Second look to the Polyakov Loop Nambu-Jona-Lasinio model at finite baryonic density

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1. Motivation

- **Φ loop is (approximate) order parameter of quark-hadron transition**

  $\Downarrow$

  Should be included to effective approaches

- **$2M_{\odot}$ limit for compact stars**

  $\Downarrow$

  quark EoS should be stiffened

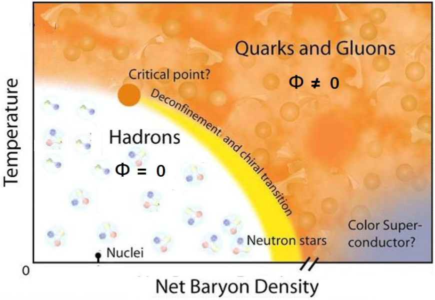
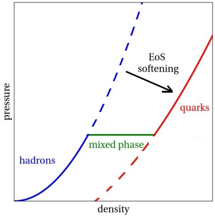
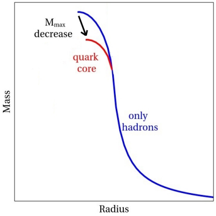


Figure from G. A. Contrera et al., arXiv:1612.09485 [nucl-th]





- Proper chiral dynamics in NJL model:

  Effective masses of chiral partners (σ and π) get equal $\Rightarrow$ restoration of chiral symmetry

  S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992)

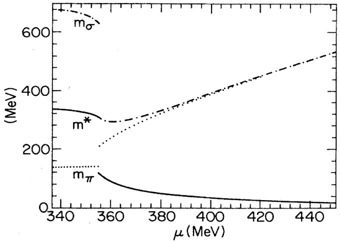
- Effective scalar theory of Polyakov loop for deconfinement in pure Yang-Mills:

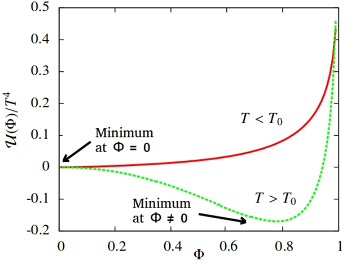
  Discontinuous change of minima of effective potential $\mathcal{U}(\Phi)$ $\Rightarrow$ deconfinement transition

  R. D. Pisarski, Phys. Rev. D 74, 121703 (2006)

- Hybrid model: NJL + Φ = PNJL

  C. Ratti, S. Roessner, M. A. Thaler and W. Weise, Eur. Phys. J. C 49, 213 (2007)





3. New PNJL @ $T = 0$ (main results hold at $T \neq 0$)

- Lagrangian

$$\mathcal{L} = \bar{q}\left(i\not{D} + \gamma^0 \frac{\mathcal{V}(\Phi,\Phi^*)}{3} - \hat{m}\right) q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\right] - \mathcal{U}(\Phi,\Phi^*)$$

- Thermodynamic potential

$$\frac{\Omega}{V} = \mathcal{U}(\Phi,\Phi^*) + \frac{G}{2}\langle\bar{q}q\rangle^2 - 6\int_f \left[\omega_f + (\mu_f^* - \omega_f)\theta(\mu_f^* - \omega_f)\right]$$

$\omega_f^2 = \vec{p}^2 + (m_f - G\langle\bar{q}q\rangle)^2$ – single particle energy of quark flavour f

$\mu_f^* = \mu_f + \frac{\mathcal{V}(\Phi,\Phi^*)}{3}$ – effective mass of quark flavour f

- Polyakov loop potential

$$\mathcal{U}(\Phi,\Phi^*) = -\frac{b_2(T,n_B)}{2}\Phi^*\Phi + b_4(T,n_B)\ln\left[1 - 6\Phi^*\Phi + 4(\Phi^{*3}+\Phi^3) - 3(\Phi^*\Phi)^2\right]$$

$$b_l(0,n_B) = a_l T_0^4 \left(\frac{n_B}{T_0^3}\right)^{\kappa_l}, \quad l = 2,\ 4,\ a_l,\ \kappa_l - \text{parameters}$$

4. Why $\mathcal{U}$ should depend on density?

- Originat model: $\mathcal{U}$ independent on n_B

C. Ratti, S. Roessner, M. A. Thaler and W. Weise, Eur. Phys. J. C 49, 213 (2007)

$$T \to 0 \quad \Rightarrow \quad \mathcal{U} \simeq b_4(T)\ln(1-6\Phi^*\Phi) \sim \mathcal{O}(T\Phi^*\Phi)$$

$$\frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\Phi^*} = 0 \quad \Rightarrow \quad \Phi \simeq -\frac{T}{b_4(T)}\int_f \frac{e^{2\beta(\mu_f-\omega_f)}}{1-e^{3\beta(\mu_f-\omega_f)}} \sim \mathcal{O}(e^{-\beta(\mu-m)})$$

$\Phi = 0$ **at** $T = 0 \Rightarrow$ **Confinement at all baryonic densities?**

No. Polyakov loop potential should depend on density

- Originat model: $\mathcal{U}$ independent on n_B

OI, M. Angeles Perez-Garcia, V. Sagun and C. Albertus PRD 100, 103020 (2019)

$$T \to 0 \quad \Rightarrow \quad b_4 = b_4(n_B) \quad \Rightarrow \quad \mathcal{U} \neq 0 \quad \Rightarrow \quad \Phi \neq 0$$

5. Why $\mathcal{V}$ should be introduced to the model?

- Baryonic density

$$\begin{aligned} n_B &= -\frac{1}{V}\frac{\partial\Omega}{\partial\mu_B} \\ &= \underbrace{2\int_f \theta(\mu_f^* - \omega_f)}_{single\ \ quark\ \ contribution} + \underbrace{\left[2\int_f \theta(\mu_f^* - \omega_f)\frac{\partial\mathcal{V}}{\partial n_B} - \frac{\partial\mathcal{U}}{\partial n_B}\right]\frac{\partial n_B}{\partial\mu_B}}_{contribution\ \ of\ \ Polyakov\ \ loop} \end{aligned}$$

- Contribution of Polyakov loop in absence of dynamical quarks

$m_q \to \infty \Rightarrow n_B = -\frac{\partial\mathcal{U}}{\partial\mu_B} \neq 0$ - baryonic charge of gluons?

Naive modification of $\mathcal{U}$ = assigning a baryonic charge to gluons

- Contribution of Polyakov loop must be zero

$$n_B\frac{\partial\mathcal{V}}{\partial n_B} - \frac{\partial\mathcal{U}}{\partial n_B} = 0, \quad n_B = 2\int_f \theta(\mu_f^* - \omega_f)$$

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- Baryonic density

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$$= \underbrace{2\int_f \theta(\mu_f^* - \omega_f)}_{single\ \ quark\ \ contribution} + \underbrace{\cancel{\left[2\int_f \theta(\mu_f^* - \omega_f)\frac{\partial \mathcal{V}}{\partial n_B} - \frac{\partial \mathcal{U}}{\partial n_B}\right]}\frac{\partial n_B}{\partial \mu_B}}_{contribution\ \ of\ \ Polyakov\ \ loop}$$

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6. Values of κ_2 and κ_4

$$\mathcal{V}(\Phi,\Phi^*) = -\frac{c_2(T,n_B)}{2}\Phi^*\Phi + c_4(T,n_B)\ln\left[1-6\Phi^*\Phi+4(\Phi^{*3}+\Phi^3)-3(\Phi^*\Phi)^2\right]$$

$$c_l(T,n_B) = \frac{\kappa_l b_l(T,n_B)}{(\kappa_l-1)n_B}, \quad b_l(0,n_B) = a_l T_0^4\left(\frac{n_B}{T_0^3}\right)^{\kappa_l}$$

- Quark-antiquark symmetry (effective chemical potential $\mu_q^* = \mu_q + \frac{\mathcal{V}}{3}$)

$$\mu_q^*(-n_B) = -\mu_q^*(n_B) \quad\Rightarrow\quad \mathcal{V}(0)=0 \quad\Rightarrow\quad \kappa_l = 0 \quad or \quad \kappa_l > 1$$

- Quark-antiquark symmetry (pressure $p = \ldots - \mathcal{U}$)

$$p(-n_B) = p(n_B) \quad\Rightarrow\quad \mathcal{U}(-n_B) = \mathcal{U}(n_B) \quad\Rightarrow\quad \kappa_l = 0,\ \frac{4}{3},\ \frac{6}{3},\ \frac{8}{3},\ \ldots$$

- Asymptotic of pressure

$$p|_{n_B\to\infty} = \underbrace{const\cdot n_B^{4/3}}_{\substack{contribution\\ of\ quarks}} + \underbrace{const\cdot n_B^{max(\kappa_2,\kappa_4)}}_{\substack{contribution\\ of\ Polyakov\ loop}} \sim \mu_B^4 \sim n_B^{4/3} \quad\Rightarrow\quad \kappa_2 = \frac{4}{3}, \quad \kappa_4 = 0$$

7. Effective chemical potential of quarks

$$\mu_q^* = \mu_q + \frac{\mathcal{V}(\Phi, \Phi^*)}{3}, \quad \mathcal{V}(\Phi, \Phi^*) = -\frac{c_2(0, n_B)}{2}\Phi^*\Phi$$

- Quark-gluon interaction term from the QCD Lagrangian

$$\overline{q}(\mu_q \gamma^0 + g \not{A})q \to q^+(\mu_q + gA^0)q \to q^+(\mu_q + g\langle A^0 \rangle)q$$

- Temporal gluon field and Polyakov loop operator

$$L = \mathcal{T} \exp\left(-g \int_0^\beta d\tau A^0\right) \quad \Rightarrow \quad gA^0 = -L^+ \frac{\partial}{\partial \beta} L$$

- Ensemble average (can not be calculated directly due to complexity of QCD)

$$\langle L \rangle = \Phi, \quad \Rightarrow \quad g\langle A^0 \rangle = -\left\langle L^+ \frac{\partial}{\partial \beta} L \right\rangle \to \chi \cdot \langle L^+ \rangle \langle L \rangle = \chi \cdot \Phi^* \Phi$$

- Effective chemical potential of quarks

$$\mu_q^* = \mu_q + g\langle A^0 \rangle = \mu_q + \chi \cdot \Phi^* \Phi$$

8. High density asymptotic

- High density asymptotic

$$\left.\frac{p}{p_{SB}}\right|_{\mu_B\to\infty} = \left[1 + \frac{2\tilde{a}_2}{3}\left(\frac{N_f}{3\pi^2}\right)^{\frac{1}{3}}\right]^{-3}$$

p_{SB} – Stefan-Boltzman pressure
N_f – number of flavours

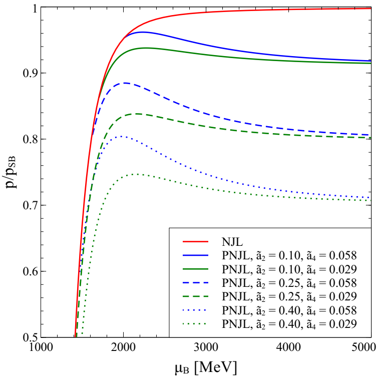
- Perturbative calculations at $\mathcal{O}(\alpha_s^2)$ and partial $\mathcal{O}(\alpha_s^3 \ln^2 \alpha_s)$

$\left.\frac{p}{p_{SB}}\right|_{\mu_B\to\infty} = 0.8 \Rightarrow a_2 = 0.25$ for $N_f = 3$

T. Gorda et al., Phys. Rev. Lett. 121, 202701 (2018).

The same $\frac{p}{p_{SB}}$ at $T \to \infty$ is given by lQCD

F. Karsch, Nucl. Phys. A 698, 199 (2002)



9. Hybrid EoS

- **Order parameter**

  $\Phi = 0 \Rightarrow$ hadron matter

  $\Phi \neq 0 \Rightarrow$ quark matter

- **Gibbs criterion**

  $\mu_B < \mu_c \Rightarrow p_h(\mu_B) > p_q(\mu_B)$ – hadron matter

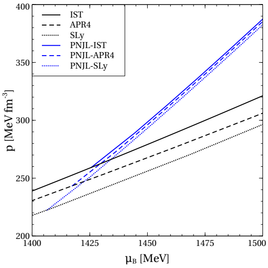
  $\mu_B > \mu_c \Rightarrow p_h(\mu_B) < p_q(\mu_B)$ – quark matter

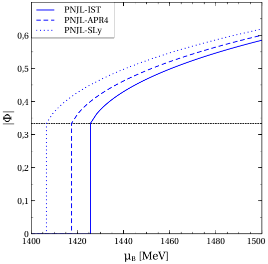
  $\mu_B > \mu_c \Rightarrow p_h(\mu_B) = p_q(\mu_B)$ – phase transition

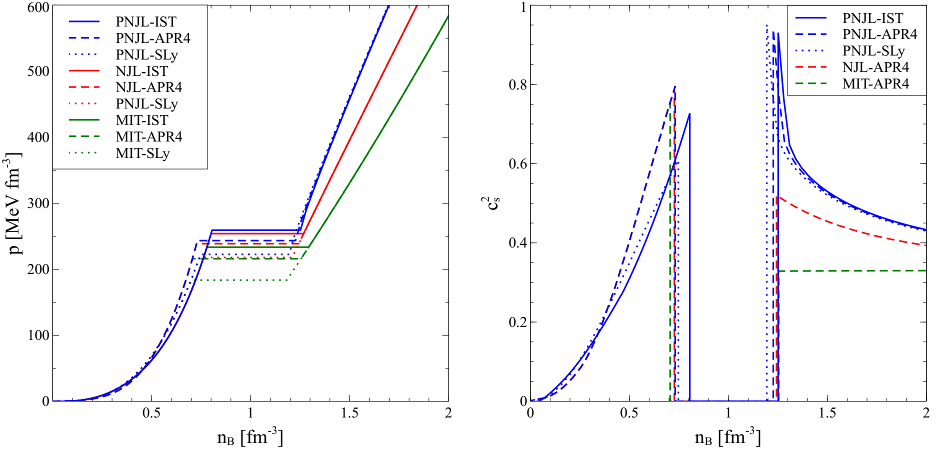
- **Polyakov-Gibbs construction:**

  phase transition happens exactly when Φ attains non zero value and phases are in equilibrium

  a_4 is tuned to provide this







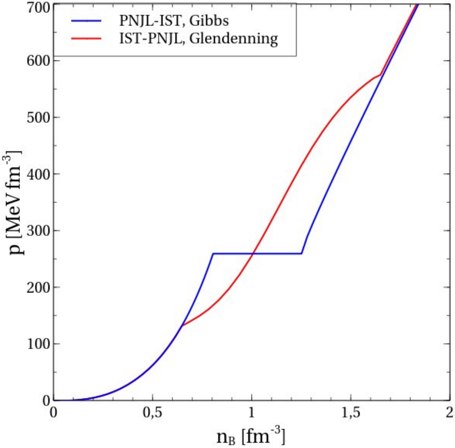
p [MeV fm^-3]
nB [fm^-3]
PNJL-IST
PNJL-APR4
PNJL-SLy
NJL-IST
NJL-APR4
PNJL-SLy
MIT-IST
MIT-APR4
MIT-SLy
c_s^2
nB [fm^-3]
PNJL-IST
PNJL-APR4
PNJL-SLy
NJL-APR4
MIT-APR4

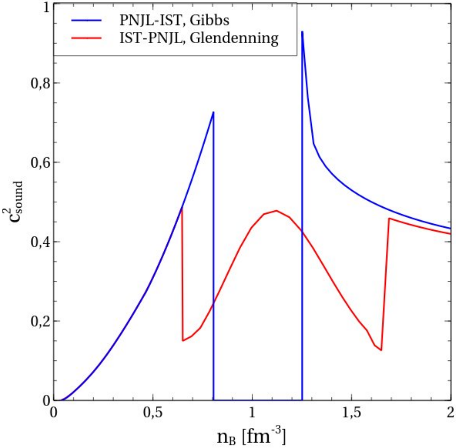
11. Conclusions

- The Polyakov loop potential of the PNJL model is generalized to the case of finite baryonic densities in order to incorporate to the model dynamics of the Polyakov loop
- Spurious contribution of gluons to the baryon charge density is removed from the model by introducing the effective chemical potential of quarks
- The model is applied to construct a hybrid quark-hadron EoS
- The Polyakov loop potential is shown to stiffen such an EoS

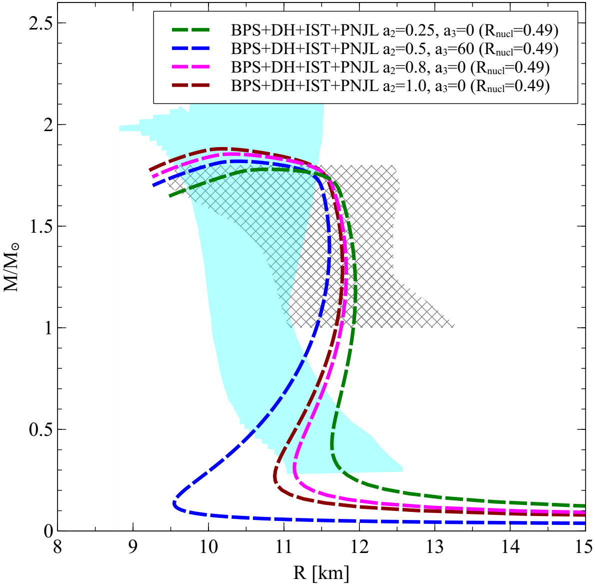
Thank you for attention!

EoS of hybrid star: Glendenning construction





M-R relation



BPS+DH+IST+PNJL a2=0.25, a3=0 (Rnucl=0.49)
BPS+DH+IST+PNJL a2=0.5, a3=60 (Rnucl=0.49)
BPS+DH+IST+PNJL a2=0.8, a3=0 (Rnucl=0.49)
BPS+DH+IST+PNJL a2=1.0, a3=0 (Rnucl=0.49)
M/M⊙
R [km]