Vector mesons spectrum in a medium with a chiral imbalance induced by the vacuum of fermions

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CP violation in QCD

Vafa-Witten theorem: vector-like global symmetries such as parity, charge conjugation, isospin and baryon number in vector-like gauge theories like QCD cannot be spontaneously broken while the θ angle is zero

However this theorem does not apply to dense QCD matter where the partition function is not any more positive definite due to the presence of a highly non-trivial fermion determinant. In addition, out-of-equilibrium symmetry-breaking effects driven by finite temperatures are not forbidden by the Vafa-Witten theorem.

Lorentz–non-invariant P -odd operators are allowed to have non-zero expec-
tation values at finite density $\mu > 0$ and finite temperature if the system is out of Equilibrium.

P − and CP − odd bubbles may appear in a finite volume due to large topological fluctuations in a hot medium

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^{\mu\nu,a} G^{a}_{\mu\nu} + \bar{q}(i\gamma^{\mu} D_{\mu} - m) q,$$

$$D_{\mu} = \partial_{\mu} - i g G^{a}_{\mu} \lambda^{a}, \quad G^{a}_{\mu\nu} = \partial_{\mu} G^{a}_{\nu} - \partial_{\nu} G^{a}_{\mu} + g f^{abc} G^{b}_{\mu} G^{c}_{\nu}$$

$$\theta \lesssim 10^{-9}.$$

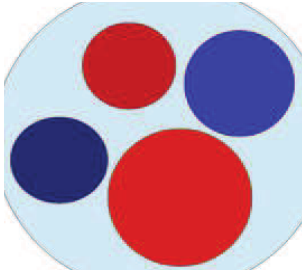
$$\theta\text{-term} \qquad \Delta\mathcal{L}_{\theta} = \theta \frac{g^2}{16\pi^2} \mathrm{Tr}\left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right)$$

CP violation in QCD

- In QCD topologically non-trivial configurations of gauge fields can exist (instantons)

- Gauge field configurations can be characterized by an integer topological (invariant) charge

$$T_5 = \frac{g^2}{16\pi^2} \int_{t_i}^{t_f} dt \int_{\text{vol.}} d^3x \, \mathrm{Tr}\left(G^{\mu\nu} \widetilde{G}_{\mu\nu}\right) \in \mathbb{Z}$$



It must survive for a sizeable lifetime in a heavy-ion fireball,

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10 \text{ fm/c};$$

Topological fluctuations as a source for parity breaking. Quasi-equilibrium treatment

The local partial conservation of the axial current relation is afflicted with the gluon anomaly

$$\partial^\mu J_{5,\mu} - 2i\bar{q}\hat{m}_q\gamma_5 q = \frac{N_f g^2}{8\pi^2}\mathrm{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right);$$

$$K_\mu = \frac{g^2}{2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(G^\nu\partial^\rho G^\sigma - i\frac{2}{3}gG^\nu G^\rho G^\sigma\right), \qquad \partial_\mu K^\mu = \frac{g^2}{4}\mathrm{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

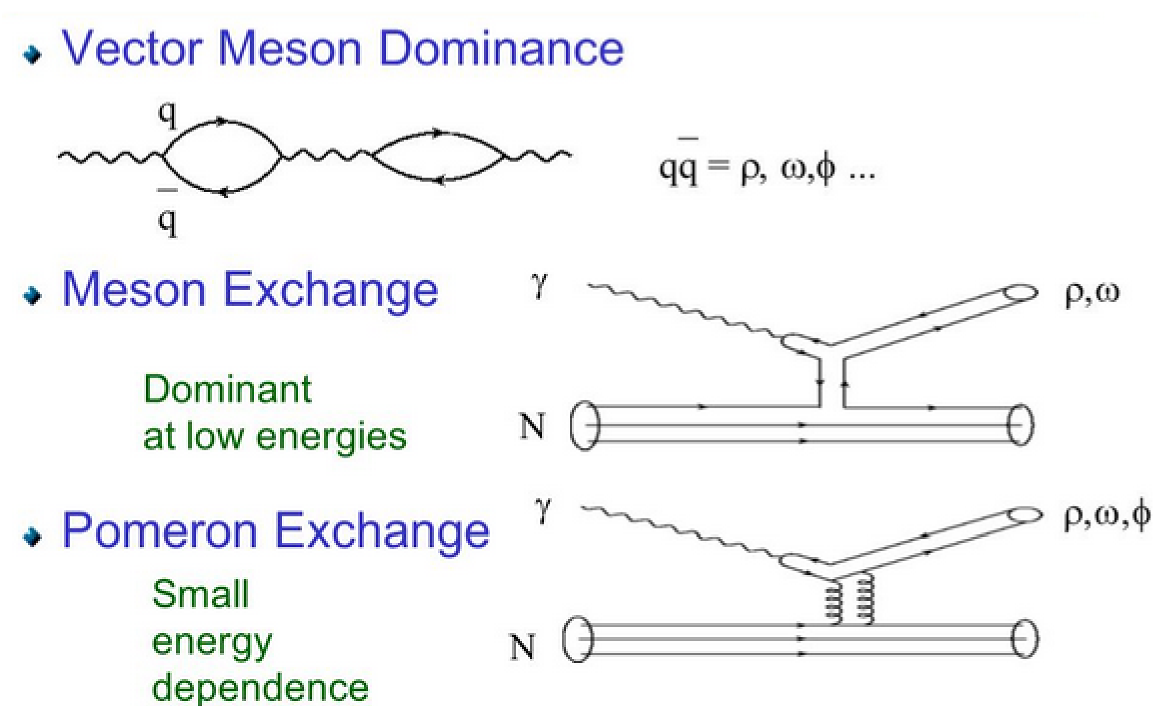
$$T_5 = \frac{g^2}{16\pi^2}\int_{t_i}^{t_f} dt\int_{\mathrm{vol.}} d^3x\,\mathrm{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right) \in \mathbb{Z};$$

$$\frac{d}{dt}(Q_5^q - 2N_f T_5) \simeq 2i\int_{\mathrm{vol.}} d^3x\,\bar{q}\hat{m}_q\gamma_5 q\;, \quad Q_5^q = \int_{\mathrm{vol.}} d^3x\,\bar{q}\gamma_0\gamma_5 q.$$

$$\langle T_5\rangle = \frac{1}{2N_f}\langle Q_5^q\rangle \quad\Longleftrightarrow\quad \mu_5 = \frac{1}{2N_f}\mu_\theta.$$

Vector Meson Dominance approach to local parity breaking

Quark loops appearing in the vacuum polarization of the photon can be described exclusively by vector mesons



QFT formulation of VMD model = effective field theory

$$j_\mu^{\mathrm{em}} = \frac{m_\rho^2}{g_\rho}\rho_\mu^0 + \frac{m_\omega^2}{g_\omega}\omega_\mu + \frac{m_\phi^2}{g_\phi}\phi_\mu$$

$$j_\mu^{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s,$$

VMD bosonization

$$j_\mu^{\mathrm{em}} = \frac{m_\rho^2}{g_\rho}\rho_\mu^0 + \frac{m_\omega^2}{g_\omega}\omega_\mu + \frac{m_\phi^2}{g_\phi}\phi_\mu$$

$$j_\mu^{\mathrm{em}} = \frac{1}{\sqrt{2}} j_\mu^\rho + \frac{1}{3\sqrt{2}} j_\mu^\omega - \frac{1}{3} j_\mu^\phi,$$

$$j_\mu^{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s,$$

$$j_\mu^\rho = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),$$

$$j_\mu^\omega = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right),$$

$$j_\mu^\phi = \bar{s}\gamma_\mu s.$$

With this definition the matrix element can be expressed as

$$\langle 0 | j_\mu^{\mathrm{em}} | V \rangle = \frac{M_V^2}{g_V}\epsilon_\mu^{(V)}$$

where $\epsilon_\mu^{(V)}$ is a polarization vector

$g_\omega \simeq g_\rho \equiv g \simeq 6 < g_\phi \simeq 7.8$ and $M_V^2 = 2 g_V^2 f_\pi^2$.

Alternative description of VMD with a mixing of vector mesons with photons in the mass term

Quark-meson interactions are described

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu V^\mu q; \quad V_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega\omega_\mu \mathbf{I}_q + \frac{1}{2}g_\rho\rho_\mu\lambda_3 + \frac{1}{\sqrt{2}}g_\phi\phi_\mu\mathbf{I}_s,$$

where $Q = \frac{\lambda_3}{2} + \frac{1}{6}\mathbf{I}_q - \frac{1}{3}\mathbf{I}_s$

λ_3 is the corresponding Gell-Mann matrix

the Maxwell and mass terms of the VMD lagrangian

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu} + \phi_{\mu\nu}\phi^{\mu\nu}\right) + \frac{1}{2}V_{\mu,a}\hat{m}^2_{ab}V^\mu_b,$$

$$\hat{m}^2 = m_V^2 \begin{pmatrix} \frac{4e^2}{3g^2} & -\frac{e}{3g} & -\frac{e}{g} & \frac{\sqrt{2}eg_\phi}{3g^2} \\ -\frac{e}{3g} & 1 & 0 & 0 \\ -\frac{e}{g} & 0 & 1 & 0 \\ \frac{\sqrt{2}eg_\phi}{3g^2} & 0 & 0 & \frac{g_\phi^2}{g^2} \end{pmatrix}, \ \det\left(\hat{m}^2\right) = 0,$$

where $(V_{\mu,a}) \equiv \left(A_\mu,\, \omega_\mu,\, \rho^0_\mu \equiv \rho_\mu,\, \phi_\mu\right)$

and $m_V^2 \equiv m_\rho^2 = 2g_\rho^2 f_\pi^2 \simeq m_\omega^2$.

Constant axial-vector background in QED theory

J. Alfaro, A. A. Andrianov, M. Cambiaso, P. Giacconi and R. Soldati, Phys. Lett. B 639, 586 (2006) [arXiv:hep-th/0604164].

constant axial–vector appears

$$b_\mu = \langle B_\mu \rangle = \langle \partial_\mu \theta \rangle$$

$$\begin{aligned}
\mathcal{L}_{\rm eff} = & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \left(1 + \frac{2\alpha b^2}{3\pi m^2}\right) + \frac{\alpha b^2}{3\pi} A^\mu A_\mu \\
& + \frac{\alpha}{3\pi m^2} b_\nu b^\rho F^{\nu\lambda} F_{\rho\lambda} - \frac{\alpha}{2\pi} b_\lambda A_\mu \epsilon^{\lambda\mu\rho\sigma} F_{\rho\sigma} \\
= & -\tfrac{1}{4}(1+\varepsilon) F^{\mu\nu} F_{\mu\nu} + \tfrac{1}{2} \varepsilon m^2 A^\mu A_\mu + \varepsilon \frac{b^\lambda b^\nu}{2b^2} F_{\lambda\rho} F_\nu{}^\rho \\
& - \frac{\alpha}{\pi} b_\mu A_\nu \widetilde{F}^{\mu\nu} \qquad \left(\varepsilon = \frac{2\alpha b^2}{3\pi m^2}\right) .
\end{aligned}$$

Effective LIV Lagranginan for VDM fields

The VDM Lagrangian in the SU(2) flavor sector reads:

$$\mathcal{L}_{\rm int} = \bar{q}\gamma_\mu V^\mu q$$

$$V_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega \omega_\mu \mathbf{I}_q + \frac{1}{2}g_\rho \rho_\mu \lambda_3$$

where $Q = \frac{\lambda_3}{2} + \frac{1}{6}\mathbf{I}_q$, $g_\omega \simeq g_\rho \equiv g \simeq 6$; $\mathbf{I}_q$ is the identity matrix in isospin space and λ_3 corresponding Gell-Mann matrix. The Maxwell and mass terms are

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu}\right) + \frac{1}{2}V_{\mu,a}m^2_{ab}V^\mu_b$$

$$m^2_{ab} = m^2_V \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det\left(m^2_{ab}\right) = 0$$

where $(V_{\mu,a}) \equiv \left(A_\mu, \omega_\mu, \rho^0_\mu \equiv \rho_\mu\right)$ and $m^2_V = m^2_\rho$, $g_\rho \simeq g_\omega \equiv g \simeq 6$ [3].

Effective LIV Lagranginan for VDM fields

Let us replace A_μ by V_μ:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm kin} - \frac{1}{4}\varepsilon V_{\mu\nu}\varepsilon V^{\mu\nu} + \frac{1}{2}\varepsilon m^2 V_\nu V^\nu + \varepsilon\frac{b^\lambda b^\nu}{2b^2}V_{\lambda\rho}V_\nu^\rho - \frac{N_c}{8\pi^2}b_\mu V_\nu \epsilon^{\mu\nu\lambda\nu}V_{\lambda\nu}$$

where

$$\varepsilon = \frac{b^2 N_c}{6\pi^2 m}^{1} \text{ and } V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

Trace over isospin matrices is assumed.

For the massive fremion mass we use m as a dynamical quark mass.

As an intermediate result,

$$\begin{aligned}\mathcal{L}_{\rm eff} = & -\frac{F_{\mu v}F^{\mu v}}{4} - \frac{\omega_{\mu v}\omega^{\mu v}}{4} - \frac{\rho_{\mu v}\rho^{\mu v}}{4} \\ & + \frac{5m_V^2 e^2 A_\mu A^\mu}{9g^2} - \frac{m_V^2 e A^\mu \omega_\mu}{3g} - \frac{m_V^2 e A^\mu \rho_\mu}{g} + \frac{b^2 N_c V^v V_v}{24\pi^2 m^2} \\ & - \frac{N_c \epsilon_{\delta\gamma\mu v} b^\mu V^v V^{\delta\gamma}}{8\pi^2} + \frac{V_{\gamma\lambda} N_c V_\mu^\lambda b^\gamma b^\mu}{12\pi^2 m^2} + \frac{1}{2}m_V^2\omega_\mu\omega^\mu + \frac{1}{2}m_V^2\rho_\mu\rho^\mu\end{aligned}$$

Orthogonal transformation in the space of three vector mesons

Let us diagonalize the vector meson mass quadratic from. The LIV-terms will also be diagonalized

$$U = \begin{pmatrix} \frac{3g}{\sqrt{\frac{9g^2}{e^2}+10e}} & -\frac{10e}{\sqrt{\frac{100e^2}{g^2}+90g}} & 0 \\ \frac{1}{\sqrt{\frac{9g^2}{e^2}+10}} & \frac{3}{\sqrt{\frac{100e^2}{g^2}+90}} & -\frac{3\sqrt{10}}{10} \\ \frac{3}{\sqrt{\frac{9g^2}{e^2}+10}} & \frac{9}{\sqrt{\frac{100e^2}{g^2}+90}} & \frac{\sqrt{10}}{10} \end{pmatrix}$$

mass matrix eignvalues

$$0, \left(1 + \frac{10e^2}{9g^2}\right) m_V, m_V$$

correspond to masses of photon, ω and ρ mesons

new vector vield $V'^{\,\mu}$ so that $V_a{}^{\mu} = U_{ab} V_b'{}^{\mu}$.

Compact form of vector meson Lagrangian

Resulting Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \left(-\frac{1}{4} - \frac{5b^2N_ce^2}{216\pi^2m^2} - \frac{g^2b^2N_c}{48\pi^2m^2}\right)\omega^{\delta\gamma}\omega_{\delta\gamma} + \left(-\frac{1}{4} - \frac{g^2b^2N_c}{48\pi^2m^2}\right)\rho^{\delta\gamma}\rho_{\delta\gamma}$$
$$+ \left(\frac{1}{2}m_V^2 + \frac{5e^2m_V^2}{9g^2} + \frac{5N_cb^2e^2}{108\pi^2} + \frac{g^2b^2N_c}{24\pi^2}\right)\omega^\mu\omega_\mu + \left(\frac{1}{2}m_V^2 + \frac{g^2b^2N_c}{24\pi^2}\right)\rho^\mu\rho_\mu$$
$$+ \left(\frac{5N_cb^\delta b^\lambda e^2}{108\pi^2m^2} + \frac{g^2N_cb^\delta b^\lambda}{24\pi^2m^2}\right)\omega_{\delta\gamma}\omega^\gamma_\lambda + \frac{g^2N_cb^\delta b^\lambda\rho_{\delta\gamma}\rho^\gamma_\lambda}{24\pi^2m^2}$$
$$+ \left(-\frac{5N_ce^2b^\lambda\epsilon_{\delta\gamma\lambda\mu}}{72\pi^2} - \frac{g^2N_c\epsilon_{\delta\gamma\lambda\mu}b^\lambda}{16\pi^2}\right)\omega^\mu\omega^{\delta\gamma} - \frac{g^2N_c\epsilon_{\delta\gamma\lambda\mu}b^\lambda\rho^\mu\rho^{\delta\gamma}}{16\pi^2}$$

Vector meson can be rewritten to

$$\mathcal{L}_V = -\frac{1}{4}\left(1+\xi\frac{b^2}{m^2}\right)V^{\mu\nu}V_{\mu\nu} + \xi\frac{b_\nu b^\rho}{2m^2}V^{\nu\lambda}V_{\rho\lambda}$$
$$-\frac{1}{2}\zeta b_\nu V_\lambda\frac{1}{2}\epsilon^{\nu\lambda\rho\sigma}V_{\rho\sigma} + \frac{1}{2}\bar{m}^2V_\nu V^\nu$$

$$\xi = \begin{cases} \xi_\omega = \dfrac{g^2N_c}{12\pi^2} + \dfrac{5e^2N_c}{54\pi^2} \\ \xi_\rho = \dfrac{g^2N_c}{12\pi^2} \end{cases} \qquad \zeta = \begin{cases} \zeta_\omega = \dfrac{g^2N_c}{4\pi^2} + \dfrac{5e^2N_c}{18\pi^2} \\ \zeta_\rho = \dfrac{g^2N_c}{4\pi^2} \end{cases}$$

$$\bar{m}^2 = \begin{cases} \bar{m}_\omega^2 = m_V^2 + \dfrac{10e^2}{9g^2}m_V^2 + \dfrac{g^2N_c}{12\pi^2}b^2 + \dfrac{5N_ce^2}{54}b^2 \\ \bar{m}_\rho^2 = m_V^2 + \dfrac{g^2N_c}{12\pi^2}b^2 \end{cases}$$

Dispersion relations for ρ and ω mesons

The modified Maxwell's equations read

$$\left(1+\xi\frac{b^2}{m^2}\right)\partial_\lambda V^{\lambda\nu}-\frac{\xi}{m^2}\left(b^\rho b_\lambda\partial_\rho V^{\lambda\nu}-b^\nu b_\lambda\partial_\rho V^{\lambda\rho}\right)+\bar{m}^2V^\nu-\zeta b_\lambda\tilde{V}^{\nu\lambda}=0$$

$$\partial_\nu V^\nu=0$$

In the momentum representation:

$$K^{v\sigma}A_\sigma(k)=0,\quad k^\sigma A_\sigma(k)=0$$

$$K^{v\sigma}\equiv\left(k^2-\bar{m}^2\right)g^{v\sigma}-k^vk^\sigma-\xi\left(\mathrm{D}/m^2\right)\mathrm{e}^{v\sigma}+i\zeta\epsilon^{v\lambda\rho\sigma}b_\lambda k_\rho$$

$$\mathrm{D}\equiv(b\cdot k)^2-b^2k^2$$

and the projector onto the two-dimensional hyperplane orthogonal to bv and kv

$$\mathrm{e}^{v\sigma}\equiv g^{v\sigma}-\frac{b\cdot k}{\mathrm{D}}\left(b^vk^\sigma+b^\sigma k^v\right)+\frac{k^2}{\mathrm{D}}b^vb^\sigma+\frac{b^2}{\mathrm{D}}k^vk^\sigma$$

Dispersion relations for ρ and ω mesons

Longitudinal polarization remains unchaged.
For the doubly transversal modes dispersion relations reads

$$\left\{k^2 - \tfrac{\xi}{m^2}\left[(b\cdot k)^2 - b^2k^2\right] - \bar{m}^2\right\}^2 - \zeta^2\left[(b\cdot k)^2 - b^2k^2\right] = 0$$

Then, for a genuine time-lik $b_\mu = (b_0, 0, 0, 0)$

$$\left\{k_0^2 - |\vec{k}|^2 - \tfrac{\xi}{m^2}\left[b_0^2k_0^2 - b_0^2(k_0^2 - |\vec{k}|^2)\right] - \bar{m}^2\right\}^2 - \zeta^2\left[b_0^2k_0^2 - b_0^2(k_0^2 - |\vec{k}|^2)\right] = 0$$

With the solution $$k_0^2 - |\vec{k}|^2 \equiv m*^2 = \bar{m}^2 \pm \zeta b_0|\vec{k}| + \xi b_0^2|\vec{k}|^2/m^2$$

Numerical estimations

We use:

$$N_c = 3, \quad g = 6, \quad e^2 = 4\pi \cdot 1/137, \quad m = 0.3\text{GeV} \ [4], \quad m_V = m_\rho = 0.7755\text{GeV}$$

Then

$$\xi = \begin{cases} \xi_\omega = 0.9119 + 0.0026 = 0.9145 \\ \xi_\rho = 0.9119 \end{cases} \qquad \zeta = \begin{cases} \zeta_\omega = 2.7435 \\ \zeta_\rho = 2.7357 \end{cases}$$

$$\bar{m}^2 = \begin{cases} \bar{m}_\omega^2 = (0.7766\text{GeV})^2 + 0.9145 \ b^2 \\ \bar{m}_\rho^2 = (0.7755\text{GeV})^2 + 0.9119 \ b^2 \end{cases}$$

The mass spectrum of vector mesons in the presense of the chiral imbalance induced by a fermionic vacuum for the transverse polarisations is

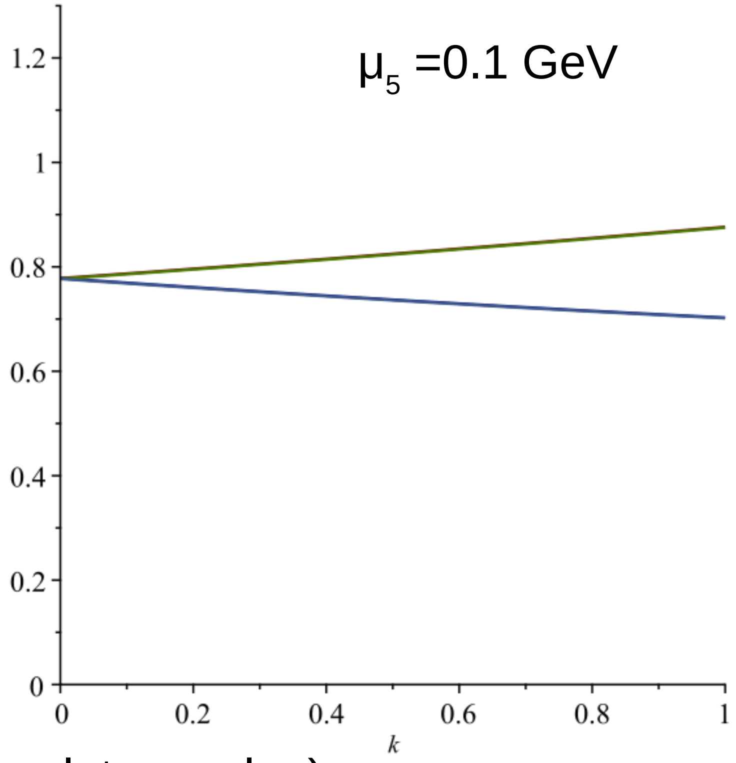
$$m_\omega^{*\,2} = (0.7766\text{GeV})^2 + 0.9145 \ b^2 \pm 2.7435 \ b \ |\vec{k}| + 10.16(\text{GeV}^{-2}) \ b^2 \ |\vec{k}|^2$$

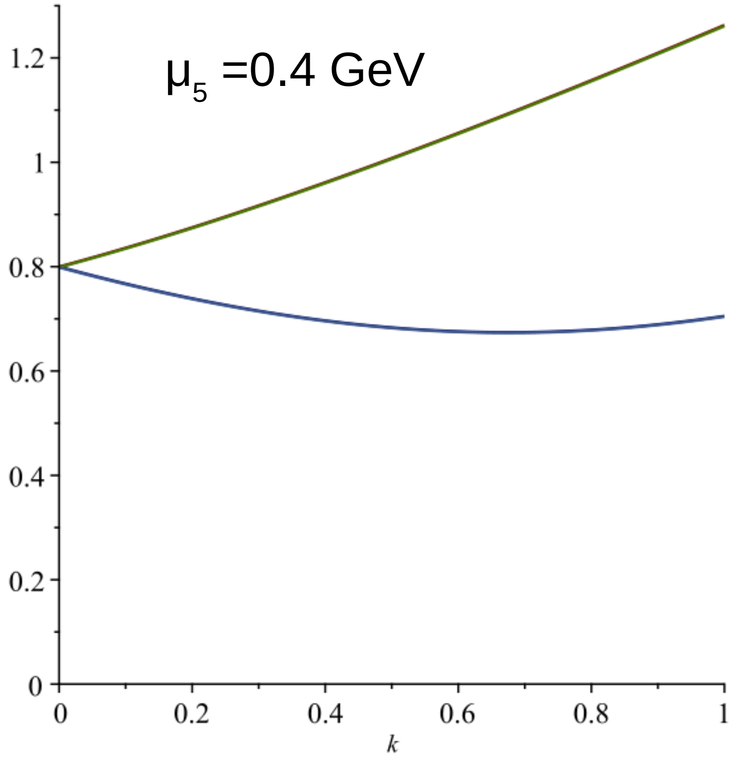
$$m_\rho^{*\,2} = (0.7755\text{GeV})^2 + 0.9119 \ b^2 \pm 2.7357 \ b \ |\vec{k}| + 10.13(\text{GeV}^{-2}) \ b^2 \ |\vec{k}|^2$$

Numerical estimations

Applying the relation $\zeta b_0 = N_c g^2 \mu_5 / 8\pi^2$ $\qquad$ $b_0 \simeq 0.5\mu_5$

Vector meson spectrum as a function of momentum

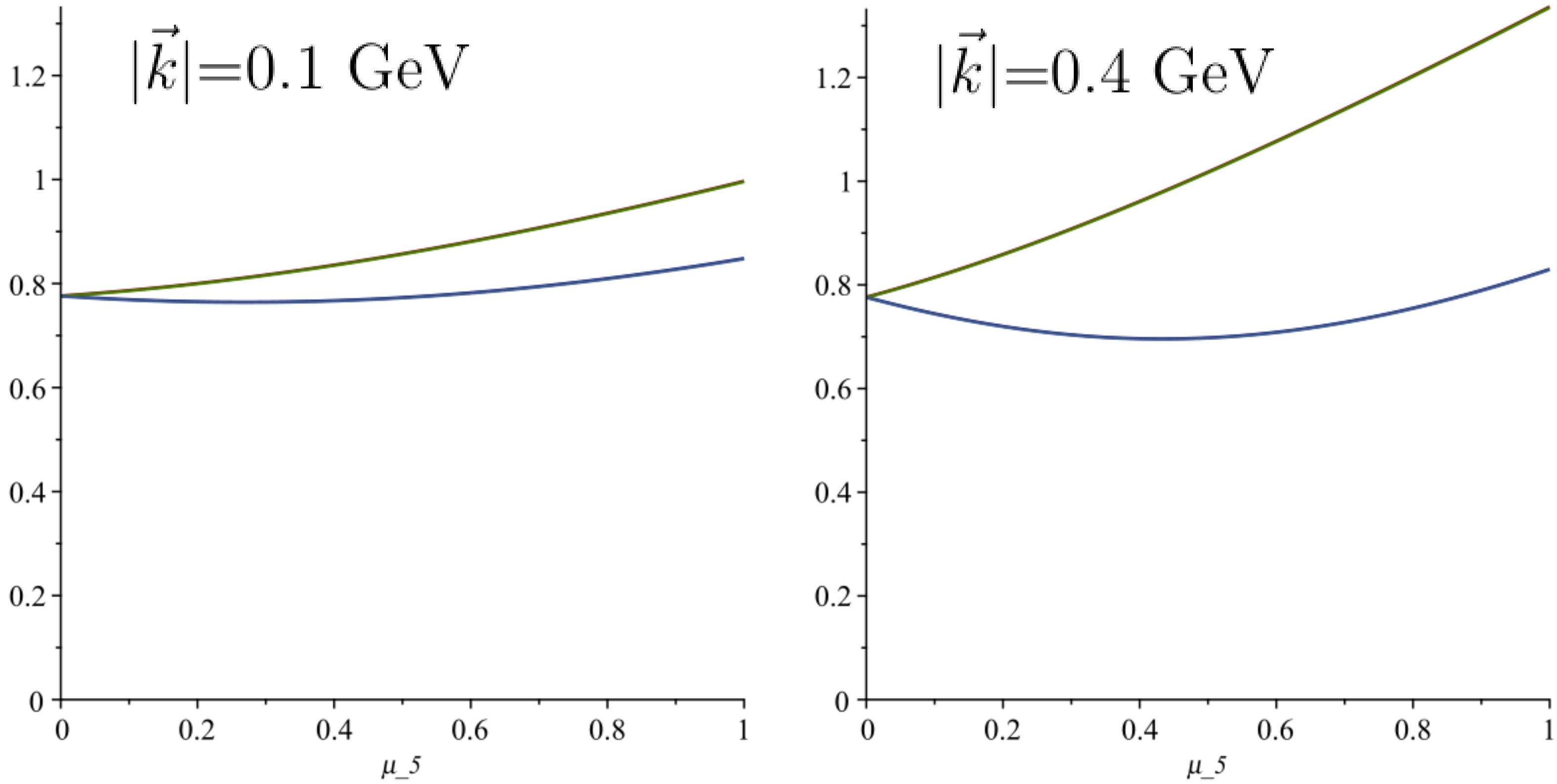




(ρ and ω plots overlap)

Numerical estimations

Vector meson spectrum as a function of chiral chemical potential



Summary and outline

The chiral imbalance can be considered as a time-like axial-vector $b\mu$ coupled to a fermion field with its zero component associated with $\mu 5$.

The properties of light vector mesons in the presence of LPB in a fireball, the vector-meson dominance model is applied in the lightest SU(2) sector.

We obtained to the vector ρ and ω meson mass spectrum as a function of momentum and chiral chemical potential $\mu 5$.

We showed that in addition to the Chern–Simons term, splitting the transverse polarisations of the mesons, there is an additional contribution that becomes important at momentum and $\mu 5$ around a few hundred MeV.

The polarisation slpitting can be detected experimentally using di-lepton decay chanel of vector mesons, using angular analysis.

The developed formalism can be coupled to the thermal or blast-wave-like model to provide the predictions of ρ and ω spectral functions.

It also can be applied in more detailed relativistic hydrodynamic models

References

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