On perturbative unitarity in the extended MSSM Higgs sector

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The SM of particle physics works extremely well. The discovery of Higgs boson at the LHC
But it has problems

- ‘SM fine-tuning problem’
- sources of CP-violation
- DM candidate ...

SM is an effective theory at low-energy

Higgs boson properties are SM-like, but investigations continue. The Higgs sector can be nonminimal
THDM

$$\Phi_i = \left(\frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i)^+\right), \quad i = 1, 2$$

$$v_1 = v \cos \beta, \ v_2 = v \sin \beta, \ v = \sqrt{v_1^2 + v_2^2} = 246 \ \text{GeV}$$

An elegant solution provides SUSY$^1$. Among SUSY-models, the most popular and investigated model is MSSM (THDM-II). The SUSY must be broken, so the low-energy theory is THDM.

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As any effective potential, THDM-Higgs potential can be presented as

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U(\Phi_1, \Phi_2) = U^{(2)} + U^{(4)} + U^{(6)} + U^{(8)} + \ldots
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U^{(2)} = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - [\mu_{12}^2(\Phi_1^\dagger \Phi_2) + h.c.],
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\[
U^{(4)} \supset \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \ldots
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\[
[\mu_i^2] = \text{GeV}^2, \quad [\lambda_j] = 1, \quad [\kappa_n] = \text{GeV}^{-2}
\]

\[i = 1, 2, \quad j = 1, \ldots, 7, \quad n = 1, \ldots, 13\]


How do the perturbative unitarity constraints change in this case?
What values of free model parameters can be allowed by perturbative unitarity?
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Perturbative unitarity constraints (PUCs)

We will study the unitarity constraints by computing the scalar scattering processes $S_1S_2 \rightarrow S_3S_4$

$$M(s, t, u) = 16\pi \sum_{J=0}^{\infty} (2J + 1) P_J(\cos \theta) a_J(s),$$

(5)

where $s, t, u$ are Mandelstam variables, $P_J$ are Legendre Polynomials

The cross section

$$d\sigma/d\Omega = |M|^2/64\pi^2 s, \quad \sigma = \frac{16\pi}{s} \sum (2J + 1) |a_J(s)|^2,$$

is proportional to the imaginary part of the amplitude in the forward direction

$$\sigma = \frac{1}{s} \text{Im}[M(\theta = 0)]$$

(6)

$$|a_l|^2 = \text{Im}(a_J) \quad \text{for all } J.$$

From

$$\text{Re}(a_l)^2 + \text{Im}(a_l)^2 = |a_J|^2$$

the famous unitarity constraint of the partial wave amplitude

$$|a_J|^2 \leq \frac{1}{2}$$

(7)
\[ J = 0 \]

\[ a_0 = - \frac{2^{-\frac{1}{2}(\delta_{12}+\delta_{34})}}{16\pi} \left\{ \left[ \lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2) \right] \frac{1}{4} \right\} \left[ \lambda^{1234} + \kappa^{125}\kappa^{345} \frac{1}{s-m_5^2} \right] \]

\[ - \kappa^{135}\kappa^{245} f_t(s, m_1^2, ... 5) - \kappa^{145}\kappa^{235} f_u(s, m_1^2, ... 5) \right\}, \]

(8)

where the factor \( \delta_{12} (\delta_{34}) \) is 1 if particles \{1, 2\} (\{3, 4\}) are identical, and zero otherwise, \( s = (p_1 + p_2)^2 \) is the Mandelstam variable, \( m_5 \) is the particle mass in a propagator, \( \lambda^{1234} \) and \( \kappa^{ijk} \) are quartic and trilinear couplings of scalars,

\[ f_t(s, m_1^2, ... 5) \equiv \frac{1}{s} \frac{1}{\left[ \lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2) \right] \frac{1}{4}} \log \left( \frac{m_1^2 + m_3^2 - m_5^2 - 2E_1E_3 + 2|p_1||p_3|}{m_1^2 + m_3^2 - m_5^2 - 2E_1E_3 - 2|p_1||p_3|} \right), \]

\[ f_u(s, m_1^2, ... 5) \equiv f_t(s, m_1^2, m_2^2, m_4^2, m_3^2, m_5^2), \]

\[ \lambda(s, m_i^2, m_j^2) \equiv \frac{1}{s^2} \left( s^2 + m_i^4 + m_j^4 - 2m_i^2m_j^2 - 2sm_i^2 - 2sm_j^2 \right), \]

(9)

\[ |p_1| = \frac{1}{2} \sqrt{s\lambda(s, m_1^2, m_2^2)}, \quad |p_3| = \frac{1}{2} \sqrt{s\lambda(s, m_3^2, m_4^2)} \]

(10)

are the centre of mass three-momenta,

\[ E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_3 = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}} \]

(11)

are the energies of the particles in the center of mass frame.

Goldstone-boson equivalence theorem
External longitudinal gauge bosons can be replaced by corresponding Goldstone modes

Lee-Quigg-Thacker theorem
In high-energy limit ($\sqrt{s} \gg m_5$) the amplitudes for two-body scattering processes are equivalent to those with longitudinal gauge bosons up to terms of $O(m_5^2/s)$

Lee, Quigg, Thacker, Phys. Rev. D 16, 1519 (1977)

For example:

$$a_0^{\text{THDM}} = \frac{1}{16\pi} \text{Diag}(X_{4\times4}, Y_{4\times4}, Z_{3\times3}, Z_{3\times3})$$

(12)

$$Y_{4\times4} = \begin{pmatrix} 
\lambda_1 & \lambda_4 & \sqrt{2}\text{Re}\lambda_6 & \sqrt{2}\text{Im}\lambda_6 \\
\lambda_4 & \lambda_2 & \sqrt{2}\text{Re}\lambda_7 & \sqrt{2}\text{Im}\lambda_7 \\
\sqrt{2}\text{Re}\lambda_6 & \sqrt{2}\text{Re}\lambda_7 & \lambda_3 + \text{Re}\lambda_5 & \text{Im}\lambda_5 \\
\sqrt{2}\text{Im}\lambda_6 & \sqrt{2}\text{Im}\lambda_7 & \text{Im}\lambda_5 & \lambda_3 - \text{Re}\lambda_5 
\end{pmatrix}, \ldots$$


It is not always a valid approximation: the full calculation including all tree-level contributions at finite energy ($\sqrt{s} \sim m_5$) can lead to much more stringent constraints
Analytical formula for trilinear and quartic couplings of the extended Higgs potential have derived. They are rather cumbersome. Explicit expressions are presented in Mathematica-code and will be open access

For example:

\[ \kappa_{hhh} = c_1 v + c_2 v^3, \]

where

\[ c_1 = -\lambda_1 s_\alpha^3 c_\beta + \lambda_2 c_\alpha^3 s_\beta - \frac{\lambda_{345}}{4} s_\alpha c_{\alpha+\beta} + \]
\[ + \frac{\text{Re}\lambda_6}{2} s_\alpha^2 (c_\beta - \alpha + 2c_\beta + \alpha) + \frac{\text{Re}\lambda_7}{2} c_\alpha^2 (c_\alpha c_\beta - 3s_\alpha s_\beta), \]
\[ c_2 = \frac{5}{2} [-\kappa_1 s_\alpha^3 c_\beta + \kappa_2 c_\alpha^3 s_\beta + (\text{Re}\kappa_8 s_\alpha^2 c_\beta^2 + \]
\[ + \text{Re}\kappa_{12} c_\alpha^2 s_\beta) c_{\alpha+\beta}] + \frac{1}{16} [(\kappa_3 + \kappa_5 + 2\text{Re}\kappa_9) s_\alpha c_\beta - \]
\[ - (\kappa_4 + \kappa_6 + 2\text{Re}\kappa_{10}) c_\alpha s_\beta] (c_2 (\beta - \alpha) - \]
\[ - 5c_2 (\alpha + \beta) - 4) + \frac{1}{32} \text{Re}(\kappa_7 + \kappa_{11} + \kappa_{13}) \times \]
\[ \times [5c_3 (\alpha + \beta) - 3(c_\beta - 3\alpha + c_\beta - \alpha - 3c_{\alpha+\beta})], \]
\[ \lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}\lambda_5. \]
Assume that

1. only 3d generation of squarks are important
2. \( M\tilde{Q} \approx M\tilde{U} \approx M\tilde{D} \approx M_{SUSY} \)
3. the Higgs-boson couplings to the heavier SM particles are SM-like (alignment limit)

**Free parameters:** \( m_A \), \( \tan\beta \) (tree-level) and \( M_{SUSY}, A_t, A_b, \mu \) (loop-level)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( M_S ), GeV</th>
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<td>1000</td>
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Table: Benchmark scenarios* specified in 1302.7033 [hep-ph]. Varied parameters adjust in such a way that \( m_h = 125 \) GeV in alignment limit. Here \( m_t = 173.2 \) GeV, \( m_b = 4.2 \) GeV, \( m_Z = 91.1876 \) GeV, \( m_W = 80.385 \) GeV, \( \alpha_S(m_t) = 0.118 \), \( G_F = 1.16639 \times 10^{-5} \) GeV\(^{-2} \);

\[
\cos\theta_W = m_W / m_Z, \quad g^2 = 8m_Z^2 G_F / \sqrt{2}, \quad g_2 = g \cos\theta_W, \quad g_1 = g_2 \tan\theta_W, \quad v = 1 / \sqrt{\sqrt{2} G_F},
\]

\[
M_S = \sqrt{m_{t_1} m_{t_2}}, \quad M_{SUSY} = \sqrt{M_S^2 - m_t^2}, \quad A_t = A_b
\]
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$M_S = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}, \quad M_{SUSY} = \sqrt{M_S^2 - m_t^2}, \quad A_t = A_b$
Table: BPs for $m_h=125$ GeV in alignment limit

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<td>6</td>
<td>3000</td>
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2. We analyzed the parameter space which can satisfy the PUCs and shown that the values of $A_{t,b,\mu}$ are acceptable up to 10 TeV.

3. Dimension-six operators change significantly PUCs in the MSSM regime with $\mu \sim 9$ TeV.

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\[ + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c.] \]  

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\[ + \kappa_4(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)^2 + \kappa_5(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \]
\[ + \kappa_6(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + [\kappa_7(\Phi_1^\dagger \Phi_2)^3 + \]
\[ + \kappa_8(\Phi_1^\dagger \Phi_1)^2(\Phi_1^\dagger \Phi_2) + \kappa_9(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)^2 + \]
\[ + \kappa_{10}(\Phi_1^\dagger \Phi_2)^2(\Phi_2^\dagger \Phi_2) + \kappa_{11}(\Phi_1^\dagger \Phi_2)^2(\Phi_2^\dagger \Phi_1) + \]
\[ + \kappa_{12}(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2)^2 + \]
\[ + \kappa_{13}(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2) + h.c.] \]
\[ m_{h}^{\text{tree}} \leq m_{Z} | \cos 2\beta | \]


\[
\frac{\Delta \lambda_{1}}{2} [1 - \text{loop}] = - \frac{3}{32\pi^{2}} \left[ h_{b}^{4} \frac{|A_{b}|^{2}}{M_{\text{SUSY}}^{2}} \left( 2 - \frac{|A_{b}|^{2}}{6M_{\text{SUSY}}^{2}} \right) - h_{t}^{4} \frac{|\mu|^{4}}{6M_{\text{SUSY}}^{4}} + \right.
\]
\[
+ 2h_{b}^{4}l + \frac{g_{2}^{2} + g_{1}^{2}}{4M_{\text{SUSY}}^{2}} (h_{t}^{2} |\mu|^{2} - h_{b}^{2} |A_{b}|^{2}) \left] - \frac{1}{768\pi^{2}} \left( 11g_{1}^{4} + 9g_{2}^{4} - 36 (g_{1}^{2} + g_{2}^{2}) h_{b}^{2} \right) l, \right.
\]

\[
\Delta \lambda_{1} [2 - \text{loop}] = - \frac{3}{16\pi^{2}} h_{b}^{4} \frac{1}{16\pi^{2}} \left( \frac{3}{2} h_{b}^{2} + \frac{1}{2} h_{t}^{2} - 8g_{S}^{2} \right) (X_{b} l + l^{2}) + \frac{3}{192\pi^{2}} h_{t}^{4} \frac{1}{16\pi^{2}} \frac{|\mu|^{4}}{M_{\text{SUSY}}^{4}} (9h_{t}^{2} - 5h_{b}^{2} - 16g_{S}^{2}) l
\]

where \( l = \log \left( \frac{M_{S}^{2}}{m_{\text{top}}^{2}} \right) \), \( X_{b} = \frac{2A_{b}^{2}}{M_{\text{SUSY}}^{2}} \left( 1 - \frac{A_{b}^{2}}{12M_{\text{SUSY}}^{2}} \right) \).
\[ \Delta \kappa_{1}^{\text{thr}} = \frac{h_b^6}{32M_S^2\pi^2} \left( 2 - \frac{3|A_b|^2}{M_S^2} + \frac{|A_b|^4}{2M_S^4} - \frac{|A_b|^6}{10M_S^6} \right) \]

\[ - h_b^4 \frac{g_1^2 + g_2^2}{128M_S^2\pi^2} \left( 3 - \frac{3|A_b|^2}{2M_S^2} + \frac{|A_b|^4}{4M_S^4} \right) + \frac{h_b^2}{512M_S^2\pi^2} \]

\[ \times \left( \frac{5}{3}g_1^4 + 2g_1^2g_2 + 3g_2^4 \right) \left( 1 - \frac{|A_b|^2}{2M_S^2} \right) - h_t^6 \frac{|\mu|^6}{320M_S^8\pi^2} + h_t^4 \frac{(g_1^2 + g_2^2)|\mu|^4}{256M_S^6\pi^2} \]

\[ - h_t^2 \frac{(17g_1^4 - 6g_1^2g_2^2 + 9g_2^4)|\mu|^2}{3072M_S^4\pi^2} + \frac{g_1^2}{1024M_S^2\pi^2} (g_1^4 - g_2^4), \]

(21)

where \( l \equiv \ln \left( \frac{M_S^2}{\sigma^2} \right) \), \( \sigma = m_{\text{top}} \) is the renormalization scale, \( h_t = \frac{g_2 m_{\text{top}}}{\sqrt{2}m_W \sin \beta} \) and \( h_b = \frac{g_2 m_b}{\sqrt{2}m_W \cos \beta} \) are the Yukawa couplings.

\[ m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v^2}{2} (\text{Re}\Delta \lambda_5 - \Delta \lambda_4) + \frac{v^4}{4} [c^2 (2\text{Re}\kappa_9 - \kappa_5) \]

\[ + s^2 (2\text{Re}\kappa_{10} - \kappa_6) - s^2 \beta (\text{Re}\kappa_{11} - 3\text{Re}\kappa_7)]. \]

(22)