

# Cosmological baryon/lepton assymetry in terms of Kaluza-Klein extra dimensions

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5<sup>th</sup> International Conference on Particle Physics and Astrophysics  
October 5-9, 2020





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- 5 Momentum (lepton number) transfer from a scalar field to fermions
- 6 Conclusion





## Internal symmetries in common 4-dim theories

$$S[\phi(x) \rightarrow \hat{g} \phi(x)] = \text{inv}, \quad \hat{g} = e^{i\hat{t}\alpha}, \quad (1)$$

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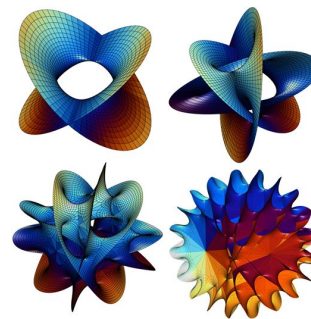
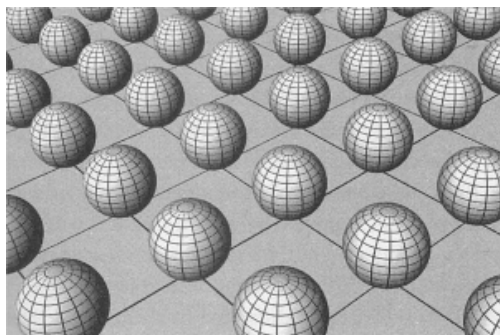
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## Conserved currents and numbers/charges in common 4-dim theories

$$\begin{aligned} \implies \quad \partial_\mu j^\mu &= 0, \quad j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \hat{t} \phi, \\ Q &= \int j^0 d^3x = \text{const}. \end{aligned} \quad (2)$$

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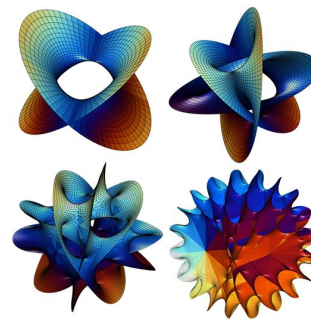
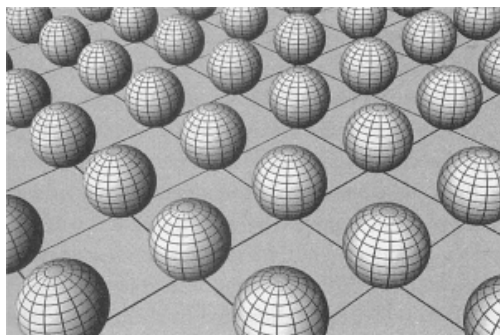


## Isometries of extra space

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + k_{mn}(y)dy^m dy^n, \quad (3)$$

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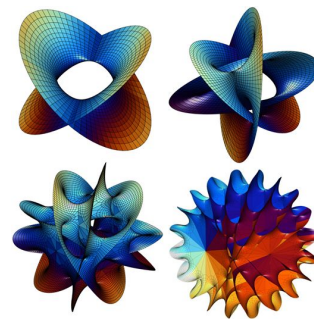
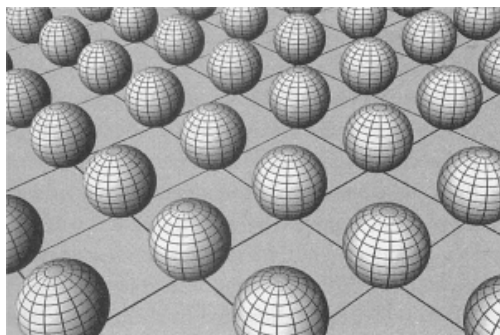
## Extra spatial symmetries in Kaluza-Klein theories

$$S[\Phi(x, y \rightarrow y')] = \text{inv}, \quad y'_m = y_m + \xi_m(y) \alpha, \quad (5)$$

$$\hat{g} = e^{i\alpha \xi_m \partial^m} \in G - \text{isometry group},$$

$$\Phi(x, y) = \phi_n(x) Y_n(y) - \text{extra coordinate decomposition}.$$

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In terms of higher-dim theory

$$\implies \quad \partial_a J^a = 0, \quad J^a = \frac{\partial \mathcal{L}}{\partial(\partial_a \Phi)} \xi^b \partial_b \Phi - \xi^a \mathcal{L}, \quad (6)$$

$$Q = \int J^0 \sqrt{|g|} \sqrt{|k|} d^3 x d^d y = \text{const}. \quad (7)$$



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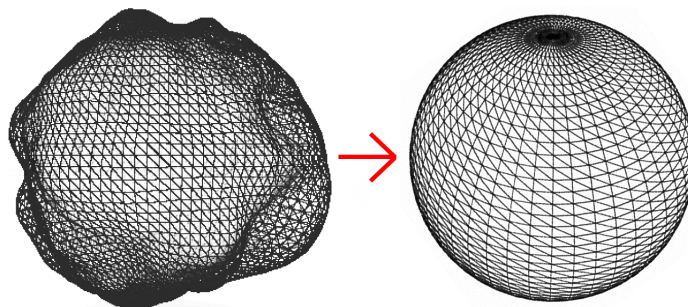
In terms of effective 4-dim theory

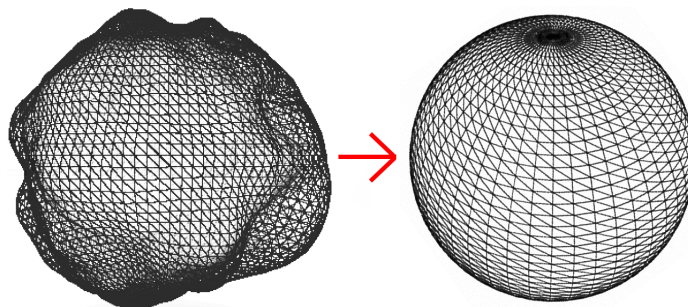
$$\implies \quad \partial_\alpha j^\alpha = 0, \quad j^\alpha = \frac{\partial \mathcal{L}_4}{\partial(\partial_\alpha \phi^n)} (t)_m^n \phi^m, \quad (8)$$

$$(t)_m^n = \int Y_n(\xi^a \partial_a) Y_m \sqrt{|k|} d^d y,$$

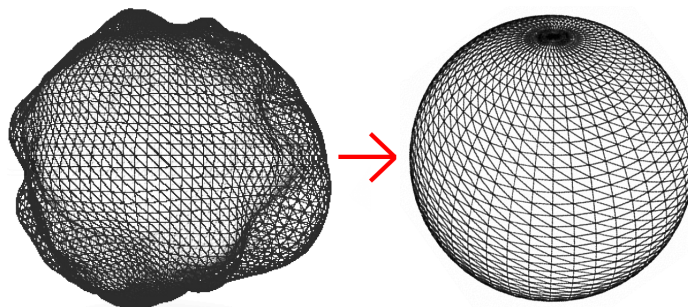
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# Violation of KK-symmetries in early Universe and lepton asymmetry

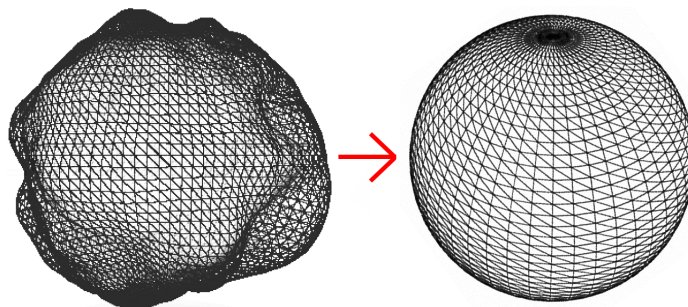




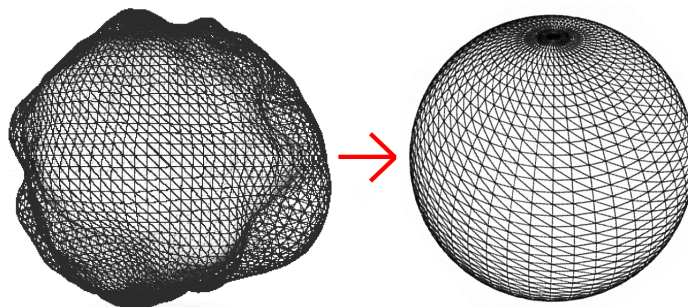
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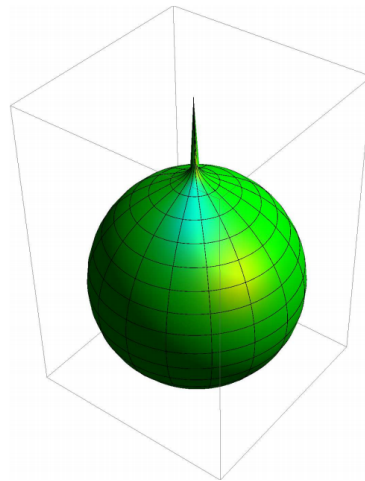
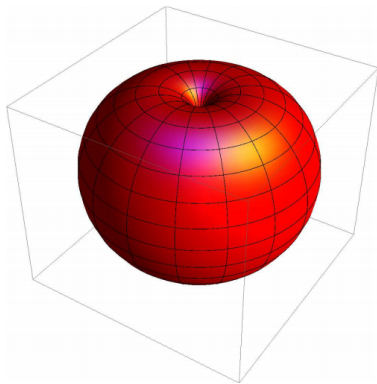


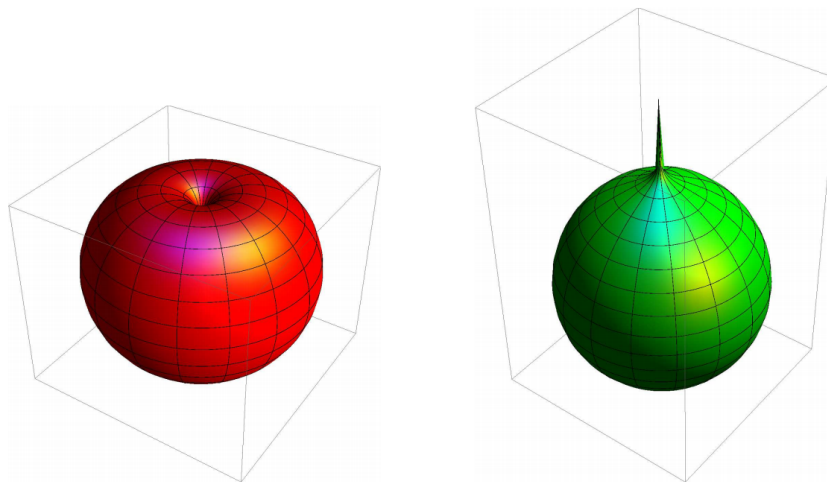
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- After the inflation (in low-energetic physics) relaxation of the metric occurs and symmetry is restored. The corresponding numbers are conserved again.

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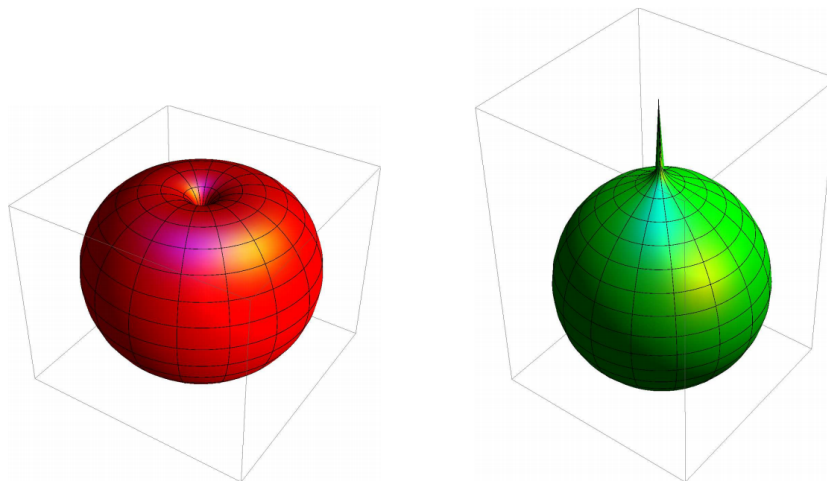


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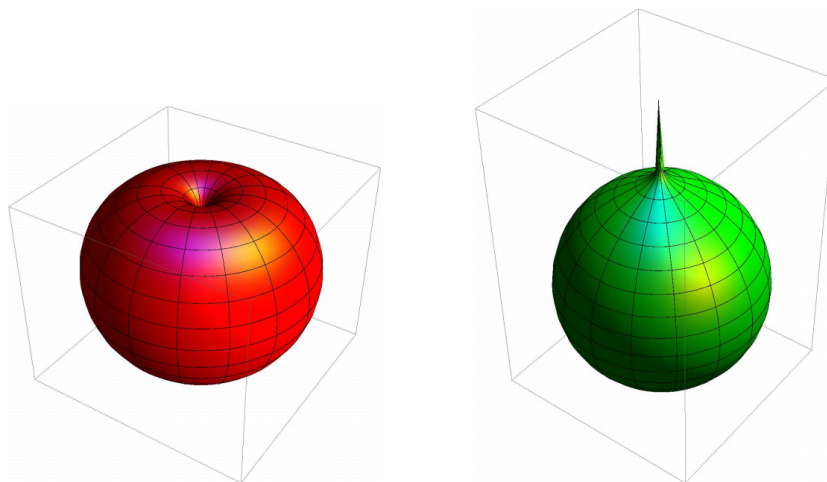




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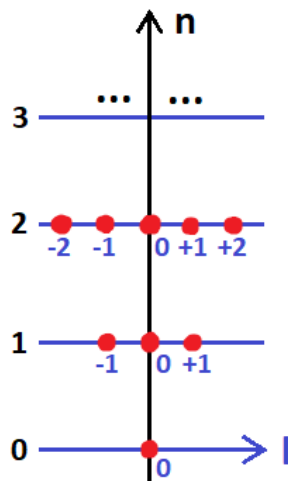


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- Such configurations considered in the framework of warped extra dimensions. It is developed in different works including ours: [arxiv:0706.0676](https://arxiv.org/abs/0706.0676), [arxiv:2006.01329](https://arxiv.org/abs/2006.01329), [arxiv:hep-th/0302067](https://arxiv.org/abs/hep-th/0302067), etc.

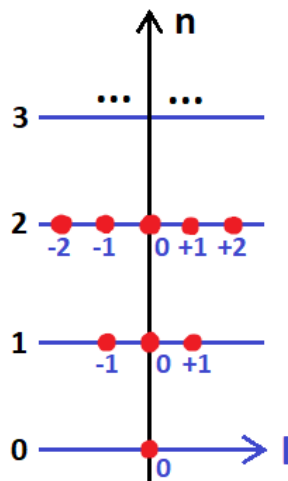


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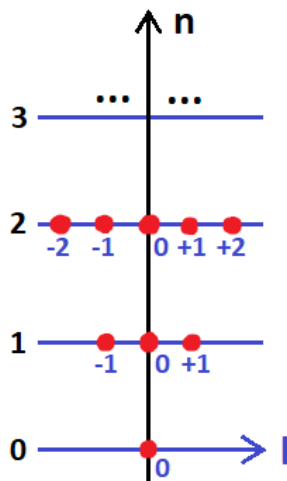
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- Field states  $\phi_{nl}$  with non-zero angular momentum  $l$  along the coordinate  $\varphi$  have a large mass  $M \sim n/r_0$ , with  $n > 0$ . It is into these states that the angular momentum will eventually pass.

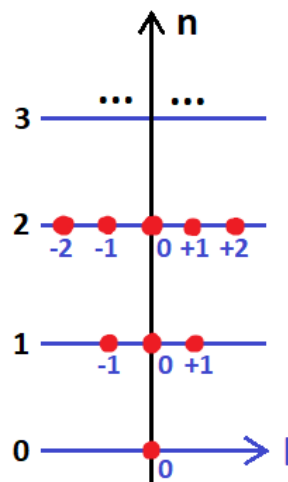


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- However, to observe low-energy lepton asymmetry, we need to have massless (ground-level  $n = 0$ ) states with a nonzero angular momentum  $l$ .
- The idea of possible such configurations is used in work [arxiv:0706.0676](https://arxiv.org/abs/0706.0676) for other purposes. It is based on compact extra spaces with an excess of angle.

# Splitting of the ground fermionic KK-level in spaces with angle excess

Angle profuse extra space

$$ds^2 = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu - r^2(\theta) (d\theta^2 + b^2 \sin^2 \theta d\varphi^2) , \quad (10)$$

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## Higher-dimensional fermionic action

$$S_\Psi = \int d^6 X \sqrt{|G|} i \bar{\Psi} h_{\tilde{A}}^B \Gamma^{\tilde{A}} \nabla_B \Psi, \quad (11)$$

where  $\Gamma^{\tilde{A}}$  is flat  $8 \times 8$  (in 6-dim) gamma matrices

$$\Gamma_\nu = \begin{pmatrix} \gamma_\nu & 0 \\ 0 & -\gamma_\nu \end{pmatrix}, \quad \Gamma_\theta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma_\varphi = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (12)$$

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- You can find a detailed derivation of fermion splitting in the paper [arxiv: 0706.0676](https://arxiv.org/abs/0706.0676).

## Kaluza-Klein decomposition

$$\Psi(x^A) = \sum_j Y_j(\theta, \varphi) \Psi_j(x) = \sum_{nl} e^{il\varphi} \begin{pmatrix} Y_{nl}^+(\theta) \psi_{nl}(x) \\ Y_{nl}^-(\theta) \xi_{nl}(x) \end{pmatrix}, \quad (14)$$

where the  $Y_{nl}^+$  and  $Y_{nl}^-$  is the eigenfunctions are computed from Dirac equation:

$$ih \frac{B}{A} \hat{\Gamma}^{\tilde{A}} \nabla_B \Psi = ih \frac{\nu}{\tilde{\mu}} \hat{\Gamma}^{\tilde{\mu}} \nabla_\nu \Psi + \underbrace{ih \frac{n}{\tilde{m}} \hat{\Gamma}^{\tilde{m}} \nabla_n}_{\text{mass operator}} \Psi = 0. \quad (15)$$

Limitation on the number  $l$  comes from the normalization condition:

$$\int |Y_{nl}|^2 \sqrt{|k|} d^2y < +\infty \quad \xRightarrow{n=0} \quad l = -[b/2], \dots, 0, \dots, +[b/2] \in \mathbb{Z}. \quad (16)$$

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## Low-energy (n=0) effective 4-dim action

$$S = \int d^6 X \sqrt{|G|} i \bar{\Psi} h \frac{B}{A} \hat{\Gamma}^{\tilde{A}} \nabla_B \Psi \sim \int \sqrt{-g} d^4 x \sum_{l=-1}^{+1} \left( i \bar{\psi}_l \gamma^\mu \partial_\mu \psi_l + i \bar{\xi}_l \gamma^\mu \partial_\mu \xi_l \right). \quad (17)$$



## Correspondence of currents

$$J_{\Psi}^m = \frac{\partial \mathcal{L}}{\partial (\partial_m \Psi)} \partial_\varphi \Psi = i \bar{\Psi} h_{\tilde{A}}^m \Gamma^{\tilde{A}} \partial_\varphi \Psi \implies$$
$$j_{\psi}^{\mu} = \frac{\partial \mathcal{L}_4}{\partial (\partial_\mu \psi_l)} t_{ll'} \psi_{l'} = i \sum_l l \bar{\psi}_l \gamma^\mu \psi_l, \quad t_{ll'} = \int Y_l \partial_\varphi Y_{l'} \sqrt{k} d^2 y = i l \delta_{ll'}, \quad (18)$$

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## U(1)-number

Take the angle profuse parameter  $b = 4$  for example.  
Then we have triplet splitting:  $l = -1, 0, +1$ .

$$Q_{\Psi} = \int J_{\Psi}^0 \sqrt{|G|} d^3 x d^2 y = \int j_{\psi}^0 \sqrt{|g|} d^3 x =$$
$$= \int i (\psi_{+1}^{\dagger} \psi_{+1} - \psi_{-1}^{\dagger} \psi_{-1}) d^3 x = N_{\psi_{+1}} - N_{\psi_{-1}} = \text{const}. \quad (19)$$

## Scalar field action and interaction

The Lagrangian of the scalar field and its interaction with the fermion field can be chosen as simple as possible:

$$S_{\Phi} + S_{\text{int}} = \int d^6x \sqrt{|G|} \left[ \frac{1}{2} \partial_M \Phi \partial^M \Phi + f \Phi \bar{\Psi} \Psi \right] \xrightarrow{\int d^2y} \quad (20)$$

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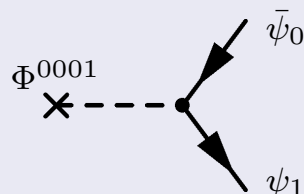
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## Internal momentum (leptonic number) conservation

$$Q = \int J^0 \sqrt{|G|} d^3x d^2y = Q_{\Psi} + Q_{\Phi} = N_{\psi_{+1}} - N_{\psi_{-1}} + Q_{\Phi} = \text{const}. \quad (22)$$



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- The accumulated charges (numbers) are then conserved due to symmetry restoration. However, in the usual case, the charge is realized in massive KK-modes with  $l \neq 0$ , which is stable due to the momentum conservation. Such a mechanism is suitable for generating stable massive dark matter particles, but is not suitable for explaining the low-energetic lepton assymetry.



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- But this charge cannot accumulate in the fermion field. Due to Fermi-Dirac statistics, fermions cannot exist in the form of coherent oscillations in which charge (number) is accumulated. Therefore, we need to introduce an interaction with a scalar field. Then, the condensate of the scalar field carrying the internal momentum (lepton number) will decay into leptons, forming a lepton asymmetry.

Thanks for you attention!

Any questions?