Neutrino Masses and Nature from the Point of View of Economy and Simplicity

S. Bilenky
JINR(Dubna)

October 5, 2020

We will discuss

I. Neutrino oscillations

II. Origin of neutrino masses and mixing

One of the most important recent discoveries in particle physics was discovery of neutrino oscillations in the atmospheric Super-Kamiokande, solar SNO and in the reactor KamLAND experiment (1998-2002)

Small neutrino masses, driving neutrino oscillations, is the only evidence (in particle physics) of a new beyond the Standard Model physics

In 2015 the Nobel Prize was awarded to T. Kajita and A. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have masses"

Idea of neutrino oscillations was put forward by B.Pontecorvo in Dubna in 1957-58 and further was developed by B.Pontecorvo and V.Gribov (1969) and B.Pontecorvo and S.Bilenky (1975-1989)

Neutrino Oscillations

We know from experiments on the investigation of the "invisible" decay $Z^0 \to \nu_l + \bar{\nu}_l$ (LEP, SLC) that three flavor left-handed neutrinos ν_e, ν_μ, ν_τ (and their antineutrinos) exist in nature

The fields of flavor neutrinos enter into leptonic CC and NC

$$j_\alpha^{CC} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L$$

$$j_\alpha^{NC} = \sum_{l=e,\mu,\tau} \bar{\nu}_L \gamma_\alpha \nu_L$$

$\gamma_5 \nu_{lL} = -\nu_{lL}$ is left-handed field

From the observation of neutrino oscillations it follows that flavor neutrino fields are mixed

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\ \nu_{iL}$$

ν_i is the field of neutrino with mass m_i

U is 3×3 Pontecorvo-MNS mixing matrix

It follows from mixing of neutrino fields that if all neutrino masses are small states of flavor neutrinos with definite momentum, produced in $\pi^+ \to \mu^+ + \nu_\mu$, $(A,Z) \to (A,Z+1) + e^- + \bar{\nu}_e$,...

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^* \; |\nu_{iL}\rangle \quad (l = e, \mu, \tau)$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i, momentum $\vec{p}$ and energy $E_i \simeq p + \frac{m_i^2}{2E}$ $(p^2 \gg m_i^2)$.

If at $t = 0$ flavor neutrino ν_l was produced, the state of neutrino at the time t will be a coherent superposition

$$|\nu_l\rangle_t = \sum_i U_{li}^* \; e^{-iE_i t} |\nu_{iL}\rangle = \sum_{l'} \mathcal{A}(\nu_l \to \nu_{l'}) |\nu_{l'}\rangle$$

$$\mathcal{A}(\nu_l \to \nu_{l'}) = e^{-ipL} \sum_i U_{l'i} e^{-i\frac{m_i^2 L}{2E}} U_{li}^*$$

is the amplitude of transition $\nu_l \to \nu_{l'}$, $L \simeq t$ is a source-detector distance, $\mathcal{A}(\nu_l \to \nu_{l'})$ is a coherent sum of products of amplitudes of transition $(\nu_l \to \nu_i)$ of propagation in the state ν_i of transition $(\nu_i \to \nu_{l'})$

Neutrino oscillations in vacuum are the result of interference between different i-amplitudes

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\mathcal{A}(\nu_l \to \nu_{l'})|^2 = |\delta_{l'l} - 2i \sum_{i \neq r} e^{-i\Delta_{ri}} U_{l'i} U^*_{li} \sin \Delta_{ri}|^2$$

$$\Delta_{ri} = \frac{\Delta m^2_{ri} L}{4E}, \quad \Delta m^2_{ik} = m^2_k - m^2_i$$

r is an arbitrary, fixed index

In the simplest case of two flavors, say, μ, τ $(r = 1, i = 2)$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\delta_{l'l} - 2ie^{-i\Delta_{12}} U_{l'2} U^*_{l2} \sin \Delta_{12}|^2$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\mathrm{P}(\nu_\mu \to \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\Delta m^2_{12} L}{4E}, \mathrm{P}(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2_{12} L}{4E}$$

Periodic functions of L/E (oscillations)

LBL Accelerator, Reactor, Atmospheric and Solar data are described by the three-neutrino mixing. Six parameters: Δm^2_S, Δm^2_A, mixing angles θ_{12}, θ_{23}, θ_{13}, CP phase δ. From solar data $\Delta m^2_S = \Delta m^2_{12} > 0$. For m_3 two possibilities (two neutrino mass spectra)

1. Normal ordering. $m_3 > m_2 > m_1, \quad \Delta m^2_A = \Delta m^2_{23}$
2. Inverted ordering. $m_2 > m_1 > m_3, \quad \Delta m^2_A = |\Delta m^2_{13}|$

Values of neutrino oscillation parameters, obtained from the global fit of the data

Parameter	Normal Ordering	Inverted Ordering
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.310^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.582^{+0.015}_{-0.019}$	$0.582^{+0.015}_{-0.018}$
$\sin^2\theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	$0.02263^{+0.00065}_{-0.00066}$
δ (in $^\circ$)	(217^{+40}_{-28})	(280^{+25}_{-28})
Δm^2_S	$(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$	$(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$
Δm^2_A	$(2.525^{+0.033}_{-0.031}) \cdot 10^{-3}\ \mathrm{eV}^2$	$(2.512^{+0.034}_{-0.031}) \cdot 10^{-3}\ \mathrm{eV}^2$

Problems for future neutrino oscillation experiments

- What is neutrino mass ordering?
- What is the value of CP phase δ?
- Measurement of oscillation parameters with higher accuracy

There are anomalies (sterile neutrino with mass ~ 1 eV) observed in SBL LSND, MiniBooNE, Galium, Reactor and Neutrino-4 experiments. Not confirmed in different recent experiments.

Crucial experiments are still ahead

Absolute values of neutrino masses

From recent tritium experiment KATRIN

$$m_\beta = (\sum_i |U_{ei}|^2 m_i^2)^{1/2} < 1.1 eV$$

From cosmology recent bound (model dependent)

$$\sum_i m_i < 0.12 \text{ eV}$$

What is the origin of small neutrino masses?

Particles with spin 1/2 can be Dirac or Majorana. Dirac field $\psi(x)$ is a complex (non hermitian) four-component field which satisfies the Dirac equation. If a Lagrangian is invariant under a global invariance $\psi(x) \to e^{i\Lambda}\psi(x)$ (Λ is a constant) a charge is conserved and $\psi(x)$ is a field of particles and antiparticles, which have opposite charges, same masses (due to the CPT invariance) and helicities ± 1. Majorana field $\chi(x)$ is a two-component field which satisfies the Dirac equation and the Majorana condition

$$\chi(x) = \chi^c(x) = C\bar{\chi}^T(x), \quad C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \; C^T = -C.$$

There is no global invariance of a Lagrangian in the Majorana case. Majorana field is two-component field of truly neutral particles with helicities ± 1.

Quarks (Q=2/3,-1/3) and leptons (Q=-1) are Dirac particles Neutrinos can be Dirac particles or Majorana particles. This is the most fundamental difference between quark and lepton and neutrinos

Neutrino masses and mixing are generated by a neutrino mass term (sum of Lorenz-invariant products of L and R components of neutrino fields)

The first neutrino mass term was proposed by V. Gribov and B. Pontecorvo (1969)

At that time it was established that the lepton CC has the form
$$j^{CC}_{\alpha} = 2(\bar{\nu}_{eL}\gamma_{\alpha}e_L + \bar{\nu}_{\mu L}\gamma_{\alpha}\mu_L)$$

G-P: is it possible to introduce neutrino masses and mixing in the case if neutrino fields are left-handed ν_{eL}, $\nu_{\mu L}$?

It was a common opinion at that time that left-handed neutrinos are massless

G-P understood that if we assume that the total lepton number L is not conserved it is possible to built neutrino mass term with L-handed neutrino fields The main point was that the C-conjugated field $\nu^c_{lL} = C\bar{\nu}^T_{lL}$ is right-handed. In the general three-flavor case (S.B. S. Petcov, 1986) possible L-violating mass term has the form

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l}\bar{\nu}_{l'L}M^{\mathrm{M}}_{l'l}\nu^c_{lL} + \mathrm{h.c.}, \quad M^{\mathrm{M}} = (M^{\mathrm{M}})^T$$

After the standard diagonalization of the matrix M^{M} we have

$$\mathcal{L}^{\mathrm{M}}(x) = -\frac{1}{2}\sum_{i=1}^{3} m_i\ \bar{\nu}_i(x)\nu_i(x)$$

$\nu_i(x)$, the field of neutrino with the mass m_i, satisfies the Majorana condition

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x).$$

Flavor field $\nu_{lL}(x)$ is a "mixed" field

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x).$$

The mass term $\mathcal{L}^{\mathrm{M}}$ is called the Majorana mass term

- The Majorana mass term is the most economical mass term
- Neutrino masses and mixing angles are parameters. No any clues why m_i are much smaller then lepton and quarks masses

Modern approach to the origin of neutrino masses

Origin of neutrino masses and neutrino mixing is an open problem.

There are many, many models.

We can claim (practically in a model independent way) that the Standard Model neutrinos are massless particles

Masses of leptons (and quarks) are of Standard Model origin.

$SU_L(2) \times U_Y(1)$ invariant Yukawa interaction

$$\mathcal{L}_l^Y = -\sqrt{2} \sum_{l',l} \bar{\psi}_{l'L} Y_{l'l} l_R \phi + \mathrm{h.c.}$$

$$\psi_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

are lepton and Higgs doublets, l_R is a singlet, Y is a dimensionless complex matrix. After spontaneous symmetry breaking and diagonalization of Y we come to the Dirac mass term

$$\mathcal{L}_l^Y(x) = -\sum_{l=e,\mu,\tau} m_l \; \bar{l}(x) \; l(x)$$

$l(x) = l_L(x) + l_R(x)$ is the Dirac field of l^- (L=1) and l^+ (L=-1)

Lepton mass

$$m_l = y_l \; v \quad l = e, \mu, \tau$$

y_l is the Yukawa coupling (parameter)
$v = (\sqrt{2} G_F)^{-1/2} \simeq 246$ GeV (electroweak scale)
All SM masses (masses of quarks, leptons, $W^{\pm}$, Z^0, Higgs boson) are proportional to v (the only SM parameter which has a dimension M)

If neutrino masses are of the SM origin

- right-handed singlets ν_{lR} enter into Lagrangian
- L is conserved and neutrinos with definite masses ν_i are Dirac particles
- $m_i = y_i^{\nu} \; v$

Yukawa constants are determined by masses For the quarks and lepton of the third family we have

$$y_t \simeq 7 \cdot 10^{-1}, \quad y_b \simeq 2 \cdot 10^{-2}, \quad y_\tau \simeq 7 \cdot 10^{-3}.$$

For the largest neutrino mass m_3 we have the following ranges
$(5 \cdot 10^{-2} \lesssim m_3 \lesssim 3 \cdot 10^{-1})\; eV, \quad 2 \cdot 10^{-13} \lesssim y_3^{\nu} \lesssim \cdot 10^{-12}$

Yukawa couplings of quarks and lepton of the same family differ by about two orders of magnitude. Neutrino Yukawa coupling is about ten orders of magnitude smaller then Yukawa couplings of quarks and lepton

It is very unlikely that neutrino masses are of the same Standard Model origin as masses of leptons and quarks.

It is a general opinion that the Standard model neutrinos are left-handed massless particles.

The Standard Model with left-handed, massless ν_e, ν_μ, ν_τ is a minimal theory, originally proposed by Weinberg and Salam

In order to generate small neutrino masses, observed in neutrino oscillation experiments, we need a new beyond the Standard Model mechanism.

A general method which allow to describe effects of a beyond the Standard Model physics is a method of the effective Lagrangian.

Effective Lagrangian is a dimension five or more non renormalizable Lagrangian built from the Standard Model fields and invariant under $SU(2)_L \times U(1)_Y$ transformations.

Effective Lagrangians are generated by beyond the Standard model interactions of SM particles with heavy particles with masses much larger than v. In the electroweak region such interactions induce processes with virtual heavy particles, which are described by effective Lagrangians (fields of heavy particles are "integrated out"). Typical example is the four-fermion, dimension six, Fermi effective Lagrangian of the weak interaction.

In order to built an effective Lagrangian which generate a neutrino mass term , consider dimension $M^{5/2}$, $SU_L(2) \times U_Y(1)$ invariant

$$(\tilde{\phi}^{\dagger}\ \psi_{lL})$$

$\tilde{\phi} = i\tau_2\phi^*$ is a conjugated Higgs doublet

It is obvious that (after spontaneous symmetry breaking) this invariant is proportional to the flavor neutrino field

$$(\tilde{\phi}^{\dagger}\ \psi_{lL}) \to \frac{v}{\sqrt{2}}\ \nu_{lL}$$

It is obvious that (like in the Gribov-Pontecorvo case) we can built an effective Lagrangian which (after spontaneous symmetry breaking) generates a neutrino mass term only if the total lepton number is not conserved. Unique expression for the effective Lagrangian (Weinberg)

$$\mathcal{L}_I^{\mathrm{W}} = -\frac{1}{\Lambda} \sum_{l',l} \overline{(\tilde{\phi}^\dagger \psi_{l'L})} X_{l'l} (\tilde{\phi}^\dagger \; \psi_{lL})^c + \mathrm{h.c.}$$

X is a dimensionless, symmetrical matrix, parameter Λ has dimension M (the operator in $\mathcal{L}_I^{\mathrm{eff}}$ has a dimension M^5). Λ characterizes a scale of a beyond the SM physics.

Remark: global invariance and conservation of L is not a fundamental symmetry of QFT. Local gauge symmetry ensure conservation of L by SM Lagrangian. Natural to expect that a beyond the SM theory does not conserve L

After SSB we come to Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'L} \, \frac{v^2}{\Lambda} \, X_{l'l} \; \nu_{lL}^c + \mathrm{h.c.} = -\frac{1}{2} \sum_{i=1}^{3} m_i \; \bar{\nu}_i \nu_i$$

$\nu_i = \nu_i^c$ is the field of the Majorana neutrino with "seesaw mass"

$$m_i = \frac{v^2}{\Lambda}\, x_i = \frac{v}{\Lambda} \cdot (x_i v)$$

$(x_i v)$ is a "typical" SM mass. The generation of neutrino masses via the effective Lagrangian mechanism leads to an additional suppression factor

$$\frac{v}{\Lambda} = \frac{\text{EW scale}}{\text{scale of a new physics}}$$

x_i and Λ are unknown. Values of neutrino masses can not be predicted

However, if $\Lambda \gg v$ Majorana neutrino masses m_i are naturally much smaller than masses of quarks and leptons

If $x_3 \simeq 1$ (like y_{top}) we have

$$\Lambda \simeq (10^{14} - 10^{15})\ \mathrm{GeV}.$$

On the origin of the Weinberg effective Lagrangian

The simplest and most economical (seesaw type-I) scenarios

Assume that lepton-Higgs pairs interact with heavy Majorana leptons N_i ($i = 1, 2, ...n$), $SU_L(2)$ singlets, via $SU_L(2) \times U_Y(1)$ interaction

$$\mathcal{L}_I = -\sqrt{2}\sum_{l,i}(\bar{\psi}_{lL}\tilde{\phi})y_{li}\ N_{iR} + \mathrm{h.c.} \quad (1)$$

$$N_i = N_i^c = C(\bar{N}_i)^T$$

For low-energy processes with virtual heavy leptons at $Q^2 \ll M_i^2$ in the tree approximation we obtain $\mathcal{L}_I^{\mathrm{W}}$ in which

$$\frac{1}{\Lambda}X_{l'l} = \sum_{i=1}^{n} y_{l'i}\ \frac{1}{M_i}\ y_{li}.$$

Λ is determined by masses of heavy Majorana leptons

The Weinberg effective Lagrangian can be also generated by interaction of heavy triplet scalar bosons with a Higgs pair and lepton pair (seesaw type-II) and by interaction of lepton-Higgs pairs with heavy Majorana triplet leptons (seesaw type-III)

There exist numerous radiative neutrino mass models which lead to Weinberg effective Lagrangian. In these models values of neutrino masses m_i are suppressed by loop mechanisms which require existence of different beyond the Standard Model particles with masses which could be much smaller than 10^{15} GeV

Our general assumptions

1. There exist a beyond SM interaction(s) of SM lepton and Higgs doublets and new particles whose masses are much larger than the electroweak v
2. SM neutrinos are massless left-handed particles

Beyond the SM interactions generate an effective Lagrangian. Independently on a model, tree-level or radiative, the only possible effective Lagrangian is Weinberg Lagrangian and the only possible mass term is the Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i + \mathrm{h.c.}, \quad \nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}.$$

the smallness of m_i can be explained, values of m_i (and mixing matrix U) can not be predicted in a model independent way

However, the following features are common for all such type of models (in this sense model independent)

1. The number of neutrinos with definite masses ν_i is equal to the number of lepton flavors (three)
2. Neutrinos with definite masses ν_i are Majorana particles

Indications in favor of fourth (sterile) neutrino m_4 with mass in the range $(10^{-1} \lesssim m_4 \lesssim 10)$ eV were obtained in different short baseline neutrino experiments: $\bar\nu_\mu \to \bar\nu_e$ LSND and $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ MiniBooNE experiments, in $\bar\nu_e \to \bar\nu_e$ reactor experiments and in $\nu_e \to \nu_e$ source Gallium experiments

Many SBL reactor, accelerator and source neutrino experiments are going on or in preparations at present. From existing data it is not possible to make a definite conclusion on sterile neutrinos

Combined analysis of the data of reactor Daya Bay and Bugey-3 experiments and accelerator MINOS+ experiment allows to exclude at 90 % CL LSND and MiniBooNE allowed regions for $\Delta m^2_{14} < 5\ \mathrm{eV}^2$, in reactor DANSS experiment it was found that the best-fit point of previous reactor experiments is excluded at 5σ,...

The study of neutrinoless double β-decay
$(A, Z) \to (A, Z+2) + e^- + e^-$ is the most sensitive way to reveal the Majorana nature of neutrinos with definite masses ν_i
In recent experiments the following lower limits were reached:

$$T_{1/2}(^{76}\mathrm{Ge}) > 9 \cdot 10^{25} \text{ yr (GERDA)},$$

$$T_{1/2}(^{136}\mathrm{Xe}^{136}) > 10.7 \cdot 10^{25} \text{ yr (KamLAND-Zen)},$$

$$T_{1/2}(^{130}\mathrm{Te}) > 3.2 \cdot 10^{25} \text{ yr (CUORE)}$$

About one-two orders of magnitude larger half-lives are expected. In future $0\nu\beta\beta$- experiments such sensitivities are planned to be reached.

Summarizing, we discussed a plausible (apparently, most plausible) scenarios of the origin of neutrino masses, based on such fundamental hypotheses as a violation of L in a beyond the SM physics. Crucial tests of this scenarios will be realized in experiments on

- The search for light sterile neutrinos
- The search for neutrinoless double β-decay