Photon damping in a strongly magnetized plasma

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Introduction

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Conclusion
Magnetars are highly interesting objects in the Universe. Recent observations give ground to believe that some astrophysical objects (SGR and AXP) are magnetars, a distinct class of isolated neutron stars with magnetic field strength of \( B \gg B_e \), where

\[
B_e = \frac{m^2}{e} \simeq 4.41 \times 10^{13} \text{ G}.
\]

The magnetic field in magnetars for the different models may reach the values up to

\[
B \sim 10^{14} - 10^{16} \text{ G}.
\]

In addition, analysis of the emission spectrum of some of these objects indicates the presence in their vicinity of a relatively hot and dense electron-positron plasma with temperature \( T \sim 1 \text{ MeV} \).
Main goal
Investigation of the photon decay process due to the absorption of a photon by an electron (positron), $\gamma e^{\pm} \rightarrow e^{\pm}$, and $e^+e^-$ - pair creation, $\gamma \rightarrow e^+e^-$. 

The problem
The expression for the decay width of photon in these processes in the limit of a strongly magnetized plasma contains singularities of the root type at points of a cyclotron resonances.
Photon propagation in the magnetized medium

To describe the evolution of the electromagnetic wave $A_\alpha(x)$, $x_\mu = (t, x)$, in time, we use the technique detailed in (Chistyakov, Mikheev 2001). We consider the linear response of the system ($A_\alpha(x)$ and vacuum polarized in a magnetized plasma) to an external source, which is adiabatically switched on at $t = -\infty$ and at time $t = 0$ turns off. At $t > 0$, the electromagnetic wave will propagate independently. For this, the source function should be selected in the form:

$$J_\alpha(x) = j_\alpha \, e^{ikx} \, e^{\varepsilon t} \theta(-t), \quad \varepsilon \to 0^+.$$

$j_\alpha = (0, j)$, $j \cdot k = 0$ is the current conservation, $q^\mu = (q_0, k)$. For simplicity, we consider the evolution of a monochromatic wave.
The dependence of $A_\alpha(x)$ on time is determined by the equation

$$
(g_{\alpha\beta} \partial_\mu^2 - \partial_\alpha \partial_\beta) A_\beta(x) + 
+ \int d^4x' \mathcal{P}_{\alpha\beta}(x - x') A_\beta(x') = J_\alpha(x),
$$

where $\mathcal{P}_{\alpha\beta}(x - x')$ is the polarisation operator in the magnetized plasma.
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Charge-symmetric plasma, $\mu = 0$

The physical polarization vectors of the photons

\[ \varepsilon^{(1)}_{\alpha}(q) = \frac{(q\varphi)_\alpha}{\sqrt{q^2}} , \quad \varepsilon^{(2)}_{\alpha}(q) = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q^2}} \]

are just as in the pure magnetic field.

The four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1, 2\}$-subspace and the Minkowski $\{0, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along third axis; \((ab)_\perp = (a\varphi\varphi b) = a_\alpha \varphi^\rho_\alpha \varphi^\rho_\beta b_\beta , \ (ab)_\parallel = (a\tilde{\varphi}\tilde{\varphi} b) = a_\alpha \tilde{\varphi}^\rho_\alpha \tilde{\varphi}^\rho_\beta b_\beta . \]

The tensors $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu} \varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.
Photon propagation in the magnetized medium

The solution of the wave equation for photons of the $\lambda = 1, 2$ modes can be represented as:

$$A_\alpha^\lambda(x) = V_\alpha^\lambda(0, x) \, \text{Re} F_\alpha^\lambda(t),$$

where

$$V_\alpha^\lambda(0, x) = 2 e^{i k x} \varepsilon_\alpha^\lambda (\varepsilon^\lambda j),$$

The function $F^\lambda(t)$ can be represented in the form of two terms

$$F^\lambda(t) = F^\lambda_{\text{pole}}(t) + F^\lambda_{\text{cut}}(t).$$

The first term is determined by the residue at the point $q_0 = \omega$, which is solution of the dispersion equation $q^2 - \mathcal{P}^\lambda(q) = 0$, in the kinematic region, where the value of the photon polarization operator, $\mathcal{P}^\lambda(q)$, is real.
The second term determines the dependence of the electromagnetic field on time in the region between the cyclotron resonances thresholds and has the form of a Fourier integral:

\[
F_{\text{cut}}^{(\lambda)}(t) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} F_{\text{cut}}^{(\lambda)}(q_0) e^{-iq_0 t},
\]

\[
F_{\text{cut}}^{(\lambda)}(q_0) = \frac{2 \theta(q_0 - 2m_e) I^{(\lambda)}}{q_0 \left[ q_0^2 - k^2 - R^{(\lambda)} \right]^2 + [I^{(\lambda)}]^2},
\]

where \( R \equiv \text{Re}\mathcal{P}^{(\lambda)}(q_0) \) is the real part of polarization operator, \( l \equiv -\text{Im}\mathcal{P}^{(\lambda)}(q_0 + i\varepsilon) \) is the imaginary part of polarization operator.
The imaginary part of polarization operator can be obtained from the photon absorption coefficient

\[ W_{\text{abs}}^{(\lambda)} = W_{\gamma(\lambda)\rightarrow e^+e^-} + W_{\gamma(\lambda)e^\pm\rightarrow e^\pm} \]

and represented in the form (Shabad 1988)

\[ \text{Im}\mathcal{P}^{(\lambda)}(\lambda) = -2q_0[1 - \exp(-q_0/T)]W_{\text{abs}}^{(\lambda)} \]

The real part of the polarization operator can be reconstructed from its imaginary part using the dispersion relation with one subtraction:

\[ \mathcal{P}^{(\lambda)}(t) = \int_0^\infty \frac{\text{Im}(\mathcal{P}^{(\lambda)}(t')) dt'}{t' - t - io} - \mathcal{P}^{(\lambda)}(0), \quad t = q_0^2. \]
Photon propagation in the magnetized medium

This equations, solve the problem on finding the time dependence of the photon wave function in the presence of strongly magnetized plasma. Strictly speaking, due to the threshold behavior of the Fourier transform $F^{(\lambda)}(q_0)$ character of time decay of the function $F^{(\lambda)}(t)$, and hence the wave function $A^{(\lambda)}_{\mu}(t)$, differs from the exponential one. However, during a certain characteristic time interval ($\sim [W_{abs}^{(\lambda)}]^{-1}$), the dependence of the wave function can be approximately described as exponentially decaying harmonic oscillations

$$A^{(\lambda)}_{\mu}(t) \sim e^{-\gamma^{(\lambda)}_{\text{eff}} t/2} \cos(\omega^{(\lambda)}_{\text{eff}} t + \phi_0).$$

Here $\omega_{\text{eff}}$ and $\gamma^{(\lambda)}_{\text{eff}}$ is the effective frequency and ratio photon absorption of the $\lambda$ mode, respectively, which should be found for each value of the impulse $k$, which determines the effective dispersion law of a photon in the region of its instability.
The results of numerical calculations of the frequency dependence of the decay width in the near-threshold regions at $B = 200B_e$, $T = 1$ MeV and $\mu = 0$ are presented in the following figures. Line 1 - coefficient photon absorption $W_{abs}^{(\lambda)}$, calculated in the tree approximation and containing root singularities; line 2 - decay width obtained from the complex solution of the dispersion equation on the second Riemannian sheet (Shabad 1988); line 3 matches the width decay $\gamma_{eff}^{(\lambda)}$ calculated based on the approximation in this paper.
1 – $W_{abs}^{(\lambda)}$, containing root singularities; 2 – the complex solution of the dispersion equation on the second Riemannian sheet (Shabad 1988); 3 – approximation of this paper.
Numerical analysis

\[
\gamma_{\text{eff}}^{(2)} / 2m
\]

\[
\omega / 2m
\]
Conclusion

The process of propagation of an electromagnetic wave in a highly magnetized, charge-symmetric plasma is investigated. Taking into account the change in the dispersion properties of a photon in a magnetic field and plasma, it has been established that, similar to the case of a pure magnetic field, the process of photon decay in a magnetized plasma has a nonexponential character.

It is shown that the effective absorption width of a photon is significantly smaller in comparison with the results known in the literature.
Thank you!!!