QCD corrections for double charmonia production in $e^+e^-$ annihilation

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**J/ψ η_c** production study at B-factories

Measurements of $J/ψ \eta_c$, $J/ψ \chi_c$ production near the threshold revealed the failure of theoretic predictions: predicted cross sections were at least 5 times lower.

- A large number of works devoted to perturbative and relativistic corrections and EFT quarkonia models (for example [2],[9],[10],[3],[5])
- Now this process is studied at two loops accuracy [6]

Cross sections for $e^+e^- \rightarrow J/ψ \eta_c$ measured in Belle and BaBar at $\sqrt{s} = 10.6$ GeV. Corrections up to $O(\alpha_S^2 v^2)$ are performed by [6].
\( J/\psi \eta_c \) production at higher energies

Studying charmonia physics in \( e^+e^- \) collisions is encouraged by FCC-ee project with \( \sqrt{s} = 90 \div 400 \text{ GeV} \) and ILC project with \( \sqrt{s} = 250 \text{ GeV} \).

Double charmonia production:

\[
\begin{align*}
    e^+e^- & \rightarrow \eta_c \eta_c \\
    e^+e^- & \rightarrow J\psi \eta_c \\
    e^+e^- & \rightarrow J/\psi J/\psi
\end{align*}
\]

Pair \( B_c \) production:

\[
\begin{align*}
    e^+e^- & \rightarrow \gamma^*,Z_0^* B_c^{(*)} B_c^{(*)} \\
    \gamma \gamma & \rightarrow B_c^{(*)} B_c^{(*)}
\end{align*}
\]

( see works [1] and [4] )

- At energies \( \sim M_{Z0} \) annihilation with \( Z_0^* \) exchange may become dominant
- Careful consideration: interference between \( \gamma^* \) and \( Z_0^* \) is needed
- QCD corrections:

\[
|A|^2 = |A_{\gamma}\text{LO}|^2 + |A_{Z}\text{LO}|^2 + 2\text{Re} \left( A_{\gamma}\text{LO}A_{Z}\text{LO}^* \right) + \\
+ 2\text{Re} \left( A_{\gamma}\text{LO}A_{\gamma}\text{NLO}^* \right) + 2\text{Re} \left( A_{Z}\text{LO}A_{Z}\text{NLO}^* \right) + 2\text{Re} \left( A_{Z}\text{LO}A_{\gamma}\text{NLO}^* \right) + \ldots
\]

- No corrections for real gluon radiation
Our approximation

Convolution with the wave functions of the quarkonia:

\[ A^{S Jjz} = \int T^{S s \bar{s} \bar{c} c}_{c \bar{c} \bar{c} c} (p_i, k(q_1), k(q_2)) \cdot \left( \Psi^L_{c \bar{c}}(q_1) \Psi^L_{c \bar{c}}(q_2) \right)^* \cdot C^{J \bar{s} \bar{j} l} \frac{dq_1}{(2\pi)^3} \frac{dq_2}{(2\pi)^3} \]

For unpolarized \( S \)-wave states:

\[ A = \frac{1}{4\pi} R_{J/\psi}(0) R_{\eta_c}(0) \cdot T_{c \bar{c} \bar{c} c}(p_i) \bigg|_{q_1,2=0} \]

Projection onto the bound states:

\[ \Pi_{J/\psi}(P, m) = \frac{\hat{P} - m}{2\sqrt{2}} \frac{1}{\sqrt{3}} \gamma^\mu \varepsilon_{J/\psi}^\mu \times \frac{1}{\sqrt{3}} \]
\[ \Pi_{\eta_c}(Q, m) = \frac{\hat{Q} - m}{2\sqrt{2}} \gamma^5 \times \frac{1}{\sqrt{3}} \]

- Colour singlet states
- No internal motion: \( \delta \)-approximation
- Velocities of quarks in the quarkonium are directly fixed equal
Production of $c\bar{c}c\bar{c}$ in $Z_0$ decay

Some sample diagrams at NLO.

FeynArts: analytic expressions for 4 + 86 diagrams

FeynArts
Generation and visualization of feynman diagrams
Computation strategy

ToolChain:

**FeynArts → FeynCalc (FCFormLink, TIDL) → Apart → FIRE → X-package**

- **FeynCalc**: algebraic calculations with Dirac and colour matrices
- **FeynCalcFormLink**: taking traces through FORM, significantly gains the time
- **TIDL library (within FC)**: Passarino-Veltman reduction, decomposition of tensor expressions with loop momentum \((k_\mu, k_\mu k_\nu, k_\mu \varepsilon_\mu \ldots)\); only \(k^2\) in numerator afterwards
- **Apart function**: partial fractioning for IR-divergent integrals
- **FIRE**: complete reduction to master integrals
- **X-package**: analytical evaluation of master integrals \((A_0, B_0, C_0\) integrals in our case)
Expression for box diagram (Pic. a) simplified to master integrals.

- Triangle diagrams do not contribute to $\gamma^* \to J/\psi \eta_c$ process.
Regularization and renormalization technique

CDR regularization scheme:

\[ D = 4 - 2\varepsilon \quad \text{for all momenta (loop and external)} \]

\[ \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}, \ g_{\mu\nu}g^{\mu\nu} = D \]

\[ \gamma^5 \] interpretation:

Traces with an odd number of \( \gamma^5 \) are left with one \( \gamma^5 \) to the right and

\[ \gamma^5 = \frac{-i}{24} \varepsilon_{\alpha\beta\sigma\rho} \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho, \]

where \( \varepsilon_{\alpha\beta\sigma\rho} \) is either 4-dim or \( D \)-dim

"On shell" scheme for mass and spinors renormalization,

\( \overline{MS} \) scheme for coupling constant:

\[ Z_{m}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\varepsilon \left[ \frac{3}{\varepsilon_{UV}} + 4 \right] + O(\alpha_s^2), \]

\[ Z_{2}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_\varepsilon \left[ \frac{1}{\varepsilon_{UV}} + \frac{2}{\varepsilon_{IR}} + 4 \right] + O(\alpha_s^2), \]

\[ \overline{Z}_{g}^{MS} = 1 - \frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[ \frac{1}{\varepsilon_{UV}} - \gamma_E + \ln(4\pi) \right] + O(\alpha_s^2), \]

\[ A^{CT} = Z_{2}^{2} A^{LO} \bigg|_{m \rightarrow Z_{m} m, g_s \rightarrow Z_{g} g_s} \]

- Automatic tools do not distinguish \( \varepsilon_{IR} \) and \( \varepsilon_{UV} \)
- Singular parts carry poles \( \sim 1/\varepsilon \) only
Technical features and cross-checks

- Triangle diagrams are relevant only for $Z_0^*$ case (axial-vector structure).

- Amplitude terms $\sim \frac{1}{D-4}$ arising after FIRE are cancelled with each other — no necessity to include extra terms $\sim O(\varepsilon)$ in $A_0, B_0$ expansion (comp. to [1]).

- For NLO $Z_0^*$ contribution no matter whether $\gamma^5$ is taken with $\varepsilon_{\alpha\beta\sigma\rho}$ as $D$-dim or 4-dim — the renormalized amplitudes coincide.

Cross-checks already done:

- $\sigma_{LO}^\gamma (J/\psi \eta_c)$ with $\gamma^*$ exchange is fixed.

- $\sigma_{LO} (J/\psi \eta_c)$ reproduces analytically $\sigma_{LO} (B_c^* B_c)$ in the limit $m_b \to m_c, \ e_b \to e_c = +\frac{2}{3}$ (see [7] as well).

- $\sigma_{NLO} (B_c^* B_c)$ calculation is reproduced and checked numerically; proceeding from this code $\sigma_{NLO} (J/\psi \eta_c)$ is obtained.
Near the threshold:
\[ \sqrt{s_{max}} = \sqrt{5.5} \ m \approx 6 \div 7 \ \text{GeV} \]

At high energies:
\[ \sigma_{LO} \sim 1/s^3 \]
\[ \sigma_{NLO} \sim 1/s^3 \]
Preliminary results: cross sections

\[ \sigma(e^+e^- \rightarrow J/\psi \eta_c), \text{ pb} \]
Starting from $\sqrt{s} \approx 60$ GeV:

$$\frac{\sigma_{NLO}}{\sigma_{LO}} = 1.6 ÷ 1.8 = \text{const},$$

however

$$\frac{\sigma_{NLO}^\gamma}{\sigma_{LO}^\gamma} \neq \text{const},$$

since $\sigma_{LO}^\gamma \sim 1/s^4$ and

$$\mathcal{O}(1/s^4) < \sigma_{NLO}^\gamma < \mathcal{O}(1/s^3)$$
Preliminary results: angular distributions

\[ \mu = \sqrt{s} = \frac{M_Z}{2} \]

\[ \mu = \sqrt{s} = \frac{2M_Z}{3} \]

\[ \cos(\theta) \frac{d\sigma}{d\cos(\theta)}, \text{pb} \]

\[ \mu = \sqrt{s} = \frac{M_Z}{2} \]

\[ \mu = \sqrt{s} = \frac{2M_Z}{3} \]

\[ \langle \cos \theta \rangle, \text{GeV} \]
Conclusions

• Calculation of the process $e^+e^- \xrightarrow{\gamma^*, Z_0^*} B_c^* B_c$ is reproduced at next-to-leading order precision

• QCD corrections $O(\alpha_S^3)$ for associative $J/\psi \eta_c$ production in $e^+e^-$ annihilation are presented; interference between virtual $\gamma$ and $Z_0$ is considered

• At energies $\sim M_{Z_0}$ cross sections are enhanced as $\sigma_{NLO} \approx 1.7 \sigma_{LO}$

• Cross-checks to do:
  fix $\sigma_{NLO}^{\gamma}$ contribution comparing with work [5] diagram by diagram, compare $\Gamma (Z_0 \rightarrow J/\psi \eta_c)$ with work [8]

Upon the code for associative production

\[
\begin{align*}
\left\{ \begin{array}{c}
  e^+e^- & \xrightarrow{Z_0^*} J/\psi \ J/\psi \\
  e^+e^- & \xrightarrow{\eta_c} \eta_c \ \eta_c
\end{array} \right. 
\end{align*}
\]

is refined we proceed with publication.
Thank you for attention!

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