# Vibrations of thick domain walls: How to avoid no-go theorem

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## Abstract

The discrete part of the domain wall excitation spectrum, the profile of which is described by a kink solution with one or both power-law asymptotics, cannot contain levels other than the zero (translational) mode. Nevertheless, it can be shown that scenarios are quite possible when long-lived vibrations will be excited on the domain wall. This, in turn, can affect the processes of interaction of two or more domain walls.

## Why are such models interesting?

- For example, higher-order polynomial potentials has kinks with power-law tails.
- Such models, in turn, may arise in cosmology, condensed matter, and so on.

## The Model

Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - V(\varphi).$$

Equation of motion:

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{dV}{d\varphi} = 0.$$

The static kink solution $\varphi_\mathrm{K}(x)$ of the ODE $d\varphi/dx = \sqrt{2V}$ is topologically non-trivial, i.e.

$$\lim_{x\to-\infty}\varphi_\mathrm{K}(x) < \lim_{x\to+\infty}\varphi_\mathrm{K}(x).$$

## Power-Law Asymptotics of Kink

Polynomial or non-polynomial potential at $\varphi \approx \varphi_0$:

$$V(\varphi) = \frac{1}{2}(\varphi-\varphi_0)^{2k}\, v(\varphi_0),$$

where $v(\varphi_0) > 0$ is a constant.

- Power-law asymptotics ($k > 1$) at $x \to +\infty$:

$$\varphi_\mathrm{K}(x) \approx \varphi_0 - \frac{A_k}{x^{1/(k-1)}},$$

where

$$A_k = \left[(k-1)\sqrt{v(\varphi_0)}\right]^{1/(1-k)}.$$

## Kink's Stability Potential

The standard procedure of obtaining the kink's stability potential includes the following steps:

- add a small perturbation $\delta\varphi(x,t)$ to the static kink $\varphi_\mathrm{K}(x)$,
$$\varphi(x,t) = \varphi_\mathrm{K}(x) + \delta\varphi(x,t);$$
- assume that
$$\delta\varphi(x,t) = \eta(x)\cos\omega t;$$
- substituting into the equation of motion, we obtain the eigenvalue problem
$$\hat{H}\eta(x) = \omega^2\eta(x),$$
where
$$\hat{H} = -\frac{d^2}{dx^2} + U(x);$$
- the stability potential then reads
$$U(x) = \left.\frac{d^2V}{d\varphi^2}\right|_{\varphi_\mathrm{K}(x)}$$

Note that in many cases the kink $\varphi_\mathrm{K}(x)$ is known only in the implicit form $x = x_\mathrm{K}(\varphi)$.

## Important Results on the Stability Potential

- 1. The stability potential is volcano-like.

  Symmetric or not depending on the symmetry of the kink.
- 2. Asymptotic behavior of the kink's stability potential is universal.
$$U(x) \approx \frac{B_k}{x^2} \quad \text{at} \quad x \to +\infty,$$
where
$$B_k = \frac{k\,(2k-1)}{(k-1)^2\, v(\varphi_0)}.$$
- 3. No-go theorem for vibrational modes:

  there is only the zero mode in the discrete part of the kink's excitation spectrum, which lies on the boundary of the continuous spectrum.
- 4. A way to avoid the above no-go theorem:
  In the case of asymmetric kinks with one power-law and one exponential asymptotics, asymmetry of the stability potential can lead to that closely placed kink and antikink can form a mutual stability potential in the form of a potential well, in which, in addition to the zero level, there will also be levels of the discrete spectrum (vibrational modes).

## An Example currently being studied

Consider the polynomial potential

$$V(\varphi) = \frac{1}{2}(1-\varphi)^{2k}(1+\varphi)^{2m}.$$

At $k = m = 1$ this leads to the well-known $\varphi^4$ model with kink solution $\varphi_\mathrm{K}(x) = \tanh x$.
At $m = 1$ and $k \geq 2$ the kink solution can be found in the implicit form

$$x = \frac{1}{2^k}\ln\frac{1+\varphi}{1-\varphi} + \sum_{n=1}^{k-1}\frac{1}{n\cdot 2^{k-n}}\frac{1}{(1-\varphi)^n}.$$

It enables us to investigate properties of this kink and its stability potential. The results will be reported in the forthcoming publication.

## Conclusion

We have studied the asymptotic behavior of the kink stability potential, which defines the spectrum of the kink small excitations. We were interested in the case of power-law asymptotics of the kink. In this case, we found that the stability potential has volcano-like shape, approaching to zero at large distances. Very important result is that the asymptotic behavior of the kinks stability potential is universal in the case of power-law kink's decay: $U(x)$ approaches zero as $1/x^2$. Moreover, despite the absence of the vibrational modes in the solitary kink's excitation spectrum, there could be positive eigenvalues (vibrational modes) in the mutual stability potential of the 'kink+antikink' system as a whole.

This study is in progress now, final results will be reported in the near future.

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## Acknowledgements

This work was supported by the Russian Foundation for Basic Research under Grant No. 19-02-00930.

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