Superheavy dark matter in $R^2$-cosmology

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**Dark matter:**
- electrically neutral, since doesn’t scatter light
- properties are practically unknown

Particles of many different types can be DM candidates.

**SUSY particles**
- No manifestation at LHC $\implies$ restricted parameter space open for SUSY

The number density of thermal relics (Zeldovich, 1965):

$$\frac{n_X}{n_\gamma} \approx \frac{1}{m_{Pl} m_X \sigma_{ann} \nu}; \quad \text{S-wave: } \sigma_{ann} \nu = \frac{\alpha^2}{m_X^2}, \quad \alpha_{SUSY} \approx 0.01$$

$n_X$ and $n_\gamma$ are the contemporary number densities of $X$-particles and CMB photons.

The LSP’s energy density

$$\varrho_{LSP} \sim \varrho_{DM}^{(obs)} (m_{LSP}/1 \text{ TeV})^2, \quad \varrho_{DM}^{(obs)} \approx 1 \text{ keV}/cm^3$$

- For $m_{LSP} \sim 1 \text{ TeV}$, $\varrho_{LSP}$ is of the order of the observed DM energy density
- For larger masses LSPs would overclose the universe.

In $(R + R^2)$-gravity the energy density of LSPs may be much lower $\implies$ it reopens for them the chance to be the dark matter, if $m_{LSP} \geq 1000\text{TeV}$. 
$R^2$ Modified Gravity

General Relativity (GR):

$$S_{EH} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \ R$$

describes basic properties of the universe in very good agreement with observations.

- $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass

Beyond the frameworks of GR:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{R^2}{6m^2} \right] + S_m$$

- $R^2$-term was suggested by V.Ts. Gurovich and A.A. Starobinsky for elimination of cosmological singularity (JETP 50 (1979) 844).
- It was found that the addition of the $R^2$-term leads to inflationary cosmology. (A. A. Starobinsky, Phys. Lett. B91, 99 (1980))

Curvature $R(t)$ can be considered as an effective scalar field (scalaron) with the mass $m_R$ and with the decay width $\Gamma$. 
Cosmological evolution in $R^2$ theory: 4 distinct epochs

I. The inflationary stage: the curvature was sufficiently large, and the universe expanded exponentially with slowly decreasing $R(t)$.

II. Scalaron dominated regime:
- $R(t)$ approached zero and started to oscillate around it as
  \[ R = -\frac{4m_R \cos(m_R t + \theta)}{t}, \quad m_R = 3 \times 10^{13} \text{ GeV} \]
- The Hubble parameter:
  \[ H \equiv \frac{\dot{a}}{a} = \frac{2}{3t} [1 + \sin(m_R t + \theta)] \]
- Energy density of matter drops down as
  \[ \rho_{R^2} = \frac{m_R^3}{120\pi t} \text{ instead of } \rho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2} \]

III. The transition period from scalaron domination to domination of the usual (relativistic) matter.

IV. After complete decay of scalaron we arrive to the cosmology governed by GR.
Cosmological energy density for different decay channels

Scalaron decay into 2 massless scalars minimally coupled to gravity

\[ \Gamma_s = \frac{m_R^3}{24 m_{Pl}^2}, \quad \varrho_s = \frac{m_R^3}{120 \pi t} \]

Scalaron decay into pair of fermions or conformally coupled scalars with mass \( m_f \):

\[ \Gamma_f = \frac{m_R m_f^2}{24 m_{Pl}^2}, \quad \varrho_f = \frac{m_R m_f^2}{120 \pi t} \]

Scalaron decay induced by the conformal anomaly:

\[ \Gamma_{an} = \frac{\beta_1^2 \alpha^2 N}{96 \pi^2} m_R^3, \quad \varrho_{an} = \frac{\beta_1^2 \alpha^2 N}{4 \pi^2} \frac{m_R^3}{120 \pi t} \]

\( \beta_1 \) is the first coefficient of the beta-function, \( N \) is the rank of the gauge group \( \alpha \) is the gauge coupling constant (at high energies it depends upon the theory).

Much slower decrease of the energy density of matter than normally for relativistic matter is ensured by the influx of energy from the scalaron decay.

- Normally for relativistic matter: \( \varrho \sim 1/a^4(t) \sim 1/t^{8/3} \), since \( a(t) \sim t^{(2/3)} \) at SD.
Connection of the temperature with time: $GR \iff R^2$

In thermalized plasma with $\varrho_{\text{therm}} = \pi^2 g_* T^4 / 30$

$$\varrho_{GR} = \frac{3m_{Pl}^2}{32\pi t^2} = \frac{\pi^2 g_* T^4}{30} \implies (tT^2)_{GR} = \left(\frac{90}{32\pi^3 g_*}\right)^{1/2}$$

- $g_*$ is the number of relativistic species in the plasma, $g_* \sim 100$.

$R^2$-theory:

$$\varrho_s = \frac{m_R^3}{120\pi t} = \frac{\pi^2 g_* T^4}{30} \implies (tT^4)_s = \frac{m_R^3}{4\pi^3 g_*} = \text{const}$$

$$\varrho_{an} = 2.6 \cdot 10^{-2} \alpha_R^2 \frac{M_R^3}{t} \implies (tT^4)_{an} = 0.78 \frac{\alpha_R^2 m_R^3}{\pi^2 g_*} = \text{const}$$

- The coupling $\alpha_R$ is taken at the energies equal to the scalaron mass.

Correspondingly

$$\left(\frac{\dot{T}}{T}\right)_{GR} = -\frac{1}{2t} \quad \left(\frac{\dot{T}}{T}\right)_{s;an} = -\frac{1}{4t}$$
Evolution of $X$-particles in thermal plasma

Freezing of massive species $X \implies$ Zeldovich Eq., 1965 (Lee-Weinberg, 1977):

$$\dot{n}_X + 3Hn_X = -\langle \sigma_{ann} v \rangle (n_X^2 - n_{eq}^2), \quad n_{eq} = g_s \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

- $\langle \sigma_{ann} v \rangle$ is the thermally averaged annihilation cross-section of $X$-particles
- $n_{eq}$ is their equilibrium number density, $g_s$ is the number of spin states.

For annihilation of the non-relativistic particles:

$$\langle \sigma_{ann} v \rangle = \sigma_{ann} v = \frac{\alpha^2 \beta_{ann}}{M_X^2} \quad \text{(S-wave)},$$

$$\langle \sigma_{ann} v \rangle = \frac{\pi \alpha^2 \beta_{ann}}{M_X^2} \frac{T}{M_X} \quad \text{(P-wave, Majorana fermions)}$$

- $M_X$ is a mass of $X$-particle, $\alpha$ is a coupling constant, in SUSY theories $\alpha \sim 0.01$
- $\beta_{ann}$ is a numerical parameter $\sim$ the number of annihilation channels, $\beta \sim 10$.

We assume that direct $X$-particle production by $R(t)$ is suppressed in comparison with inverse annihilation of light particles into $X\bar{X}$-pair.
Some comments

Two possible channels to produce massive stable $X$-particles:
- Directly through the scalaron decay into a pair $X\bar{X}$,
- By inverse annihilation of relativistic particles in thermal plasma.

Direct production of $X\bar{X}$-pair by scalaron gives

$$\rho_X^{(0)} \approx \rho_{DM} \approx 1\text{keV/cm}^3, \text{ if } M_X \approx 10^7 \text{ GeV}$$

"Catch-22":
- For such small mass thermal production results in too large $\rho_X$.
- For larger masses $\rho_X^{(0)}$ would be unacceptably larger than $\rho_{DM}$.

A possible way out:
- Since oscillating curvature scalar creates particles only in symmetric state, the direct production of $X$-particles is forbidden, if they are Majorana fermions, which must be in antisymmetric state.
Scalaron decay into massless non-conformal scalars

Dimensionless Zeldovich equation

\[
\frac{df}{dx} = -\frac{0.12g_s\alpha^2\beta_{ann}}{\pi^3g_*} \left( \frac{m_R}{M_X} \right)^3 \frac{f^2 - f_{eq}^2}{x^5}, \quad n_X = n_{in} \left( \frac{a_{in}}{a} \right)^3 f, \quad x = \frac{M_X}{T}
\]

For \( g_* = 100, \alpha = 0.01, \beta_{ann} = 10, \) \( m_R = 3 \times 10^{13} \) GeV, and \( n_\gamma = 412/\text{cm}^3 \)

The present day energy density of the \( X \)-particles:

\[
\rho_X = M_X n_\gamma f_{\text{fin}} \approx 1.7 \times 10^8 \left( \frac{10^{10}\text{Gev}}{M_X} \right) \text{keV/cm}^3
\]

To compare with the observed energy density of DM: \( \rho_{DM} \approx 1 \) keV/cm\(^3\).

- \( X \)-particles must have huge mass \( M_X \gg m_R \) to make reasonable DM density.
- If \( M_X > m_R \), then classical scalaron field can still create \( X \)-particles, but the probability of their production would be strongly suppressed \( \Rightarrow \) such LSP with the mass somewhat larger than \( m_R \) could successfully make the cosmological DM.
Scalaron decay into fermions or massive conformal scalars

If bosons are conformally invariant due to non-minimal coupling to curvature \( \Rightarrow \) they are not produced if their mass is zero.

- The probability of production of both bosons and fermions \( \sim m^2_{\text{particle}} \)
- In what follows we confine ourselves to consideration of fermions only.

The width and the energy density of the scalaron decay into a pair of fermions:

\[
\Gamma_f = \frac{m_R m_f^2}{24 m_{Pl}^2}, \quad \varrho_f = \frac{m_R m_f^2}{120 \pi t}
\]

The largest contribution into the cosmological energy density at scalaron dominated regime is presented by the decay into the heaviest fermion species.

We assume:

- The mass of the LSP is considerably smaller than the masses of the other decay products, \( m_X \ll m_f \), at least as \( m_X \lesssim 0.1 m_f \).
- The direct production of \( X \)-particles by \( R(t) \) can be neglected.

In such a case LSPs are dominantly produced by the secondary reactions in the plasma, which was created by the scalaron production of heavier particles.
Kinetic equation for freezing of fermionic species

\[
\frac{df}{dx} = -\frac{\alpha^2 \beta_{ann}}{\pi^3 g_*} \frac{n_{in} m_R m_f^2}{m_X^6} \frac{f^2 - f_{eq}^2}{x^5}
\]

\(n_{in} = 0.09 g_s m_X^3\) is the initial number density of \(X\)-particles at \(T \sim m_X\).

\[\varrho_X = m_X n_\gamma \left( \frac{n_X}{n_{rel}} \right)_{\text{now}} = 7 \cdot 10^{-9} \frac{m_f^3}{m_X m_R} \text{ cm}^{-3}\]

- \(\alpha = 0.01\), \(\beta_{ann} = 10\), \(g_* = 100\), \(n_\gamma \approx 412/\text{cm}^3\), \(n_{rel} \approx \varrho_{rel} / 3T\)
- If we take \(m_f = 10^5\) GeV and \(m_X = 10^4\) GeV, then \(\varrho_X \ll \varrho_{DM}\).

\(\varrho_X\) becomes comparable with the energy density of the cosmological DM, \(\varrho_{DM} \approx 1\) keV/cm\(^3\), if \(m_X \sim 10^6\) GeV, \(m_f \sim 10^7\) GeV:

\[\varrho_X = 0.23 \left( \frac{m_f}{10^7\text{ GeV}} \right)^3 \left( \frac{10^6\text{ GeV}}{m_X} \right) \text{ keV/cm}^3\]
Scalaron decay into gauge bosons due to conformal anomaly

- $X, \bar{X}$ are Majorana fermions $\implies$ direct production by scalaron is forbidden.
- $X\bar{X}$-pairs are produced through the inverse annihilation of relativistic particles in the thermal plasma.

Log of the ratio of the energy density of $X$-particles to the observed energy density of DM as a function of $\mu = m_R/M_X$ calculated through the Zeldovich equation.

Log of the ratio of the energy density of $X$-particles to the observed energy density of DM as a function of $\mu = m_R/M_X$ calculated through the Zeldovich equation.

$X$-particles may be viable candidates for the carriers of the cosmological dark matter, if their mass $M_X \approx 5 \cdot 10^{12}$ GeV.
Possible observations

According to our results, the mass of DM particles, with the interaction strength typical for supersymmetric ones, can be in the range from $10^6$ to $10^{13}$ GeV.

Possibilities to make X-particles visible:


   the density of DM is many times higher than DM cosmological density.

2. The decay of superheavy DM particles, which could have a lifetime long enough to manifest themselves as stable DM, but at the same time lead to the possibly observable contribution to the UHECR spectrum.

3. Furthermore, instability of superheavy DM particles can arise due to Zeldovich mechanism through virtual black holes formation.
Conclusions

The existence of stable particles with interaction strength typical for SUSY and heavier than several TeV is in tension with the conventional Friedmann cosmology.

Starobinsky inflationary model opens a way to save life of such X-particles, because in this model the density of heavy relics could be significantly reduced.

If the epoch of the domination of the curvature oscillations (the scalaron domination) lasted after freezing of massive species, their density with respect to the plasma entropy could be noticeably diluted by radiation from the scalaron decay.

The range of allowed masses depends upon the dominant decay mode of scalaron.

- If the scalaron is minimally coupled to scalar particles $X_S$, the decay amplitude does not depend upon the scalar particle mass and an acceptably low density of $X_S$ can be achieved if $M_{X_S} \gtrsim m_R \approx 3 \cdot 10^{13}$ GeV.

- If the scalaron predominantly decays into fermions or conformally coupled scalars then the probability of the decay is proportional to the particle mass squared, and the allowed mass of $X$-particles to form DM could be $m_X \sim 10^6$ GeV.

- If the scalaron decays into gauge bosons due to conformal anomaly and $X$-particles are Majorana fermions, they are thermally produced in plasma and could make proper amount of DM, if their mass is about $5 \cdot 10^{12}$ GeV.
The END

Thank You for Your Attention