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Interactive, $F(R)$ and other cosmological models, recent observational data and H_0 tension

◇ S. Pan, G. S. and W. Yang, *Field theoretic interpretations of interacting dark energy scenarios and recent observations*, PRD **101** 103533 (2020), arXiv:2001.03120.

◇ S. D. Odintsov, D. Sáez-Gómez and G. S., *Testing logarithmic corrections on R^2-exponential gravity by observational data*, PRD **99** 024003 (2019), arXiv:1807.02163.

◇ S. D. Odintsov, D. Sáez-Gómez and G. S. *Testing the equation of state for viscous dark energy*, PRD **101** 044010 (2020), arXiv:2001.07945.

◇ G. S. and E. S. Sinyakov. *Cosmological models, observational data and tension in Hubble constant*, Math. Modelling and Geometry. 8, No 1, (2020) 1 arXiv:2002.03599

Problems:

1. Search of the most adequate cosmological model describing all recent observational data (SNe Ia, $H(z)$, BAO, CMB, lensing etc.)

2. Tension in Hubble constant estimations

Planck collaboration (2018) $H_0 = 67.37 \pm 0.54$ km /(s·Mpc)

SH0ES (Hubble Space Telescope) group (2019) $H_0 = 74.03 \pm 1.42$ km /(s·Mpc).

Models:

- **Interactive models**

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -Q,$$
$$\dot{\rho}_x + 3H(p_x + \rho_x) = Q;$$

- **Exponential $F(R)$ model (+ log)**

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,F(R) + S_{matter},$$

.

$$F(R) = R - 2\Lambda(1 - e^{-bR})\cdot A + F_{\mathbf{inf}} \quad A = 1 - aR\ln\frac{R}{R_0};$$

- **Scenarios with bulk viscosity**

$$p_x = f(\rho_x) - 3\zeta(H)\cdot H.$$

Models:

1. Interacting dark energy scenarios with field theoretic interpretation[1]

Interaction between dark matter (ρ_{dm}) and dark energy (ρ_x) is described as

$$H^2 = \frac{8\pi G}{3}\rho, \qquad \dot{H} = -4\pi G \sum_i (p_i + \rho_i), \tag{1}$$

where $8\pi G \equiv \kappa^2$, $\rho = \rho_{dm} + \rho_x + \rho_b + \rho_r$,

$$\dot{\rho}_i + 3H(p_i + \rho_i) = 0, \quad i = b,\, r; \qquad \begin{array}{l} \dot{\rho}_{dm} + 3H\rho_{dm} = -Q, \\ \dot{\rho}_x + 3H(p_x + \rho_x) = Q. \end{array} \tag{2}$$

Interaction is interpreted via the model with 2 scalar fields ϕ_1 (DM) and ϕ_2 (DE)

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{\epsilon_1}{2}(\nabla\phi_1)^2 - \frac{\epsilon_2}{2}(\nabla\phi_2)^2 - V(\phi_1,\phi_2)\right] + S^m.$$

Dynamics $R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G\left[\sum_{j=1}^2 \epsilon_j\Big(\partial_\mu\phi_j\partial_\nu\phi_j - \frac{1}{2}(\nabla\phi_j)^2 g_{\mu\nu}\Big) - V g_{\mu\nu} + T^m_{\mu\nu}\right]$, $\nabla^\mu\nabla_\mu\phi_j = \epsilon_j\frac{\partial V}{\partial\phi_j}$,
for the FLRW flat universe $(k = 0)$ may be reduced to Eqs. (1) with

$$\rho_{tot} = \frac{\epsilon_1}{2}\dot{\phi}_1^2 + \frac{\epsilon_2}{2}\dot{\phi}_2^2 + V(\phi_1,\phi_2) + \rho_b + \rho_r, \quad p_{tot} = \frac{\epsilon_1}{2}\dot{\phi}_1^2 + \frac{\epsilon_2}{2}\dot{\phi}_2^2 - V(\phi_1,\phi_2) + p_b + p_r. \tag{3}$$

$$\ddot{\phi}_j + 3H\dot{\phi}_j = -\epsilon_j\frac{\partial V}{\partial\phi_j}, \qquad j = 1, 2. \tag{4}$$

[1] S. Pan, G. S. Sharov and W. Yang, *Field theoretic interpretations of interacting dark energy ...*, PRD **101** 103533 (2020), arXiv:2001.03120.

Eqs. (1) are reproduced and Eqs. (4) $\Longrightarrow$ (2), if we divide (and define)

$$V(\phi_1,\phi_2) = V_1(\phi_1,\phi_2) + V_2(\phi_1,\phi_2); \qquad \rho_j = \frac{\epsilon_j}{2}\dot{\phi}_j^2 + V_j, \quad p_j = \frac{\epsilon_j}{2}\dot{\phi}_j^2 - V_j, \tag{5}$$

field Eqs. (4) $\Longrightarrow$

$$\begin{aligned} \dot{\rho}_1 + 3H(\rho_1 + p_1) &= -Q, \\ \dot{\rho}_2 + 3H(\rho_2 + p_2) &= Q, \end{aligned} \tag{6}$$

where the interacting term

$$Q = \dot{\phi}_1 \frac{\partial V_2}{\partial \phi_1} - \dot{\phi}_2 \frac{\partial V_1}{\partial \phi_2}. \tag{7}$$

$\Longrightarrow$ non-interacting potential $V(\phi_1,\phi_2) = V_1(\phi_1) + V_2(\phi_2)$.

Examples:

$$\Lambda\text{CDM}: \qquad V_1(\phi_1) = \frac{\Lambda}{16\pi G}\sinh^2\left[\sqrt{6\pi G}\,(\phi_1 - \phi_0)\right], \quad V_2 = \frac{\Lambda}{8\pi G} = \text{const}, \quad \phi_2 = 0.$$

The model[2] with the Big Rip singularity

$$V(\phi_1,\phi_2) = \frac{\phi_0^2}{2t_0^2}\left[(\theta-1)\,e^{-2\phi_1/\phi_0} + (\theta+1)\,e^{-2\phi_2/\phi_0} + 2\theta e^{-(\phi_1+\phi_2)/\phi_0}\right],$$

$$H = \frac{\theta}{3}\left(\frac{1}{t} + \frac{1}{t-t_s}\right), \quad \phi_1 = \phi_0 \log\frac{t}{t_0}, \quad \phi_2 = \phi_0 \log\frac{t_s - t}{t_0}, \qquad \epsilon_2 = -1.$$

$$\Longrightarrow \qquad Q = 3\xi H \frac{\rho_1 \rho_2}{\rho_1 + \rho_2}, \qquad \xi = -1/\theta.$$

[2] S. Nojiri, S. D. Odintsov and S. Tsujikawa, *Properties of singularities in (phantom) dark energy universe,* PRD **71**, 063004 (2005)

The potential (under condition $p_1 = 0$, where $p_1 \equiv \frac{1}{2}\dot{\phi}_1^2 - V_1(\phi_1, \phi_2)$, $\epsilon_1 = 1$)

$$V = V_1(\phi_1) + V_2(\phi_1, \phi_2) = \frac{\phi_0^2}{2t_1^2} e^{-2\phi_1/\phi_0} + A_2 t_1^{\gamma_1} t_2^{\gamma_2} e^{\gamma_1\phi_1/\phi_0 + \gamma_2\phi_2/\psi_0}.$$

and the solution

$$\phi_1 = \phi_0 \log\frac{t}{t_1}, \quad \phi_2 = \psi_0 \log\frac{t}{t_2}, \quad H = \frac{h_0}{t}, \quad \phi_0,\ h_0,\ t_j \text{ are constants}$$

yields

$$Q = 3H\xi\rho_1, \qquad \text{IVS0}, \qquad \xi = \frac{2 - 3h_0}{3h_0},$$

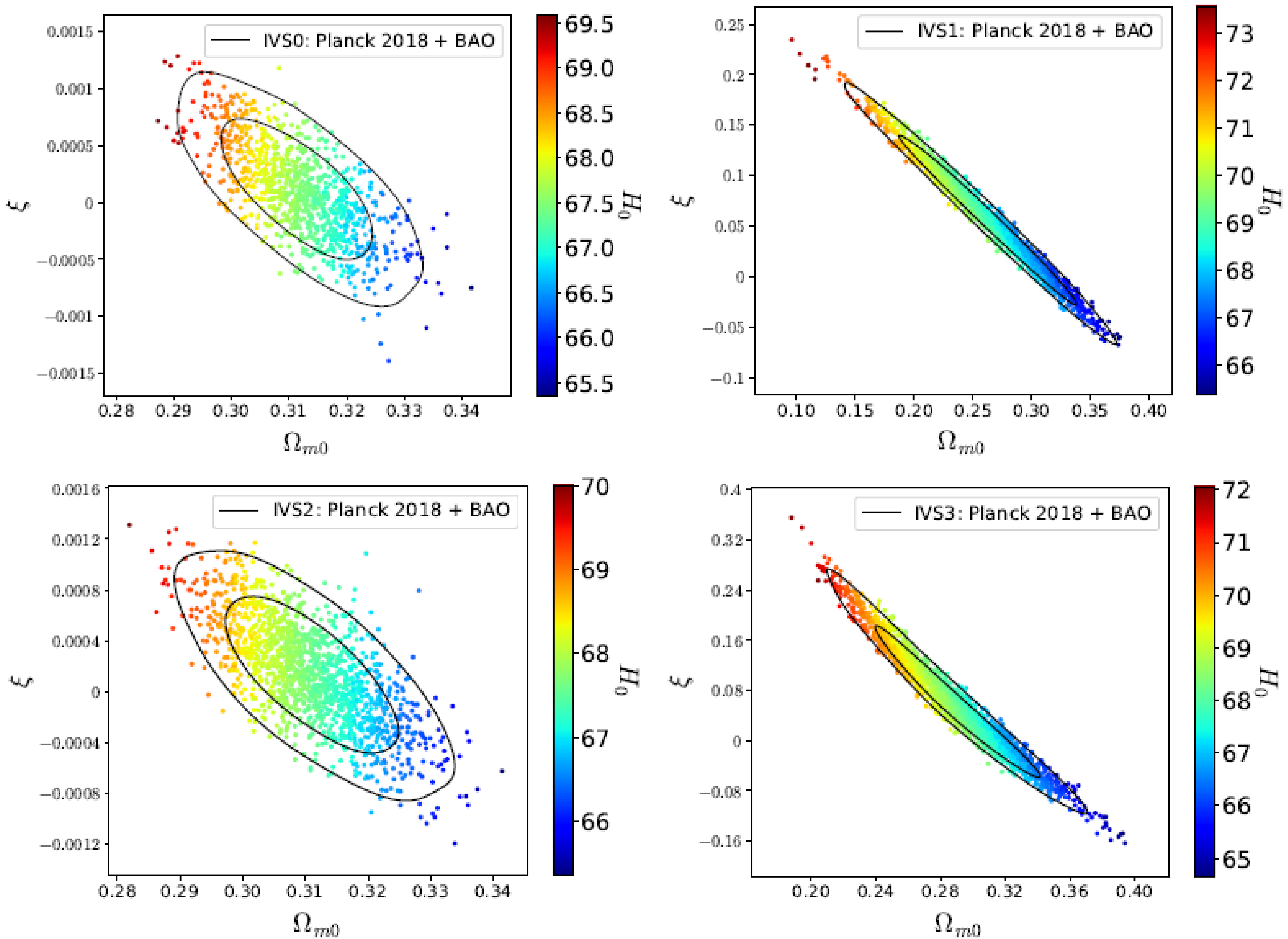
$$Q = 3H\xi\rho_2, \qquad \text{IVS1}, \qquad \xi = \frac{\gamma_1(3h_0 - 1)}{3h_0(3h_0 + \gamma_1/2)}.$$

Other interacting vacuum scenarios:

$$Q = 3H\xi(\rho_1 + \rho_2), \qquad \text{IVS2},$$

$$Q = 3H\xi\frac{\rho_1\rho_2}{\rho_1 + \rho_2}, \qquad \text{IVS3}, \qquad \xi = -1/\theta.$$

Scenarios IVS0 – IVS2 ($w_x = -1$) are tested with Planck 2018 (CMB power spectra, lensing and BAO); IVS1 and IVS3 are the most preferable in comparison with ΛCDM.



l plots in 68% and 95% CL for all the IVS models in the $\xi - \Omega_{m0}$ plane colored by the H_0

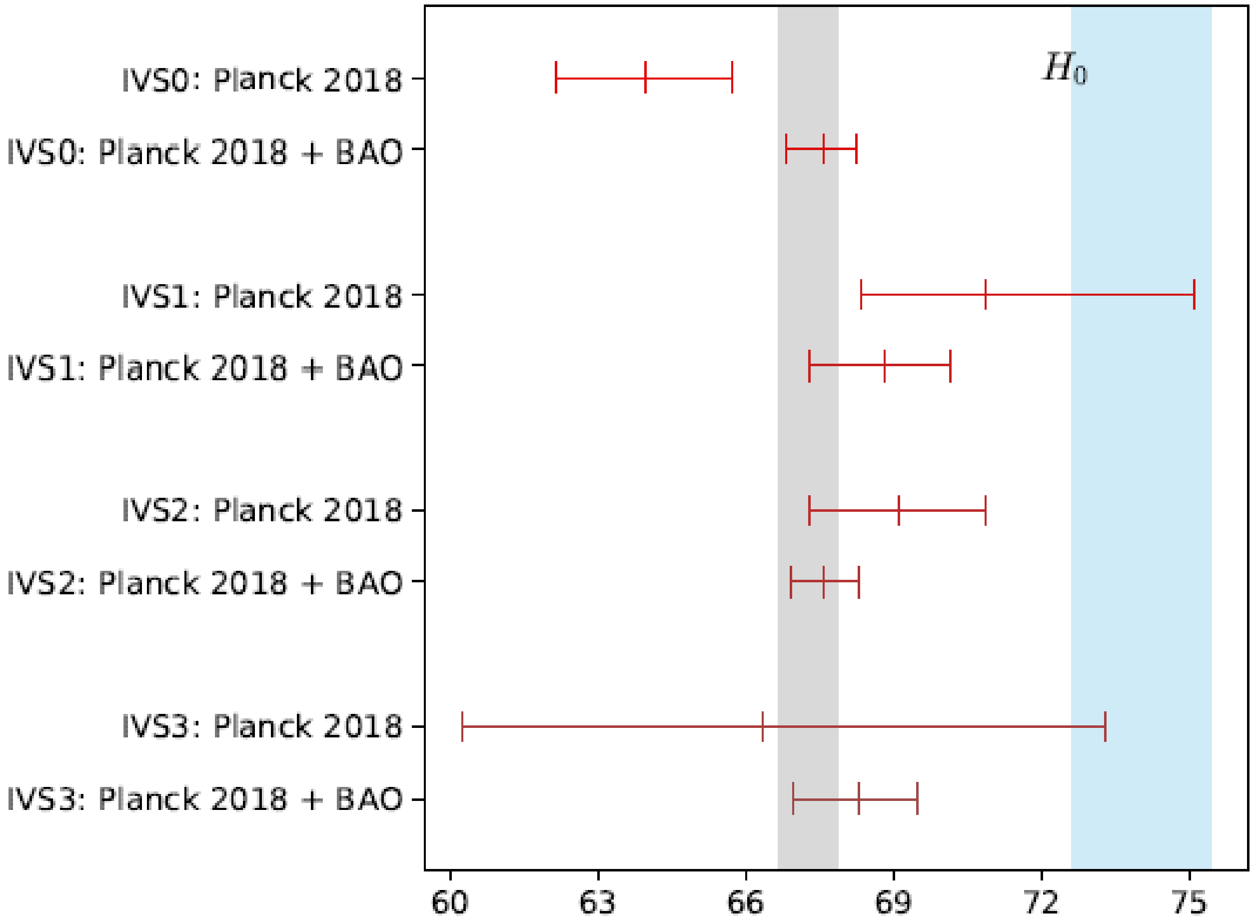


FIG. 6: Whisker plot showing the 68% CL constraints on H_0

with Planck 2018 $H_0 = 67.37 \pm 0.54$ and HST $H_0 = 74.03 \pm 1.42$ km/(s Mpc)

2. Exponential $F(R)$ model with log corrections[1] motivated in QG

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,F(R) + S_{matter},$$

$$F(R) = R - 2\Lambda\Big[1-\exp\Big(-\beta\frac{R}{2\Lambda}\Big)\Big]\left(1-\alpha\frac{R}{2\Lambda}\log\frac{R}{4\Lambda}\right) + \gamma_0\Big(1+\gamma_1\log\frac{R}{R_0}\Big)R^2. \quad (5)$$

R is the Ricci scalar, Λ is the cosmological constant, $\kappa^2 = 8\pi G$;

$$F(R) \to \Lambda\text{CDM}, \ \text{if} \ \beta\to\infty, \ \alpha\to 0.$$

Inflation at $z \geq z_i \sim 10^{25}$, $R \geq R_i \sim 10^{90}\Lambda$ slow-roll scenario with graceful exit.

At middle times $10^{25} \gg z \geq 10^4$, the model tends to ΛCDM with corrections:

$$F(R) \quad \Longrightarrow \quad R - 2\Lambda\left(1-\alpha\frac{R}{2\Lambda}\log\frac{R}{4\Lambda}\right).$$

Late time $z \leq 10^3$: observational limitations (SNe Ia, $H(z)$, BAO, CMB).

[1] S. D. Odintsov, D. Sáez-Gómez and G. S. Sharov, PRD **99**, 024003 (2019)

Dynamical equations

$F_R \equiv F'(R), \quad F_{RR} \equiv F''(R)$

$$F_R R_{\mu\nu} - \frac{F}{2} g_{\mu\nu} + \big(g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \nabla_\beta - \nabla_\mu \nabla_\nu\big) F_R = \kappa^2 T_{\mu\nu}$$

(at middle and late time) for the flat FLRW space-time $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x}^2$ are reduced to the system for R and $H = \dot{a}/a$:

$$\frac{dH}{d\log a} = \frac{R}{6H} - 2H, \qquad \frac{dR}{d\log a} = \frac{1}{F_{RR}} \left(\frac{\kappa^2 \rho}{3H^2} - F_R + \frac{RF_R - F}{6H^2} \right), \tag{6}$$

$$\rho = \rho_m^0 a^{-3} + \rho_r^0 a^{-4} = \rho_m^0 \big(a^{-3} + X_r a^{-4}\big), \qquad (X_r = \rho_r^0/\rho_m^0) \tag{7}$$

or

$$\frac{dE}{d\log a} = \Omega_\Lambda^* \frac{\mathcal{R}}{E} - 2E, \qquad E = \frac{H}{H_0^*}, \quad \mathcal{R} = \frac{R}{2\Lambda}$$

$$\frac{d\log \mathcal{R}}{d\log a} = \frac{\big\{E_{\Lambda CDM}^2 + \Omega_\Lambda^* \big[\alpha \mathcal{R} \big(1 - e^{-\beta\mathcal{R}} (1 - \beta\mathcal{R}\ell)\big) - e^{-\beta\mathcal{R}} (1 + \beta\mathcal{R})\big]\big\}/E^2 - 1 + \beta e^{-\beta\mathcal{R}} - \alpha\Phi}{\alpha + \alpha\, e^{-\beta\mathcal{R}} \big\{-1 + \beta\mathcal{R}\big[2 + (2 - \beta\mathcal{R})\ell\big]\big\} + \beta^2 \mathcal{R}\, e^{-\beta\mathcal{R}}}.$$

where $\ell = \log(\mathcal{R}/2)$, $\Phi = 1 + \ell - e^{-\beta\mathcal{R}}\big[1 + (1 - \beta\mathcal{R})\ell\big]$, $\quad E_{\Lambda CDM}^2 = \Omega_m^* \big(a^{-3} + X_r a^{-4}\big) + \Omega_\Lambda^*$.

This system is integrated “into the future” from a point at $10^5 \le z \ll 10^{15}$ (“quasi ΛCDM limit”) with: $\quad E^2 = E_{\Lambda CDM}^2 \big[1 + O(\alpha)\big], \quad \mathcal{R} \simeq A a^{-4}$,

$$H_0^* \equiv H_0^{\Lambda CDM}, \quad \Omega_m^* \equiv \Omega_m^{\Lambda CDM}, \quad \Omega_\Lambda^* \equiv \Omega_\Lambda^{\Lambda CDM}; \qquad (H_0^* \ne H_0, \quad \Omega_m^* \ne \Omega_m^0, \dots)$$

Free parameters in the $F(R)$ model (5): $\quad \alpha, \ \beta, \ \Omega_m^*, \ \Omega_\Lambda^*, \ H_0^*$.

3. Two scenarios with bulk viscosity[2]

$p_x = f(\rho_x) - 3\zeta H, \quad \zeta(H) \sim H^{2\beta-1}$

$$\rho = \rho_m + \rho_r + \rho_x; \qquad \rho_m = \rho_m^0 a^{-3}, \qquad \rho_r = \rho_r^0 a^{-4}.$$

a) **power-law model**

$$p_x = -\rho_x + A\rho_x^\alpha + BH^{2\beta} \ .$$

b) **logarithmic model**

$$p_x = A\rho_x \log\frac{\rho_x}{\rho_*} + BH^{2\beta} \ .$$

Dynamics for flat FLRW $\begin{cases} H^2 = \frac{\kappa^2}{3}\rho, \\ \dot{\rho}_i + 3H(p_i + \rho_i) = 0 \end{cases}$ is reduced to

$$H^2/H_0^2 = \Omega_m^0\left(a^{-3} + X_r a^{-4}\right) + \Omega_x(a),$$

$$\frac{d\Omega_x}{d\log a} = \begin{cases} -3\Big[A\Omega_x^\alpha + B\big(H/H_0\big)^{2\beta}\Big], & \text{(power-law)}, \\ -3\Big[\Omega_x + A\Omega_x \log\frac{\Omega_x}{\Omega_*} + B\big(H/H_0\big)^{2\beta}\Big], & \text{(logarithmic)}. \end{cases}$$

Free model parameters: $\Omega_m^0 = \frac{\kappa^2\rho_m^0}{3H_0^2}$

$$\Omega_m^0,\ A,\ \alpha,\ B,\ \beta,\ H_0, \qquad \text{(power-law)},$$
$$\Omega_m^0,\ A,\ B,\ \beta,\ \Omega_*,\ H_0, \qquad \text{(logarithmic)}.$$

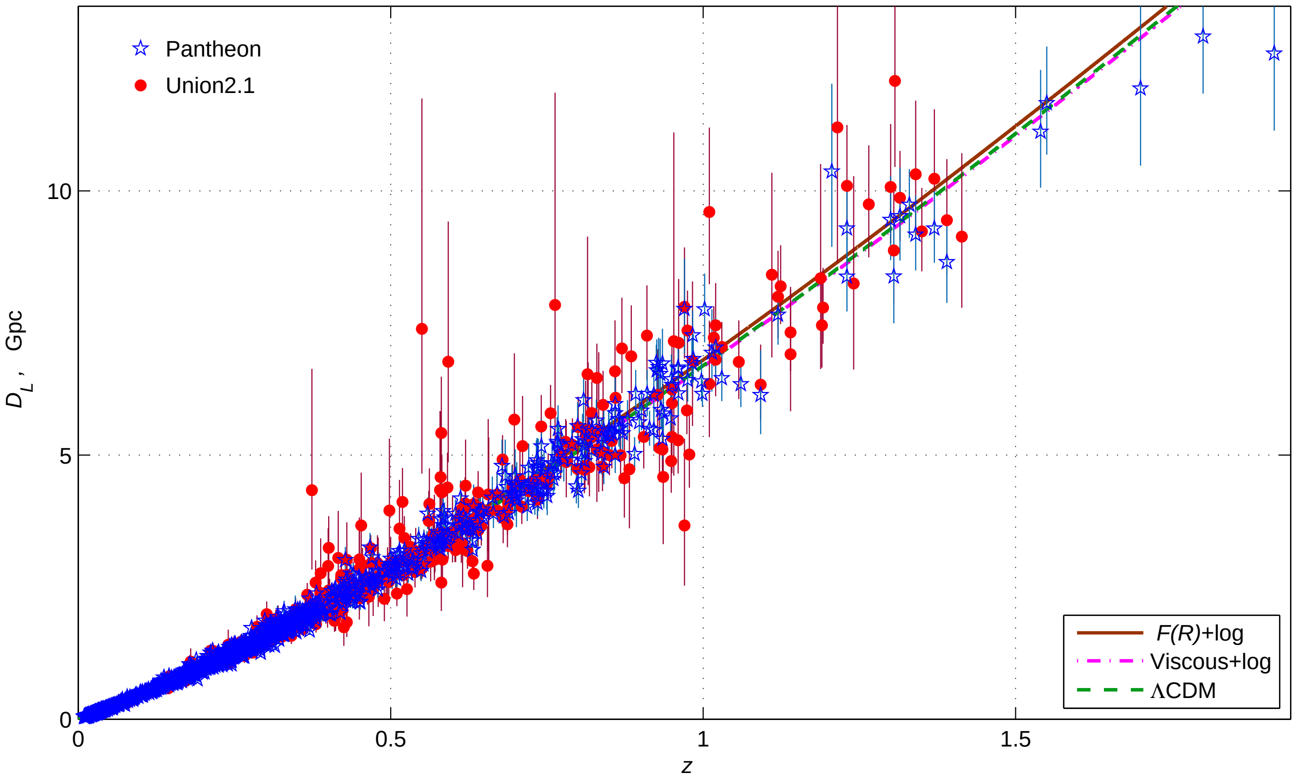
[2] S. D. Odintsov, D. Sáez-Gómez and G. S. Sharov, PRD **101**, 044010 (2020), arXiv:2001.07945

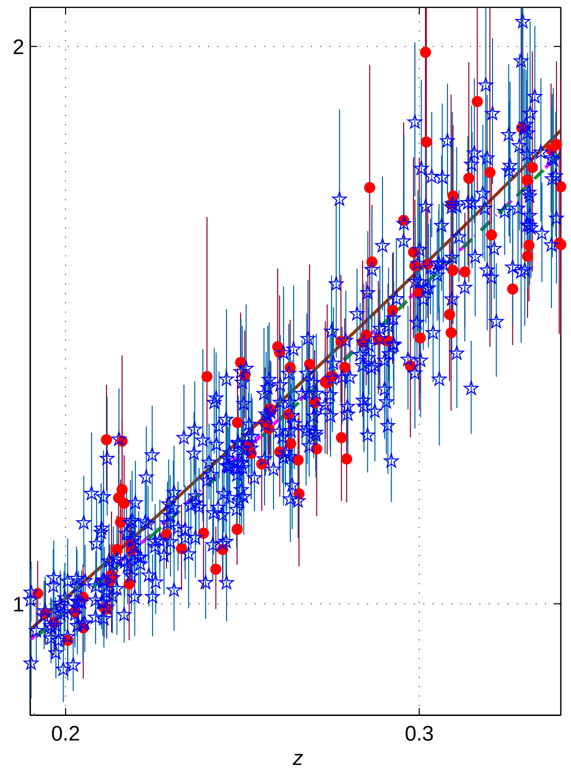
The observational constraints include:

1. The Pantheon Supernovae Ia with $N_{\rm SN} = 1048$ data points (D.M. Scolnic *et al.*, *Ap. J.* **859** (2018) 101). The observed SNe Ia distance moduli μ_i^{obs} are compared with $\mu^{th}(z_i)$:

$$\mu^{th}(z) = 5\log_{10}\frac{D_L(z)}{10\text{pc}}, \quad D_L(z) = (1+z)D_M(z), \quad D_M(z) = \frac{c}{H_0}S_k\bigg(\int\limits_0^z \frac{H_0\,d\tilde z}{H(\tilde z)}\bigg),$$

$$\chi^2_{\rm SN} = \min_{H_0^*}\sum_{i,j=1}^{N_{\rm SN}}\Delta\mu_i\big(C_{\rm SN}^{-1}\big)_{ij}\Delta\mu_j, \quad \Delta\mu_i = \mu^{th}(z_i) - \mu_i^{obs}, \quad S_k(x) = \begin{cases} \frac{\sinh\big(x\sqrt{\Omega_k}\big)}{\sqrt{\Omega_k}}, & \Omega_k > 0,\\ x, & \Omega_k = 0,\\ \frac{\sin\big(x\sqrt{|\Omega_k|}\big)}{\sqrt{|\Omega_k|}}, & \Omega_k < 0.\end{cases}$$



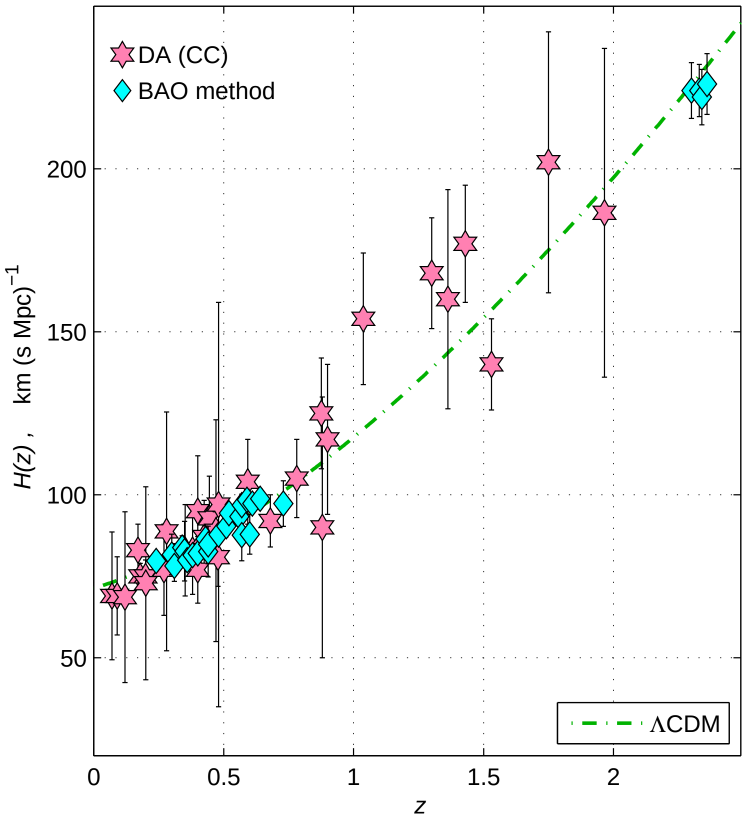


2. The Hubble parameter estimations $H(z)$:

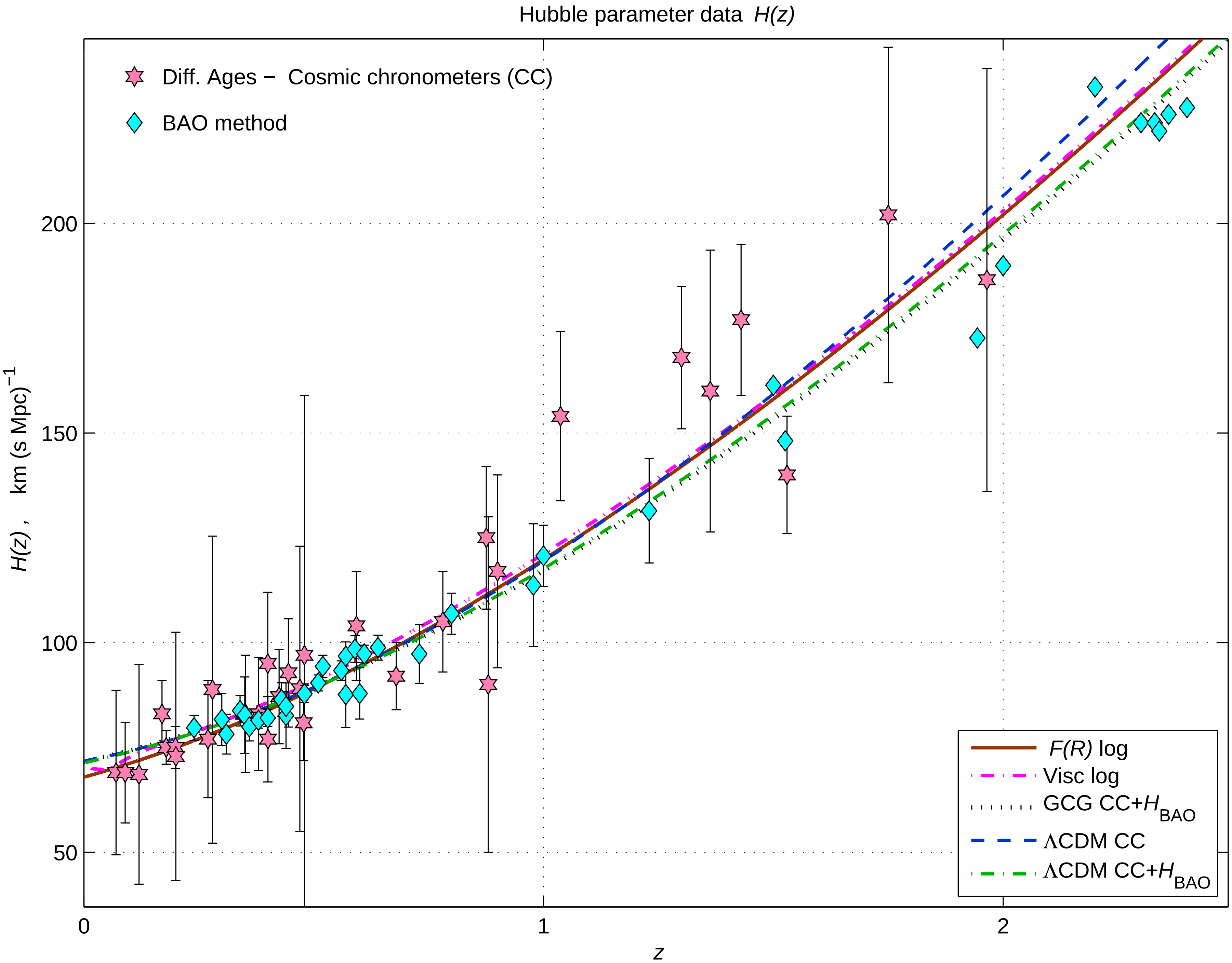
a) differential ages of galaxies (cosmic chronometers)

$$H(z) = \frac{\dot{a}}{a} \simeq -\frac{1}{1+z}\frac{\Delta z}{\Delta t},$$

b) line-of-sight BAO data.



DA method				BAO method			
z	$H(z)$	σ_H	Refs	z	$H(z)$	σ_H	Refs
0.070	69	19.6	Zhang12	0.24	79.69	2.99	Gaztañaga09
0.090	69	12	Simon05	0.30	81.7	6.22	Oka13
0.120	68.6	26.2	Zhang12	0.31	78.18	4.74	Wang17
0.170	83	8	Simon05	0.34	83.8	3.66	Gaztañaga09
0.1791	75	4	Moresco12	0.35	82.7	9.1	ChuangW12
0.1993	75	5	Moresco12	0.36	79.94	3.38	Wang17
0.200	72.9	29.6	Zhang12	0.38	81.5	1.9	Alam16
0.270	77	14	Simon05	0.40	82.04	2.03	Wang17
0.280	88.8	36.6	Zhang12	0.43	86.45	3.97	Gaztañaga09
0.3519	83	14	Moresco12	0.44	82.6	7.8	Blake12
0.3802	83	13.5	Moresco16	0.44	84.81	1.83	Wang17
0.400	95	17	Simon05	0.48	87.79	2.03	Wang17
0.4004	77	10.2	Moresco16	0.51	90.4	1.9	Alam16
0.4247	87.1	11.2	Moresco16	0.52	94.35	2.64	Wang17
0.4497	92.8	12.9	Moresco16	0.56	93.34	2.3	Wang17
0.470	89	34	Ratsimbazafy17	0.57	87.6	7.8	Chuang13
0.4783	80.9	9	Moresco16	0.57	96.8	3.4	Anderson14
0.480	97	62	Stern10	0.59	98.48	3.18	Wang17
0.593	104	13	Moresco12	0.60	87.9	6.1	Blake12
0.6797	92	8	Moresco12	0.61	97.3	2.1	Alam16
0.7812	105	12	Moresco12	0.64	98.82	2.98	Wang17
0.8754	125	17	Moresco12	0.73	97.3	7.0	Blake12
0.880	90	40	Stern10	2.30	224	8.6	Busca12
0.900	117	23	Simon05	2.33	224	8	Bautista17
1.037	154	20	Moresco12	2.34	222	8.5	Delubac14
1.300	168	17	Simon05	2.36	226	9.3	Font-Ribera13
1.363	160	33.6	Moresco15	0.8	106.9	4.9	Zhu18
1.430	177	18	Simon05	0.978	113.72	14.63	Zhao18
1.530	140	14	Simon05	1.0	120.7	7.3	Zhu18
1.750	202	40	Simon05	1.230	131.44	12.42	Zhao18
1.965	186.5	50.4	Moresco15	1.5	161.4	30.9	Zhu18


Hubble parameter data H(z)
Diff. Ages − Cosmic chronometers (CC)
BAO method
F(R) log
Visc log
GCG CC+H_BAO
ΛCDM CC
ΛCDM CC+H_BAO
H(z) , km (s Mpc)^−1
z
0
1
2
50
100
150
200

3. Baryon acoustic oscillations (BAO) data include observable parameters

$$d_z(z) = \frac{r_s(z_d)}{D_V(z)} \text{ (17 data points)} \quad \text{and} \quad A(z) = \frac{H_0\sqrt{\Omega_m^0}\,D_V(z)}{cz} \text{ (7 data points);}$$

$r_s(z_d)$ is the sound horizon at the end of the baryon drag era, $\quad D_V(z) = \left[\frac{cz D_M^2(z)}{H(z)}\right]^{1/3}$.

z	$d_z(z)$	σ_d	Refs	z	$d_z(z)$	σ_d	Refs
0.106	0.336	0.015	Beutler 11	0.35	0.1161	0.0146	ChuangW 12
0.15	0.2232	0.0084	Ross 14	0.44	0.0916	0.0071	Blake 11
0.20	0.1905	0.0061	Percival 09	0.57	0.0739	0.0043	Chuang 13
0.275	0.1390	0.0037	Percival 09	0.57	0.0726	0.0014	Anderson 14
0.278	0.1394	0.0049	Kazin 09	0.60	0.0726	0.0034	Blake 11
0.314	0.1239	0.0033	Blake 11	0.73	0.0592	0.0032	Blake 11
0.32	0.1181	0.0026	Anderson 14	2.34	0.0320	0.0021	Delubac 14
0.35	0.1097	0.0036	Percival 09	2.36	0.0329	0.0017	Font-Ribera13
0.35	0.1126	0.0022	Padmanabhan12				

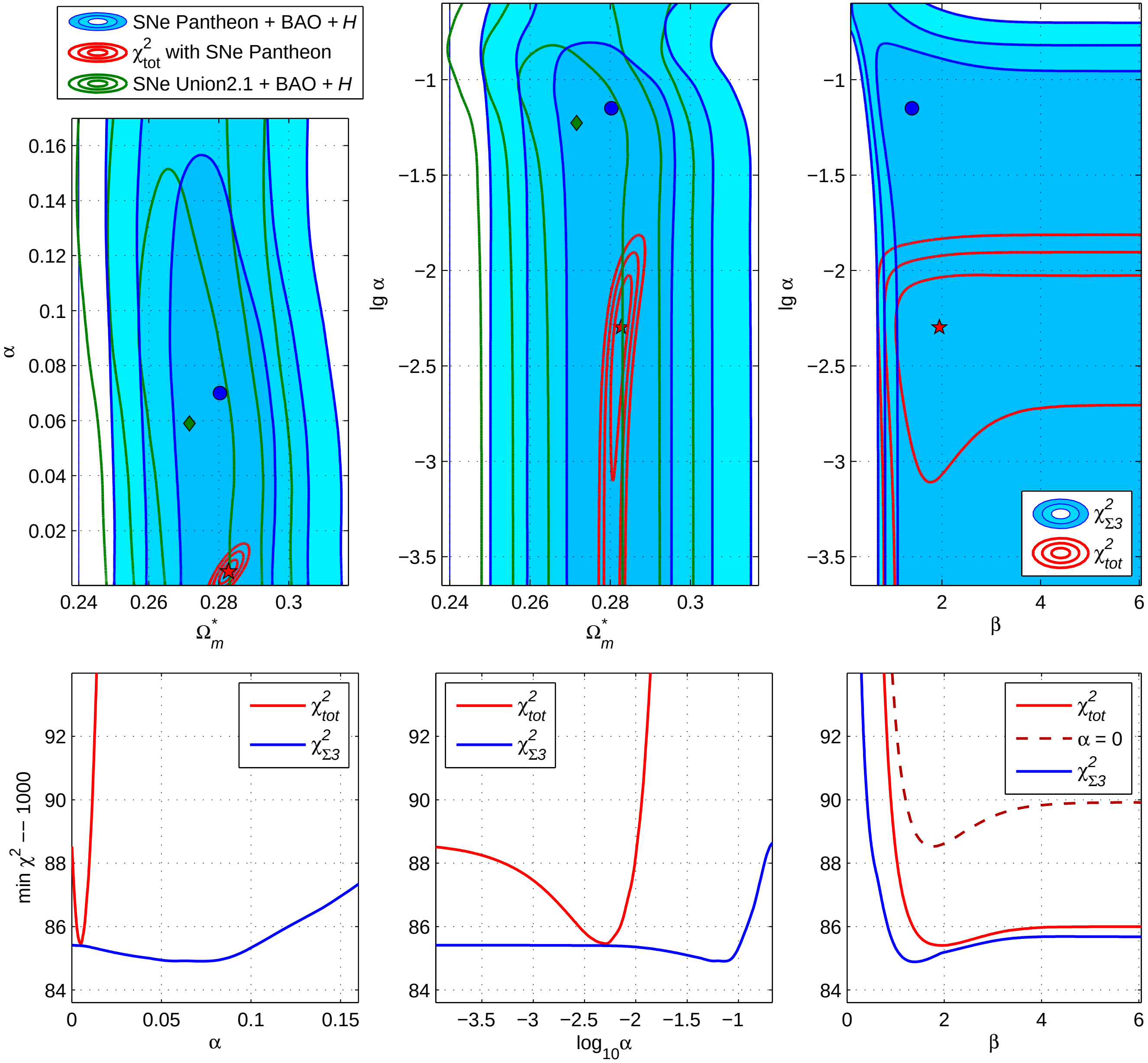
4. The CMB parameters

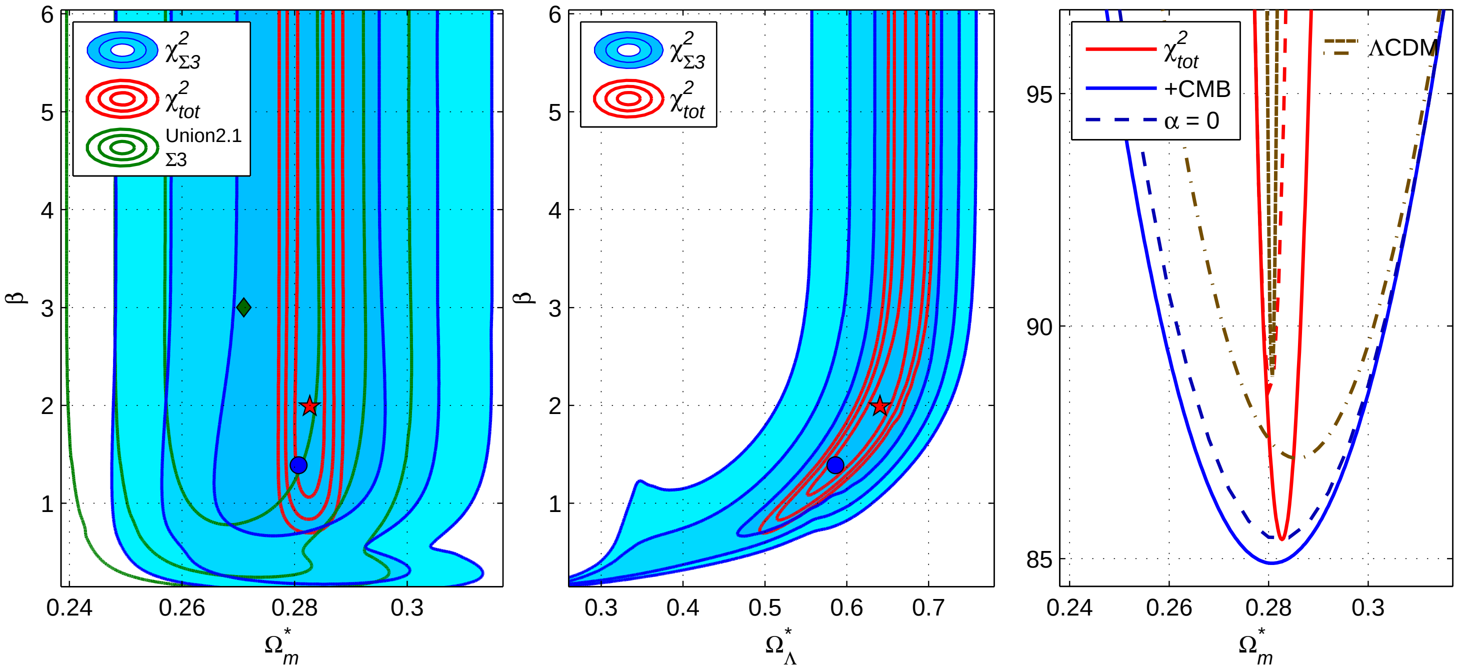
$$\mathbf{x} = (R, \ell_A, \omega_b) = \left(\sqrt{\Omega_m^0}\frac{H_0 D_M(z_*)}{c}, \frac{\pi D_M(z_*)}{r_s(z_*)}, \Omega_b^0 h^2\right)$$

($z_* \leftrightarrow$ recombination epoch) are compared with the estimations Q.-G. Huang, K. Wang, S. Wang, JCAP, 1512 (2015) 022 (Planck 2015) and L. Chen, Q.-G. Huang and K. Wang, JCAP, 2019 (2019) 028 (Planck 2018):

$$R^{Pl} = 1.7428 \pm 0.0053, \quad \ell_A^{Pl} = 301.406 \pm 0.090, \quad \omega_b^{Pl} = 0.02259 \pm 0.00017.$$

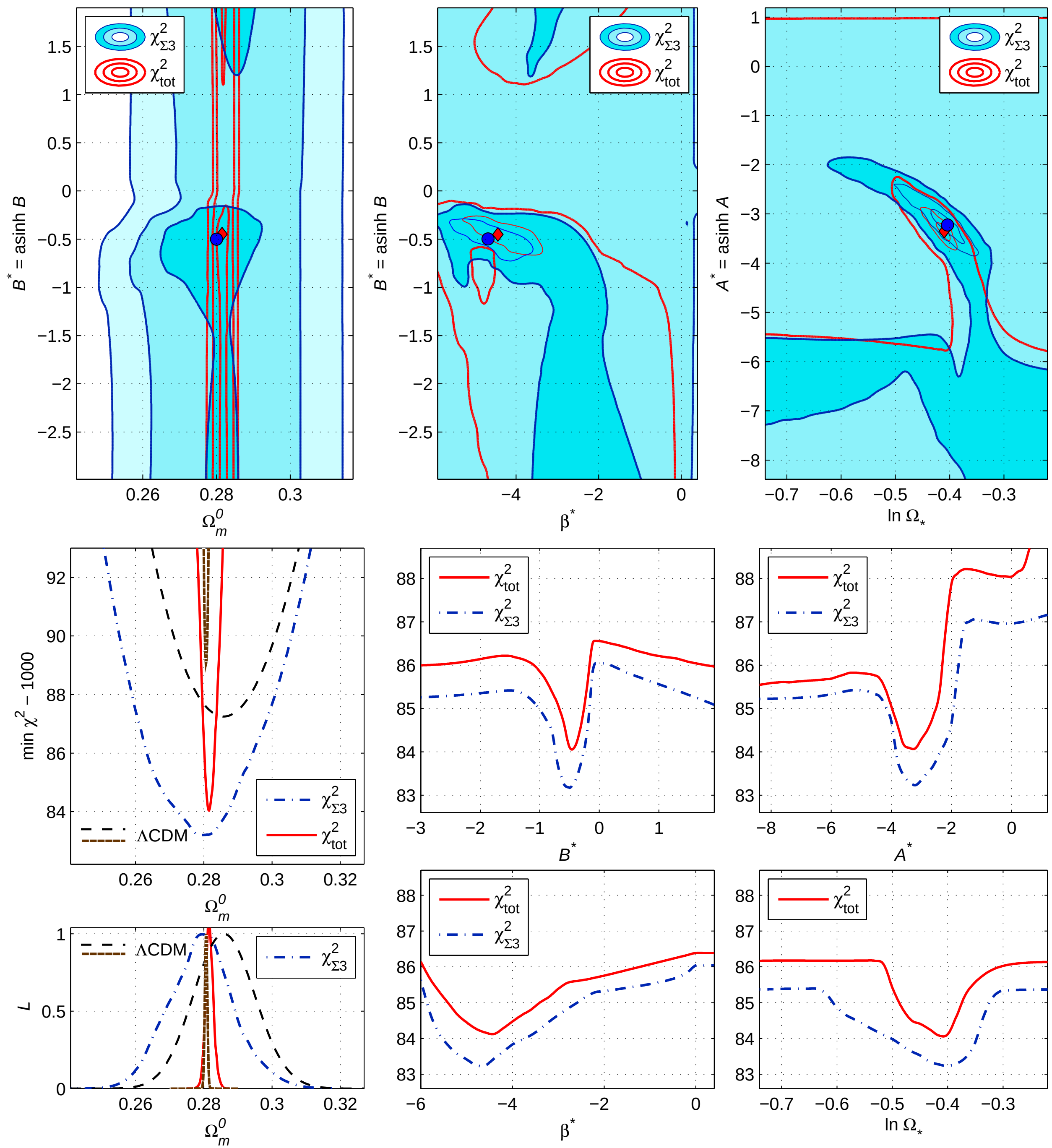
$F(R)$ exp+log model (1): 1σ, 2σ and 3σ contour plots for $\chi^2_{\Sigma 3} = \chi^2_{SN} + \chi^2_H + \chi^2_{BAO}$ (SNe+H+BAO) and for $\chi^2_{tot} = \chi^2_{\Sigma 3} + \chi^2_{CMB}$.





Model	data	$\min \chi^2/d.o.f$	$\Omega_m^0,\ \Omega_m^*$	other parameters
Exp $F(R)$+log	SN+H+BAO	1084.90 / 1099	$0.2807^{+0.010}_{-0.010}$	$\alpha = 0.070^{+0.048}_{-0.070}$, $\beta = 1.39^{+\infty}_{-0.53}$, $\Omega_\Lambda^* = 0.587^{+0.106}_{-0.074}$
Viscous+log	SN+H+BAO	1083.20 / 1098	$0.280^{+0.008}_{-0.009}$	$A^* = -3.22^{+0.99}_{-0.66}$, $B^* = -4.68^{+1.24}_{-0.95}$, $\beta^* = -0.50^{+0.27}_{-0.26}$
ΛCDM	SN+H+BAO	1087.25 / 1102	$0.2859^{+0.009}_{-0.009}$	$\Omega_\Lambda = 0.714^{+0.009}_{-0.009}$, $H_0 = 68.75^{+2.01}_{-1.98}$
Exp $F(R)$+log	SN+H+BAO+CMB	1085.41 / 1102	$0.2827^{+0.0017}_{-0.0018}$	$\alpha = 0.0051^{+0.0027}_{-0.0030}$, $\beta = 1.95^{+\infty}_{-0.70}$, $\Omega_\Lambda^* = 0.654^{+0.017}_{-0.046}$
Exp $F(R)$	SN+H+BAO+CMB	1088.53 / 1103	$0.2803^{+0.001}_{-0.001}$	$\alpha = 0$, $\beta = 1.76^{+1.33}_{-0.49}$, $\Omega_\Lambda^* = 0.655^{+0.014}_{-0.042}$
Viscous+power	SN+H+BAO+CMB	1088.98 / 1102	$0.2815^{+0.0019}_{-0.0018}$	$A^* = -9.2^{+5.04}_{-\infty}$, $B^* = 8.15^{+\infty}_{-4.75}$, $\beta^* = -0.068^{+0.068}_{-0.082}$
Viscous+log	SN+H+BAO+CMB	1084.05 / 1101	$0.2815^{+0.0012}_{-0.0009}$	$A^* = -3.35^{+0.84}_{-0.65}$, $B^* = -4.44^{+1.16}_{-0.89}$, $\beta^* = -0.45^{+0.20}_{-0.24}$
ΛCDM	SN+H+BAO+CMB	1089.03 / 1105	$0.2807^{+0.0003}_{-0.0004}$	$\Omega_\Lambda = 0.7193^{+0.0004}_{-0.0003}$, $H_0 = 69.72^{+1.60}_{-1.59}$

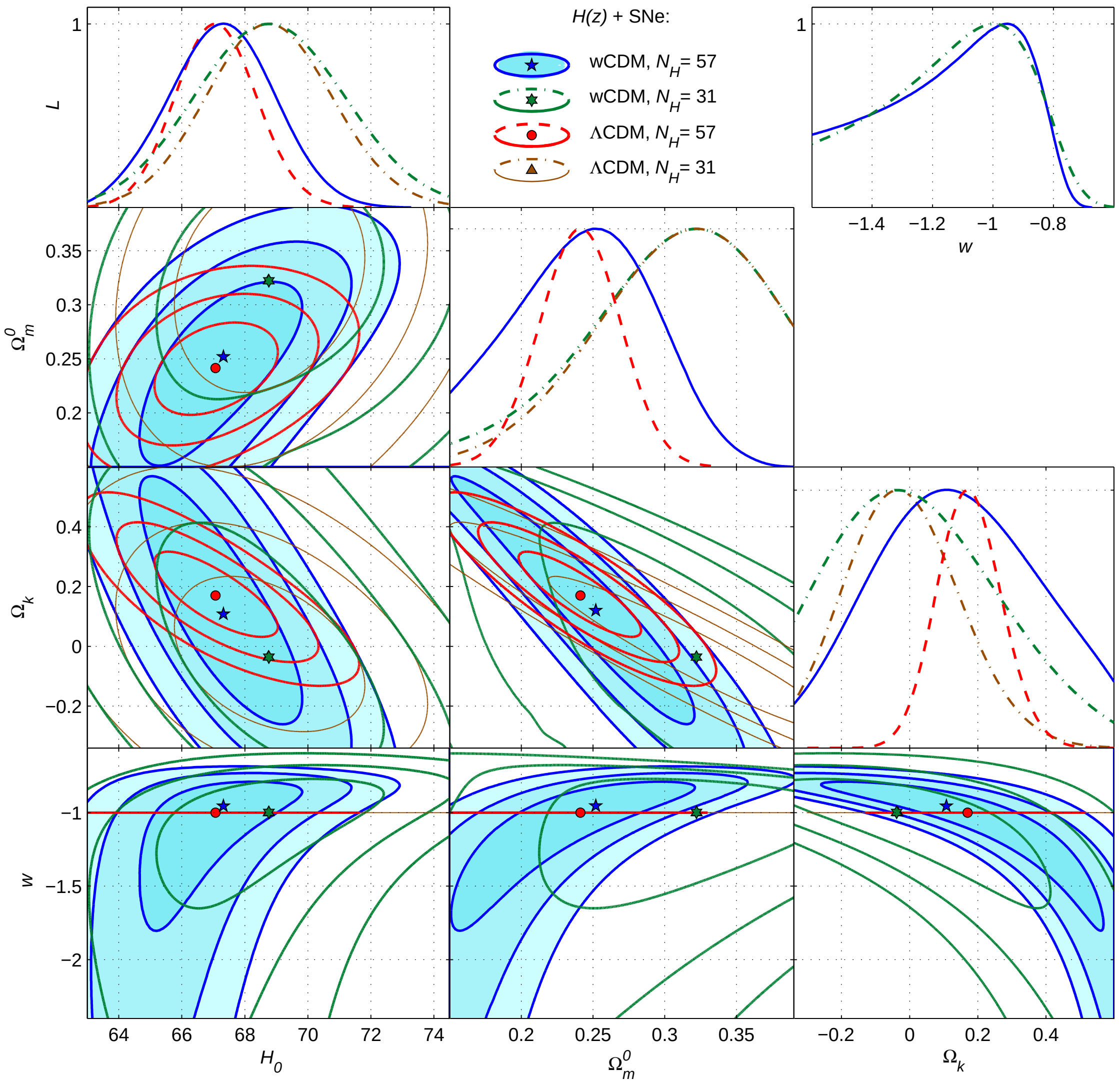
Viscous + logarithmic model $p_x = A\rho_x \log\frac{\rho_x}{\rho_*} + BH^{2\beta}$



4. Tension in Hubble constant estimations[3]

Planck collaboration (2018) $H_0 = 67.37 \pm 0.54$ km/(s·Mpc)

SH0ES (Hubble Space Telescope) (2019) $H_0 = 74.03 \pm 1.42$ km/(s·Mpc) (R19).



wCDM model:

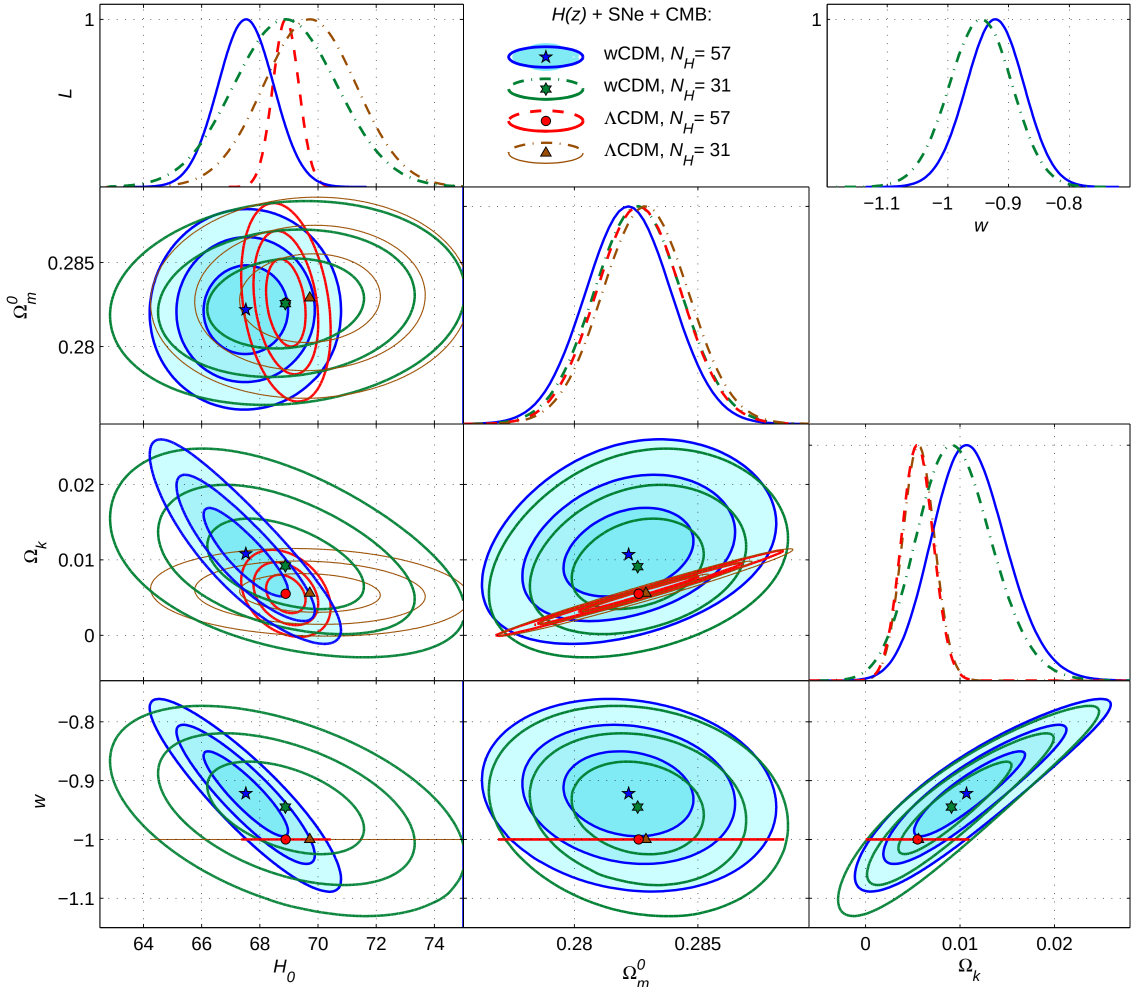
$$\frac{H^2}{H_0^2} = \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \\ + \Omega_k(1+z)^2 + \Omega_x(1+z)^{3(1+w)}.$$

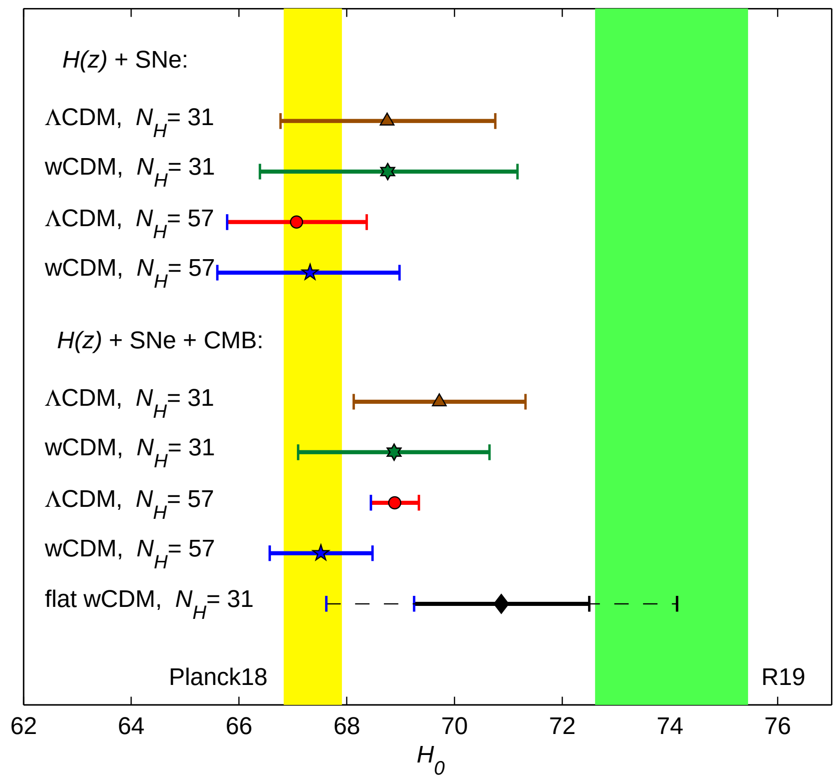
4 free parameters: H_0, Ω_m^0, Ω_k, w

If $w = -1$, wCDM $\longrightarrow$ ΛCDM.

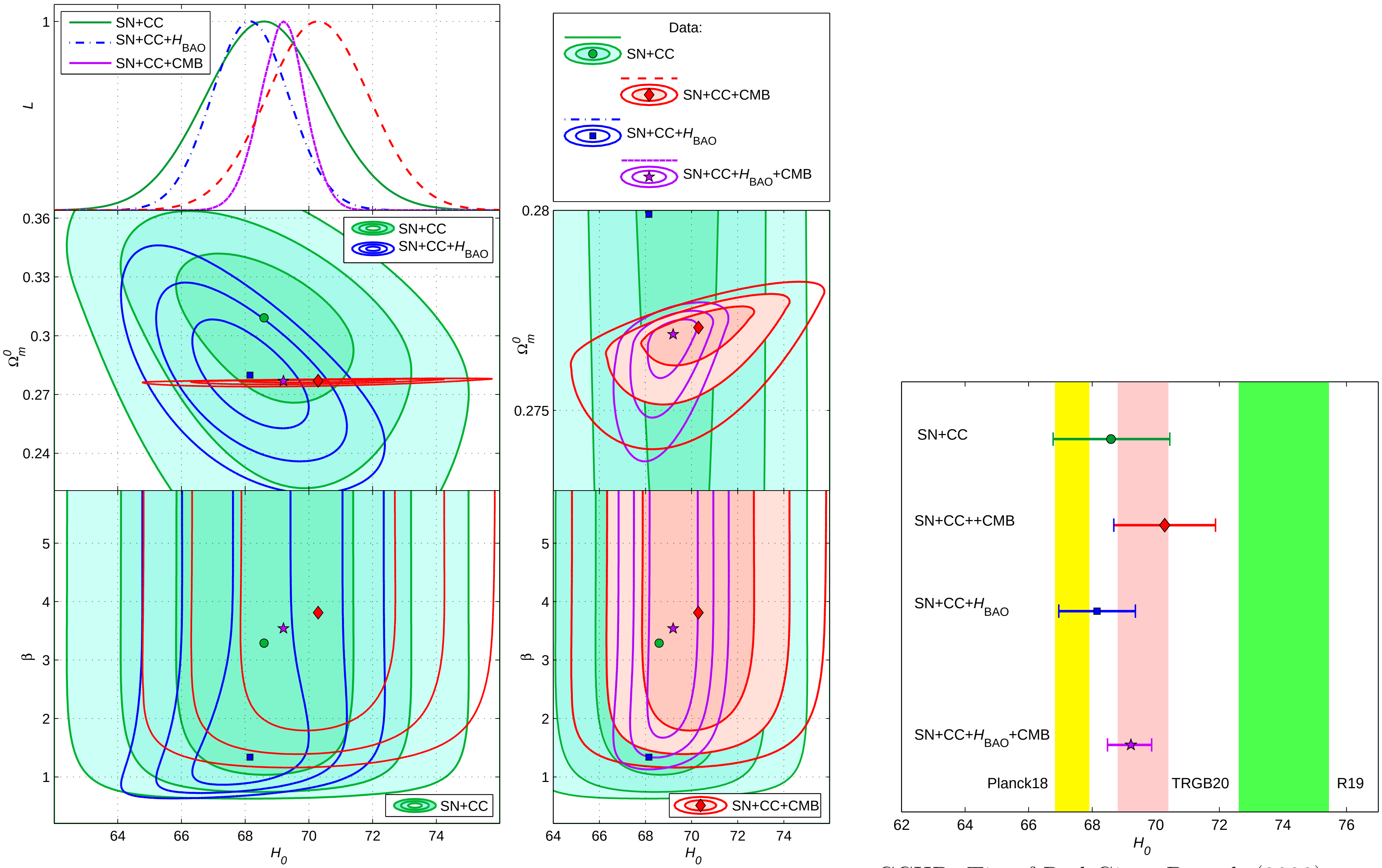
[3] G.Ṡ. Sharov and E. S. Sinyakov. MMG. 8, No 1, (2020) 1, arXiv:2002.03599.

Contour plots and whisker plots for wCDM, H + SNe Ia + CMB data





Exponential $F(R) = R - 2\Lambda(1 - e^{-\beta\mathcal{R}})$ model , H + SNe Ia + CMB data



CCHP Tip of Red Giant Branch (2020)

Thank you for attention!