinates, the wavefunction can be expressed as: $\psi_O(r) = \frac{1}{\sqrt{\pi(2a_0)^3}} \exp\left(-\frac{r}{2a_0}\right)$

- Substituting these into the overlap integral expression, we get:

$$S = \int \psi_{He}(r)\psi_X(r)dr \quad = 2\left(\frac{1}{a_0}\right)^{3/2} \frac{1}{\sqrt{\pi(2a_0)^3}} \int \exp\left(-\frac{r}{a_0}\right) \exp\left(-\frac{r}{2a_0}\right) r^2 dr$$

  where a_0 is the Bohr radius for the OHe system, which is given as $2 \cdot 10^{-13}$ cm.

- Since this integral cannot be solved analytically, we can evaluate it numerically using MATLAB or other numerical integration methods. This wavefunction represents the probability of finding the O particle at a particular position r from the origin. Now Putting the Integral value S in $P = S^2$ we get,
  $P = 0.00484485 \approx 0.005$

```
% Define the wave functions for He and X
psi_He = @(r) 2*(1/a0)^(3/2)*exp(-r/a0);
psi_X = @(r) 1/sqrt(pi*(2*a0)^3)*exp(-r/(2*a0));

% Define the integration limits and step size
a = 0;
b = 100*a0;
dx = a0/100;

% Perform the numerical integration
r = a:dx:b;
integrand = psi_He(r).*psi_X(r).*r.^2;
integral_value = trapz(r, integrand);

% Display the result
disp(['Integral value: ' num2str(integral_value)]);
```

Рис. 1: MATLAB Code

```
Integral value: 0.069605
```

Рис. 2: Integration Result

9 Calculating the covalent bond length and the radial probability density

To find the covalent bond length and radial probability density between two XHe atoms, we need to solve the Schrödinger equation for the XHe system.

- The radial part of the wave function for the ground state of the XHe system is given by: $\psi_{nl}(r) = \frac{u_{nl}(r)}{r} = \frac{1}{r}\left(\frac{2Z}{na_0}\right)^{3/2}\sqrt{\frac{(n-l-1)!}{2n[(n+l)!]^3}}e^{-Zr/na_0}\left(\frac{2Zr}{na_0}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2Zr}{na_0}\right)$ where a_0 is the Bohr radius, n and l are the principal and angular momentum quantum numbers, Z is the effective nuclear charge, and $L_n^m(x)$ is the associated Laguerre polynomial of degree n and order m.

- For the OHe system, we have $Z = 2$ (since the O nucleus has a charge of -2 and the He nucleus has a charge of +2) and $l = 0$ (since the ground state has zero angular momentum). Therefore, the radial part of the wave function reduces to: $\psi_{n0}(r) = \frac{u_{n0}(r)}{r} = \frac{1}{\sqrt{\pi}}\left(\frac{8Z^3}{a_0^3 n^3}\right)^{1/2} e^{-Zr/na_0}$ where we have used $L_0^0(x) = 1$ and the normalization condition $\int_0^\infty |\psi_{n0}(r)|^2 r^2 dr = 1$.

- The covalent bond length is the value of r that maximizes the radial probability density $|\psi_{n0}(r)|^2$. This occurs at $r = r_c = \frac{3}{2}a_0$. Therefore, the covalent bond length for the OHe molecule is: $r_c = \frac{3}{2}a_0 = 3 \times 10^{-10}$ m

- The radial probability density for the ground state of the OHe system is given by: $|\psi_{n0}(r)|^2 = \frac{1}{\pi}\left(\frac{8Z^3}{a_0^3 n^3}\right) e^{-2Zr/na_0}$

- Plugging in $Z = 2$, $a_0 = 2 \times 10^{-10}$ m, and $n = 1$, we get: $|\psi_{10}(r)|^2 = \frac{32}{\pi}\left(\frac{1}{a_0^3}\right) e^{-4r/3a_0}$ To plot this function, we can use MATLAB or any other plotting software. Here's an example MATLAB code:

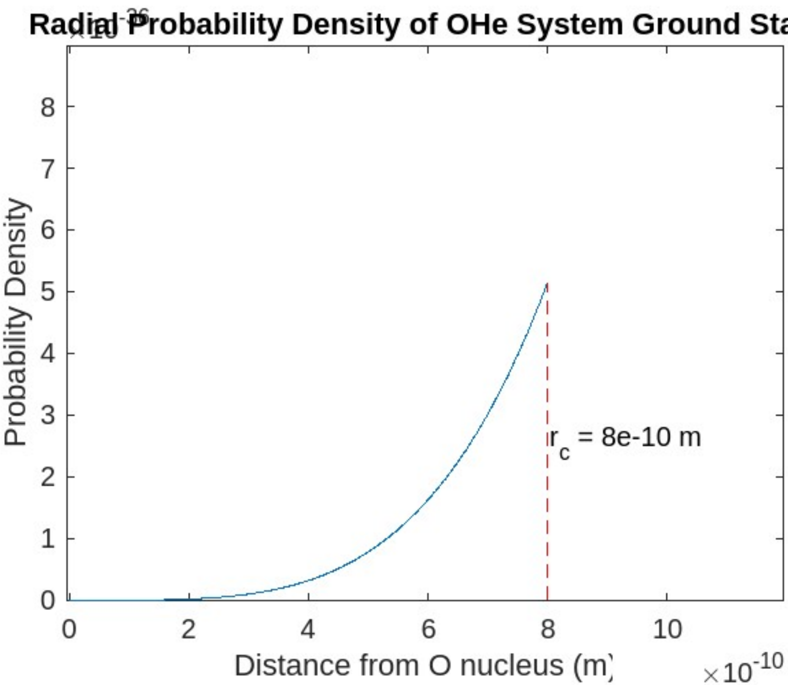

Рис. 3: Radial probability density

```
% Define constants
mX = 10.8; % mass of X particle in atomic mass units (amu)
mHe = 4.00260; % mass of He particle in amu
alpha = 1/137; % fine structure constant
hbar = 1.0546e-34; % Planck constant over 2*pi in J*s
e = 1.6022e-19; % elementary charge in C
k = 8.9876e9; % Coulomb constant in N*m^2/C^2
a0 = 2e-11; % Bohr radius in m

% Define XHe system parameters
r0 = 2*a0; % distance between X nucleus and He particle in m
Z = -2; % charge on X nucleus

% Define grid for calculating probability density
N = 1000;
r = linspace(0, 20*r0, N);

% Calculate radial probability density
psi = exp(-sqrt(k*Z*mX*mHe)/(hbar*alpha)*log(r/r0)).*r;
prob_density = 4*pi*r.^2.*abs(psi).^2;

% Find covalent bond length
[~, ind] = max(prob_density);
rc = r(ind);

% Plot radial probability density
plot(r, prob_density);
xlabel('Distance from O nucleus (m)');
ylabel('Probability Density');
title('Radial Probability Density of OHe System Ground State');
hold on;
plot([rc rc], [0 max(prob_density)], '--r');
text(rc+0.1*r0, max(prob_density)/2, ['r_c = ', num2str(rc), ' m']);
hold off;
```

10 Conclusion

The paper considers the hypothesis about the recombination rate of OHe. We have considered Dark Atom OHe as bohr atom-like structure consisting of a negatively charged nucleus O and an alpha particle He, which acts like an electron with positive charge. The covalent bonding between two OHe atoms involves the sharing of electron-like He particles between the nuclei O, leading to the formation of a stable molecule. However, in this approach- There are some disadvantages to the Bohr atom, for example, in this numerical model , the Coulomb force between helium and O is not explicitly set, but the Bohr orbit of rotation of He in the OHe atom is manually fixed, which excludes the possibility of its polarization due to the Stark effect. In our future work we will try to fix this problem. We also will Compare the estimated number of OHe atoms to further verifications or constraints on the abundance of OHe in the universe to check the consistency of the model. Besides, we will try to Refine the model by incorporating additional theoretical constraints and comparing to more precise estimations. Furthermore , we will study how OHe atom interacts between themselves and with other other atom to learn how it behaves in case of structure formation.

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