Equivalent photons approximation: survival factor

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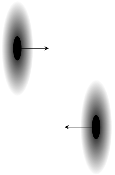
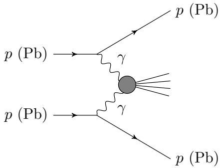
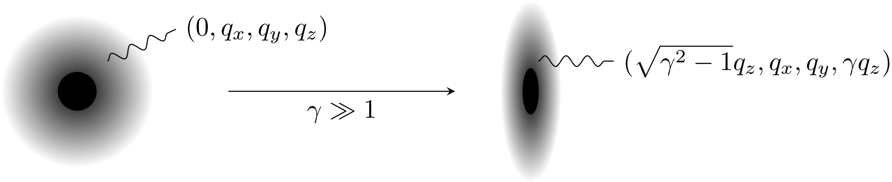
Ultraperipheral collisions



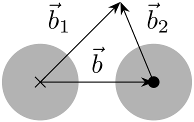




Photon virtuality: $-q^2 = q_x^2 + q_y^2 + q_z^2 \ll (\gamma q_z)^2 \equiv \omega^2$

UPC spectrum and cross section

With the sizes of the colliding particles neglected:



$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int_0^\infty \left[\frac{F(\sqrt{q_\perp^2 + (\omega/\gamma)^2})}{q_\perp^2 + (\omega/\gamma)^2} \right]^2 q_\perp^3 \mathrm{d}q_\perp,$$

$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_A(\omega_1) \, n_B(\omega_2).$$

With the sizes of the colliding particles taken into account:

$$n(\omega) = \int n(b,\omega) \, \mathrm{d}^2 b, \quad n(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega} \left[\int_0^\infty \frac{F(\sqrt{q_\perp^2 + (\omega/\gamma)^2})}{q_\perp + (\omega/\gamma)^2} J_1(bq_\perp) \, q_\perp^2 \mathrm{d}q_\perp \right]^2,$$

$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, n_A(b_1,\omega_1) \, n_B(b_2,\omega_2) \, P_{AB}(|\vec{b}_1 - \vec{b}_2|).$$

$F(q)$ is the electromagnetic form factor, $P_{AB}(b)$ is the probability for the colliding particles to survive after the collision with the impact parameter b.

Photon-photon luminosity and survival factor

Let $x = \omega_1/\omega_2$, $s = 4\omega_1\omega_2$. Then

$$\sigma(AB \to ABX) = \int_0^\infty \sigma(\gamma\gamma \to X) \cdot \frac{\mathrm{d}L_{AB}}{\mathrm{d}s}\,\mathrm{d}s,$$

where

$$\frac{\mathrm{d}L_{AB}}{\mathrm{d}s} = \int_0^\infty \frac{\mathrm{d}x}{8x} \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2\, n_A\left(b_1, \sqrt{\tfrac{sx}{4}}\right)\, n_B\left(b_2, \sqrt{\tfrac{s}{4x}}\right)\, P_{AB}(|\vec{b}_1 - \vec{b}_2|)$$

is the $\gamma\gamma$ luminosity in the collision of the particles A and B. When $P_{AB}(b) \equiv 1$,

$$\left.\frac{\mathrm{d}L_{AB}}{\mathrm{d}s}\right|_{P=1} = \int_0^\infty \frac{\mathrm{d}x}{8x}\, n_A\left(\sqrt{\tfrac{sx}{4}}\right)\, n_B\left(\sqrt{\tfrac{s}{4x}}\right).$$

The survival factor:

$$S_{AB} = \frac{\mathrm{d}L_{AB}/\mathrm{d}s}{\left.\mathrm{d}L_{AB}/\mathrm{d}s\right|_{P=1}}.$$

$\sqrt{s}$ is the invariant mass of the system produced.

pp UPC

Proton form factor:

$$F_p(q) = \frac{1}{(1+(q/\Lambda)^2)^2}\left[1 + \frac{(\mu_p - 1)\tau}{1+\tau}\right] \approx \frac{1}{(1+(q/\Lambda)^2)^2}$$

$\Lambda = 0.84$ GeV, $\mu_p = 2.73$ is the proton magnetic moment, $\tau = q^2/4m_p^2$. Since $q^2 \lesssim \Lambda^2_{\text{QCD}} \ll 4m_p^2$, $\tau \ll 1$.

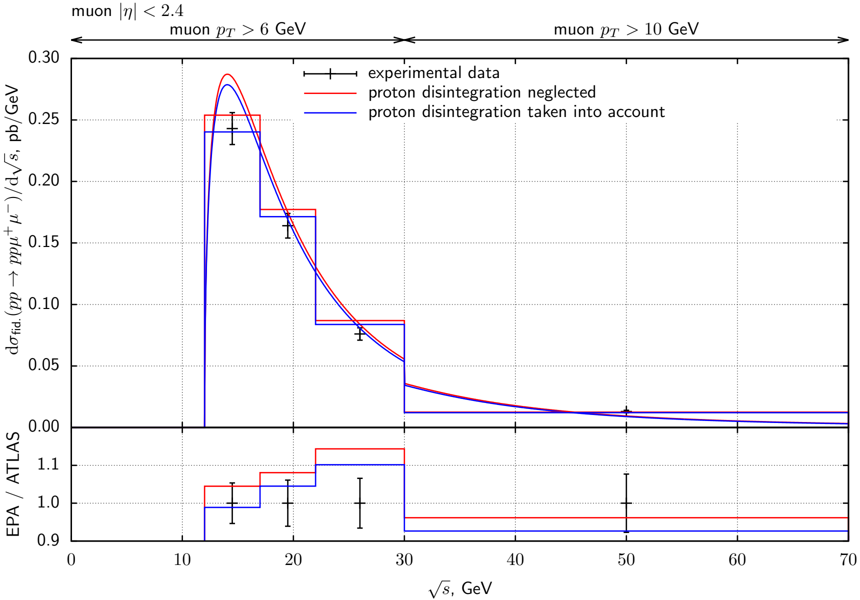
Proton EPA spectrum (dipole approximation):

$$n_2(b,\omega) = \frac{\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right) - \sqrt{\Lambda^2 + \left(\frac{\omega}{\gamma}\right)^2}K_1\left(b\sqrt{\Lambda^2 + \left(\frac{\omega}{\gamma}\right)^2}\right) - \frac{b\Lambda^2}{2}K_0\left(b\sqrt{\Lambda^2 + \left(\frac{\omega}{\gamma}\right)^2}\right)\right]^2.$$

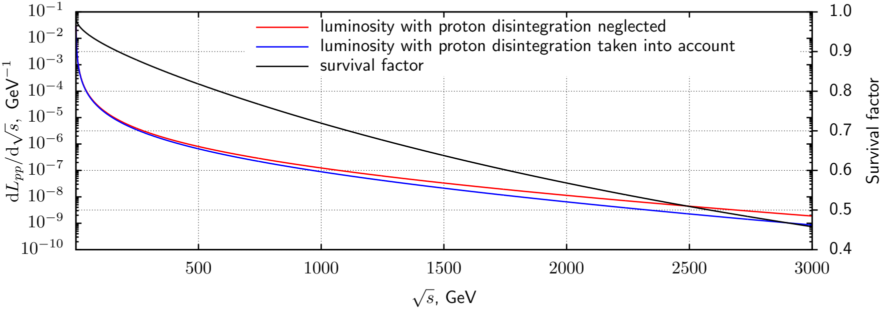
Proton survival probability [hep-ph/0608271, 1408.5778]:

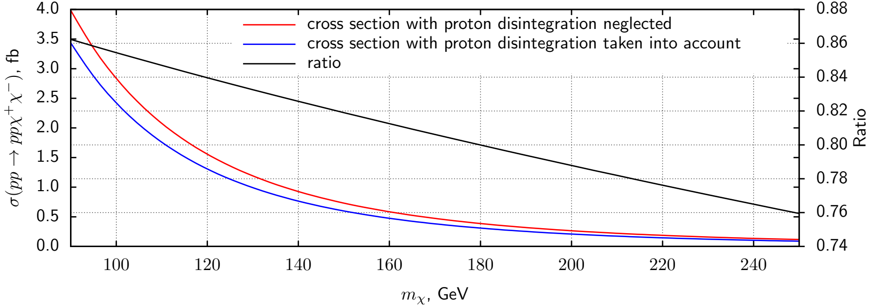
$$P_{pp}(b) = \left(1 - \mathrm{e}^{-\frac{b^2}{2B}}\right)^2, \quad B = 19.7 \text{ GeV}^2.$$

$pp \to pp\,\mu\mu$ [1708.04053]



pp survival factor





Pb Pb UPC

Heavy nucleus form factor (Fourier-Bessel decomposition):

$$F_{\mathsf{Pb}}(q) = \frac{\sin qR}{qR} \frac{\sum\limits_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2 - (qR)^2}}{\sum\limits_{k=1}^{N} \frac{(-1)^k a_k}{(k\pi)^2}}.$$

Monopole approximation:

$$F_1(q) = \frac{1}{1 + (q/\Lambda)^2}, \quad \Lambda = 50 \text{ MeV [1995] or } 80 \text{ MeV [1987].}$$

Monopole EPA spectrum:

$$n_1(b, \omega) = \frac{Z^2 \alpha}{\pi^2 \omega} \left[\frac{\omega}{\gamma} K_1\left(\frac{b\omega}{\gamma}\right) - \sqrt{\Lambda^2 + \left(\frac{\omega}{\gamma}\right)^2} K_1\left(b\sqrt{\Lambda^2 + \left(\frac{\omega}{\gamma}\right)^2}\right)\right]^2.$$

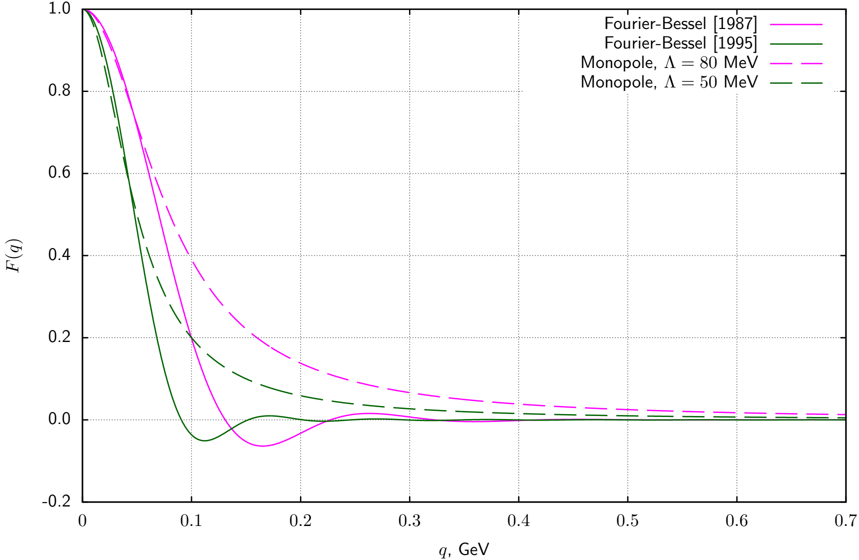
Pb survival probability [nucl-ex/0302016, 1607.03838]

$$P_{\mathsf{Pb}}(b) = \exp\left[-\sigma_{NN} \iint T(b_1) T(b_2) \delta^{(2)}(\vec{b} - \vec{b}_1 + \vec{b}_2) \mathrm{d}^2 b_1 \mathrm{d}^2 b_2\right],$$

$$T(b) = \int\limits_{-\infty}^{\infty} \rho_N(\sqrt{b^2 + z^2}) \mathrm{d}z$$

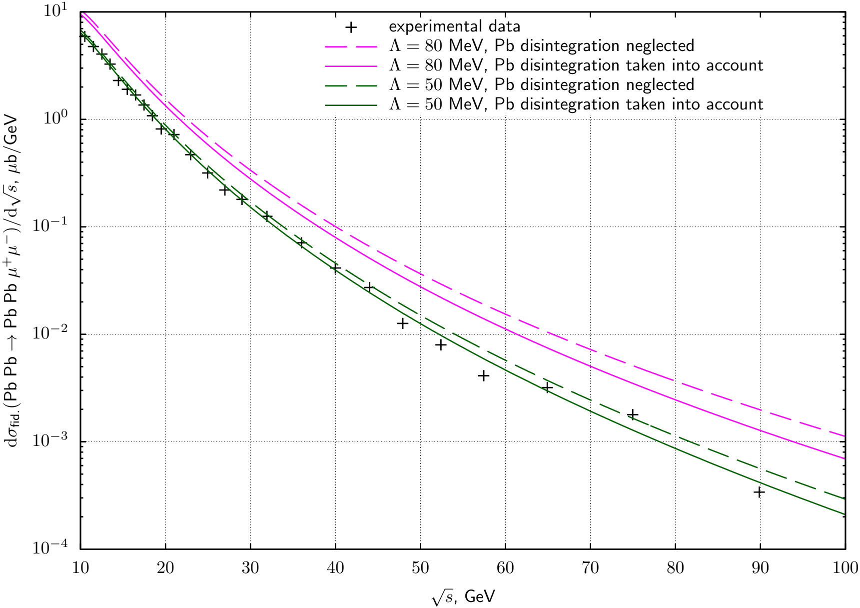
^{208}Pb form factor

[1987]: At.Data and Nucl.Data Tabl. 36, 495 (1987)
[1995]: At.Data and Nucl.Data Tabl. 60, 177 (1995)

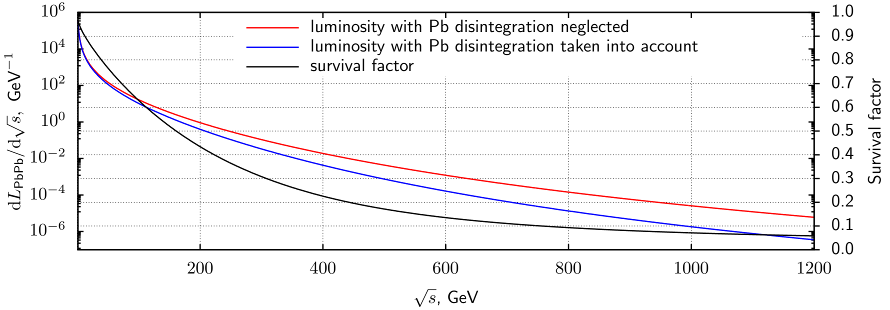


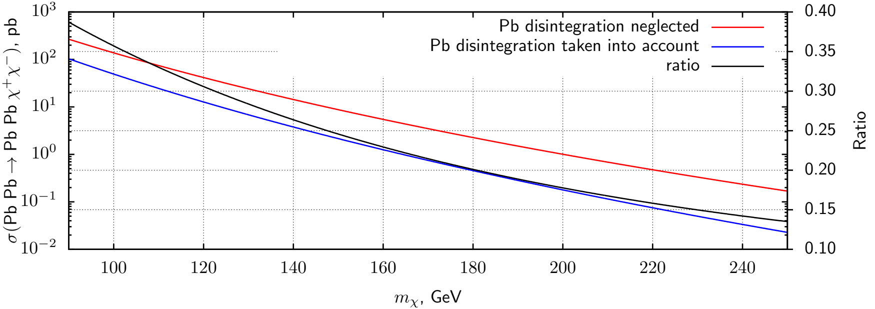
PbPb → PbPb $\mu\mu$ [ATLAS-CONF-2016-025]

muon $|\eta| < 2.4$, $p_T > 4$ GeV



^{208}Pb^{208}Pb survival factor



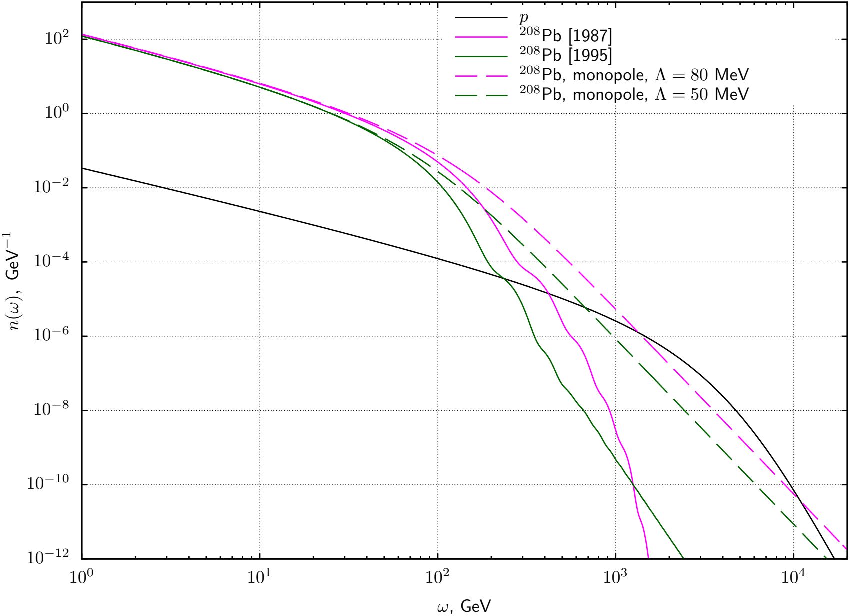


Conclusion

- Protons:
  - The spectrum can be calculated analytically.
  - EPA approximation describes well the experimental data.
  - Taking into account the survival factor reduces by $\sim 5\%$ the cross section for the production of light particles (muons) and by $\sim 20\%$ the cross section for the production of particles with the mass 200 GeV.
- ^{208}Pb:
  - So far the spectrum can only be calculated for the monopole approximation.
  - EPA approximation depends strongly on the nucleus form factor.
  - Calculation with the older form factor overshoots the experiment by 50%; the newer form factor fits the experiment perfectly but has some inconsistencies (see backup). This is the subject of future investigations.
  - Taking into account the survival factor reduces by $\sim 6\%$ the cross section for the production of light particles (muons) and over 80% the cross section for the production of particles with mass 200 GeV.

Thank you for your attention

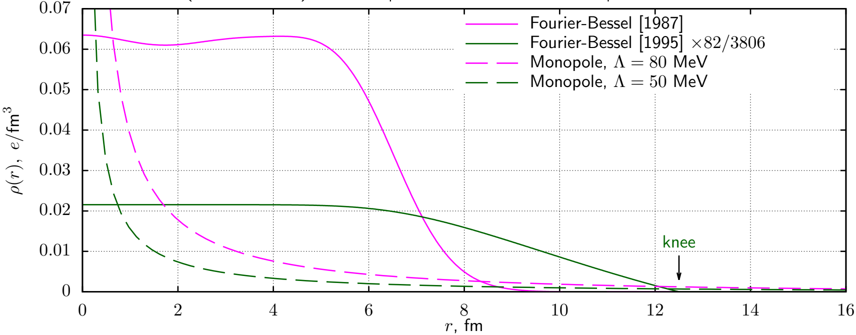
EPA spectra


p
^{208}Pb [1987]
^{208}Pb [1995]
^{208}Pb, monopole, $\Lambda = 80$ MeV
^{208}Pb, monopole, $\Lambda = 50$ MeV
$n(\omega)$, GeV^{-1}
ω, GeV
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^0
10^1
10^2
10^3
10^4

^{208}Pb charge density

$$\rho(r) = \sum_{k=1}^{N} a_k\, j_0(k\pi r/R)\,\theta(R-r), \quad F(q) = \frac{\int \rho(r) e^{i\vec{q}\vec{r}} d^3 r}{\int \rho(r) d^3 r}$$

	[1987]		[1995]	
	Quoted	Actual	Quoted	Actual
$Z = \int \rho(r) d^3 r$	82	82.03	82	3806
$\langle r^2 \rangle^{1/2} = \left(\frac{\int \rho(r) r^2 d^3 r}{\int \rho(r) d^3 r} \right)^{\frac{1}{2}}$, fm	5.499(1)	5.501	5.4785	8.0850



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