From thin to thick domain walls: An example of the φ^8 model

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Abstract

We study an example of higher-order field-theoretic model with an eighth-degree polynomial potential – the φ^8 model. We show that for some certain ratios of constants of the potential, the problem of finding kink-type solutions in (1+1)-dimensional space-time reduces to solving algebraic equations.
Based on the explicit formulas found for the kink solutions, we show that for certain values of the constants, kink-like solutions with power-law asymptotics arise in the model, describing, in particular, thick domain walls. Objects of this kind could be of interest for modern cosmology.

The φ^8 model

The Lagrangian:

$$\mathscr{L} = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - V(\varphi) \qquad (1)$$

with the potential in the form of eighth degree polynomial

$$V(\varphi) = \frac{1}{2}\left(\varphi^2 - a^2\right)^2\left(\varphi^2 - b^2\right)^2, \qquad (2)$$

where a and b are constants, $0 < a < b$.

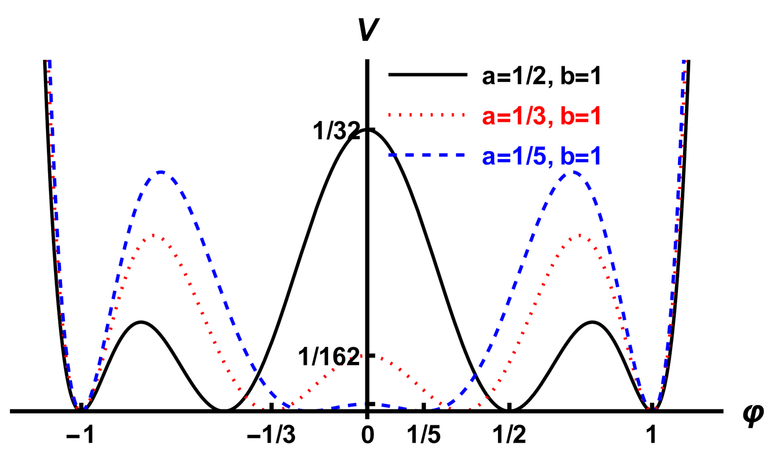

Figure 1: The potential (2) of the φ^8 model for $b = 1$ and $a = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}$.

Algebraic Equations for Kinks

Topological sectors (a, b) **and** $(-b, -a)$.
An implicit kink solution:

$$x = \frac{1}{2\left(b^2 - a^2\right)}\ln\left[\left(\frac{\varphi - a}{\varphi + a}\right)^{1/a}\left(\frac{b + \varphi}{b - \varphi}\right)^{1/b}\right]. \qquad (3)$$

Denote $b/a = n$ and set $b = 1$, then we get

$$\left(\frac{n\,\varphi - 1}{n\,\varphi + 1}\right)^n \frac{1 + \varphi}{1 - \varphi} = \alpha_n(x), \qquad (4)$$

where

$$\alpha_n(x) = \exp\left[2\left(1 - \frac{1}{n^2}\right)x\right]. \qquad (5)$$

Topological sector $(-a, a)$:

$$\left(\frac{1 + n\,\varphi}{1 - n\,\varphi}\right)^n \frac{1 - \varphi}{1 + \varphi} = \alpha_n(x), \qquad (6)$$

Case $n = 2$

Topological sectors $(-1, -1/2)$**,** $(-1/2, 1/2)$ **and** $(1/2, 1)$:

$$\varphi_{\mathrm{K}}(x) = \cos\left(\frac{1}{3}\arccos\left[\tanh\left(\frac{3}{4}x\right)\right] + \frac{\pi m}{3}\right), \qquad (7)$$

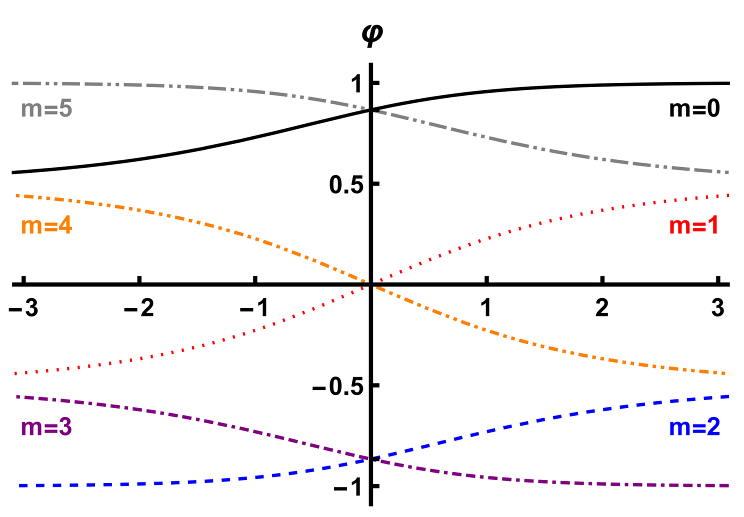

Figure 2: All kinks and antikinks at $n = 2$, Eq. (7), for different values of m.

Case $n = 3$

For $n = 3$:
Topological sector $(1/3, 1)$ **and symmetrical to it** $(-1, -1/3)$:

$$\varphi_{\mathrm{K}}^{(3)}(x) = \begin{cases} \dfrac{1}{3}\left(-\sqrt{1 - \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right)} + \sqrt{2 + \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right) - \dfrac{2\tanh\left(\frac{8}{9}x\right)}{\sqrt{1 - \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right)}}}\right), & x < 0, \\ \dfrac{1}{3}\left(\sqrt{1 - \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right)} + \sqrt{2 + \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right) + \dfrac{2\tanh\left(\frac{8}{9}x\right)}{\sqrt{1 - \operatorname{sech}^{\frac{2}{3}}\left(\frac{8}{9}x\right)}}}\right), & x > 0. \end{cases} \qquad (8)$$

Topological sector $(-1/3, 1/3)$:

$$\varphi_{\mathrm{K}}^{(3)}(x) = \begin{cases} \dfrac{1}{3}\left(\sqrt{1 + \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right)} - \sqrt{2 - \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right) - \dfrac{2\coth\left(\frac{8}{9}x\right)}{\sqrt{1 + \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right)}}}\right), & x < 0, \\ \dfrac{1}{3}\left(-\sqrt{1 + \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right)} + \sqrt{2 - \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right) + \dfrac{2\coth\left(\frac{8}{9}x\right)}{\sqrt{1 + \operatorname{csch}^{\frac{2}{3}}\left(\frac{8}{9}x\right)}}}\right), & x > 0, \end{cases} \qquad (9)$$

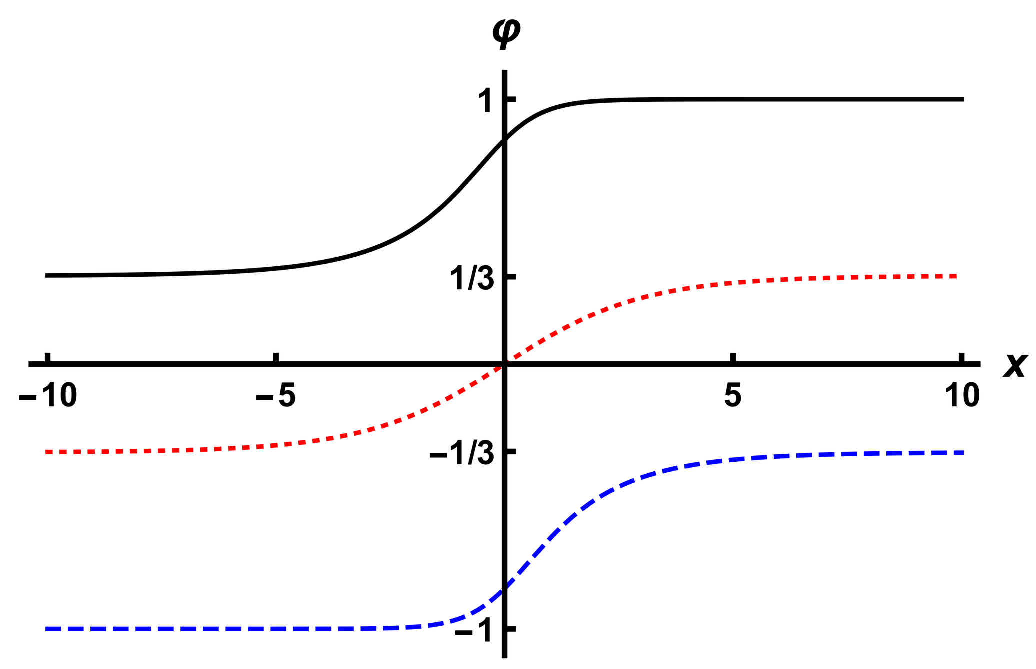

Figure 3: All kinks at $n = 3$: Eq. (8) in the sector $(\frac{1}{3}, 1)$ — black solid line, Eq. (??) in the sector $(-1, -\frac{1}{3})$ — blue dashed line, and Eq. (9) in the sector $(-\frac{1}{3}, \frac{1}{3})$ — red dotted line.

Asymptotics of Kinks and Limit $n \to \infty$

Topological sector $(\frac{1}{n}, 1)$:

$$\varphi_{\mathrm{K}}^{(n)}(x) \approx \begin{cases} \dfrac{1}{n} + \dfrac{2}{n}\left(\dfrac{n-1}{n+1}\right)^{\frac{1}{n}} \exp\left[\dfrac{2}{n}\left(1 - \dfrac{1}{n^2}\right)x\right] & \text{at} \quad x \to -\infty, \\ 1 - 2\left(\dfrac{n-1}{n+1}\right)^n \exp\left[-2\left(1 - \dfrac{1}{n^2}\right)x\right] & \text{at} \quad x \to +\infty. \end{cases} \qquad (10)$$

Topological sector $(-\frac{1}{n}, \frac{1}{n})$.:

$$\varphi_{\mathrm{K}}^{(n)}(x) \approx \begin{cases} -\dfrac{1}{n} + \dfrac{2}{n}\left(\dfrac{n-1}{n+1}\right)^{\frac{1}{n}} \exp\left[\dfrac{2}{n}\left(1 - \dfrac{1}{n^2}\right)x\right] & \text{at} \quad x \to -\infty, \\ \dfrac{1}{n} - \dfrac{2}{n}\left(\dfrac{n-1}{n+1}\right)^{\frac{1}{n}} \exp\left[-\dfrac{2}{n}\left(1 - \dfrac{1}{n^2}\right)x\right] & \text{at} \quad x \to +\infty. \end{cases} \qquad (11)$$

In the limit $n \to \infty$ (i.e., for $a \to 0$) at $x \to -\infty$ it can be seen that for any finite x the argument of the exponent tends to zero, which corresponds to the transition from exponential to power-law asymptotic behavior of the kink in the topological sector $(0, 1)$.

Conclusion

We have shown that in the case of a ratio of constants $b/a = n$ equal to positive integers, in order to obtain explicit formulas for kinks, it is necessary to solve an algebraic equation of degree $n + 1$. As an example, we have considered cases of $n = 2$ and 3 and obtained analytical formulas for kinks in all topological sectors of the model. For $n = 3$, the expressions for kinks look rather cumbersome; nevertheless, this is a significant step forward in the study of topological solitons of the φ^8 model.

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