QCD corrections for double charmonia production in e^+e^- annihilation

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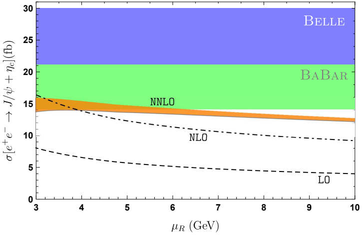
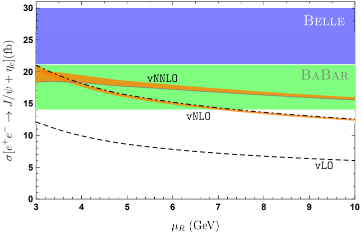
$J/\psi\ \eta_c$ production study at B-factories





Cross sections for $e^+e^- \to J/\psi\ \eta_c$ measured in Belle and BaBar at $\sqrt{s} = 10.6$ GeV. Corrections up to $\mathcal{O}(\alpha_S^2 v^2)$ are performed by [6].

Meaurements of $J/\psi\ \eta_c$, $J/\psi\ \chi_c$ production near the threshold revealed the failure of theoretic predictions: predicted cross sections were at least 5 times lower.

- A large number of works devoted to perturbative and relativistic corrections and EFT quarkonia models (for example [2],[9],[10],[3],[5])

- Now this process is studied at two loops accuracy [6]

Studying charmonia physics in e^+e^- collisions is encouraged by FCC-ee project with $\sqrt{s} = 90 \div 400$ GeV and ILC project with $\sqrt{s} = 250$ GeV.

Double charmonia production:

$$\begin{cases} e^+e^- \xrightarrow{Z_0^*} \eta_c\ \eta_c \\ e^+e^- \xrightarrow{\gamma^*, Z_0^*} J\psi\ \eta_c \\ e^+e^- \xrightarrow{Z_0^*} J/\psi\ J/\psi \end{cases}$$

Pair B_c production:

$$e^+e^- \xrightarrow{\gamma^*, Z_0^*} B_c^{(*)} B_c^{(*)}$$

$$\gamma\ \gamma \longrightarrow B_c^{(*)} B_c^{(*)}$$

(see works [1] and [4])

- At energies $\sim M_{Z0}$ annihilation with Z_0^* exchange may become dominant
- Careful consideration: interference between γ^* and Z_0^* is needed
- QCD corrections:

$$|A|^2 = |A_\gamma^{LO}|^2 + |A_Z^{LO}|^2 + 2\mathrm{Re}\left(A_\gamma^{LO} A_Z^{LO*}\right) +$$

$$+2\mathrm{Re}\left(A_\gamma^{LO} A_\gamma^{NLO*}\right) + 2\mathrm{Re}\left(A_Z^{LO} A_Z^{NLO*}\right) + 2\mathrm{Re}\left(A_\gamma^{LO} A_Z^{NLO*}\right) + 2\mathrm{Re}\left(A_Z^{LO} A_\gamma^{NLO*}\right) + \ldots$$

- No corrections for real gluon radiation

Convolution with the wave functions of the quarkonia:

$$A^{SJj_z} = \int T^{Ss_z}_{c\bar{c}c\bar{c}}(p_i, k(\boldsymbol{q}_1), k(\boldsymbol{q}_2)) \cdot \left(\Psi^{Ll_z}_{c\bar{c}}(\boldsymbol{q}_1)\Psi^{Ll_z}_{c\bar{c}}(\boldsymbol{q}_2)\right)^* \cdot C^{Jj_z}_{s_z l_z} \frac{d\boldsymbol{q}_1}{(2\pi)^3}\frac{d\boldsymbol{q}_2}{(2\pi)^3}$$

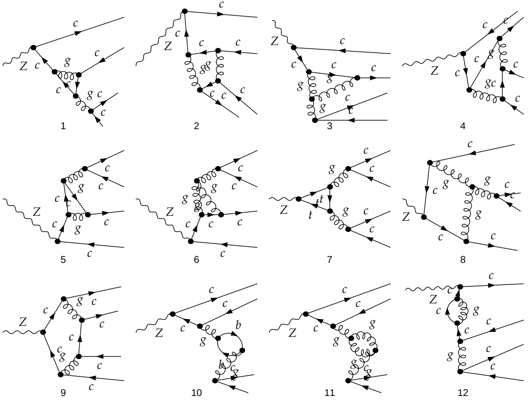
For unpolarized S-wave states:

$$A = \frac{1}{4\pi} R_{J/\psi}(0) R_{\eta_c}(0) \cdot T_{c\bar{c}c\bar{c}}(p_i)\Big|_{\boldsymbol{q}_{1,2}=0}$$

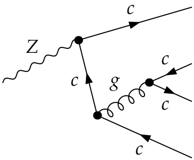
Projection onto the bound states:

$$\Pi_{J/\psi}(P, m) = \frac{\widehat{P} - m}{2\sqrt{2}}\,\gamma^\mu \varepsilon^\mu_{J/\psi} \times \frac{\mathbf{1}}{\sqrt{3}} \qquad \Pi_{\eta_c}(Q, m) = \frac{\widehat{Q} - m}{2\sqrt{2}}\,\gamma^5 \times \frac{\mathbf{1}}{\sqrt{3}}$$

- Colour singlet states
- No internal motion: δ-approximation
- Velocities of quarks in the quarkonium are directly fixed equal



Some sample diagrams at NLO.



- FeynArts

  Generation and visualization of feynman diagrams

FeynArts: analytic expressions for 4 + 86 diagrams

ToolChain:

FeynArts → FeynCalc (FCFormLink, TIDL) → Apart → FIRE → X-package

- FeynCalc: algebraic calculations with Dirac and colour matrices
- FeynCalcFormLink: taking traces through FORM, significantly gains the time
- TIDL library (within FC): Passarino-Veltman reduction, decomposition of tensor expressions with loop momentum ($k^\mu, k^\mu k^\nu, k^\mu \varepsilon^\mu \ldots$); only k^2 in numerator aftewards
- $Apart function: partial fractioning for IR-divergent integrals
- FIRE: complete reduction to master integrals
- X-package: analytical evaluation of master integrals ($\boldsymbol{A}_0, \boldsymbol{B}_0, \boldsymbol{C}_0$ integrals in our case)

$$
\frac{1}{3\, m^5 (s-4)\, s^2} i\, e\, C_F\, g_s^4\, \epsilon^{\gamma\, \mu\, \overline{\mathrm{P1}}\, \overline{\mathrm{P2}}} \left(\frac{m^2 \left(3 D^2 s - 2 D (17 s + 1) + 60 s + 6\right) \left\| \frac{1}{k^2 \left(-k\cdot \mathrm{P1} - 2 (k\cdot \mathrm{P2}) + k^2 + 2 m^2 s\right)} \right\|}{D-2} - \right.
$$

$$
\frac{m^2 \left(D^2 (s-4) + D (36 - 11 s) + 20 s - 64\right) \left\| \frac{1}{(k\cdot \mathrm{P1} + k^2)(k^2 - k\cdot \mathrm{P2})} \right\|}{(D-4)(D-2)} +
$$

$$
\frac{2 m^2 \left(D^3 s - 15 D^2 s + 2 D (35 s - 6) - 90 s + 28\right) \left\| \frac{1}{(k\cdot \mathrm{P1} + k^2)\left(-k\cdot \mathrm{P1} - 2 (k\cdot \mathrm{P2}) + k^2 + 2 m^2 s\right)} \right\|}{D-2} +
$$

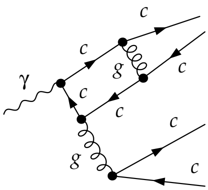
$$
\frac{m^2 \left(D^3 s - 13 D^2 s + D (47 s - 12) - 49 s + 28\right) \left\| - \frac{1}{(k\cdot \mathrm{P2} + k^2)\left(2 (k\cdot \mathrm{P2} + m^2 s) + k\cdot \mathrm{P1} + k^2\right)} \right\|}{D-2} +
$$

$$
\frac{1}{D-2} m^4 \left(D^3 s^2 + 4 D^2 (1 - 4 s) s + 4 D \left(20 s^2 - 15 s + 8\right) - 4 \left(26 s^2 - 25 s + 16\right)\right)
$$

$$
\left\| \frac{1}{k^2 (k\cdot \mathrm{P1} + k^2)\left(-k\cdot \mathrm{P1} - 2 (k\cdot \mathrm{P2}) + k^2 + 2 m^2 s\right)} \right\| +
$$

$$
\frac{m^4 \left(D^3 s^2 - 13 D^2 s^2 + 4 D \left(12 s^2 - 3 s - 4\right) - 52 s^2 + 32 s + 32\right) \left\| \frac{1}{k^2 (k^2 - k\cdot \mathrm{P2})\left(-k\cdot \mathrm{P1} - 2 (k\cdot \mathrm{P2}) + k^2 + 2 m^2 s\right)} \right\|}{D-2} -
$$

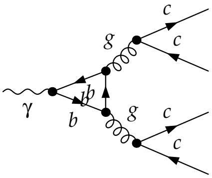
$$
\left. \frac{\left(D^4 s - 18 D^3 s + 2 D^2 (57 s - 5) + D (54 - 296 s) + 264 s - 72\right) \left\| \frac{1}{k^2 - k\cdot \mathrm{P1}} \right\|}{2 (D-4)(D-3)} \right)
$$



(a) box

Expression for box diagram (Pic. a) simplified to master integrals.

- Triangle diagrams do not contribute to $\gamma^* \to J/\psi\ \eta_c$ process.



(b) triangle

CDR regularization scheme:

$D = 4 - 2\varepsilon$ for all momenta (loop and external)

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g_{\mu\nu}g^{\mu\nu} = D$$

γ^5 interpretation:

Traces with an odd number of γ^5 are left with one γ^5 to the right and

$$\gamma^5 = \frac{-i}{24}\varepsilon_{\alpha\beta\sigma\rho}\gamma^\alpha\gamma^\beta\gamma^\sigma\gamma^\rho,$$

where $\varepsilon_{\alpha\beta\sigma\rho}$ is either 4-dim or D-dim

"On shell" scheme for mass and spinors renormalization, $\overline{MS}$ scheme for coupling constant:

$$Z_m^{OS} = 1 - \frac{\alpha_s}{4\pi}C_F C_\epsilon\left[\frac{3}{\epsilon_{UV}} + 4\right] + O(\alpha_s^2),$$

$$Z_2^{OS} = 1 - \frac{\alpha_s}{4\pi}C_F C_\epsilon\left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4\right] + O(\alpha_s^2),$$

$$Z_g^{\overline{MS}} = 1 - \frac{\beta_0}{2}\frac{\alpha_s}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi)\right] + O(\alpha_s^2),$$

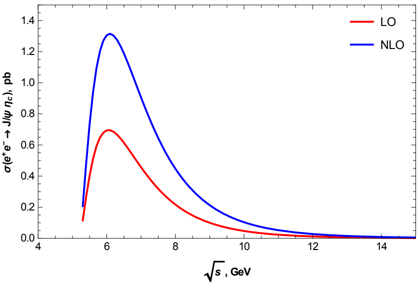
$$\mathcal{A}^{CT} = Z_2^2 \mathcal{A}^{LO}\Big|_{\substack{m \to Z_m m \\ g_s \to Z_g g_s}}$$

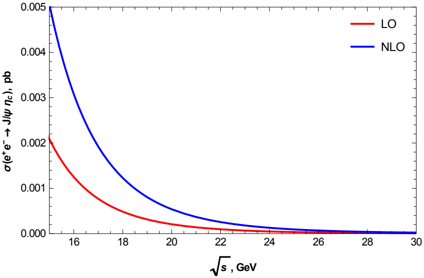
- Automatic tools do not distinguish ε_{IR} and ε_{UV}
- Singular parts carry poles $\sim 1/\epsilon$ only

- Triangle diagrams are relevant only for Z_0^* case (axial-vector structure)
- Amplitude terms $\sim \frac{1}{D-4}$ arising after FIRE are cancelled with each other — no necessarity to include extra terms $\sim O(\varepsilon)$ in $\boldsymbol{A}_0, \boldsymbol{B}_0$ expansion (comp. to [1])
- For NLO Z_0^* contribution no matter whether γ^5 is taken with $\varepsilon_{\alpha\beta\sigma\rho}$ as D-dim or 4-dim — the renormalized amplitudes coincide

Cross-checks already done:

- $\sigma_{LO}^{\gamma}\,(J/\psi\ \eta_c)$ with γ^* exchange is fixed
- $\sigma_{LO}\,(J/\psi\ \eta_c)$ reproduces analitically $\sigma_{LO}\,(B_c^* B_c)$ in the limit $m_b \to m_c,\ e_b \to e_c = +\frac{2}{3}$ (see [7] as well)
- $\sigma_{NLO}\,(B_c^* B_c)$ calculation is reproduced and checked numerically; proceeding from this code $\sigma_{NLO}\,(J/\psi\ \eta_c)$ is obtained



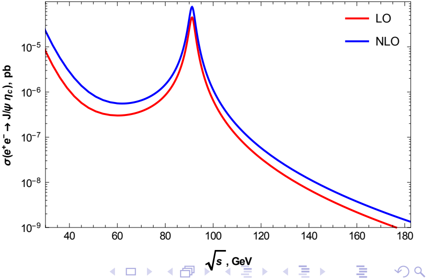


Near the threshold:

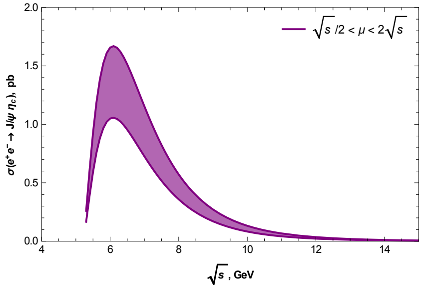
$$\sqrt{s}_{max} = \sqrt{5.5}\ m \approx 6 \div 7 \text{ GeV}$$

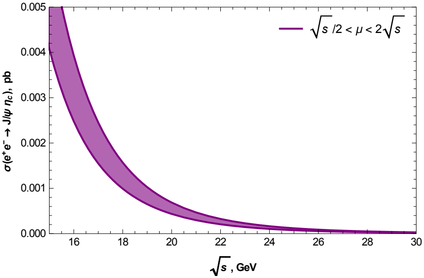
At high energies:

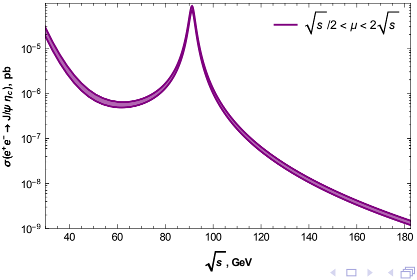
$$\sigma_{LO} \sim 1/s^3$$
$$\sigma_{NLO} \sim 1/s^3$$



Preliminary results: cross sections







Starting from $\sqrt{s} \approx 60$ GeV:

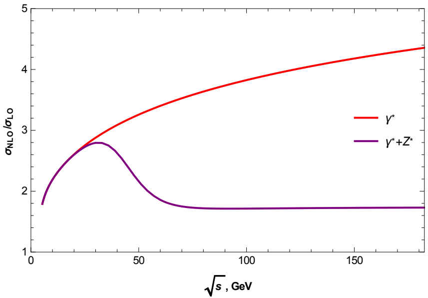
$$\frac{\sigma_{NLO}}{\sigma_{LO}} = 1.6 \div 1.8 = const,$$

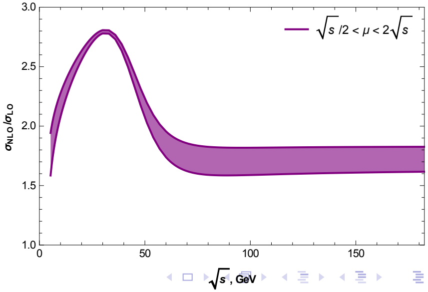
however

$$\frac{\sigma^{\gamma}_{NLO}}{\sigma^{\gamma}_{LO}} \neq const,$$

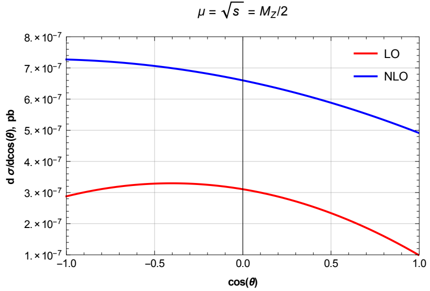
since $\sigma^{\gamma}_{LO} \sim 1/s^4$ and

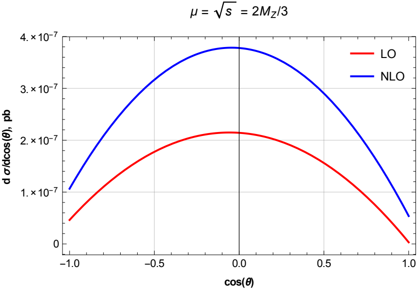
$\mathcal{O}(1/s^4) < \sigma^{\gamma}_{NLO} < \mathcal{O}(1/s^3)$

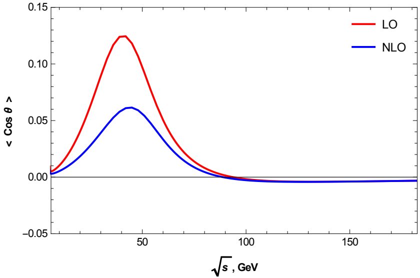




Preliminary results: angular distributions







- Calculation of the process $e^+e^- \xrightarrow{\gamma^*, Z_0^*} B_c^* B_c$ is reproduced at next-to-leading order precision
- QCD corrections $\mathcal{O}(\alpha_S^3)$ for associative $J/\psi\ \eta_c$ production in e^+e^- annihilation are presented; interference between virtual γ and Z_0 is considered
- At energies $\sim M_{Z0}$ cross sections are enhanced as $\sigma_{NLO} \approx 1.7\ \sigma_{LO}$
- Cross-checks to do:
  fix σ^{γ}_{NLO} contribution comparing with work [5] diagram by diagram,
  compare $\Gamma\left(Z_0 \to J/\psi\ \eta_c\right)$ with work [8]

Upon the code for associative production $\begin{cases} e^+e^- \xrightarrow{Z_0^*} J/\psi\ J/\psi \\ e^+e^- \xrightarrow{Z_0^*} \eta_c\ \eta_c \end{cases}$

is refined we proceed with publication.

Thank you for attention!

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