Domain wall thickness and deformations of the field model

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Abstract

We consider the change in the asymptotic behavior of solutions of the type of flat domain walls (kink solutions) in field-theoretic models with a real scalar field. We show that when the model is deformed by a bounded deforming function, the exponential asymptotics of the corresponding kink solutions remain exponential, while the power-law ones remain power-law. However, the parameters of these asymptotics, which are related to the wall thickness, can change.

1. Kink Solutions

The Lagrangian:

$$\mathscr{L} = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - V(\varphi). \qquad (1)$$

Equation of motion:

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{dV}{d\varphi} = 0. \qquad (2)$$

The kink solution $\varphi_\mathrm{K}(x)$ of the ODE $\dfrac{d\varphi}{dx} = \sqrt{2V}$ is topologically non-trivial, that is

$$\lim_{x\to-\infty}\varphi_\mathrm{K}(x) < \lim_{x\to+\infty}\varphi_\mathrm{K}(x). \qquad (3)$$

2. Deformation Procedure

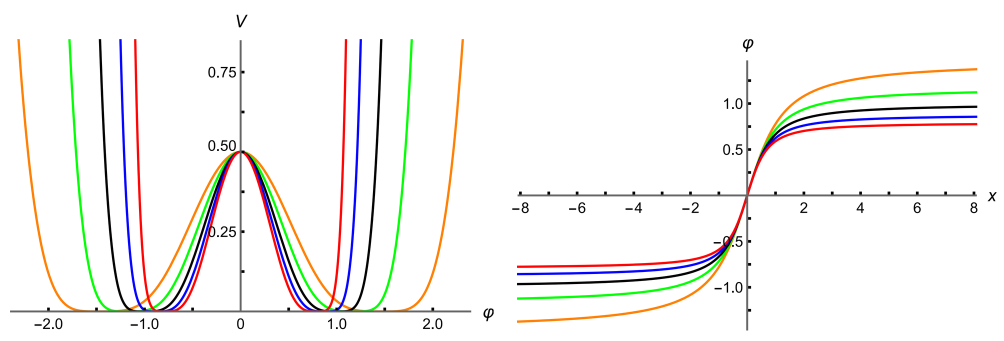
If the initial potential and kink are $V_0(\varphi)$ and $\varphi_\mathrm{K}^{(0)}(x)$, respectively, then the f-deformed potential

$$V_1(\varphi) = \frac{V_0[f(\varphi)]}{[f'(\varphi)]^2} \qquad (4)$$

and deformed kink

$$\varphi_\mathrm{K}^{(1)}(x) = f^{-1}[\varphi_\mathrm{K}^{(0)}(x)]. \qquad (5)$$

Many deformed potentials and kinks:



3. Kink's Asymptotics

Polynomial or non-polynomial potential at $\varphi \approx \varphi_0$:

$$V_0(\varphi) = \frac{1}{2}(\varphi-\varphi_0)^{2k}\, v_0(\varphi_0), \qquad (6)$$

Asymptotics at $x\to+\infty$

- Exponential asymptotics ($k=1$):

$$\varphi_\mathrm{K}^{(0)}(x) \approx \varphi_0 - \exp\left[-\sqrt{v_0(\varphi_0)}\, x\right]. \qquad (7)$$

- Power-law asymptotics ($k>1$):

$$\varphi_\mathrm{K}^{(0)}(x) \approx \varphi_0 - \frac{A_k^{(0)}}{x^{1/(k-1)}}, \qquad (8)$$

where

$$A_k^{(0)} = \left[(k-1)\sqrt{v_0(\varphi_0)}\right]^{1/(1-k)}. \qquad (9)$$

4. Soft Deformation

Deforming function $f(\varphi)$ is strictly monotonic with a finite derivative.

Deformed potential at $\varphi \approx \varphi_1 = f^{-1}(\varphi_0)$:

$$V_1(\varphi) = \frac{\frac{1}{2}[\varphi_0 - f(\varphi)]^{2k}\, v_0(f(\varphi))}{[f'(\varphi)]^2}. \qquad (10)$$

Asymptotics at $x\to+\infty$

- Exponential asymptotics ($k=1$):

$$\varphi_\mathrm{K}^{(1)}(x) \approx \varphi_1 - \exp\left[-\sqrt{v_0(\varphi_0)}\, x\right]. \qquad (11)$$

- Power-law asymptotics ($k>1$):

$$\varphi_\mathrm{K}^{(1)}(x) \approx \varphi_1 - \frac{A_k^{(1)}}{x^{1/(k-1)}}, \qquad (12)$$

where

$$A_k^{(1)} = \left[(k-1)\,[f'(\varphi_1)]^{k-1}\sqrt{v_0(\varphi_0)}\right]^{\frac{1}{1-k}} \qquad (13)$$

$$\Longrightarrow \frac{A_k^{(0)}}{A_k^{(1)}} = f'(\varphi_1). \qquad (14)$$

5. Hard Deformation

Now let the function $f(\varphi)$ be such that

$$f(\varphi) \approx f(\varphi_1) - B\,(\varphi_1-\varphi)^\beta \quad \text{at } \varphi\to\varphi_1 - 0, \qquad (15)$$

where $B>0$ and $0<\beta<1$ are constants, i.e. $f'(\varphi)$ tends to infinity at $\varphi\to\varphi_1-0$. The potential of the f-deformed model at $\varphi\approx\varphi_1$:

$$V_1(\varphi) \approx \frac{1}{2}\frac{B^{2k-2}v_0(\varphi_0)}{\beta^2}(\varphi_1-\varphi)^{2+2\beta(k-1)}. \qquad (16)$$

Asymptotics at $x\to+\infty$

- Exponential asymptotics ($k=1$):

$$\varphi_\mathrm{K}^{(1)}(x) \approx \varphi_1 - \exp\left[-\frac{\sqrt{v_0(\varphi_0)}}{\beta}\, x\right]. \qquad (17)$$

- Power-law asymptotics ($k>1$):

$$\varphi_\mathrm{K}^{(1)}(x) \approx \varphi_1 - \frac{B_k^{(1)}}{x^{\frac{1}{\beta(k-1)}}}, \qquad (18)$$

where

$$B_k^{(1)} = \left[(k-1)\,B^{k-1}\sqrt{v_0(\varphi_0)}\right]^{\frac{1}{\beta(1-k)}}. \qquad (19)$$

6. Summary

1. Strictly monotonic deforming function with finite derivative:
   - the exponential asymptotics remains exponential with the same coefficient in front of x;
   - the power-law asymptotics remains power-law with the same power, but, depending on the derivative of deforming function, the numerical coefficient may change.
2. The derivative of the deforming function goes to infinity in the vacuum of the deformed model:
   - the exponential asymptotics remains exponential, however, the coefficient in front of x increases;
   - the power-law asymptotics remains power-law, however, the power of the coordinate in the denominator increases;
   - now for both asymptotics the field of the deformed kink approaches the vacuum value faster
3. We supplemented the deformation procedure with a formula for implicit kinks.

Acknowledgments

The work of the MEPhI group was supported by the MEPhI Academic Excellence Project (Contract No. 02.a03.21.0005, 27.08.2013).

V.A.G. also acknowledges the support of the Russian Foundation for Basic Research under Grant No. 19-02-00930.

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