

Vibrations of thick domain walls: How to avoid no-go theorem

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Abstract

Kink's Stability Potential

Conclusion

The discrete part of the domain wall excitation spectrum, the profile of which is described by a kink solution with one or both power-law asymptotics, cannot contain levels other than the zero (translational) mode. Nevertheless, it can be shown that scenarios are quite possible when long-lived vibrations will be excited on the domain wall. This, in turn, can affect the processes of interaction of two or more domain walls. The standard procedure of obtaining the kink's stability potential includes the following steps:

We have studied the asymptotic behavior of the kink stability potential, which defines the spectrum of the kink small excitations. We were interested in the case of power-law asymptotics of the kink. In this case, we found that the stability potential has volcano-like shape, approaching to zero at large distances. Very important result is that the asymptotic behavior of the kinks stability potential is universal in the case of power-law kink's decay: U(x)approaches zero as $1/x^2$. Moreover, despite the absence of the vibrational modes in the solitary kink's excitation spectrum, there could be positive eigenvalues (vibrational modes) in the mutual stability potential of the 'kink+antikink' system as a whole.



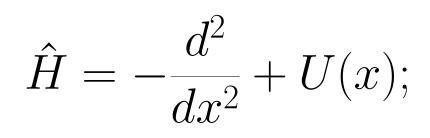
• add a small perturbation $\delta \varphi(x,t)$ to the static kink $\varphi_{\rm K}(x)$, $\varphi(x,t) = \varphi_{\rm K}(x) + \delta \varphi(x,t);$

• assume that

 $\delta \varphi(x,t) = \eta(x) \cos \omega t;$

 \bullet substituting into the equation of motion, we obtain the eigenvalue problem $\hat{H}\eta(x)=\omega^2\eta(x),$

where



Why are such models interesting?

For example, higher-order
 polynomial potentials has kinks
 with power-law tails.

• Such models, in turn, may arise in cosmology, condensed matter, and so on.

Note that in many cases the kink $\varphi_{\rm K}(x)$ is known only in the implicit form $x = x_{\rm K}(\varphi)$.

 $U(x) = \frac{d^2 V}{d\varphi^2} \Big|_{\varphi_{\mathcal{O}_{\mathcal{V}}}(x)}$

Important Results on the Stability Potential

This study is in progress now, final results will be reported in the near future.

The Model

Lagrangian:

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \varphi}{\partial x} \right)^2 - V(\varphi).$$

Equation of motion:

 $\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + \frac{dV}{d\varphi} = 0.$

The static kink solution $\varphi_{\rm K}(x)$ of the ODE $d\varphi/dx = \sqrt{2V}$ is topologically non-trivial, i.e.

 $\lim_{x \to -\infty} \varphi_{\mathbf{K}}(x) < \lim_{x \to +\infty} \varphi_{\mathbf{K}}(x).$

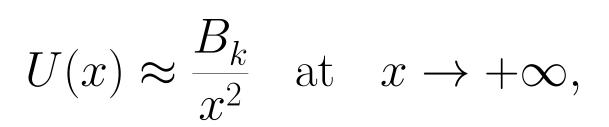
Power-Law Asymptotics

• 1. The stability potential is volcano-like.

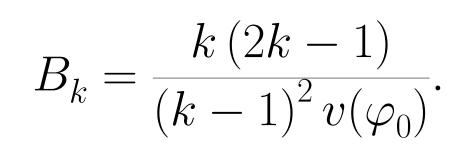
• the stability potential then reads

Symmetric or not depending on the symmetry of the kink.

• 2. Asymptotic behavior of the kink's stability potential is universal.



where



• 3. No-go theorem for vibrational modes:

there is only the zero mode in the discrete part of the kink's excitation spectrum, which lies on the boundary of the continuous spectrum.

• 4. A way to avoid the above no-go theorem:

In the case of asymmetric kinks with one power-law and one exponential asymptotics, asymmetry of the stability potential can lead to that closely placed kink and antikink can form a mutual stability potential in the form of a potential well, in which, in addition to the zero level, there will also be

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Acknowledgements

of Kink

levels of the discrete spectrum (vibrational modes).

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Polynomial or non-polynomial potential at $\varphi \approx \varphi_0$:

 $V(\varphi) = \frac{1}{2} \left(\varphi - \varphi_0 \right)^{2k} v(\varphi_0),$

where $v(\varphi_0) > 0$ is a constant.

• Power-law asymptotics (k > 1) at

 $x \to +\infty$:

 $\varphi_{\mathrm{K}}(x) pprox \varphi_{0} - rac{A_{k}}{x^{1/(k-1)}},$

where

$A_k = \left[(k-1) \sqrt{v(\varphi_0)} \right]^{1/(1-k)}.$

An Example currently being studied

Consider the polynomial potential $V(\varphi) = \frac{1}{2} (1 - \varphi)^{2k} (1 + \varphi)^{2m}.$ At k = m = 1 this leads to the well-known φ^4 model with kink solution $\varphi_{\rm K}(x) = \tanh x.$ At m = 1 and $k \ge 2$ the kink solution can be found in the implicit form

 $x = \frac{1}{2^{k}} \ln \frac{1 + \varphi}{1 - \varphi} + \sum_{n=1}^{k-1} \frac{1}{n \cdot 2^{k-n}} \frac{1}{(1 - \varphi)^{n}}.$

It enables us to investigate properties of this kink and its stability potential. The results will be reported in the forthcoming publication.