Asymmetric kink solutions of hyperbolically deformed model

Vakhid A. Gani[1,2], Aliakbar Moradi Marjaneh[3]

[1]Department of Mathematics, National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow, Russia

[2]Theory Department, Institute for Theoretical and Experimental Physics of National Research Centre "Kurchatov Institute", Moscow, Russia

[3]Department of Physics, Quchan Branch, Islamic Azad university, Quchan, Iran

Abstract

We study some properties of kink solutions of a model with non-polynomial potential obtained by deforming the well-known φ^6 field model. We consider the excitation spectrum of the kink. We also discuss the properties of the 'kink+antikink' system as a whole that are not inherent to a solitary kink or antikink.

The φ^6 Model

Within the φ^6 model [1] the dynamics of a real scalar field $\varphi(x,t)$ in $(1+1)$ dimensions is described by the Lagrangian density

$$L = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - V_0(\varphi) \qquad (1)$$

with the potential

$$V_0(\varphi) = \frac{1}{2}\varphi^2\left(1-\varphi^2\right)^2. \qquad (2)$$

The model has kink solutions connecting the vacua $\varphi = 0$ and $\varphi = \pm 1$. All of them can be obtained by symmetry transformations from the kink

$$\varphi_{\mathrm{K}}^{(0)}(x) = \sqrt{(1+\tanh x)/2}. \qquad (3)$$

Deformation Procedure

If the initial potential and kink are $V_0(\varphi)$ and $\varphi_{\mathrm{K}}^{(0)}(x)$, respectively, then the f-deformed potential:

$$V_1(\varphi) = \frac{V_0[f(\varphi)]}{[f'(\varphi)]^2} \qquad (4)$$

and the deformed kink [2,3]:

$$\varphi_{\mathrm{K}}^{(1)}(x) = f^{-1}[\varphi_{\mathrm{K}}^{(0)}(x)]. \qquad (5)$$

Hyperbolic Deformation of the φ^6 Model

We apply the deformation procedure to the φ^6 model, using the deforming function $f(\varphi) = \sinh\varphi$. As a result, we obtain the sinh-deformed φ^6 model:

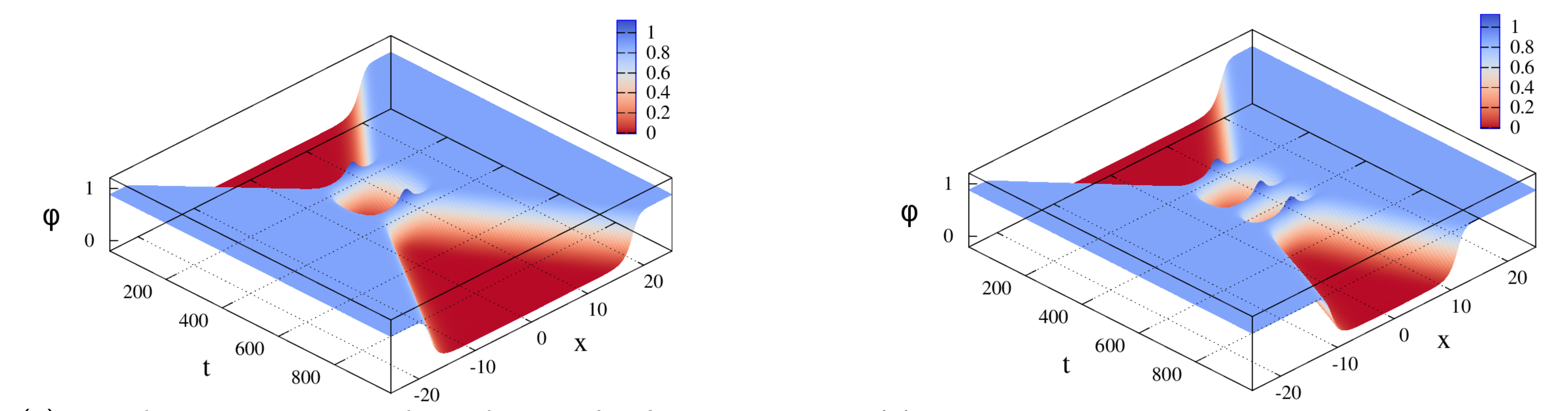
$$V_1(\varphi) = \frac{1}{2}\tanh^2\varphi(1-\sinh^2\varphi)^2 \qquad (6)$$

with the kink solution

$$\varphi_{\mathrm{K}}^{(1)}(x) = \mathrm{arsinh}\sqrt{\frac{1+\tanh x}{2}}. \qquad (7)$$

Why is this model interesting?

- Non-polynomial potential
- Asymmetric kinks



(a) two-bounce escape window, the initial velocity 0.0456

(b) three-bounce escape window, , the initial velocity 0.0441

Figure 1:Examples of resonance phenomena in the antikink-kink scattering

Stability Potential of the sinh-Deformed φ^6 Kink

First of all, the stability potential of a kink can be obtained by adding a small perturbation to the static kink. Then the linearized equation of motion leads to the Sturm-Liouville problem

$$\left[-\frac{d^2}{dx^2} + U(x)\right]\psi(x) = \omega^2\psi(x). \qquad (8)$$

The function

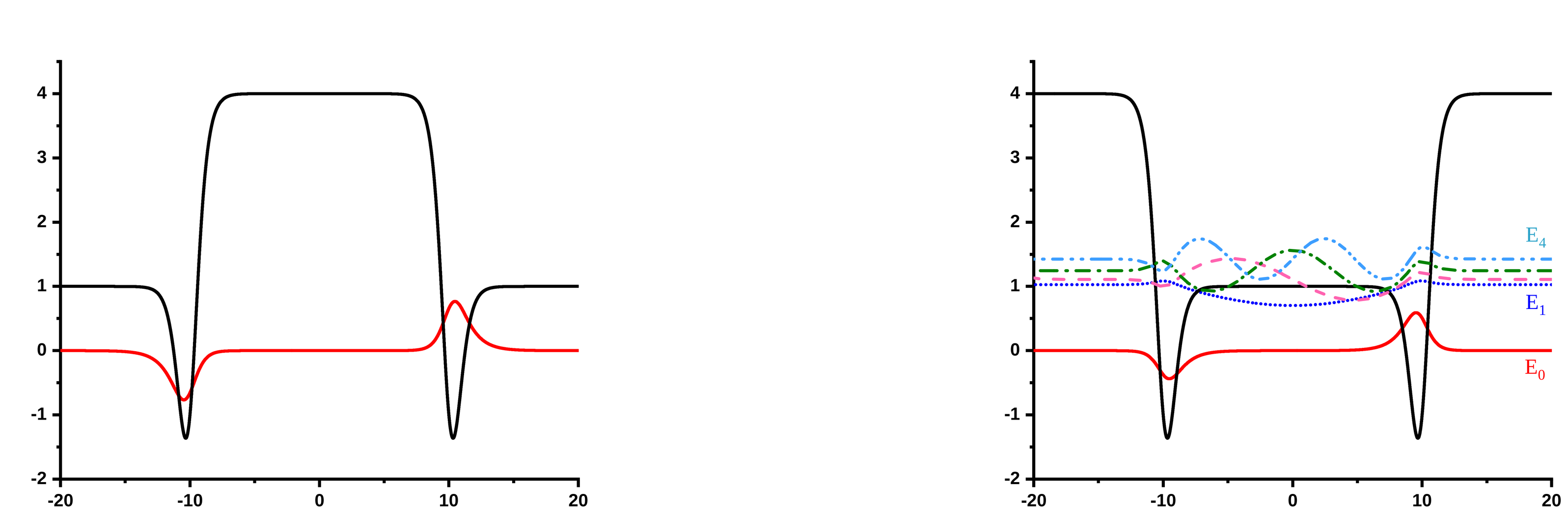
$$U(x) = \left.\frac{d^2V}{d\varphi^2}\right|_{\varphi_{\mathrm{K}}(x)} = \frac{30\tanh x + 2\,\mathrm{sech}^4 x - (16\tanh x + 45)\mathrm{sech}^2 x + 34}{(\tanh x + 3)^2} \qquad (9)$$

is the stability potential, which defines the kink's excitation spectrum.

- The spectrum always has zero level, and all eigenvalues of the problem (8) are non-negative.
- The stability potential of the φ^6 kink does not have vibrational modes.
- We found that the stability potential of the sinh-deformed φ^6 kink also does not have vibrational modes.
- It would seem that it can be assumed that resonance phenomena associated with resonant energy exchange between zero and vibrational modes are impossible in the kink-antikink collisions in both models. However, this is not quite true.
- In the antikink-kink collisions we observe escape windows, see figure 1. Appearance of the escape windows indicates the presence of resonant energy exchange between kinetic energy and (at least one) vibrational mode.
- The question is where is this vibrational mode hiding?

Stability Potential of the Antikink-Kink Pair

- The kinks of both models are asymmetric.
- This leads to the asymmetric stability potentials with different asymptotic values at $x \to \pm\infty$.
- Such asymmetry means that closely placed kink and antikink can form a mutual stability potential in the form of a potential well, in which, in addition to the zero level, there will also be levels of the discrete spectrum (vibrational modes).
- Our idea is that the situation can be similar to that found for the φ^6 [4].



(a) kink-antikink zero mode

(b) antikink-kink modes

Figure 2:Stability potentials and appropriately normalized eigenfunctions of the discrete spectrum

Conclusion

We have studied the sinh-deformed φ^6 model wich is obtained from the well-known φ^6 scalar field theory. We have shown that in this model there are no vibrational modes in the kink excitation spectrum. At the same time, in our numerical simulations of collisions of the sinh-deformed φ^6 kink and antikink at some initial velocities we observed resonance phenomena — escape windows. We suppose that these resonance phenomena may be a consequence of the resonant energy exchange between the translational modes of kinks (their kinetic energy) and the vibrational modes of the antikink-kink system as a whole. A more detailed study of this issue is of great importance and is planned for the near future.

Acknowledgements

The work of the MEPhI group was supported by the MEPhI Academic Excellence Project (Contract No. 02.a03.21.0005, 27.08.2013).

The work of V.A.G. was supported by the Russian Foundation for Basic Research under Grant No. 19-02-00930.

A.M.M. thanks the Islamic Azad University, Quchan Branch, Iran (IAUQ) for their financial support under the Grant.

References

[1] V.A. Gani, A.E. Kudryavtsev, M.A. Lizunova, Kink interactions in the (1+1)-dimensional φ^6 model, Phys. Rev. D 89 (2014) 125009 [arXiv:1402.5903].

[2] D. Bazeia, L. Losano, J.M.C. Malbouisson, Deformed defects, Phys. Rev. D 66 (2002) 101701 [hep-th/0209027].

[3] P.A. Blinov, T.V. Gani, V.A. Gani, Deformations of Kink Tails, arXiv:2008.13159.

[4] P. Dorey, K. Mersh, T. Romańczukiewicz, Ya. Shnir, Kink-antikink collisions in the ϕ^6 model, Phys. Rev. Lett. 107, 091602 (2011) [arXiv:1101.5951].

Contact Information

- Dr. Vakhid A. Gani: VAGani(at)mephi.ru
- Dr. Aliakbar Moradi Marjaneh: moradimarjaneh(at)gmail.com