Incorporating the Heisenberg and Pauli principles into the kinetic approach to neutrino oscillations

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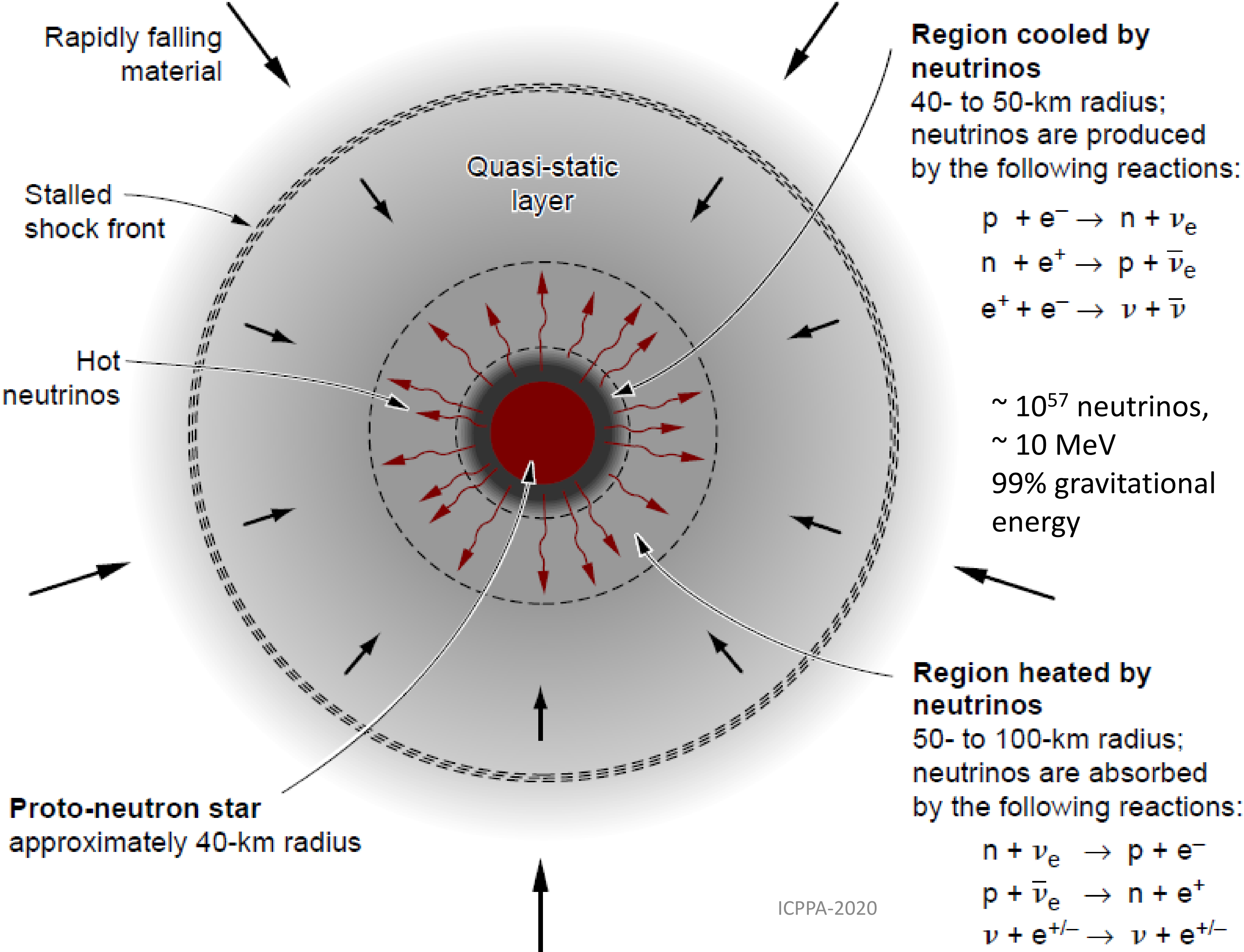
Based on ArXiv 2007.13736



~ 10^{57} neutrinos,
~ 10 MeV
99% gravitational energy

Collective neutrino oscillations

Raffelt & Tamborra
Phys. Rev. D **82**, 125004
“decoherence by dephasing of many neutrinos”

Akhmedov & Mirrizi
nuclphysb.2016.02.011
“decoherence by wave packet separation”

Outline

The effect of

decoherence by wave packet separation

can be incorporated into the

kinetic approach to neutrino oscillations

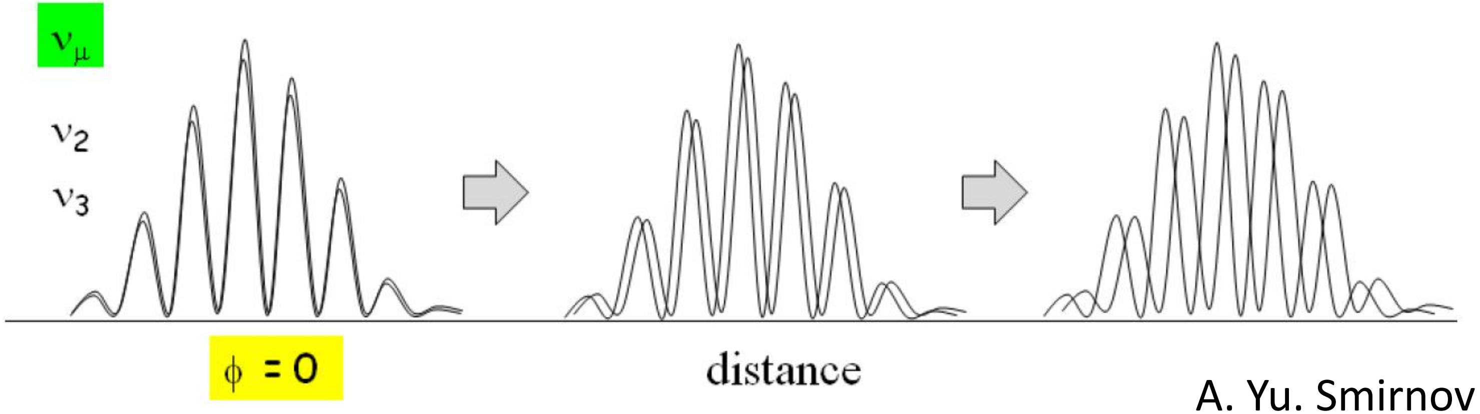
through the initial conditions consistent with the

Heisenberg principle

Decoherence by wave packet separation

$$i\partial_t \psi_i(t,\mathbf{x}) = \mathsf{H}_{ij}(t,\mathbf{x})\psi_j(t,\mathbf{x})$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$



A. Yu. Smirnov
1609.02386

$$\psi_i(0,\mathbf{p}) \propto \exp\left(-\frac{(\mathbf{p}-\mathbf{p}_w)^2}{4\sigma_p^2}\right) e^{-i\mathbf{p}\mathbf{x}_w}$$

Kinetic approach to neutrino oscillations

Liouville term

$$\partial_t \varrho(t,\mathbf{x},\mathbf{p}) + \tfrac{1}{2}\{\partial_{\mathbf{p}}\mathsf{H}(t,\mathbf{x},\mathbf{p}), \partial_{\mathbf{x}}\varrho(t,\mathbf{x},\mathbf{p})\} - \tfrac{1}{2}\{\partial_{\mathbf{x}}\mathsf{H}(t,\mathbf{x},\mathbf{p}), \partial_{\mathbf{p}}\varrho(t,\mathbf{x},\mathbf{p})\}$$
$$\approx -i\big[\mathsf{H}(t,\mathbf{x},\mathbf{p}), \varrho(t,\mathbf{x},\mathbf{p})\big] + \mathcal{C}\,.$$

Oscillation term　　Collision term

?

- In quantum mechanics coordinate and momentum are not measurable simultaneously
- In the kinetic approach the coordinate and momentum arguments have definite values
- This is frequently interpreted as a sign, that the kinetic approach is inherently classical

Heisenberg principle

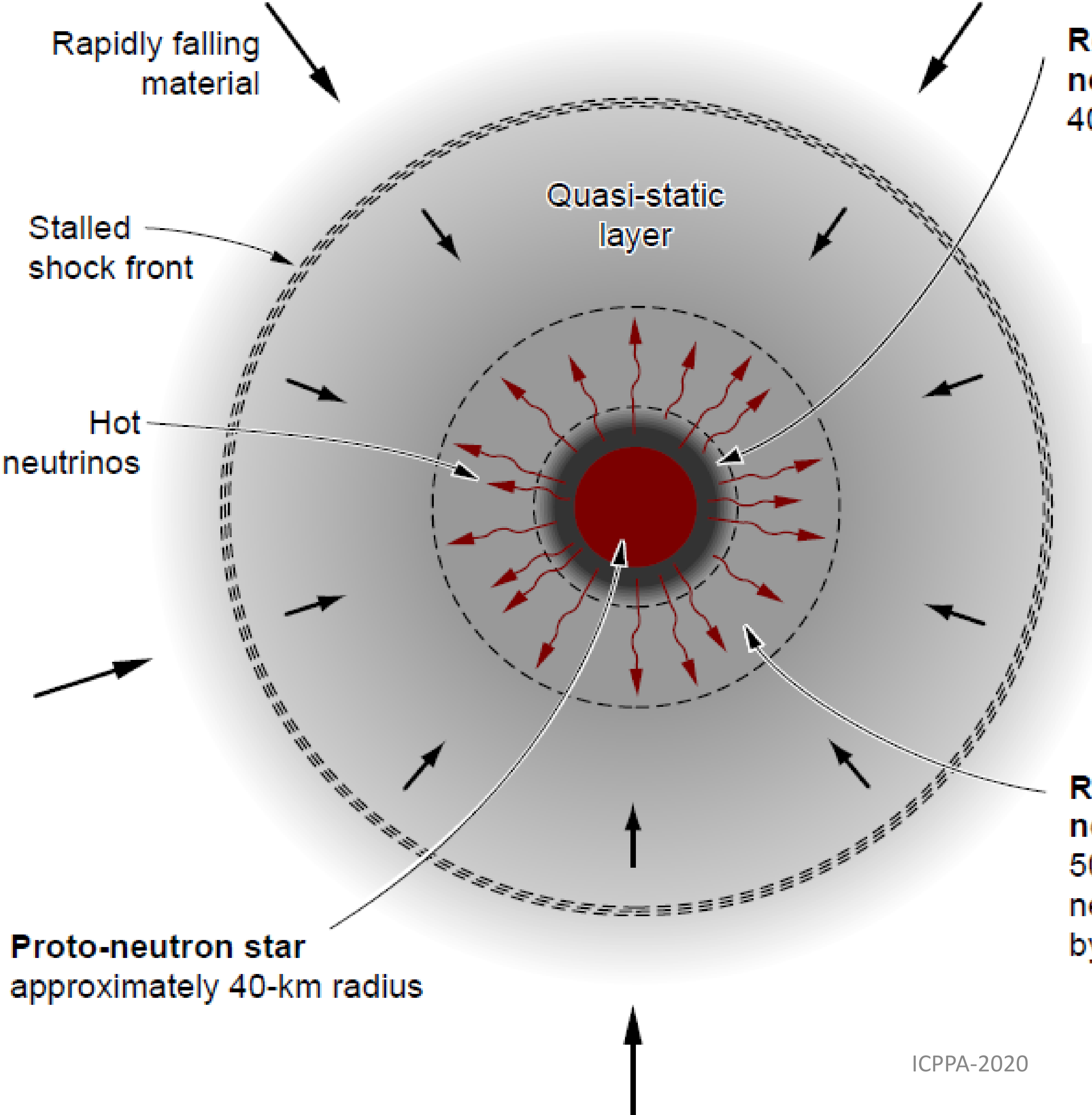
$$\hat{\varrho}_{ij}(t, \mathbf{x}, \mathbf{p}) = \int \frac{d^3 \mathbf{\Delta}}{(2\pi)^3} e^{i \mathbf{\Delta x}} \hat{a}_j^\dagger(t, \mathbf{p} - \mathbf{\Delta}/2) \hat{a}_i(t, \mathbf{p} + \mathbf{\Delta}/2)$$

The coordinate and momentum arguments are *not* the neutrino coordinate and momentum

$$|\psi\rangle = \sum_i \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \psi_i(0, \mathbf{p}) \hat{a}_i^\dagger(0, \mathbf{p}) |0\rangle$$

$$\varrho_{ij}(0, \mathbf{x}, \mathbf{p}) = \int \frac{d^3 \mathbf{\Delta}}{(2\pi)^3} e^{i \mathbf{\Delta x}} \psi_i(0, \mathbf{p} + \mathbf{\Delta}/2) \psi_j^*(0, \mathbf{p} - \mathbf{\Delta}/2)$$

$$\varrho_{ij}(0, \mathbf{x}, \mathbf{p}) \propto \exp\left(- \frac{(\mathbf{p} - \mathbf{p}_w)^2}{2\sigma_p^2} \right) \exp\left(- \frac{(\mathbf{x} - \mathbf{x}_w)^2}{2\sigma_x^2} \right)$$



$$|\psi\rangle = \frac{1}{\sqrt{N!}} \sum_{i_1 \dots i_N} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \cdots \frac{d^3\mathbf{p}_N}{(2\pi)^3} \times \psi_{i_1 \dots i_N}(0, \mathbf{p}_1 \dots \mathbf{p}_N)\, \hat{a}^\dagger_{i_1}(0, \mathbf{p}_1) \dots \hat{a}^\dagger_{i_N}(0, \mathbf{p}_N)|0\rangle$$

Akhmedov & Mirrizi
“decoherence by
wave packet separation”

$$\varrho_{ij}(0, \mathbf{x}, \mathbf{p}) \approx \sum_{l=1}^{N} \varrho_{ij}^{(l)}(0, \mathbf{x}, \mathbf{p})$$

Raffelt & Tamborra
“decoherence by
dephasing of many
neutrinos”

Summary

The effect of

decoherence by wave packet separation

can be incorporated into the

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through the initial conditions consistent with the

Heisenberg principle

Wigner approach

$$i\partial_t \psi_i(t,\mathbf{x}) = \mathsf{H}_{ij}(t,\mathbf{x})\psi_j(t,\mathbf{x})$$

$$\varrho_{ij}(t,\mathbf{x},\mathbf{p}) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{i\boldsymbol{\Delta}\mathbf{x}} \psi_i(t,\mathbf{p}+\boldsymbol{\Delta}/2)\psi_j^*(t,\mathbf{p}-\boldsymbol{\Delta}/2)$$

$$\begin{aligned}\partial_t \varrho(t,\mathbf{x},\mathbf{p}) + \tfrac{1}{2}\big\{\partial_{\mathbf{p}}\mathsf{H}(t,\mathbf{x},\mathbf{p}), \partial_{\mathbf{x}}\varrho(t,\mathbf{x},\mathbf{p})\big\} - \tfrac{1}{2}\big\{\partial_{\mathbf{x}}\mathsf{H}(t,\mathbf{x},\mathbf{p}), \partial_{\mathbf{p}}\varrho(t,\mathbf{x},\mathbf{p})\big\} \\ \approx -i\big[\mathsf{H}(t,\mathbf{x},\mathbf{p}), \varrho(t,\mathbf{x},\mathbf{p})\big]\,.\end{aligned}$$