On perturbative unitarity in the extended MSSM Higgs sector

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   But it has problems
   - 'SM fine-tuning problem'
   - sources of CP-violation
   - DM candidate ...

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2. Higgs boson properties are SM-like, but investigations continue. The Higgs sector can be nonminimal
   THDM

$$\Phi_i = \begin{pmatrix} -i\omega_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}, \qquad i = 1, 2$$

$$v_1 = v\cos\beta,\ v_2 = v\sin\beta,\ v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}$$

3. An elegant solution provides SUSY[1]. Among SUSY-models, the most popular and investigated model is MSSM (THDM-II). The SUSY must be broken, so the low-energy theory is THDM.

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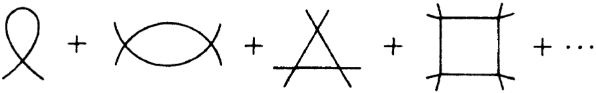

$$U(\Phi_1, \Phi_2) = U^{(2)} + U^{(4)} + U^{(6)} + U^{(8)} + ... \quad (1)$$

$$U^{(2)} = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - [\mu_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.], \quad (2)$$

$$U^{(4)} \supset \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + ... \quad (3)$$

$$U^{(6)} \supset \kappa_1(\Phi_1^\dagger\Phi_1)^3 + \kappa_2(\Phi_2^\dagger\Phi_2)^3 + \kappa_3(\Phi_1^\dagger\Phi_1)^2(\Phi_2^\dagger\Phi_2) + ... \quad (4)$$

$$[\mu_i^2] = \text{GeV}^2, \qquad [\lambda_j] = 1, \qquad [\kappa_n] = \text{GeV}^{-2}$$
$$i = 1, 2, \qquad j = 1, ...7, \qquad n = 1, ...13$$

Dubinin, Petrova, Phys. Rev. D 95, 055021 (2017)

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What values of free model parameters can be allowed by perturbative unitarity?

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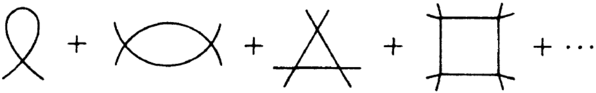

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Perturbative unitarity constraints (PUCs)

We will study the unitarity constraints by computing the scalar scattering processes $S_1 S_2 \to S_3 S_4$

$$M(s,t,u) = 16\pi \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta) a_J(s), \tag{5}$$

where s, t, u are Mandelstam variables, P_J are Legendre Polynomials
The cross section

$$d\sigma/d\Omega = |M|^2/64\pi^2 s, \qquad \sigma = \frac{16\pi}{s} \sum (2J+1)|a_J(s)|^2,$$

is proportional to the imaginary part of the amplitude in the forward direction

$$\sigma = \frac{1}{s} \mathrm{Im}[M(\theta = 0)] \tag{6}$$

$$|a_l|^2 = \mathrm{Im}(a_J) \qquad \text{for all } J.$$

From

$$Re(a_l)^2 + Im(a_l)^2 = |a_J|^2$$

the famous unitarity constraint of the partial wave amplitude

$$|a_J|^2 \leq \frac{1}{2} \tag{7}$$

$J = 0$

$$a_0 = - \frac{2^{-\frac{1}{2}(\delta_{12}+\delta_{34})}}{16\pi} \left\{ \left[\lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2)\right]^{\frac{1}{4}} \left[\lambda^{1234} + \kappa^{125}\kappa^{345} \frac{1}{s - m_5^2} \right] \right.$$
$$\left. - \kappa^{135}\kappa^{245} f_t(s, m^2_{1,\dots5}) - \kappa^{145}\kappa^{235} f_u(s, m^2_{1,\dots5}) \right\}, \quad (8)$$

where the factor δ_{12} (δ_{34}) is 1 if particles $\{1, 2\}$ ($\{3, 4\}$) are identical, and zero otherwise, $s = (p_1 + p_2)^2$ is the Mandelstam variable, m_5 is the particle mass in a propogator, λ^{1234} and κ^{ijk} are quartic and trilinear couplings of scalars,

$$f_t(s, m^2_{1,\dots5}) \equiv \frac{1}{s} \frac{1}{[\lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2)]^{\frac{1}{4}}} \log \left(\frac{m_1^2 + m_3^2 - m_5^2 - 2E_1E_3 + 2|\mathbf{p}_1||\mathbf{p}_3|}{m_1^2 + m_3^2 - m_5^2 - 2E_1E_3 - 2|\mathbf{p}_1||\mathbf{p}_3|} \right),$$
$$f_u(s, m^2_{1,\dots5}) \equiv f_t(s, m_1^2, m_2^2, m_4^2, m_3^2, m_5^2),$$
$$\lambda(s, m_i^2, m_j^2) \equiv \frac{1}{s^2} \left(s^2 + m_i^4 + m_j^4 - 2m_i^2 m_j^2 - 2s m_i^2 - 2s m_j^2 \right), \quad (9)$$

$$|\mathbf{p}_1| = \frac{1}{2}\sqrt{s\lambda(s, m_1^2, m_2^2)}, \qquad |\mathbf{p}_3| = \frac{1}{2}\sqrt{s\lambda(s, m_3^2, m_4^2)} \quad (10)$$

are the centre of mass three-momenta,

$$E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \qquad E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}} \quad (11)$$

are the energies of the particles in the center of mass frame.

Krauss, F. Staub, Phys.Rev. D98, no.1, 015041 (2018)

Goldstone-boson equivalence theorem

External longitudinal gauge bosons can be replaced by corresponding Goldstone modes

Lee-Quigg-Thacker theorem

In high-energy limit ($\sqrt{s} \gg m_5$) the amplitudes for two-body scattering processes are equivalent to those with longitudinal gauge bosons up to terms of $\mathcal{O}(m_5^2/s)$

Lee, Quigg, Thacker, Phys. Rev. D **16**, 1519 (1977)

For example:

$$a_0^{\mathrm{THDM}} = \frac{1}{16\pi}\mathrm{Diag}(X_{4\times4}, Y_{4\times4}, Z_{3\times3}, Z_{3\times3}) \tag{12}$$

$$Y_{4\times4} = \begin{pmatrix} \lambda_1 & \lambda_4 & \sqrt{2}\mathrm{Re}\lambda_6 & \sqrt{2}\mathrm{Im}\lambda_6 \\ \lambda_4 & \lambda_2 & \sqrt{2}\mathrm{Re}\lambda_7 & \sqrt{2}\mathrm{Im}\lambda_7 \\ \sqrt{2}\mathrm{Re}\lambda_6 & \sqrt{2}\mathrm{Re}\lambda_7 & \lambda_3 + \mathrm{Re}\lambda_5 & \mathrm{Im}\lambda_5 \\ \sqrt{2}\mathrm{Im}\lambda_6 & \sqrt{2}\mathrm{Im}\lambda_7 & \mathrm{Im}\lambda_5 & \lambda_3 - \mathrm{Re}\lambda_5 \end{pmatrix}, \ldots$$

Kanemura, Yagyu, Phys. Lett. B **751**, 289 (2015)

It is not always a valid approximation: the full calculation including all tree-level contributions at finite energy ($\sqrt{s} \sim m_5$) can lead to much more stringent constraints

Analytical formula for trilinear and quartic couplings of the extended Higgs potential have derived. They are rather cumbersome. Explicit expressions are presented in Mathematica-code and will be open access

For example:

$$\kappa_{hhh} = c_1 v + c_2 v^3, \tag{13}$$

where

$$\begin{aligned}
c_1 &= -\lambda_1 s_\alpha^3 c_\beta + \lambda_2 c_\alpha^3 s_\beta - \frac{\lambda_{345}}{4} s_{2\alpha} c_{\alpha+\beta} + \\
&+ \frac{\mathtt{Re}\lambda_6}{2} s_\alpha^2 \left(c_{\beta-\alpha} + 2c_{\beta+\alpha}\right) + \frac{\mathtt{Re}\lambda_7}{2} c_\alpha^2 (c_\alpha c_\beta - 3 s_\alpha s_\beta), \\
c_2 &= \frac{5}{2}[-\kappa_1 s_\alpha^3 c_\beta^3 + \kappa_2 c_\alpha^3 s_\beta^3 + (\mathtt{Re}\kappa_8 s_\alpha^2 c_\beta^2 + \\
&+ \mathtt{Re}\kappa_{12} c_\alpha^2 s_\beta^2) c_{\alpha+\beta}] + \frac{1}{16}[(\kappa_3 + \kappa_5 + 2\mathtt{Re}\kappa_9) s_\alpha c_\beta - \\
&- (\kappa_4 + \kappa_6 + 2\mathtt{Re}\kappa_{10}) c_\alpha s_\beta](c_{2(\beta-\alpha)} - \\
&- 5c_{2(\alpha+\beta)} - 4) + \frac{1}{32}\mathtt{Re}(\kappa_7 + \kappa_{11} + \kappa_{13}) \times \\
&\times [5c_{3(\alpha+\beta)} - 3(c_{\beta-3\alpha} + c_{3\beta-\alpha} - 3c_{\alpha+\beta})],
\end{aligned}$$

$\lambda_{345} = \lambda_3 + \lambda_4 + \mathtt{Re}\lambda_5$.

Assume that

1. only 3d generation of squarks are important
2. $M_{\tilde{Q}} \approx M_{\tilde{U}} \approx M_{\tilde{D}} \approx M_{SUSY}$
3. the Higgs-boson couplings to the heavier SM particles are SM-like (alignment limit)

Free parameters: m_A, $\tan\beta$ (tree-level) and M_{SUSY}, A_t, A_b, μ (loop-level)

Scenario	M_S, GeV	μ, GeV	A_t, GeV	$\tan\beta$	m_A, GeV
m_h^{max}*	1000	200	$\sqrt{6}M_S + \mu/\tan\beta$	varied	varied
τ-phobic*	1500	2000	$2.9M_S + \mu/\tan\beta$	varied	varied
low-m_A	2000	varied	varied	2	28

Table: Benchmark scenarios* specified in 1302.7033 [hep-ph]. Varied parameters adjust in such a way that m_h=125 GeV in alignment limit. Here m_t=173.2 GeV, m_b=4.2 GeV, m_Z=91.1876 GeV, m_W=80.385 GeV, $\alpha_S(m_t)$=0.118, G_F=1.16639×10^{-5} GeV^{-2}; $\cos\theta_W = m_W/m_Z$, $g^2 = 8m_Z^2 G_F/\sqrt{2}$, $g_2 = g\cos\theta_W$, $g_1 = g_2\tan\theta_W$, $v = 1/\sqrt{\sqrt{2}G_F}$, $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, $M_{SUSY} = \sqrt{M_S^2 - m_t^2}$, $A_t = A_b$

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Table: BPs for m_h=125 GeV in alignment limit

Scenario	BP	M_S, GeV	μ, GeV	A_t, GeV	$\tan\beta$	m_A, GeV
m_h^{max}	BP1	1000	200	2482	6	3000
τ-phobic	BP2	1500	2000	4635	7	3000
low-m_A	$BP3^{(4)}$	2000	6250	9000	2	28
	$BP3^{(6)}$	2000	5950	9000	2	28
	BP4	1000	9000	2350	5	90

Table: $\max|a_0|$, $\sqrt{s}$=13 TeV

Approx.	max.oper.	BP1	BP2	$BP3^{(4,6)}$	BP4
$\sqrt{s} \gg m_5$	$U^{(4)}$	0.8526	0.8298	3.8364	48.6415
$\sqrt{s} \sim m_5$	$U^{(4)}$	0.0265	0.0267	0.0017	0.1060
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Conclusions

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2. We analyzed the parameter space which can satisfy the PUCs and shown that the values of $A_{t,b}, \mu$ are acceptable up to 10 TeV
3. Dimension-six operators change significantly PUCs in the MSSM regime with $\mu \sim 9$ TeV
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U^{(4)} &= \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \\
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&+ \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.], && (16)\\
U^{(6)} &= \kappa_1(\Phi_1^\dagger\Phi_1)^3 + \kappa_2(\Phi_2^\dagger\Phi_2)^3 + \kappa_3(\Phi_1^\dagger\Phi_1)^2(\Phi_2^\dagger\Phi_2) + \\
&+ \kappa_4(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)^2 + \kappa_5(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \\
&+ \kappa_6(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + [\kappa_7(\Phi_1^\dagger\Phi_2)^3 + \\
&+ \kappa_8(\Phi_1^\dagger\Phi_1)^2(\Phi_1^\dagger\Phi_2) + \kappa_9(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)^2 + \\
&+ \kappa_{10}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_2) + \kappa_{11}(\Phi_1^\dagger\Phi_2)^2(\Phi_2^\dagger\Phi_1) + \\
&+ \kappa_{12}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2)^2 + \\
&+ \kappa_{13}(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + h.c.], && (17)
\end{aligned}$$

$$m_h^{\rm tree} \leq m_Z |\cos 2\beta|$$

Akhmetzyanova, Dolgopolov, Dubinin, Phys.Rev.D71, 2005;
Carena et al., arXiv:hep-ph/9504316v2, 1995

$$\frac{\Delta\lambda_1}{2}[1-\texttt{loop}] = -\frac{3}{32\pi^2}\Big[h_b^4 \frac{|A_b|^2}{M^2_{\rm SUSY}}\left(2 - \frac{|A_b|^2}{6M^2_{\rm SUSY}}\right) - h_t^4 \frac{|\mu|^4}{6M^4_{\rm SUSY}} + \quad (18)$$

$$+ 2h_b^4 l + \frac{g_2^2+g_1^2}{4\,M^2_{\rm SUSY}}(h_t^2|\mu|^2 - h_b^2|A_b|^2)\Big] -$$

$$- \frac{1}{768\pi^2}\left(11 g_1^4 + 9 g_2^4 - 36\,(g_1^2+g_2^2)\,h_b^2\right) l,$$

$$\Delta\lambda_1[2-\texttt{loop}] = -\frac{3}{16\pi^2} h_b^4 \frac{1}{16\pi^2}(\frac{3}{2}h_b^2 + \frac{1}{2}h_t^2 - 8g_{\rm S}^2)(X_b l + l^2) \quad (19)$$

$$+ \frac{3}{192\pi^2} h_t^4 \frac{1}{16\pi^2}\frac{|\mu|^4}{M^4_{\rm SUSY}}(9h_t^2 - 5h_b^2 - 16 g_{\rm S}^2) l \quad (20)$$

where $l = \log\left(\frac{M_S^2}{m_{top}^2}\right)$, $\qquad X_b = \frac{2A_b^2}{M^2_{\rm SUSY}}\left(1 - \frac{A_b^2}{12M^2_{\rm SUSY}}\right)$.

$$
\begin{aligned}
\Delta\kappa_1^{\rm thr} &= \frac{h_b^6}{32M_S^2\pi^2}\left(2-\frac{3|A_b|^2}{M_S^2}+\frac{|A_b|^4}{M_S^4}-\frac{|A_b|^6}{10M_S^6}\right)\\
&- h_b^4\frac{g_1^2+g_2^2}{128M_S^2\pi^2}\left(3-3\frac{|A_b|^2}{M_S^2}+\frac{|A_b|^4}{2M_S^4}\right)+\frac{h_b^2}{512M_S^2\pi^2}\\
&\times \left(\frac{5}{3}g_1^4+2g_1^2g_2^2+3g_2^4\right)\left(1-\frac{|A_b|^2}{2M_S^2}\right)-h_t^6\frac{|\mu|^6}{320M_S^8\pi^2}+h_t^4\frac{(g_1^2+g_2^2)|\mu|^4}{256M_S^6\pi^2}\\
&- h_t^2\frac{(17g_1^4-6g_1^2g_2^2+9g_2^4)|\mu|^2}{3072M_S^4\pi^2}+\frac{g_1^2}{1024M_S^2\pi^2}(g_1^4-g_2^4), \qquad (21)
\end{aligned}
$$

where $l \equiv \ln\left(\frac{M_S^2}{\sigma^2}\right)$, $\sigma = m_{top}$ is the renormalization scale, $h_t = \frac{g_2 m_{top}}{\sqrt{2}m_W \sin\beta}$ and $h_b = \frac{g_2 m_b}{\sqrt{2}m_W \cos\beta}$ are the Yukawa couplings.

$$
\begin{aligned}
m_{H^\pm}^2 &= m_W^2+m_A^2-\frac{v^2}{2}({\rm Re}\Delta\lambda_5-\Delta\lambda_4)+\frac{v^4}{4}[c_\beta^2(2{\rm Re}\kappa_9-\kappa_5)\\
&+ s_\beta^2(2{\rm Re}\kappa_{10}-\kappa_6)-s_{2\beta}({\rm Re}\kappa_{11}-3{\rm Re}\kappa_7)]. \qquad (22)
\end{aligned}
$$