Equivalent photons approximation: survival factor

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Ultraperipheral collisions

\[
(0, q_x, q_y, q_z) \xrightarrow{\gamma \gg 1} \left( \sqrt{\gamma^2 - 1} q_z, q_x, q_y, \gamma q_z \right)
\]

Photon virtuality: 

\[
-q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2
\]
UPC spectrum and cross section

With the sizes of the colliding particles neglected:

\[ n(\omega) = \frac{2Z^2\alpha}{\pi \omega} \int_0^\infty \left[ \frac{F(\sqrt{q_\perp^2 + (\omega/\gamma)^2})}{q_\perp^2 + (\omega/\gamma)^2} \right]^2 q_\perp^3 dq_\perp, \]

\[ \sigma(AB \rightarrow ABX) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sigma(\gamma\gamma \rightarrow X) n_A(\omega_1) n_B(\omega_2). \]

With the sizes of the colliding particles taken into account:

\[ n(\omega) = \int n(b, \omega) d^2b, \quad n(b, \omega) = \frac{Z^2\alpha}{\pi^2 \omega} \left[ \int_0^\infty \frac{F(\sqrt{q_\perp^2 + (\omega/\gamma)^2})}{q_\perp + (\omega/\gamma)^2} J_1(bq_\perp) q_\perp^2 dq_\perp \right]^2, \]

\[ \sigma(AB \rightarrow ABX) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sigma(\gamma\gamma \rightarrow X) \int d^2b_1 \int d^2b_2 n_A(b_1, \omega_1) n_B(b_2, \omega_2) P_{AB}(|\vec{b}_1 - \vec{b}_2|). \]

\( F(q) \) is the electromagnetic form factor, \( P_{AB}(b) \) is the probability for the colliding particles to survive after the collision with the impact parameter \( b \).
Photon-photon luminosity and survival factor

Let $x = \omega_1/\omega_2$, $s = 4\omega_1\omega_2$. Then

$$
\sigma(AB \rightarrow ABX) = \int_0^\infty \sigma(\gamma\gamma \rightarrow X) \cdot \frac{dL_{AB}}{ds} ds,
$$

where

$$
\frac{dL_{AB}}{ds} = \int_0^\infty \frac{dx}{8x} \int d^2b_1 \int d^2b_2 \, n_A \left( b_1, \sqrt{\frac{sx}{4}} \right) \, n_B \left( b_2, \sqrt{\frac{s}{4x}} \right) \, P_{AB}(|b_1 - b_2|)
$$

is the $\gamma\gamma$ luminosity in the collision of the particles $A$ and $B$. When $P_{AB}(b) \equiv 1$,

$$
\left. \frac{dL_{AB}}{ds} \right|_{P=1} = \int_0^\infty \frac{dx}{8x} \, n_A \left( \sqrt{\frac{sx}{4}} \right) \, n_B \left( \sqrt{\frac{s}{4x}} \right).
$$

The survival factor:

$$
S_{AB} = \frac{dL_{AB}/ds}{dL_{AB}/ds|_{P=1}}.
$$

$\sqrt{s}$ is the invariant mass of the system produced.
**pp UPC**

Proton form factor:

\[
F_p(q) = \frac{1}{(1 + (q/\Lambda)^2)^2} \left[ 1 + \frac{(\mu_p - 1)\tau}{1 + \tau} \right] \approx \frac{1}{(1 + (q/\Lambda)^2)^2}
\]

\(\Lambda = 0.84\ \text{GeV}, \ \mu_p = 2.73\) is the proton magnetic moment, \(\tau = q^2/4m_p^2\). Since \(q^2 \lesssim \Lambda_{QCD}^2 \ll 4m_p^2, \ \tau \ll 1\).

Proton EPA spectrum (dipole approximation):

\[
n_2(b, \omega) = \frac{\alpha}{\pi^2 \omega} \left[ \frac{\omega}{\gamma} K_1 \left( \frac{b\omega}{\gamma} \right) - \sqrt{\Lambda^2 + \left( \frac{\omega}{\gamma} \right)^2} K_1 \left( b \sqrt{\Lambda^2 + \left( \frac{\omega}{\gamma} \right)^2} \right) - \frac{b\Lambda^2}{2} K_0 \left( b \sqrt{\Lambda^2 + \left( \frac{\omega}{\gamma} \right)^2} \right) \right]^2.
\]

Proton survival probability [hep-ph/0608271, 1408.5778]:

\[
P_{pp}(b) = \left( 1 - e^{-\frac{b^2}{2B}} \right)^2, \quad B = 19.7\ \text{GeV}^2.
\]
\[ pp \to pp \mu\mu \] [1708.04053]

Muon \(|\eta| < 2.4\)
Muon \(p_T > 6\) GeV
Muon \(p_T > 10\) GeV

<table>
<thead>
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Experimental data
Proton disintegration neglected
Proton disintegration taken into account

EPA / ATLAS

\(d\sigma_{\text{fid.}}(pp \to pp\mu^+\mu^-)/d\sqrt{s},\text{ pb}/\text{GeV}\)
**pp survival factor**

![Graph showing survival factor vs. √s, GeV and σ(pp → ppχ⁺, χ⁻) vs. mₓ, GeV with proton disintegration neglected and taken into account.](image)

- Survival factor
  - Luminosity with proton disintegration neglected
  - Luminosity with proton disintegration taken into account
  - Survival factor

- Cross section with proton disintegration neglected
  - Cross section with proton disintegration taken into account
  - Ratio
Heavy nucleus form factor (Fourier-Bessel decomposition):

\[ F_{\text{Pb}}(q) = \frac{\sin qR}{qR} \sum_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2 - (qR)^2}. \]

Monopole approximation:

\[ F_1(q) = \frac{1}{1 + (q/\Lambda)^2}, \quad \Lambda = 50 \text{ MeV [1995]} \text{ or } 80 \text{ MeV [1987]}. \]

Monopole EPA spectrum:

\[ n_1(b, \omega) = \frac{Z^2 \alpha}{\pi^2 \omega} \left[ \frac{\omega}{\gamma} K_1 \left( \frac{b\omega}{\gamma} \right) - \sqrt{\Lambda^2 + \left( \frac{\omega}{\gamma} \right)^2} K_1 \left( b \sqrt{\Lambda^2 + \left( \frac{\omega}{\gamma} \right)^2} \right) \right]^2. \]

Pb survival probability [nucl-ex/0302016, 1607.03838]

\[ P_{\text{Pb}}(b) = \exp \left[ -\sigma_{NN} \int \int T(b_1)T(b_2)\delta^{(2)}(\vec{b} - \vec{b}_1 + \vec{b}_2)db_1db_2 \right], \]

\[ T(b) = \int_{-\infty}^{\infty} \rho_N(\sqrt{b^2 + z^2})dz \]
\textbf{$^{208}$Pb form factor}

[1987]: At.Data and Nucl.Data Tabl. 36, 495 (1987)
\( \text{PbPb} \rightarrow \text{PbPb} \mu\mu \) [ATLAS-CONF-2016-025]

muon \(|\eta| < 2.4, p_T > 4 \text{ GeV}\)

- Experimental data
- \( \Lambda = 80 \text{ MeV}, \) Pb disintegration neglected
- \( \Lambda = 80 \text{ MeV}, \) Pb disintegration taken into account
- \( \Lambda = 50 \text{ MeV}, \) Pb disintegration neglected
- \( \Lambda = 50 \text{ MeV}, \) Pb disintegration taken into account

\( d\sigma_{\text{fid.}}(\text{PbPb} \rightarrow \text{PbPb} \mu^+\mu^-)/d\sqrt{s}, \mu \text{b/GeV}\) vs. \( \sqrt{s}, \text{ GeV}\)
$^{208}\text{Pb}^{208}\text{Pb}$ survival factor

\[ \frac{dL_{\text{PbPb}}}{d\sqrt{s}}, \text{ GeV}^{-1} \]

\[ \sigma(Pb Pb \rightarrow Pb Pb\chi^+\chi^-), \text{ pb} \]

- Luminosity with Pb disintegration neglected
- Luminosity with Pb disintegration taken into account
- Survival factor
- Ratio

\[ m_\chi, \text{ GeV} \]

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Conclusion

▶ **Protons:**
  ▶ The spectrum can be calculated analytically.
  ▶ EPA approximation describes well the experimental data.
  ▶ Taking into account the survival factor reduces by $\sim 5\%$ the cross section for the production of light particles (muons) and by $\sim 20\%$ the cross section for the production of particles with the mass 200 GeV.

▶ **208 Pb:**
  ▶ So far the spectrum can only be calculated for the monopole approximation.
  ▶ EPA approximation depends strongly on the nucleus form factor.
  ▶ Calculation with the older form factor overshoots the experiment by 50%; the newer form factor fits the experiment perfectly but has some inconsistencies (see backup). This is the subject of future investigations.
  ▶ Taking into account the survival factor reduces by $\sim 6\%$ the cross section for the production of light particles (muons) and over 80% the cross section for the production of particles with mass 200 GeV.
Thank you for your attention
EPA spectra

\[ n(\omega), \text{GeV}^{-1} \]

\[ \omega, \text{GeV} \]

- \( p \)
- \( ^{208}\text{Pb} [1987] \)
- \( ^{208}\text{Pb} [1995] \)
- \( ^{208}\text{Pb}, \) monopole, \( \Lambda = 80 \text{ MeV} \)
- \( ^{208}\text{Pb}, \) monopole, \( \Lambda = 50 \text{ MeV} \)

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EPA: survival factor
The charge density of $^{208}$Pb is given by:

$$\rho(r) = \sum_{k=1}^{N} a_k j_0(k\pi r/R) \theta(R - r),$$

and

$$F(q) = \frac{\int \rho(r)e^{iqr}d^3r}{\int \rho(r)d^3r}.$$

<table>
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<tr>
<th>Quoted</th>
<th>Actual</th>
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<tbody>
<tr>
<td>[1987]</td>
<td>82</td>
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<tr>
<td>[1995]</td>
<td>82</td>
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The average radius is:

$$\langle r^2 \rangle^{1/2} = \left( \left( \frac{\int \rho(r)r^2d^3r}{\int \rho(r)d^3r} \right) \right)^{1/2}, \text{ fm}$$

- $Z = \int \rho(r)d^3r$
- $\langle r^2 \rangle^{1/2}$
- Fourier-Bessel [1987]
- Fourier-Bessel [1995] $\times \frac{82}{3806}$
- Monopole, $\Lambda = 80 \text{ MeV}$
- Monopole, $\Lambda = 50 \text{ MeV}$

[1987]: At.Data and Nucl.Data Tabl. 36, 495 (1987)