The strongly intensive observable in pp collisions at LHC energies in the string fusion model

V.V. Vechernin, S.N. Belokurova

St.Petersburg State University

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Physical motivation. String fusion effects.

pp → pA → AA - the increase of the string density in transverse plain leads to the string fusion (color ropes or string cluster formation)

T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B **245**, 449 (1984)
A. Bialas, W. Czyz, Nucl. Phys. B **267**, 242 (1986)
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993)

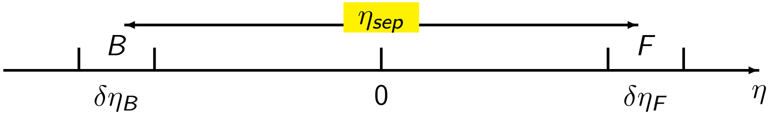
The same with increasing energy and centrality of pp collisions

LHC !

⇒ Reduction of multiplicity, increase of transverse momenta.
⇒ The influence on the Long-Range FB Correlations (LRC).
N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).

The same ideas in DIPSY:
C. Bierlich, G. Gustafson, L. Lonnblad, A. Tarasov *JHEP* **03** (2015) 148

Forward-Backward (FB) Rapidity Correlations



Forward-Backward (FB) Rapidity Correlations: $(k_z, \mathbf{k}_\perp) \Rightarrow (\eta, \mathbf{k}_\perp)$

$$\eta \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z} , \qquad \eta' \equiv \frac{1}{2} \ln \frac{|\mathbf{k}| + k_z}{|\mathbf{k}| - k_z} = - \ln \mathrm{tg} \left(\frac{\theta^*}{2} \right)$$

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F, B)}{D_F} \qquad (1)$$

Short- and long-range rapidity correlations

Traditional Observables

Traditional FB correlation:

B, $F \Rightarrow n_B$, n_F - the extensive variables $\Rightarrow b_{nn}$

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

The locality of strong interaction in rapidity $\Rightarrow$

Short-Range FB Correlations (SRC),

between particles from a same source (string).

$z - \eta$ correspondence, *X.Artru,Phys.Rept.***97***(1983)147,*

V.V.V.,arXiv:0812.0604

Event-by-event variance in the number of cut pomerons (strings) $\Rightarrow$

Long-Range FB Correlations (LRC) at large η_{sep}

(the trivial "volume" fluctuations).

We'll look for observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation in the quality of sources (e.g. to the formation of string clusters by string fusion).

Advanced Observables

The string fusion processes under consideration affects both LRC and SRC.
The LRC is sensitive to fluctuations in both quantity and type of source.
The SRC is sensitive to the modification of the properties of a single string in process of their fusion into string clusters.

Unfortunately for traditional observables
the n_F-n_B correlation is strongly influenced by the "volume" fluctuations.

We can suppress the influence of these trivial "volume" fluctuations compared to the contribution of string fusion processes:

1) for SRC going from b_{nn} to more sophisticated correlation observables, e.g. to the strongly intensive observable $\Sigma(n_F, n_B)$ (see e.g. [E. Andronov, V.V., Eur.Phys.J.A 55(2019)14, V.V., EPJ Web Conf. 191(2018)04011]).

2) for LRC going from traditional extensive variables n_F and n_B to new intensive ones, e.g. event-mean transverse momenta p_F and p_B of all particles (n_F and n_B) in the intervals $\delta\eta_F$ and $\delta\eta_B$ (see e.g. [V.V., EPJ Web of Conf. 125, 04022 (2016)]).

The strongly intensive observable $\Sigma(n_F, n_B)$

The strongly intensive quantities
[*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*].
We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows
[*E.V.Andronov,Theor.Math.Phys.185(2015)1383*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F\rangle + \langle n_B\rangle}[\langle n_F\rangle\, \omega_{n_B} + \langle n_B\rangle\, \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)]\ , \qquad (2)$$

where

$$\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle\ , \qquad (3)$$

and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n\rangle} = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}\ . \qquad (4)$$

$\Sigma(n_F, n_B)$ for symmetric reaction and symmetric windows

For symmetric reaction and symmetric observation windows $\delta\eta_F = \delta\eta_B = \delta\eta$:

$$\langle n_F \rangle = \langle n_B \rangle \equiv \langle n \rangle \ , \qquad \omega_{n_F} = \omega_{n_B} \equiv \omega_n \tag{5}$$

and

$$\Sigma(n_F, n_B) = \omega_n - \mathrm{cov}(n_F, n_B)/\langle n \rangle = \frac{\langle n^2 \rangle - \langle n_F n_B \rangle}{\langle n \rangle} =$$

$$= \frac{D_n - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} = \frac{\mathrm{cov}(n_F, n_F) - \mathrm{cov}(n_F, n_B)}{\langle n \rangle} \ . \tag{6}$$

Connection with FBC coefficient b_{nn}:

$$\Sigma(n_F, n_B) = \omega_n \, (1 - b_{nn}) \tag{7}$$

Two-particle correlation function C_2

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ ,$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{d^2 N_{ch}}{d\eta_1 \, d\eta_2}$$

This enables to extract from experimental data the absolute value of the $C_2(\eta_1, \eta_2)$ without any commonly used event mixing procedure, including the cases when a translation invariance in rapidity is absent.

If the translation invariance in rapidity is valid
(symmetric reaction, mid-rapidities, LHC energies) then:

$$\rho(\eta) = \rho_0 = const \ , \qquad \rho_2(\eta_1, \eta_2) = \rho_2(\eta_1 - \eta_2) = \rho_2(\Delta\eta)$$

$$C_2(\eta_1, \eta_2) = C_2(\Delta\eta) = \rho_2(\Delta\eta)/\rho_0^2 - 1 \ .$$

The b_{nn} in the model with identical strings

For FBC coefficient b_{nn} we have [V.V.,Nucl.Phys.A939(2015)21] (regardless of a model):

$$b_{nn} = \frac{\langle n\rangle \mathrm{cov}(n_F, n_B)}{\omega_n} = \frac{\langle n\rangle\, I_{FB}}{1+\langle n\rangle\, I_{FF}} \quad \to \frac{\langle n\rangle\, C_2(\eta_{sep})}{1+\langle n\rangle\, C_2(0)}$$

where

$$I_{FF} = \frac{1}{\delta\eta_F^2}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\ C_2(\eta_1-\eta_2) \qquad \to C_2(0)$$

$$I_{FB} = \frac{1}{\delta\eta_F\delta\eta_B}\int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2\ C_2(\eta_1-\eta_2) \qquad \to C_2(\eta_{sep})$$

The last limit is valid for the small windows: $\delta\eta_F=\delta\eta_B=\delta\eta \ll \eta_{corr}$. Then in the model we have

$$b_{nn} = \frac{\langle n\rangle\, C_2(\Delta\eta)}{1+\langle n\rangle\, C_2(0)} = \frac{\mu_0\delta\eta\,[\omega_N+\Lambda(\Delta\eta)]}{1+\mu_0\delta\eta\,[\omega_N+\Lambda(0)]}$$

$\Sigma(n_F, n_B)$ through two-particle correlation functions

Again regardless of a model:

$$\Sigma(n_F, n_B) = 1 + \langle n\rangle[I_{FF} - I_{FB}] \to 1 + \langle n\rangle[C_2(0) - C_2(\eta_{sep})]$$

The last limit is valid for the small windows: $\delta\eta_F = \delta\eta_B = \delta\eta \ll \eta_{corr}$.
But now using [V.V.,Nucl.Phys.A939(2015)21]

$$C_2(\Delta\eta) = \frac{\omega_N + \Lambda(\Delta\eta)}{\langle N\rangle}$$

we have [V.V., EPJ Web Conf. 191(2018)04011]:

$$\Sigma(\Delta\eta) = 1 + \mu_0\delta\eta[\Lambda(0) - \Lambda(\Delta\eta)]$$

We see that in model with identical strings the $\Sigma(\Delta\eta)$ really is strongly intensive quantity.
It does not depend on $\langle N\rangle$ and ω_N. It depends ONLY on string parameters:

$$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B)$$

Properties of Σ in model with independent identical strings

$$\Sigma(\Delta\eta) = 1 + \mu_0 \delta\eta [\Lambda(0) - \Lambda(\Delta\eta)]$$

The $\Sigma(0) = 1$ and increases with the gap between windows, $\Delta\eta$,
because the $\Lambda(\Delta\eta)$ decrease with $\Delta\eta$,
as the correlations in string go off with increase of $\Delta\eta$.

The rate of the $\Sigma(\Delta\eta)$ growth with $\Delta\eta$ is proportional
to the width of the observation window $\delta\eta$ and μ_0 -
- the multiplicity produced from one string.

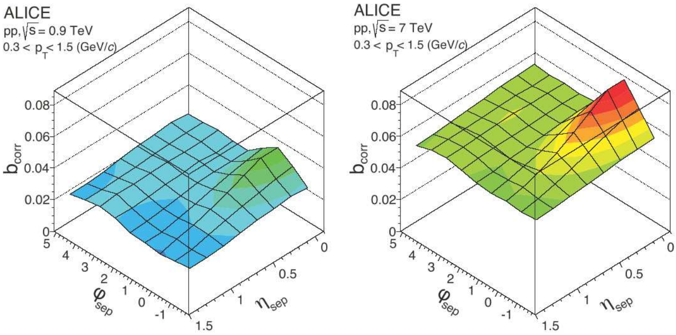
The model predicts saturation of the $\Sigma(\Delta\eta)$ on the level

$$\Sigma(\Delta\eta) = 1 + \mu_0 \delta\eta \, \Lambda(0) = \omega_\mu$$

at large $\Delta\eta$, as $\Lambda(\Delta\eta) \to 0$ at the $\Delta\eta \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

The ALICE data on b_{nn} in pp

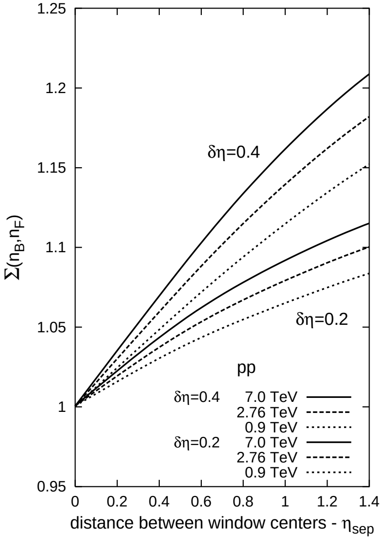
ALICE collab., JHEP 05(2015)097



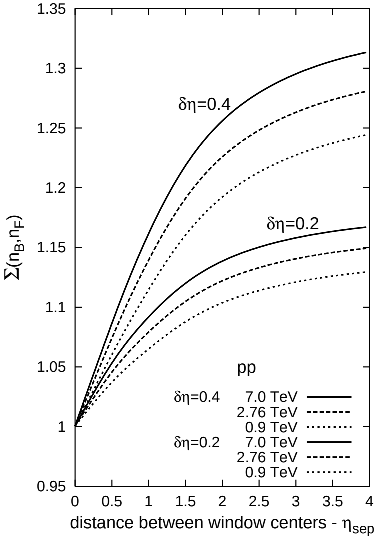
$$\Rightarrow \Lambda(\Delta\eta)$$

V.V.,Nucl.Phys.A939(2015)21; V.V., EPJ Web Conf. 191(2018)04011.
E. Andronov, V.V., Eur.Phys.J.A 55(2019)14,

Σ for 2π azimuth windows



in ALICE TPC acceptance



in wider pseudorapidity range

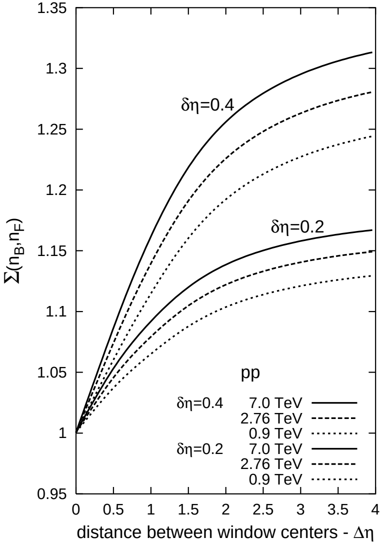
$\Sigma(n_F, n_B)$ in the model with string fusion

In the model with string fusion on transverse grid we find [*S.N. Belokurova, V.V.V., Theor.Math.Phys. 200(2019)1094*]:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \; , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} \; , \tag{8}$$

where k is a degree of string overlapping and $\langle n^{(k)} \rangle$ is a mean number of particles produced from areas with such overlapping. $\sum \alpha_k = 1$.

The similar result was obtained in the model with two types of string in [*E.V.Andronov,Theor.Math.Phys.185(2015)1383*] for the long-range part of $\Sigma(n_F, n_B)$, when at $\Delta\eta \gg \eta_{corr}$ we have $\Sigma_k(\mu_F, \mu_B) = \omega_\mu^{(k)}$ with $k = 1,2$.



Increase of the string cluster contribution to $\Sigma(n_F, n_B)$ with collision energy in pp collisions

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} ,$$

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

$$\Sigma(n_F, n_B) = 1 + \delta\eta \times$$

$$\times \sum_{k=1}^{\infty} \alpha_k \, \mu_0^{(k)} \Lambda_0^{(k)} [1 - \exp\left(-|\Delta\eta| / \eta_{corr}^{(k)}\right)]$$

$\Sigma(n_F, n_B)$ in the model with string fusion

Monte Carlo simulation of the weighting factors α_k as a function of centrality and initial energy of pp collision.

Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes.

V.V., I. Lakomov.
Proceedings of Science (Baldin ISHEPP XXI) (2013) 072).

Distribution of strings in the transverse plane

pp interactions

$$w_{str}(\vec{s},\vec{b}) \sim T(\vec{s}-\vec{b}/2)\,T(\vec{s}+\vec{b}/2)/\sigma_{pp}(b) \qquad (9)$$

$\sigma_{pp} = \int \sigma_{pp}(b)\ d^2\vec{b}$ - non-diffractive pp cross section

$T(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho(\vec{s},z)\ dz$ - parton profile function of nucleon

$$\rho(r) = \frac{1}{\pi^{3/2}\alpha^3} e^{-r^2/\alpha^2}\ , \qquad T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\ , \qquad (10)$$

$$w_{str}(\vec{s},\vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2}/\sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

b–s factorization $\Rightarrow$

$$\langle N_{str}(b)\rangle \sim e^{-b^2/2\alpha^2}/\sigma_{pp}(b) \qquad (11)$$

Dependence of the average number of cut pomerons on the impact parameter in non-diffractive *pp* collisions

$N_{str} = 2N, \quad N$ - the number of cut pomerons in a given event

$$\langle N(b) \rangle \sim e^{-b^2/2\alpha^2} / \sigma_{pp}(b)$$

Event-by-event fluctuations of the number of cut pomerons

$$\boxed{P(N,b)=e^{-\overline{N}(b)}\overline{N}(b)^N/N!}\text{ -Poisson,}\qquad P(0,b)=e^{-\overline{N}(b)}$$

$$\boxed{\widetilde{P}(N,b)=P(N,b)/[1-P(0,b)]}\text{ -modified Poisson,}\quad \sum_{N=1}\widetilde{P}(N,b)=1$$

$$\langle N(b)\rangle=\sum_{N=1}N\,\widetilde{P}(N,b)=\overline{N}(b)/[1-P(0,b)] \qquad (12)$$

$$\sigma_{pp}^{ND}(b)=1-P(0,b)=1-e^{-\overline{N}(b)} \qquad (13)$$

$$\langle N(b)\rangle=\overline{N}(b)/\sigma_{pp}^{ND}(b)$$

$$\boxed{\overline{N}(b)=N_0e^{-b^2/2\alpha^2}}\qquad \boxed{\langle N(b)\rangle=\overline{N}(b)/[1-\exp(-\overline{N}(b))]}$$

Probability to have N cut pomerons in a non-diffractive pp collision

At an integer N it can be reduced to

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N} \left[1 - e^{-N_0} \sum_{k=0}^{N-1} N_0^k / k!\right] = \frac{\sigma_N}{\sigma_{pp}^{ND}}$$

where we have introduced the σ_N by

$$\sigma_N \equiv \frac{2\pi\alpha^2}{N} \left[1 - e^{-N_0} \sum_{k=0}^{N-1} N_0^k / k!\right]$$

$$\sum_{N=1}^{\infty} \sigma_N = 2\pi\alpha^2 [E_1(N_0) + \gamma + \ln N_0] = \sigma_{pp}^{ND}$$

where σ_{pp}^{ND} is the non-diffractive pp cross section.

$$E_1(z) = \int_1^{\infty} e^{-zt} \frac{dt}{t} \,, \qquad \gamma = 0.577...$$

Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the σ_N coincides with the well known result for the cross-section σ_N of N cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches:

$$\sigma_N = \frac{4\pi\lambda}{C\,N}\left[1 - e^{-z}\sum_{k=0}^{N-1} z^k/k!\right]$$

where

$$z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \qquad \lambda = R^2 + \alpha'\xi\ , \qquad \xi = \ln(s/1GeV^2)\ .$$

Here Δ and α' are the residue and the slope of the pomeron trajectory. The parameters γ and R characterize the coupling of the pomeron trajectory with initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons.

K.A. Ter-Martirosyan Phys. Lett. B **44**, 377 (1973).

A.B. Kaidalov, K.A. Ter-Martirosyan Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984).

V.A. Abramovsky, V.N. Gribov, O.V. Kancheli Yad. Fiz. **18**, 595 (1973).

Comparison with the quasi-eikonal and Regge approaches

This enables to connect the parameters N_0 and α of our model with the parameters of the pomeron trajectory and its couplings to hadrons.
Comparing we have

$$N_0 = z = \frac{2\gamma C}{\lambda} \exp(\Delta\xi) \ , \quad \alpha = \sqrt{\frac{2\lambda}{C}} \ , \quad \lambda = R^2 + \alpha'\xi \qquad (14)$$

The numerical values of the parameters in the paper:
G.H.Arakelyan,A.Capella,A.B.Kaidalov,Yu.M.Shabelski Eur.Phys.J.C**26**,81(2002)

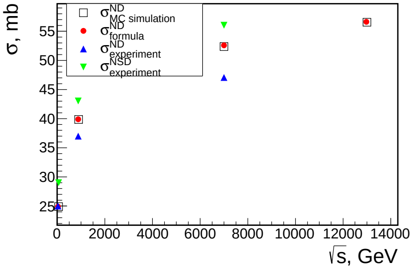
$$\Delta = 0.139, \quad \alpha' = 0.21 \ GeV^{-2},$$
$$\gamma_{pp} = 1.77 \ GeV^{-2}, \quad R^2 = 3.18 \ GeV^{-2}, \quad C = 1.5 \ ,$$

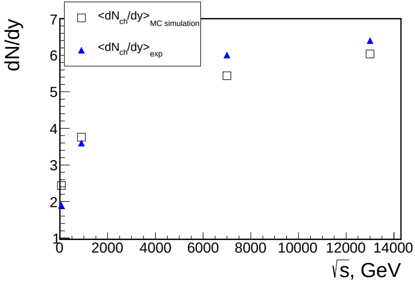
Our values of the parameters:

$$\Delta = 0.2, \quad \alpha' = 0.05 \ GeV^{-2},$$
$$\gamma_{pp} = 1.035 \ GeV^{-2}, \quad R^2 = 3.3 \ GeV^{-2}, \quad C = 1.5 \ .$$

Soft and Hard Pomeron:
J. Bleibel, L.V. Bravina, E.E. Zabrodin. Phys. Rev. D 93, 114012 (2016)

Fitting the parameters of the initial string distribution in the impact parameter plane of pp collisions





$$\sigma^{ND}_{MC\ \ simulations} = \frac{n_{sim}(N{=}0)}{n_{sim}(N{\geq}0)} S_b$$

$$\mu_0^{(k)} = \mu_0^{(1)} \sqrt{k} \text{ with } \mu_0^1 = 0.7$$

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$\langle n\rangle_k = \mu_0\sqrt{k}\ S_k/\sigma_0$, $\langle p_t^2\rangle_k = p_0^2\sqrt{k}$, $k = 1, 2, 3, ...$

global fusion (clusters)

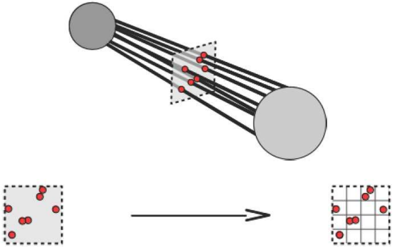
M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}$, $\langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0$, $k_{cl} = k\sigma_0/S_{cl}$

the version of SFM with the finite lattice (grid) in transverse plane

V.V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., V.V.V., Eur.Phys.J. C32 (2004) 535



Domains in transverse area

The approach with string fusion on a transverse lattice (grid) was exploited later for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions in

ALICE collaboration et al., J. Phys. G **32** 1295 (2006), [Sect. 6.5.15]
V.V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858
M.A. Braun, C. Pajares, Eur. Phys. J. C **71**, 1558 (2011)
M.A. Braun, C. Pajares, V.V.V., Nucl. Phys. A **906**, 14 (2013)
V.N. Kovalenko, Phys. Atom. Nucl. **76**, 1189 (2013)
M.A. Braun, C. Pajares, V.V.V., Eur. Phys. J. A **51**, 44 (2015)
V.V.V., Theor. Math. Phys. 184 (2015) 1271
V.V.V., Theor. Math. Phys. 190 (2017) 251

It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.

What was also considered in the CGC approach
A.Kovner., M. Lublinsky, Phys.Rev. D **83**, 034017 (2011)

$\Sigma(n_F, n_B)$ in the model with string fusion

The comparison of the model results with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV enables evaluate the increase of the string cluster contribution and their characteristics:

$$\mu_0^{(k)} = \mu_0^{(1)}\sqrt{k}\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)} = const\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)}/\sqrt{k}\ ,$$

with $\mu_0^1 = 0.7$, $\Lambda_0^{(1)} = 0.8$, $\eta_{corr}^{(1)} = 2.7$, compared with

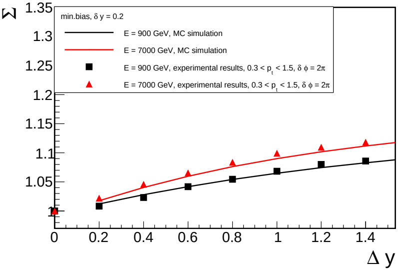
$$\mu_0^{(k)} = \mu_0^{(1)}\, k\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)}/k\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)} = const\ .$$

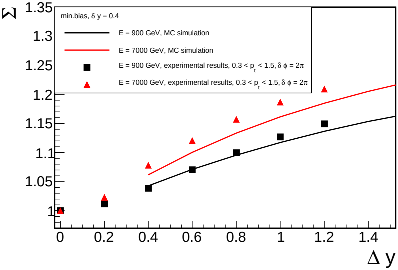
for the case without string fusion in the given transverse cell.

(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on the energy of pp collisions.)

$\Sigma(n_F, n_B)$ in the model with string fusion

The comparison of the model results with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV.





The points were obtained by Andrey Erokhin (St.Petersburg State Univ.) from ALICE data for charge particles with transverse momenta in the interval $0.3 < p_T < 1.5$ GeV (work in progress).

Conclusions

The influence of the formation of string clusters on the $\Sigma(n_F, n_B)$:

- The observable $\Sigma(n_F, n_B)$ loses the strongly intensive property, as it becomes equal to the weighted average of its values for different string clusters with the weights depending on collision conditions (the initial energy and centrality).
- Nevertheless it can be used for the extraction of the information on the properties of the string clusters (the multiplicity density and the pair correlation function of paricles, produced from a cluster decay).
- The increase of the $\Sigma(n_F, n_B)$ in pp collisions with energy can be explained by the increasing role of string fusion processes and the formation of string clusters with new properties.
- The modification of the $\Sigma(n_F, n_B)$ with the collision centrality also can be explained by the string fusion processes in the framework of the same approach (work in progress).

The research was supported by the Russian Foundation for Basic Research grant (No. 18-02-40075)

Backup

Backup slides

Two-particle correlation function of a string

We can introduce the string two-particle correlation function, $\Lambda(\eta_1, \eta_2)$, characterizing correlation between particles, produced from one string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 \ .$$

In model with identical strings we have the following connection:

$$C_2(\eta_1, \eta_2) = \frac{\omega_N + \Lambda(\eta_1, \eta_2)}{\langle N \rangle}$$

[*V.V., Nucl.Phys.A939(2015)21*]. (Note that one often looses the constant part $\omega_N / \langle N \rangle$ of C_2, using di-hadron correlation approach.)
At midrapidities, we have uniform rapidity distribution ($\lambda(\eta) = \mu_0 \delta\eta$, $\rho(\eta) = \rho_0 = \langle N \rangle \mu_0 \delta\eta$) and the correlation functions depending only on a difference of rapidities: $\eta_{sep} = \eta_1 - \eta_2 = \Delta\eta$.
We have also

$$\Lambda(\Delta\eta) \to 0, \text{ when } \Delta\eta \gg \eta_{corr} \ ,$$

where the η_{corr} is the correlation length.

The parametrization of the single correlation function

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V.,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} \ . \qquad (15)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (15)

$$|\varphi| \leq \pi \ . \qquad (16)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (16) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (17)$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

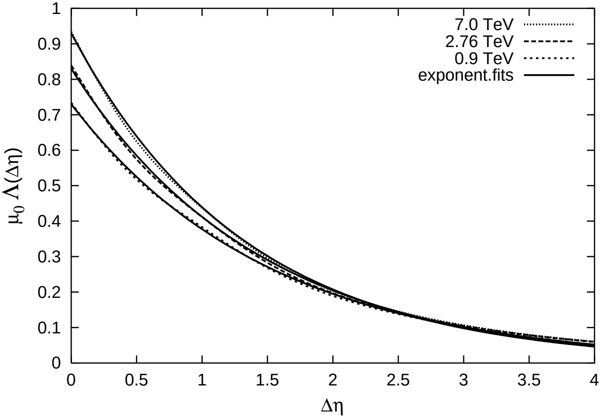
$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.

[V.V.,Nucl.Phys.A939(2015)21]

The stringt correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^{\pi} \Lambda(\eta_{sep}, \phi_{sep})\, d\phi_{sep} \ .$$



The string correlation function $\Lambda(\Delta\eta)$

The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} , \tag{18}$$

with the parameters presented in the table:

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

[V.V., EPJ Web Conf. 191(2018)04011]

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.

This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions.

$\Sigma(n_F, n_B)$ in the model with string fusion

The same value of $\Sigma(n_F, n_B)$ in AA collisions, as in pp,
if we suppose the formation of the same strings in AA and pp collisions.
Because the $\Sigma(n_F, n_B)$ does not depends on the mean value, $\langle N \rangle$,
and the event-by-event fluctuations, ω_N, in the number of strings.
It depends only on the string characteristics.

If we suppose the formation of string clusters in AA collisions
(and in central pp collisions at high energy) with some new characteristics,
due to e.g. string fusion processes, then for a source with k fused strings

$$\Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)]$$

For these fused strings we expect, basing on the string decay picture
[*V.V.*, Baldin ISHEPP XIX v.1(2008)276; arXiv:0812.0604]:
1) larger multiplicity from one string, $\mu_0^{(k)} > \mu_0$,
2) smaller correlation length, $\eta_{corr}^{(k)} < \eta_{corr}$.