Analytical reconstruction of *pp* elastic scattering amplitudes from the complete sets of experiments at the SPASCHARM facility at U70.

M.B. Nurusheva[1], A.A. Bogdanov[1], V.A.Chetvertkova[4], A.V. Kozelov[2], V.P. Ladygin[3], V.V.Mochalov[1,2], V.A. Okorokov[1], A.O. Oreshkov[1,2], V.L. Rykov[1], P.A. Semenov[1,2], A.N.Vasiliev[1,2],

[1]National Research Nuclear University (Moscow Engineering Physics Institute), Moscow, 115409, Russia
[2]NRC «Kurchatov Institute» - IHEP, Protvino, Moscow region, 142281, Russia
[3] Joint Institute for Nuclear Research, Dubna, Moscow region, 141980,Russia
[4]GSI, Plankstrasse 1, 64291, Darmstadt, Germany

Introduction

The comprehensive physics program of the fixed target SPASCHARM experiment (SPin ASymmetry in CHARMonia) is described in detail in the Conceptual Design Report [1] . It will explore spin phenomena in dozens of inclusive and exclusive hadronic reactions.

For measuring the beam polarization [2], a number of different approaches will be developed based on known spin asymmetries, including the ones in elastic scattering [3]. In turn, an availability of the polarized beams and of a number of different polarized and unpolarized targets provides an opportunity for extended studies of spin effects in elastic processes [4].

[1] Abramov V V et al. IHEP Preprint 2019-12 (in russian)
[2] Bogdanov A A et al 2017 J.Phys.Conf.Ser. 798 012179
[3] Bogdanov A A et al 2016 J.Phys.Conf.Ser. 678 012034
[4] Abramov V V et al 2017 J.Phys.Conf.Ser. 938 012006

Motivation

The scattering matrix is a canonical concept for describing the scattering processes. However, its elements cannot be directly measured. Therefore, an appropriate full set of observables needs to be defined for the subsequent unambiguous direct reconstruction of the scattering amplitudes (DRSA) from this set.

DRSA is a fully model-independent analysis using only the fundamental conservation laws.

For the last 30 years, the statistical and direct reconstructions of the pp scattering matrix have been undertaken a number of times at various energies [5]. But, direct reconstruction of amplitudes for pp-elastic scattering has been done only for energies below 6 GeV/c. So far, no any data are available for p(bar)p - elastic scattering.

A complete list of available DRSA analyzes for the observed elastic pp scattering is provided in [5] F. Legar, E.A. Strokovsky "Phenomenology and Analysis of Nucleon Scattering Data" (2010)

The main goal of this report

Earlier [6], we defined the minimal complete set of 10 observables (out of 15 available), including the differential cross section, which are required for DRSA, using the targets with longitudinal and vertical polarizations.

With the new polarized active target [7], all three polarizations are available: longitudinal, vertical and horizontal, providing the opportunity to measure 19 observables and then reconstruct all amplitudes in laboratory frame in simple way.

[6] Bogdanov A A *et al* 2020 *J.Phys.Conf.Ser.* **1435** 012044

[7] Biroth M et al 2015 “Design of the Mainz Active Polarized Proton Target”, PoS (PSTP2015) 005

Notation

According the Nucleon-Nucleon (NN) formalism and the four-index notation for observables given in [11], we use the scattering matrix in the form:

$$M(\mathbf{k}',\mathbf{k}) = 1/2\{(a+b)+(a-b)(\sigma_1,\mathbf{n})(\sigma_2,\mathbf{n})+(c+d))(\sigma_1,\mathbf{m})(\sigma_2,\mathbf{m})+$$
$$+(c-d)(\sigma_1,\mathbf{l})(\sigma_2,\mathbf{l})+e(\sigma_1+\sigma_2,\mathbf{n})\},$$

$$\mathbf{n} = [\mathbf{k}'\times\mathbf{k}]/|[\mathbf{k}'\times\mathbf{k}]|$$
$$\mathbf{l} = (\mathbf{k}'+\mathbf{k})/|\mathbf{k}'+\mathbf{k}|$$
$$\mathbf{m} = (\mathbf{k}'-\mathbf{k})/|\mathbf{k}'-\mathbf{k}|$$

where ***a*, *b*, *c*, *d*** and ***e*** are the scattering complex amplitudes, σ_1 and σ_2 are the Pauli 2 x 2 matrices, ***k*** and ***k’*** are the unit vectors in the directions of the incident and scattered particles, respectively

[11] J. Bystricky, F. Lehar, P. Winternitz. Journal de Physique, 1978, 39 (1), pp.1-32.

Notation

The subscripts of the observables, X_{srbt}, refer to the polarization states of the scattered, recoil, beam, and target particles, respectively:

$$X_{srbt}$$

The polarizations of the incident and target particles are oriented along unit vectors **n, s,** and **k** for the beam and target laboratory frame:

$$\mathbf{k}, \quad \mathbf{n} = [\,\mathbf{k} \times \mathbf{k'}],\quad \mathbf{s} = [\,\mathbf{n} \times \mathbf{k}]$$

and **n, s'',** and **k''** for the recoil particle frame:

$$\mathbf{k''},\ \ \mathbf{n},\ \ \mathbf{s''} = [\,\mathbf{n} \times \mathbf{k''}]$$

We denote by θ the CM scattering angle, and by $\theta 2$ - the laboratory angle of the recoil particle.

The observables

In the SPASCHARM setup, the following non-vanishing observables can be measured:

A_{oono}, A_{oono} - beam and target analyzing power,

A_{oonn}, A_{ooss}, A_{ookk}, A_{oosk} - spin correlations.

K_{onno}, $K_{os''so}$, $K_{os''ko}$ - three polarization transfer coefficients from the beam to the recoil particle;

D_{onon}, $D_{os''ok}$, $D_{os''os}$ - three depolarization coefficients for the target;

N_{onkk}, N_{onsk}, N_{onks}, $N_{os''ns}$ $N_{os''nk}$, $N_{os''kn}$, $N_{os''sn}$ - seven polarizations of the recoil particle.

In total, the 19 different observables will be measure at U70.

Relations between scattering amplitudes *a, b, c, d, e* and observables

$$\sigma = \frac{d\sigma}{d\Omega} = \frac{1}{2}|a|^2 + \frac{1}{2}|b|^2 + \frac{1}{2}|c|^2 + \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \quad (1)$$

$$A_3 = \sigma A_{oonn} = \frac{1}{2}|a|^2 - \frac{1}{2}|b|^2 - \frac{1}{2}|c|^2 + \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \quad (2)$$

$$K_3 = \sigma K_{onno} = \frac{1}{2}|a|^2 - \frac{1}{2}|b|^2 + \frac{1}{2}|c|^2 - \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \quad (3)$$

$$D_3 = \sigma D_{onon} = \frac{1}{2}|a|^2 + \frac{1}{2}|b|^2 - \frac{1}{2}|c|^2 - \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \quad (4)$$

$$P = \sigma A_{oono} = \sigma A_{ooon} = \mathrm{Re}(a^* e) \quad (5)$$

Relations between scattering amplitudes *a, b, c, d, e* and observables

$$N_2 = \sigma N_{onkk} = -\mathrm{Re}(d^*e)\cos(\theta) - \mathrm{Im}(a^*d)\sin(\theta) \quad (6)$$

$$A_1 = \sigma A_{oosk} = -\mathrm{Im}(d^*e)\cos(\theta) - \mathrm{Re}(a^*d)\sin(\theta) \quad (7)$$

$$N_1 = \sigma N_{onsk} = -\mathrm{Re}(d^*e)\sin(\theta) + \mathrm{Im}(a^*d)\cos(\theta) + \mathrm{Im}(b^*c) \quad (8)$$

$$N_7 = \sigma N_{onks} = -\mathrm{Re}(d^*e)\sin(\theta) + \mathrm{Im}(a^*d)\cos(\theta) - \mathrm{Im}(b^*c) \quad (9)$$

$$A_2 = \sigma A_{ookk} = \mathrm{Im}(d^*e)\sin(\theta) - \mathrm{Re}(a^*d)\cos(\theta) + \mathrm{Re}(b^*c) \quad (10)$$

$$A_4 = \sigma A_{ooss} = -\mathrm{Im}(d^*e)\sin(\theta) + \mathrm{Re}(a^*d)\cos(\theta) + \mathrm{Re}(b^*c) \quad (11)$$

$$K_1 = \sigma K_{os''ko} = \mathrm{Re}(a^*c)\sin(\theta+\theta_2) + \mathrm{Im}(c^*e)\cos(\theta+\theta_2) - \mathrm{Re}(b^*d)\sin(\theta_2) \quad (12)$$

$$D_1 = \sigma D_{os''ok} = \mathrm{Re}(a^*b)\sin(\theta+\theta_2) - \mathrm{Re}(c^*d)\sin(\theta_2) + \mathrm{Im}(b^*e)\cos(\theta+\theta_2) \quad (13)$$

Relations between scattering amplitudes *a, b, c, d, e* and observables

$$K_2 = \sigma K_{os''so} = -\mathrm{Re}(a^* c)\cos(\theta+\theta_2) + \mathrm{Im}(c^* e)\sin(\theta+\theta_2) - \mathrm{Re}(b^* d)\cos(\theta_2) \quad (14)$$

$$N_4 = \sigma N_{os''kn} = -\mathrm{Im}(a^* c)\cos(\theta+\theta_2) + \mathrm{Re}(c^* e)\sin(\theta+\theta_2) + \mathrm{Im}(b^* d)\cos(\theta_2) \quad (15)$$

$$N_5 = \sigma N_{os''sn} = -\mathrm{Im}(a^* c)\sin(\theta+\theta_2) - \mathrm{Re}(c^* e)\cos(\theta+\theta_2) - \mathrm{Im}(b^* d)\sin(\theta_2) \quad (16)$$

$$D_2 = \sigma D_{os''os} = -\mathrm{Re}(a^* b)\cos(\theta+\theta_2) - \mathrm{Re}(c^* d)\cos(\theta_2) + \mathrm{Im}(b^* e)\sin(\theta+\theta_2) \quad (17)$$

$$N_3 = \sigma N_{os''nk} = -\mathrm{Im}(a^* b)\cos(\theta+\theta_2) + \mathrm{Im}(c^* d)\cos(\theta_2) + \mathrm{Re}(b^* e)\sin(\theta+\theta_2) \quad (18)$$

$$N_6 = \sigma N_{os''ns} = -\mathrm{Im}(a^* b)\sin(\theta+\theta_2) - \mathrm{Im}(c^* d)\sin(\theta_2) - \mathrm{Re}(b^* e)\cos(\theta+\theta_2) \quad (19)$$

Simple direct reconstruction from 11 observables

We have found the most convenient set to consist of 11 quantities:

$$\sigma, P, A_3, D_3, K_3, A_1, A_2, A_4, N_1, N_2, N_7 \qquad (20)$$

Lets assume that e amplitude is real Re $e = e$, Im $e = 0$.

Using equations (1) - (5) ⇨ , we find :

$$a_1 = \frac{P}{e} \qquad (21)$$

$$a_2 = [(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2) / 2e^2]^{1/2} \qquad (22)$$

From (8) – (11): ⇨

$$\mathrm{Re}(b^{*}c) = \frac{A_4 + A_2}{2} \quad (23)$$

$$\mathrm{Im}(b^{*}c) = \frac{N_1 - N_7}{2} \quad (24)$$

We express d_1 and d_2 :

$$d_1 = \frac{-(N_1 + N_7)\sin(\theta) - 2N_2\cos(\theta)}{2e} \quad (25)$$

$$d_2 = \frac{(A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta)}{2e} \quad (26)$$

We find the amplitude e from d_1 (25) and d_2 (26) by substituting them into the expression :

$$\sigma + A_3 - D_3 - K_3 = 2|d|^2$$

$$e = [(\sigma + A_3 - D_3 - K_3)((4(A_1^2 + N_2^2) - (A_4 - A_2)^2 - $$
$$-(N_1 + N_7)^2)\cos(2\theta) + (4A_1(A_4 - A_2) + 4N_2 \cdot \quad (27)$$
$$\cdot(N_1 + N_7))\sin(2\theta) + 4(A_1^2 + N_2^2) + (A_4 - A_2)^2 +$$
$$+(N_1 + N_7)^2)]^{1/2} / 2(\sigma + A_3 - D_3 - K_3)$$

Direct reconstruction from 13 observables

Determination b_1, b_2, c_1 and c_2, by using two more observables D_1 and K_1 ⟹ to set (20) and solve linear equations:

$$\sigma, P, A_3, D_3, \boldsymbol{D_1}, K_3, \boldsymbol{K_1}, A_1, A_2, A_4, N_1, N_2, N_7 \quad (28)$$

We introduce the notation and express c_1 and c_2, in terms of b_1, b_2, L_1 and L_2 :

$$L_1 = b_1 c_1 + b_2 c_2 = \frac{A_2 + A_4}{2} \qquad L_2 = b_1 c_2 - b_2 c_1 = \frac{N_1 - N_7}{2} \quad (29)$$

$$c_1 = \frac{L_1 b_1 - L_2 b_2}{|b|^2} \qquad c_2 = \frac{L_1 b_2 + L_2 b_1}{|b|^2} \quad (30)$$

To use K_1 и D_1 and taking into account L_1, L_2 and c_1, c_2 we obtain a system of two linear equations for b_1 and b_2:

$$K_1|b|^2 = \{(a_1L_1 + a_2L_2)\sin(\theta+\theta_2) - |b|^2 d_1\sin(\theta_2) -$$

$$-eL_2\cos(\theta+\theta_2)\}b_1 + \{(a_2L_1 - a_1L_2)\sin(\theta+\theta_2) -$$

$$-|b|^2 d_2\sin(\theta_2) - eL_1\cos(\theta+\theta_2)\}b_2$$

$$D_1|b|^2 = \{|b|^2 a_1\sin(\theta+\theta_2) - (d_1L_1 + d_2L_2)\sin(\theta_2)\}b_1 +$$

$$+\{|b|^2(a_2\sin(\theta+\theta_2) - e\cos(\theta+\theta_2)) +$$

$$(d_1L_2 + d_2L_1)\sin(\theta_2)\}b_2$$

We denote the coefficients for b_1 and b_2 and solve the system of linear equations:

$$Q = (a_1L_1 + a_2L_2)\sin(\theta + \theta_2) - |b|^2 d_1 \sin(\theta_2) - eL_2 \cos(\theta + \theta_2)$$

$$R = (a_2L_1 - a_1L_2)\sin(\theta + \theta_2) - |b|^2 d_2 \sin(\theta_2) - eL_1 \cos(\theta + \theta_2)$$

$$S = |b|^2 a_1 \sin(\theta + \theta_2) - (d_1L_1 + d_2L_2)\sin(\theta_2)$$

$$T = |b|^2 (a_2 \sin(\theta + \theta_2) - e\cos(\theta + \theta_2)) + (d_1L_2 + d_2L_1)\sin(\theta_2)$$

$$b_1 = \frac{|b|^2 (D_1R - K_1T)}{SR - TQ} \qquad b_2 = \frac{|b|^2 (D_1Q - K_1S)}{TQ - SR} \qquad (31)$$

We express the coefficients in terms of the observables and the amplitude e, taking into account the fact that a_1, a_2, d_1, d_2 e , and $|b|^2$ are known , substituting the obtain coefficients into equations:

$$b_1 = \frac{|b|^2 (D_1 R - K_1 T)}{SR - TQ} \quad b_2 = \frac{|b|^2 (D_1 Q - K_1 S)}{TQ - SR}$$

we find b_1 , b_2.

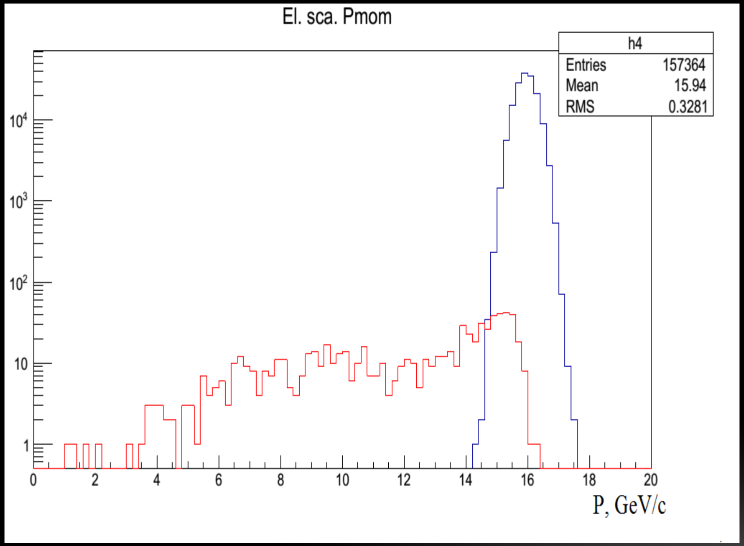
The expressions for c_1 and c_2 in terms of the amplitude e and the observables we find from equations:

$$c_1 = \frac{L_1 b_1 - L_2 b_2}{|b|^2} \qquad c_2 = \frac{L_1 b_2 + L_2 b_1}{|b|^2}$$

Thus, we obtain all complex amplitudes from 13 observables with no ambiguities at all

Possibility to study elastic scattering

The possibility to study elastic scattering at the SPASCHARM experiment was demonstrated earlier [6] with the fast Monte-Carlo. Figure 1 [6] demonstrates that, with the use of the special criteria selected, we can significantly suppress background from diffraction events with a slight suppression of the signal. The ratio of the signal to background S/(S + B) is 0.995.



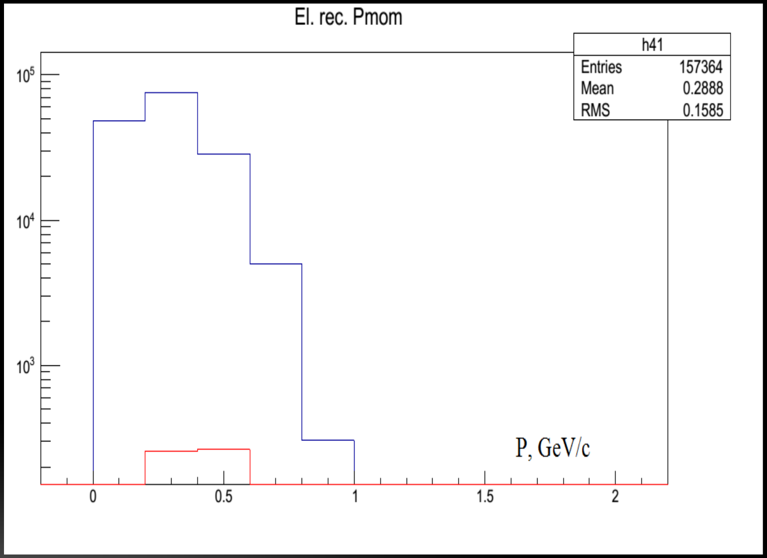


Figure 1. The momentum distributions of the scattered (left) and recoil (right) particles. Blue line is elastic processes, red line – the diffraction processes.

Discussion and conclusion

- The SPASCHARM configuration, when polarizations of protons beam and target can be oriented along any of three directions, allow us to measure 19 non-vanishing observables. In addition to the experiment with a target, polarized vertically and longitudinally only, here we can measure an additional four observables: $A_{ooss}, D_{os''os}, N_{onks}, N_{os''ns}$
- This makes it possible to choose a set of observables convenient for reconstructing amplitudes in a laboratory system, using the quite simple calculations. We have shown that the set of 13 observables (28) is complete at any given energy and angle.
- In this report, left out of scope the experimental limitations and errors, and reconstructed the amplitudes *a, b, c, d, e* analytically as f if the measured observables were exactly known. Fast MC study demonstrated the possibility to suppress background and select the quite clean elastic process. Nevertheless, a full Monte-Carlo with good description of the real SPASCHARM setup is required in order to estimate the achievable systematic and statistical errors

Acknowledgments

This work was supported in part by the RFBR, project number 18-02-00006 and by the Ministry of Science and Higher Education of the Russian Federation, Project "Fundamental properties of elementary particles and cosmology" No 0723-2020-0041.

Thanks for your attention

Back up

The status of the DRSA is as follows:

- The first DRSA analysis was 0.429 GeV by the Chicago University group(1968) [5]
- Before 1975, the direct reconstruction was possible at 90° in CM frame at a few energies only [6].
- The statistical reconstruction of scattering amplitudes was studied in reference [7], and the first direct reconstruction of the pp scattering matrix over a large angular range has been reported in references [8,9]

[5] Lemon P et al 1968 Phys. Rev. 169 1026
[6] Fontaine J. M., Experientia Suppl. 25 (1975) 464.
[7] Besset D., Favier B., Greeniaus L. G., Hess R., et al., Nucl. Instr. Methods 148 (1978) 129.
[8] Hausammann R., Ph. D. Thesis Nu. 2038, DPNC, University of Geneva (1982).
[9] E. Aprile,, et al., Phys. Rev. Lett. 46, 1047 (1981) [10]. M.W. McNaughtonet al., Phys. Rev. C 41, 2809 (1990)

The current status of the DRSA

- The complete sets measured at PSI below 0.6 GeV contained 17 linearly independent observables and allowed not only an unambigous direct reconstruction of the scattering matrix, but also a test of time-reversal invariance (TRI) [10, 11].
- Later, the direct reconstruction was carried out for PSI and LAMPF data and four solutions were found [11,12].
- The direct reconstruction of the ANL-ZGS data at 6 GeV/c at six four momentum transfer - t from 0.27 to 1.0 $(\mathbf{GeV/c})^2$ is described in references [13]

However, all the amplitudes for the elastic scattering have been never extracted for the energies above 10 GeV.

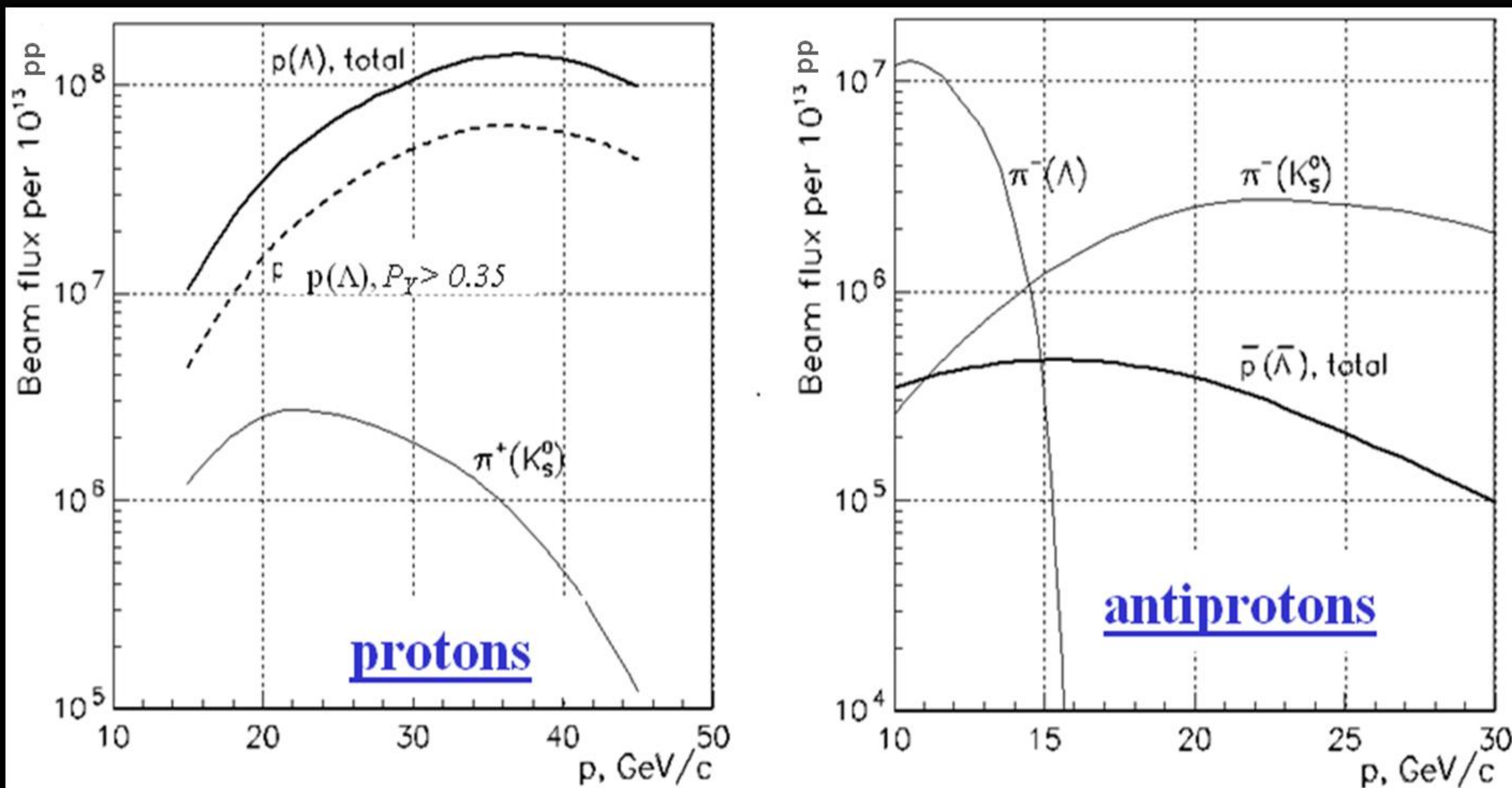
[10]. R Hausammann, et al., Phys. Rev. D 40, 22 (1989) [11]. F. Arash, F., et al., Phys. Rev. D 32, 74 (1985)
[12] C.D. Lac, J. Ball, J. Bystricky, et al. Journal de Physique, 1990, 51 (23), pp.2689-2716I
[13]. M. Matsuda, H. Suemitsu, M. Yonezawa, Phys. Rev. D 33, 2563 (1986)

The status of the DRSA

Laboratory	Energy T_{kin}, (MeV)	Number of angle values θ	Number of observables
Chicago University (1968)	429	1	8
PSI (1981)	447-579	6	12
PSI (1989)	447-579	15	10
LAMPF (1985)	800	5	10
LAMPF (1990)	730	12	15
SATURN II (1990)	834-1595	6-10	14
SATURN II (1998)	1800-2700	2-12	15
ANL 1986	5135	21	20

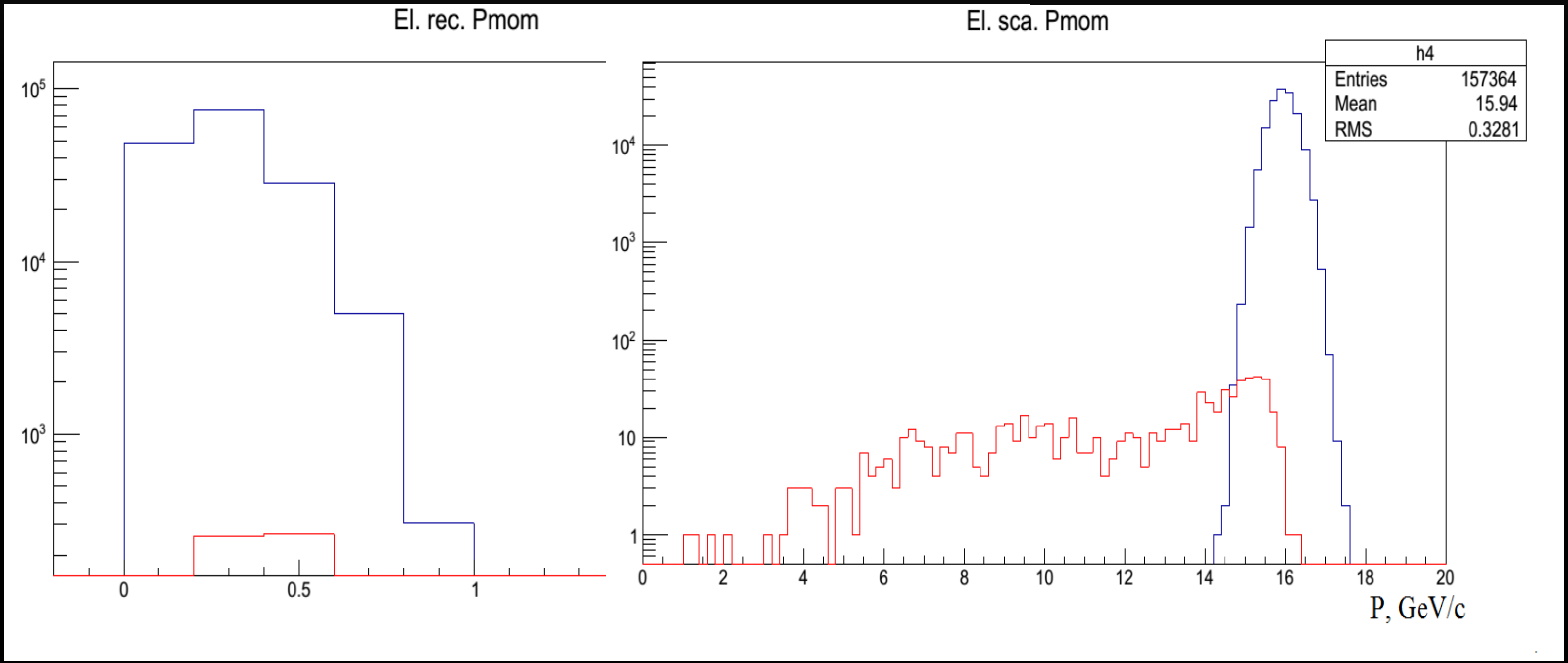
A complete list of available DRSA analyzes for the observed elastic pp scattering is provided in [5] F. Legar, E.A. Strokovsky "Phenomenology and Analysis of Nucleon Scattering Data" (2010)

The intensity of polarized proton (left frame) and antiproton (right frame) beams with the maximum Δp/p per 10^{13} of 60 GeV primary protons



V.V. Abramov et al Nuclear Inst. and Methods in Physics Research, A 901 (2018) 62–68

The momentum distributions of particles entering the recoil and scattering hodoscopes



The recoil particles

The scattered particles.

Blue line is elastic processes, red - diffraction processes

Signal **Background**

$$S / (S + B) = 0.995$$

We have found the most convenient set to consist of 11 quantities:

$$\sigma, P, A_3, D_3, K_3, A_1, A_2, A_4, N_1, N_2, N_7 \qquad (20)$$

Lets assume that e amplitude is real $\mathrm{Re}\, e = e, \quad \mathrm{Im}\, e = 0.$

$$a = a_1 + \mathrm{i}a_2$$
$$b = b_1 + \mathrm{i}b_2$$
$$c = c_1 + \mathrm{i}c_2$$
$$d = d_1 + \mathrm{i}d_2$$

Using equations (1) - (5) ⇨, we find :

$$a_1 = \frac{P}{e} \qquad (21)$$

$$a_2 = [(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2) / 2e^2]^{1/2} \qquad (22)$$

From (8) – (11): ⇒

$$\mathrm{Re}(b^* c) = \frac{A_4 + A_2}{2} \quad (23)$$

$$\mathrm{Im}(b^* c) = \frac{N_1 - N_7}{2} \quad (24)$$

We express d_1 and d_2 in terms of e , composing N_1 (8), N_7 (9) and N_2 (6), and A_2 (10), A_4(11) and A_1 (7), multiplying by $\sin(\theta)$ and $\cos(\theta)$:

$$d_1 = \frac{-(N_1 + N_7)\sin(\theta) - 2N_2\cos(\theta)}{2e} \quad (25)$$

$$d_2 = \frac{(A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta)}{2e} \quad (26)$$

We find the amplitude e from d_1 (25) and d_2 (26) by substituting them into the expression :

$$\sigma + A_3 - D_3 - K_3 = 2|d|^2$$

$$e = [(\sigma + A_3 - D_3 - K_3)((4(A_1^2 + N_2^2) - (A_4 - A_2)^2 - \\ -(N_1 + N_7)^2)\cos(2\theta) + (4A_1(A_4 - A_2) + 4N_2 \cdot \\ \cdot(N_1 + N_7))\sin(2\theta) + 4(A_1^2 + N_2^2) + (A_4 - A_2)^2 + \\ +(N_1 + N_7)^2)]^{1/2} / 2(\sigma + A_3 - D_3 - K_3) \qquad (27)$$

Determination b_1, b_2, c_1 and c_2, by using two more observables D_1 and K_1 ⟹ to set (20) and solve linear equations with respect to the sought amplitudes using (23) and (24):

$$\sigma, P, A_3, D_3, \boldsymbol{D_1}, K_3, \boldsymbol{K_1}, A_1, A_2, A_4, N_1, N_2, N_7 \qquad (28)$$

We introduce the notation and express c_1 and c_2, in terms of b_1, b_2, L_1 and L_1:

$$L_1 = b_1c_1 + b_2c_2 = \frac{A_2 + A_4}{2} \qquad L_2 = b_1c_2 - b_2c_1 = \frac{N_1 - N_7}{2} \qquad (29)$$

$$c_1 = \frac{L_1b_1 - L_2b_2}{|b|^2} \qquad c_2 = \frac{L_1b_2 + L_2b_1}{|b|^2} \qquad (30)$$

To use K_1 (14) и D_1 (16) and taking into account L_1, L_2 (28) and c_1, c_2 (29) we obtain a system of two linear equations for b_1 and b_2:

$$K_1|b|^2 = \{(a_1L_1 + a_2L_2)\sin(\theta+\theta_2) - |b|^2 d_1 \sin(\theta_2) - $$

$$-eL_2\cos(\theta+\theta_2)\}b_1 + \{(a_2L_1 - a_1L_2)\sin(\theta+\theta_2) - $$

$$-|b|^2 d_2 \sin(\theta_2) - eL_1\cos(\theta+\theta_2)\}b_2$$

$$D_1|b|^2 = \{|b|^2 a_1 \sin(\theta+\theta_2) - (d_1L_1 + d_2L_2)\sin(\theta_2)\}b_1 + $$

$$+\{|b|^2(a_2\sin(\theta+\theta_2) - e\cos(\theta+\theta_2)) + $$

$$(d_1L_2 + d_2L_1)\sin(\theta_2)\}b_2$$

We denote coefficients for b_1 and b_2 and solve the system of linear equations:

$$Q = (a_1 L_1 + a_2 L_2)\sin(\theta + \theta_2) - |b|^2 d_1 \sin(\theta_2) - eL_2 \cos(\theta + \theta_2)$$

$$R = (a_2 L_1 - a_1 L_2)\sin(\theta + \theta_2) - |b|^2 d_2 \sin(\theta_2) - eL_1 \cos(\theta + \theta_2)$$

$$S = |b|^2 a_1 \sin(\theta + \theta_2) - (d_1 L_1 + d_2 L_2)\sin(\theta_2)$$

$$T = |b|^2 (a_2 \sin(\theta + \theta_2) - e\cos(\theta + \theta_2)) + (d_1 L_2 + d_2 L_1)\sin(\theta_2)$$

$$b_1 = \frac{|b|^2 (D_1 R - K_1 T)}{SR - TQ} \qquad b_2 = \frac{|b|^2 (D_1 Q - K_1 S)}{TQ - SR} \qquad (31)$$

The expressions for the coefficients in terms of the observables and the amplitude e, taking into account, that a_1, a_2, d_1, d_2 and e known, and $|b|^2$ defined by the expression : $\sigma - A_3 + D_3 - K_3 = 2|b|^2$

$$Q = \frac{P(A_2 + A_4) + (N_1 - N_7)[(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2]^{1/2}}{2e}\sin(\theta + \theta_2) +$$

$$+(\sigma - A_3 + D_3 - K_3)\frac{(N_1 + N_7)\sin(\theta) + 2N_2\cos(\theta)}{4e}\sin(\theta_2) + \frac{e(N_7 - N_1)}{2}\cos(\theta + \theta_2)$$

$$R = \frac{(A_2 + A_4)[(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2]^{1/2} - P(N_1 - N_7)}{2e}\sin(\theta + \theta_2) -$$

$$-(\sigma - A_3 + D_3 - K_3)\frac{(A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta)}{4e}\sin(\theta_2) - \frac{e(A_2 + A_4)}{2}\cos(\theta + \theta_2)$$

$$S = \{4(\sigma - A_3 + D_3 - K_3)P\sin(\theta + \theta_2) - [((N_1 + N_7)\sin(\theta) + 2N_2\cos(\theta))(A_2 + A_4) +$$

$$+((A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta))(N_1 - N_7)]\sin(\theta_2)\}/4e$$

$$T = \left([(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2e^2]^{1/2}\sin(\theta + \theta_2) - e\cos(\theta + \theta_2)\right)\cdot$$

$$\cdot(\sigma - A_3 + D_3 - K_3) + \{[((N_7^2 - N_1^2) + (A_4^2 - A_2^2))\sin(\theta) +$$

$$+2(N_2(N_7 - N_1) + A_1(A_2 + A_4))\cos(\theta)]\sin(\theta_2)\}/4e$$