Study of the ionization efficiency for nuclear recoils[1]

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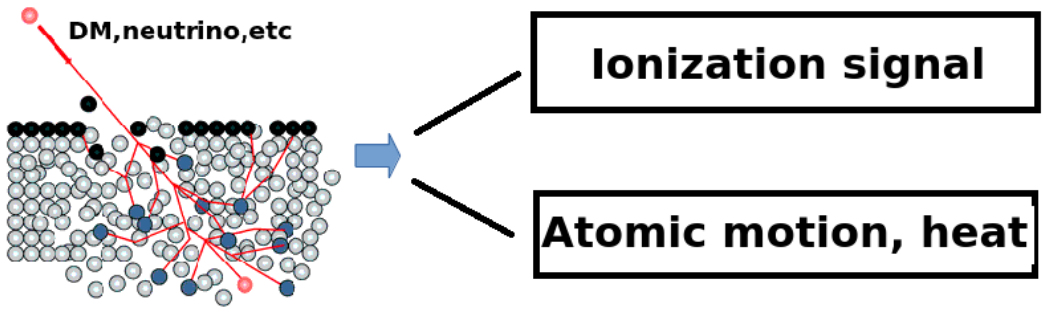
V International Conference on Particle Physics and Astrophysics
5th to 9th of October, 2020

[1] **Y.S**, A. Aguilar-Arévalo, J.C. D'Olivo,Phys. Rev. D 101, 102001 (2020)

Contents

Introduction

- In experiments for direct DM searches or detection of Coherent Elastic Neutrino Nucleus Scattering (CEνNS), ionization only detectors, the signal entails the detection of the ionization produced by the recoiling target ions following a scattering event.



- The electronic excitation produced by a recoiling ion is typically smaller than that produced by an electron of the same energy, we name this as **quenching** ($[\mathrm{eV_{nr}}] \rightarrow [\mathrm{eV_{ee}}]$).

Quenching Factor for Dark Matter and ν experiments

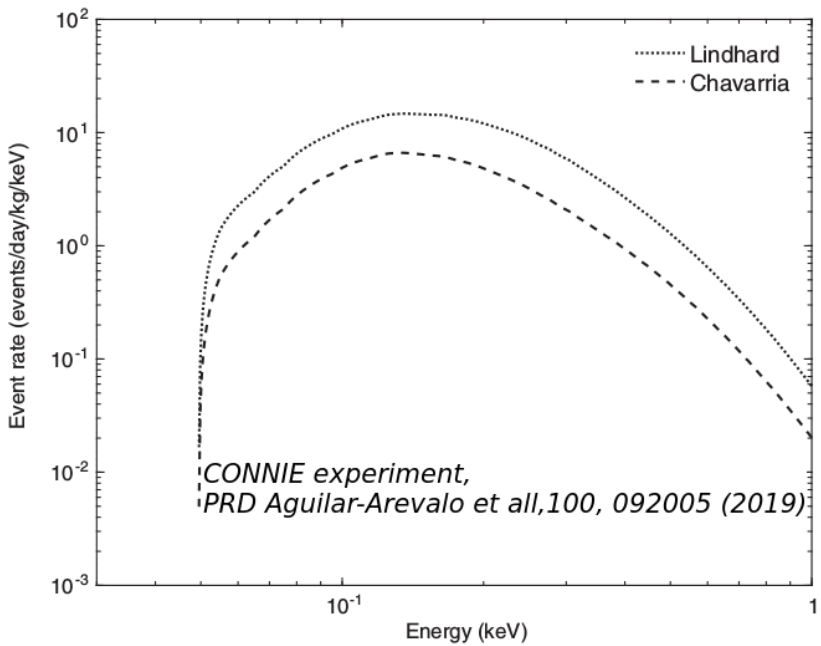
- For DM searches with pure crystals the quenching play an important role for calibration and efficiency.
- Different quenching, Lindhard and Chavarria (data), change significantly the rate for CEνNs signal.

$$\frac{dR}{dE_I} = \frac{dR}{dE_R} \underbrace{\frac{1}{Q}\left(1 - \frac{E_I}{Q}\frac{dQ}{dE_I}\right)}_{\frac{dE_R}{dE_I}}$$

$$Q = \frac{E_I}{E_R}, \quad \text{Quenching Factor}$$

E_I, Ionization energy

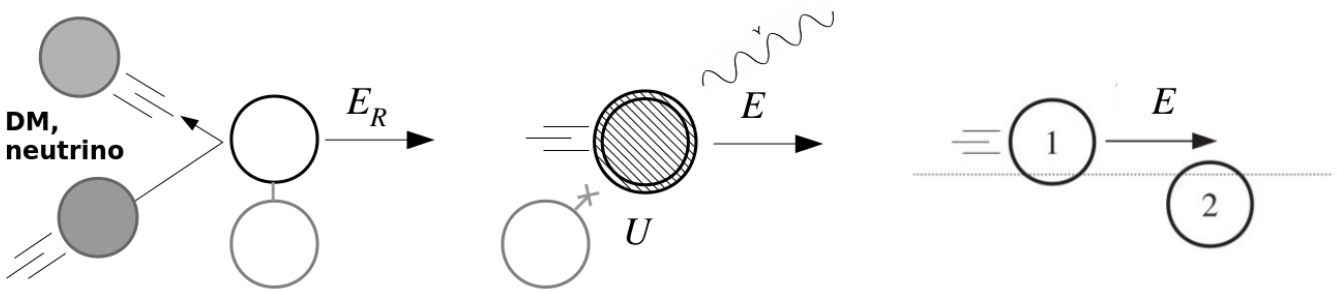
E_R, Recoil energy



Many experiments that rely on quenching factors

Nuclear recoil in a pure material

- Suppose that the ion recoils from the interaction with an energy E_R, after recoiling with an incident particle (e.g., a DM particle).
- Energy U is lost to some disruption of the atomic bonding, then $E_R = E + U$, then the ion moves with a kinetic energy E.
- The moving ion sets off a cascade of slowing-down processes that dissipate the energy E throughout the medium.



Lindhard's model

- Lindhard's theory concerns with determining the fraction of E_R which is given to electrons, H, and that which is given to atomic motion, N, with $E_R = N + H$.
- Defining reduced dimensionless quantities, $\varepsilon_R = c_Z E_R, \eta = c_Z H, \nu = c_Z N$ where $c_Z = 11.5/Z^{7/3}$keV.
- This separation is written as $\varepsilon_R = \bar{\eta} + \bar{\nu}$ ("average").
- The quenching factor (f_n) for a nuclear recoil is then defined as the fraction of E_R which is given to electrons ($u = c_Z U$):

$$\boxed{f_n = \frac{\bar{\eta}}{\varepsilon_R} = \frac{\varepsilon + u - \bar{\nu}}{\varepsilon + u}} \quad (1)$$

When u=0 one recovers the usual definition.

Basic integral equation and approximations

$$\underbrace{\int d\sigma_{n,e}}_{\text{total cross section}} \left[\underbrace{\bar{\nu}\left(E - T_n - \sum_i T_{ei}\right)}_{\text{A}} + \underbrace{\bar{\nu}\left(T_n - U\right)}_{\text{B}} + \underbrace{\bar{\nu}(E)}_{\text{C}} + \underbrace{\sum_i \bar{\nu}_e\left(T_{ei} - U_{ei}\right)}_{\text{D}} \right] = 0 \quad (2)$$

Lindhard's (five) approximations

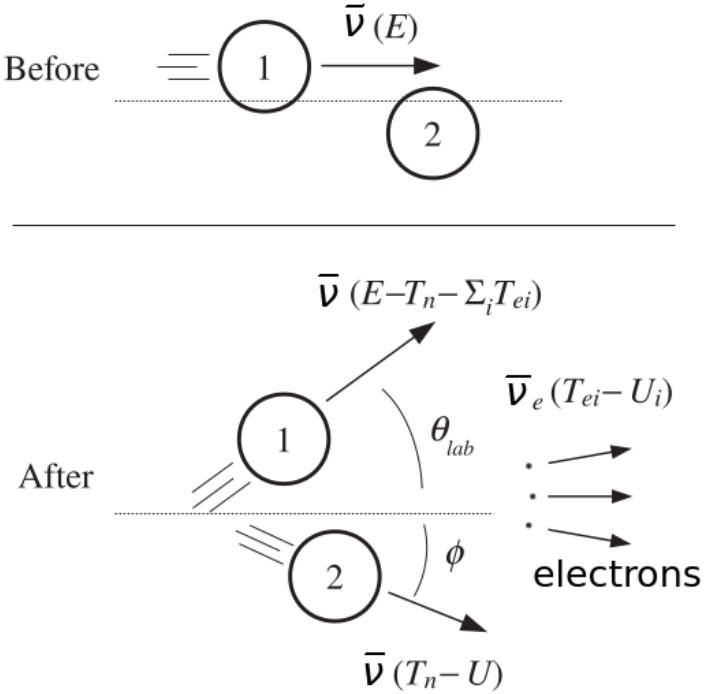
I Neglect contribution to atomic motion coming from electrons.

II **Neglect the binding energy, U = 0.**

III The energy transferred to ionized electrons is small compared to that transferred to recoiling ions.

IV Effects of electronic and atomic collisions can be treated separately.

V T_n is also small compared to the energy E.



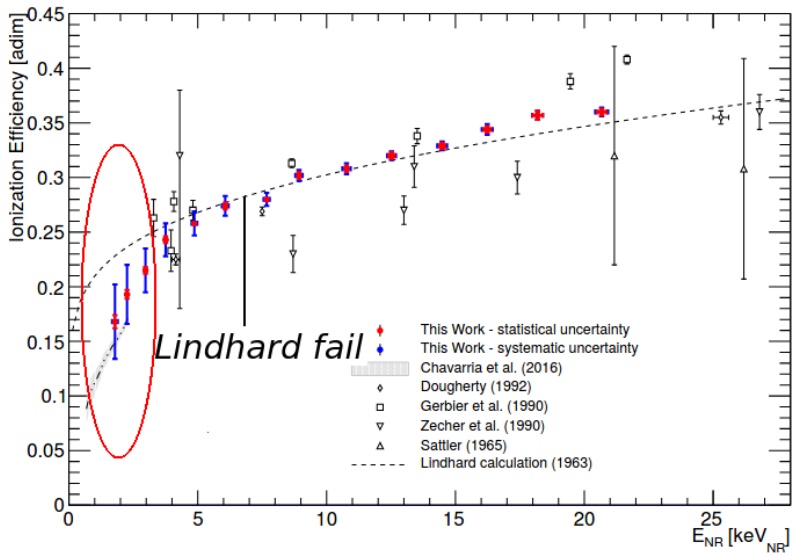
Lindhard quenching factor

- In 1963, for E.q (2) Lindhard used the above approximations, in which the most important was U=0.
- He gave a parametrization for $\bar{\nu}$, but only works for $\varepsilon_R \gtrsim 0.1$ (when U=0, $\varepsilon_R = \varepsilon$).

$$\bar{\nu}_L(\varepsilon) = \frac{\varepsilon}{1 + kg(\varepsilon)},$$
$$g(\varepsilon) = 3\varepsilon^{0.15} + 0.7\varepsilon^{0.6} + \varepsilon.$$

- First principles (e) stopping power $S_e = k\varepsilon^{1/2}$, $k = 0.133Z^{2/3}/A^{1/2}$.
- Hence the quenching factor at energies of few keV, start to deviate from measurements.

PRD Chavarria et all, 94, 082007(2016)



Simplified equation with binding energy

In order to keep the binding energy in to account a more general equation have to be build based in relaxing the Lindhard II, III and V approximations. We considered u constant and $S_e = k\varepsilon^{1/2}$, nuclear stopping $d\sigma_n(t)$ with $t = \varepsilon^2 \sin^2(\theta/2)$, so Eq.(2) transform:

$$-\frac{1}{2}k\varepsilon^{3/2}\bar{\nu}''(\varepsilon)+\underbrace{k\varepsilon^{1/2}}_{S_e}\bar{\nu}'(\varepsilon) = \int_{\varepsilon u}^{\varepsilon^2} \underbrace{dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}}_{d\sigma_n}[\bar{\nu}(\varepsilon-t/\varepsilon)+\bar{\nu}(t/\varepsilon-u)-\bar{\nu}(\varepsilon)] \tag{3}$$

We recover Lindhard's approach with u=0 and removing the red term. So the equation predicts a threshold $\varepsilon_R^{threshold} = 2u$. The function $f(t)$ is related to the inter-atomic potential (e.g Thomas-Fermi).

The equation admits a solution featuring a "kink" at $\varepsilon = u$.

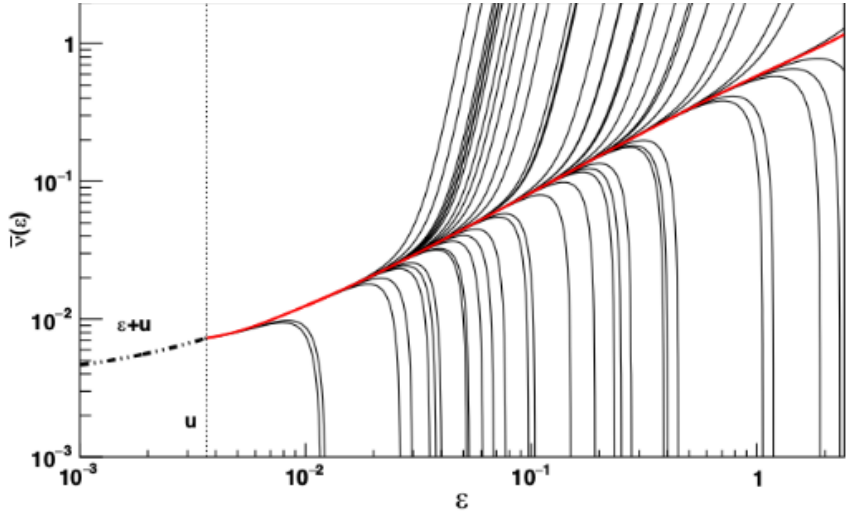
Numerical solution

Shooting method

This equation can be solved numerically from $\varepsilon \geqslant u$. Considering the physics and the properties of the model, Eq.(3) required the parametrization,

$$\bar{\nu}(\varepsilon) = \begin{cases} \varepsilon + u, & \varepsilon < u \\ \varepsilon + u - \lambda(\varepsilon), & \varepsilon \geq u \end{cases} \quad (4)$$

where λ is a continuous function. To solve for λ it's possible to implement a shooting method since there exist boundary condition for $\varepsilon \gg 1$.



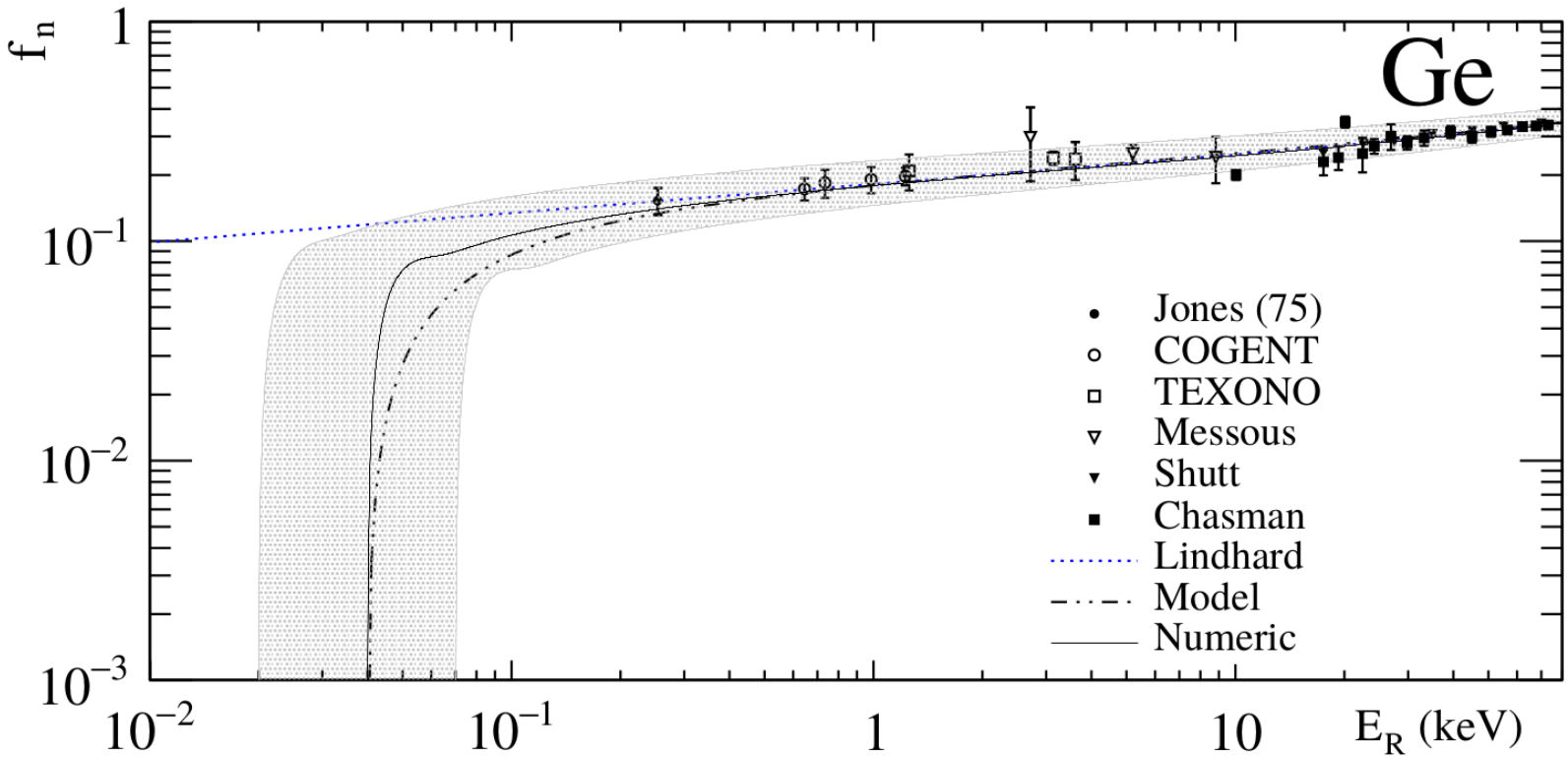
Atomic constant binding energy U model

In general, U includes both the energy needed to remove the ion from its site and contributions to excitation of bound atomic electrons, therefore incorporates the Migdal effect.

	Silicon*			Germanium*	
Shell	$U(\mathrm{eV})$	$\#e$	Shell	$U(\mathrm{eV})$	$\#e$
$[\mathrm{Ne}]^4$		4	$[\mathrm{Ar}]^{18}$		18
$2p$	100	6	$3d$	30	10
Average $e-h$	3.7	4	Average $e-h$	3.0	4
Dislocation	36		Dislocation	23	

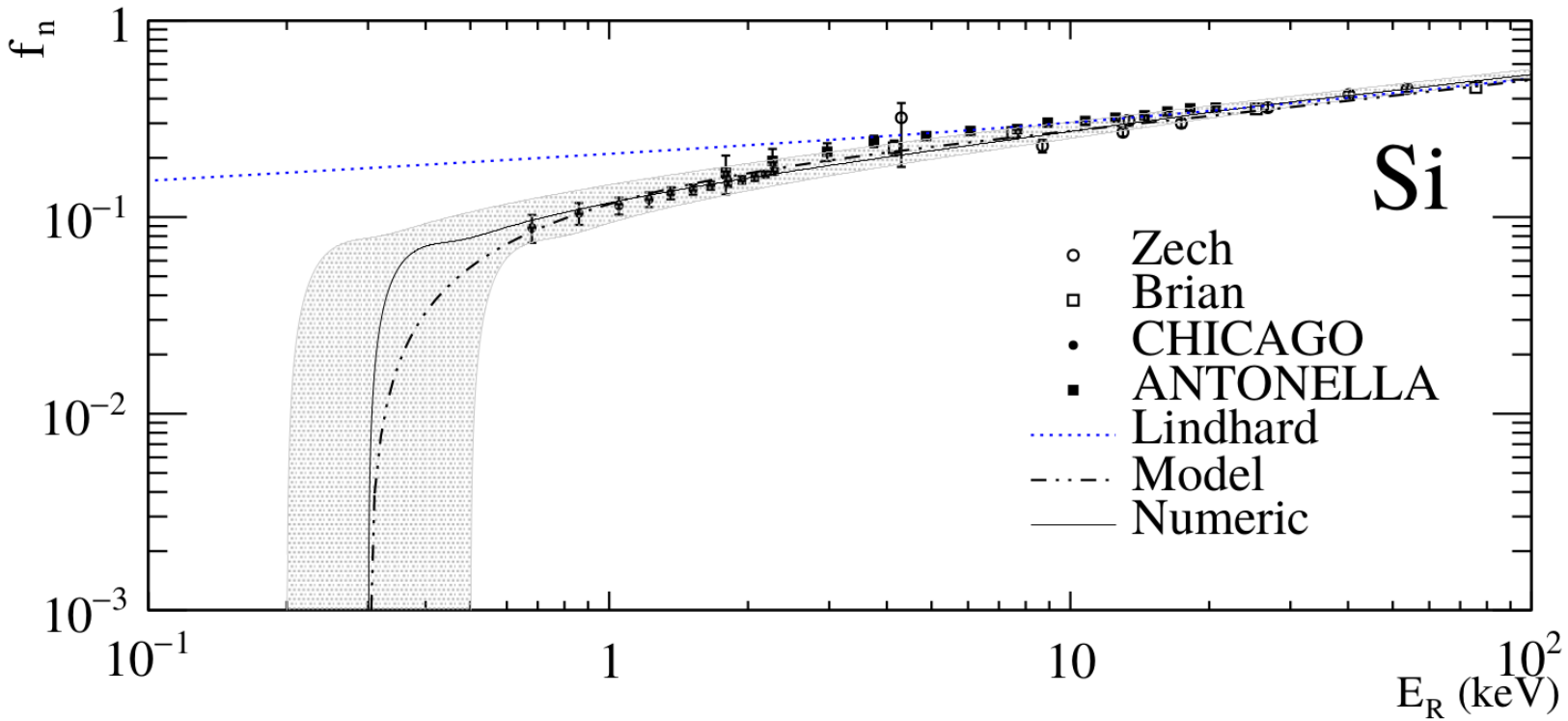
* E. Clementi and D.L.Raimondi, J. Chem. Phys. 1963, 38, 2686.

Results (Error band approximate cover the data fluctuation)



Germanium QF in good agreement with data, U = 0.02 keV and k = 0.162. (Model refers to a simple anzats, see publication)

Results



Silicon QF is in good agreement with data and a cutoff in 0.3 keV, that is consistent with Chavarria[2] measurements (0.3 ± 0.1keV), U = 0.15 keV and k = 0.161.

[2]PRD,Chavarria et all,94, 082006(2016)

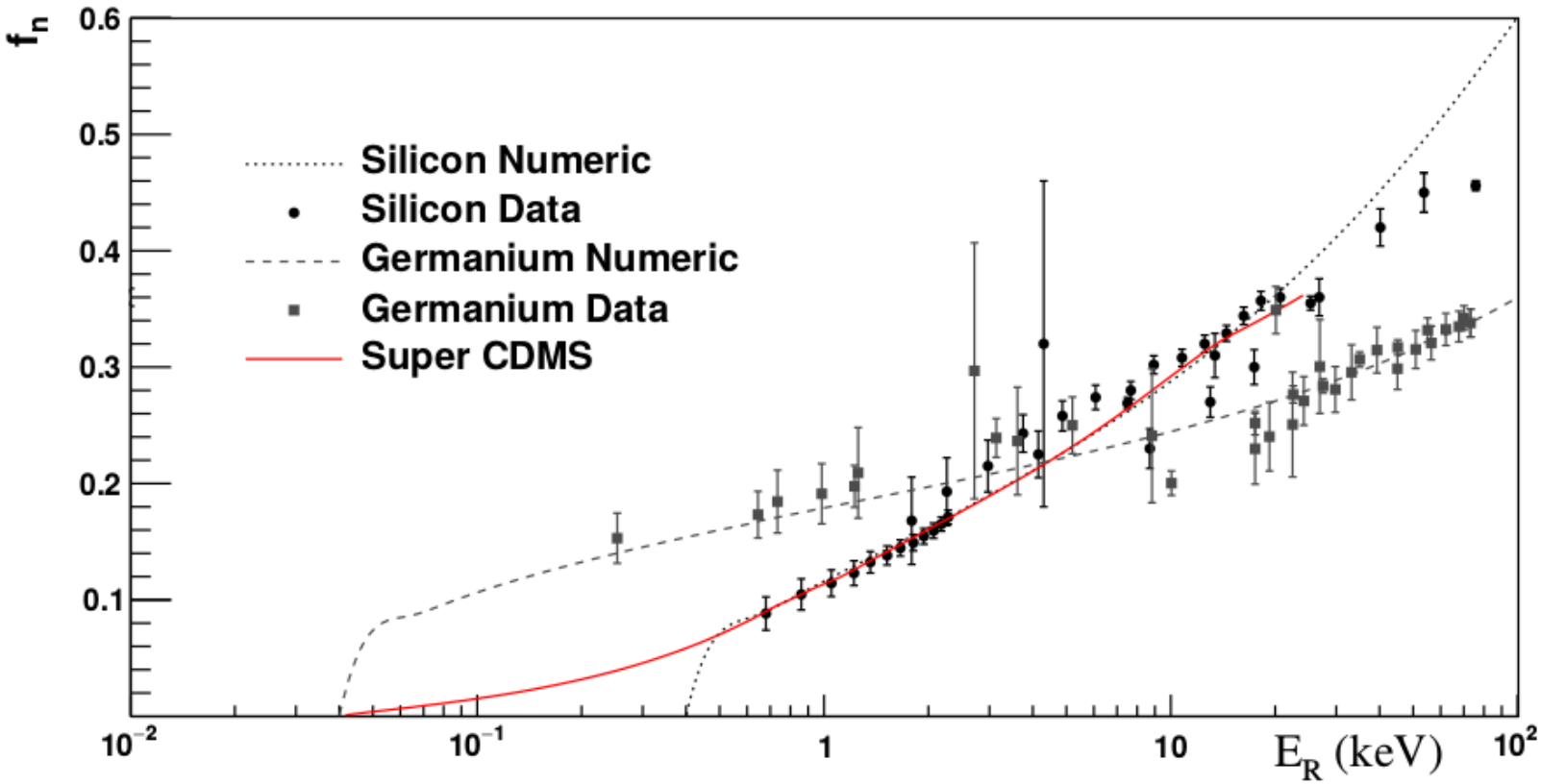


Figure: Comparison of the numerical solutions for Si, Ge, with data. Also shown is the phenomenological fit by Super-CDMS

Beyond constant U model

- Going furhter down in energy we have to considered an energy dependent binding energy.
- Inelastic interactions generally takes place against a background of elastic scattering in colliding nuclei.
- Consequently, at any point of the phase space the electron gas is expected to be only slightly excited.
- In the low energy ($< 1keV$) region the S_e, departures from velocity proportionality due to Coulomb repulsion effects.
- Several theoretical results are available now that discuss the above remarks; e.g Tilinin[3], Kishinevsky.[4]

[3] PRA,I.S Tilinin,51,3058(1995)

[4] Kishinevsky, L.M., 1962, Izv. Akad. Nauk SSSR, Ser. Fiz. 26, 1410.

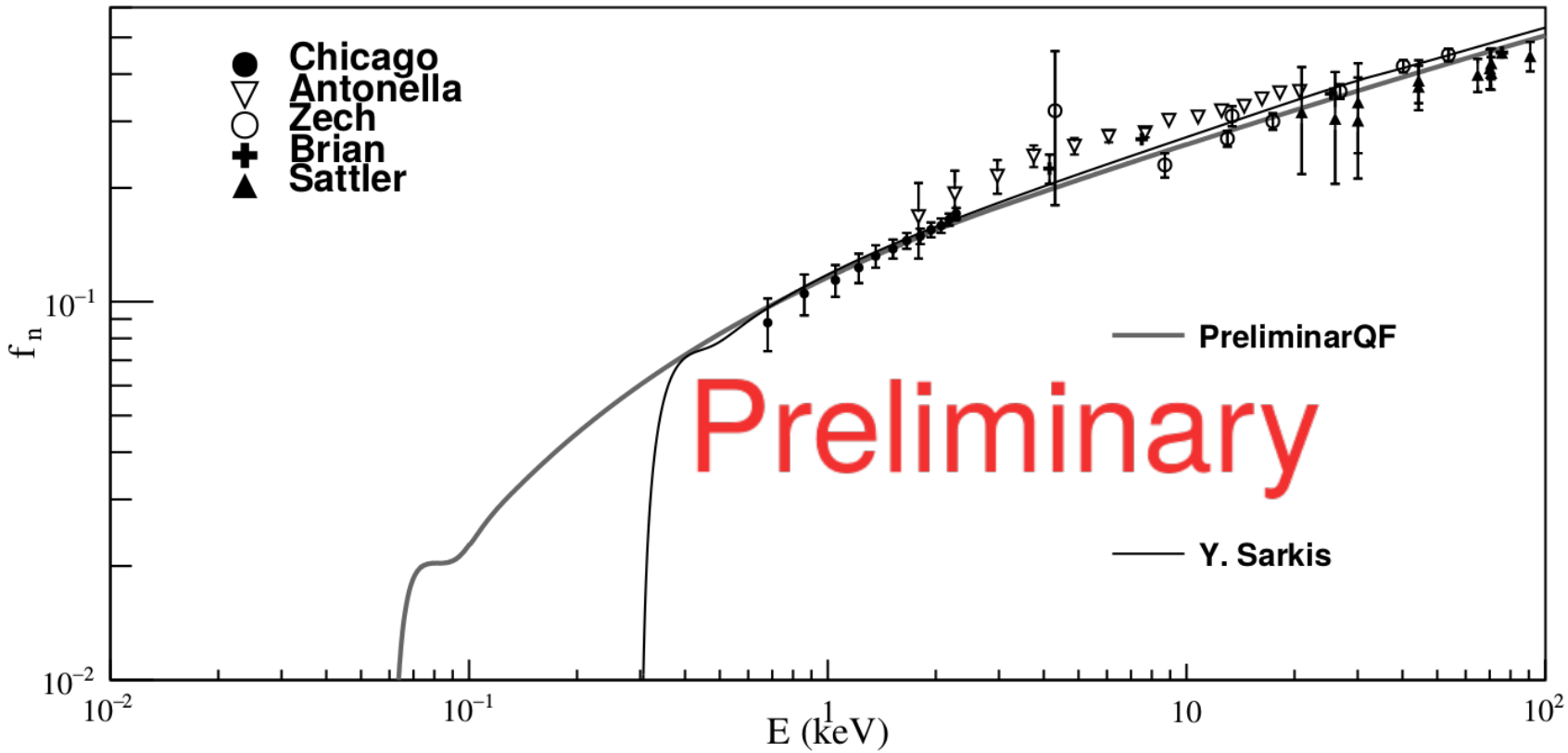


Figure: Preliminary QF in Si using a model beyond constant binding energy.

Conclusions

1. We found an appropriate form for the basic integro-differential equation describing the energy given to atomic motion by nuclear recoils in Si and Ge, when a constant binding energy is considered and $0.1 < k < 0.2$.

2. Also this model can describe the total quanta in noble liquids (Xe, Ar). Light and charge yields can be computed.

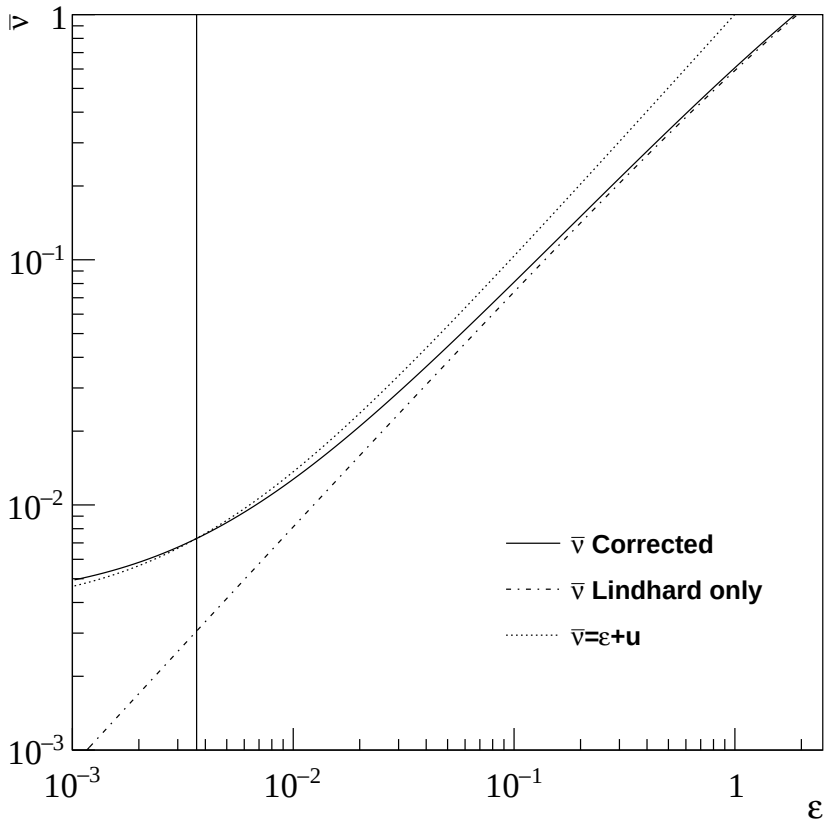
3. This model can be extended considering energy variable binding energy. Also we are working in incorporate low energy effects in the electronic stopping power.

Thanks

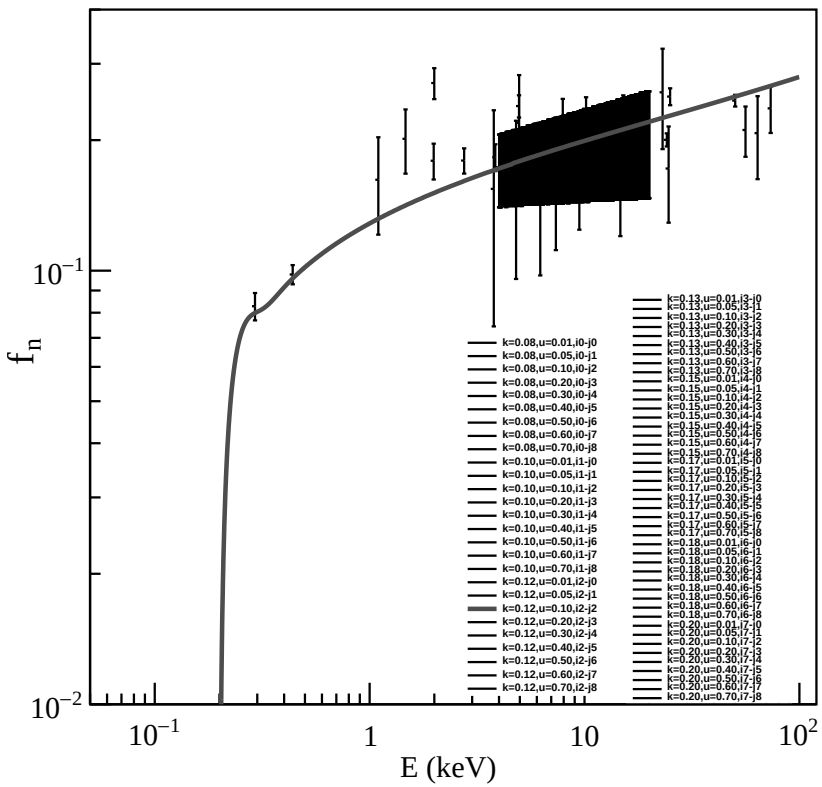
This research was supported in part by DGAPA-UNAM grant number PAPIIT-IN108917, and Consejo Nacional de Ciencia y Tecnología (CONACYT) through gran CB2014/240666.

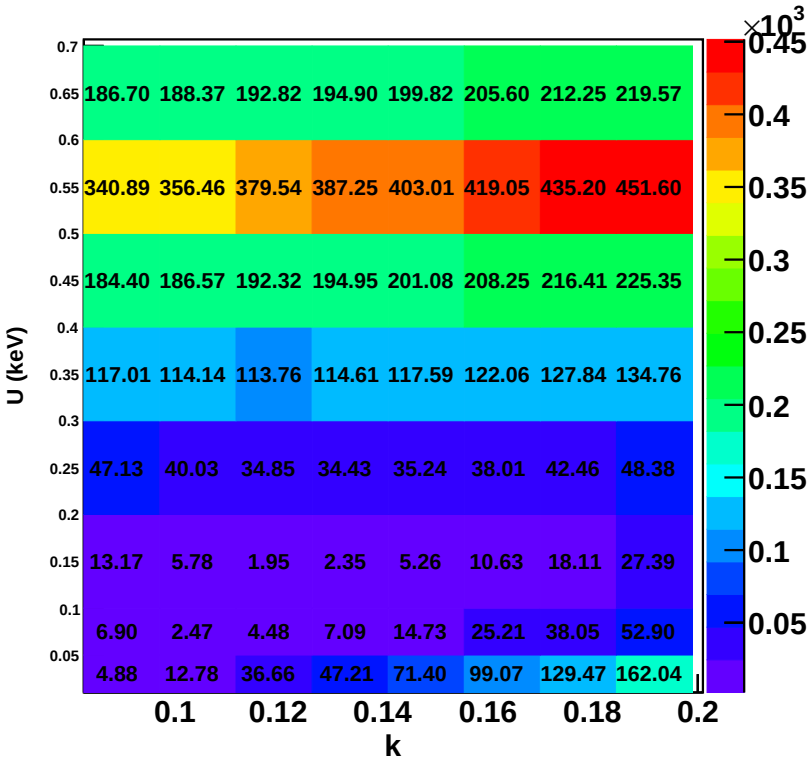
Ansatz

- We can implement a good analytical approximation to solve the integral equation.
- The idea is to lessen the ionization contribution, subtracting a fraction of the electronic stopping power.
- $\bar{\eta} = \bar{\eta}_{lind} - c\varepsilon^{1/2} - c'$ where c, c' and u are estimated from a fit to the available data.
- Where $\bar{\eta} = \varepsilon - \bar{\nu}$.

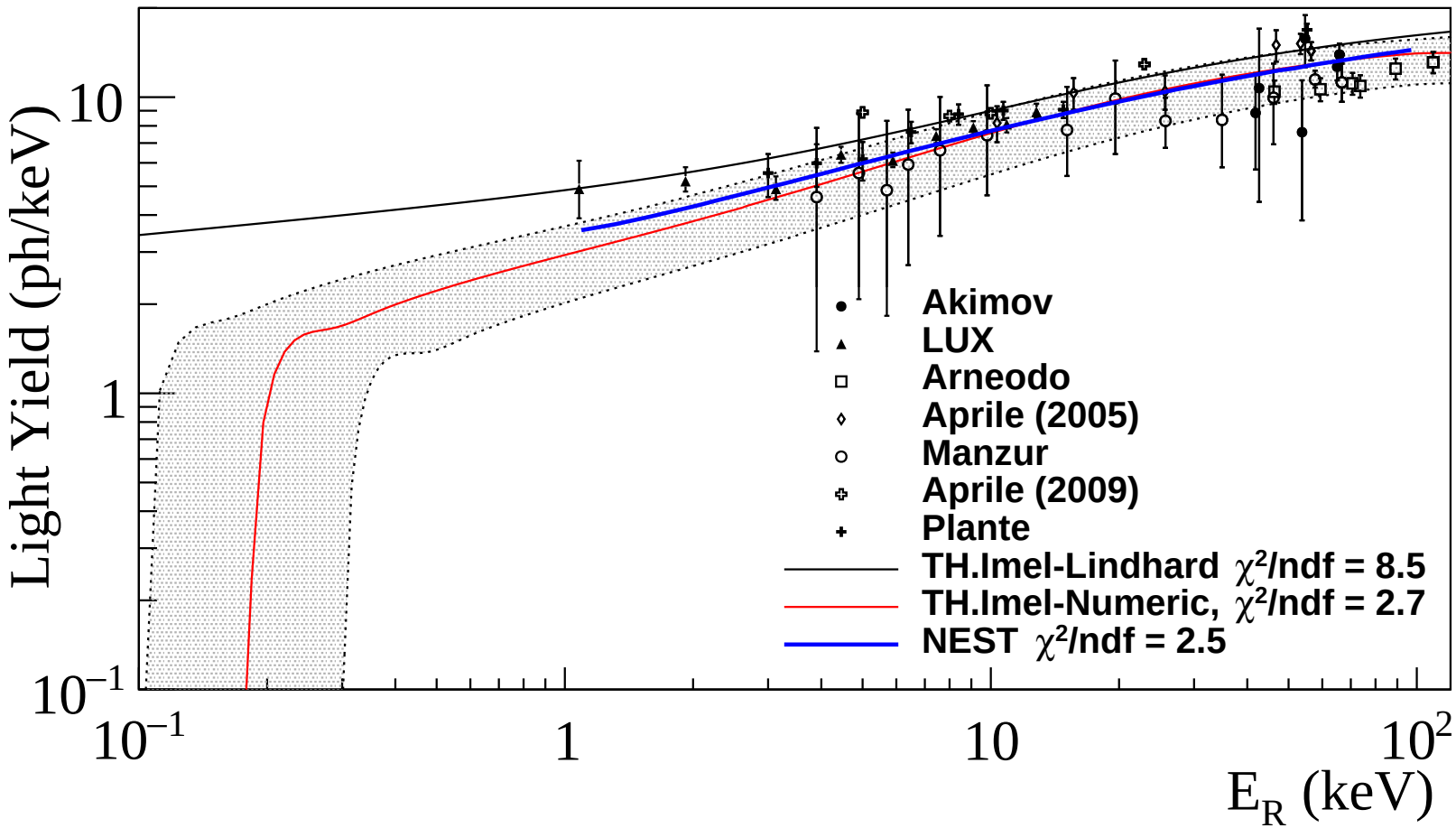


Xenon

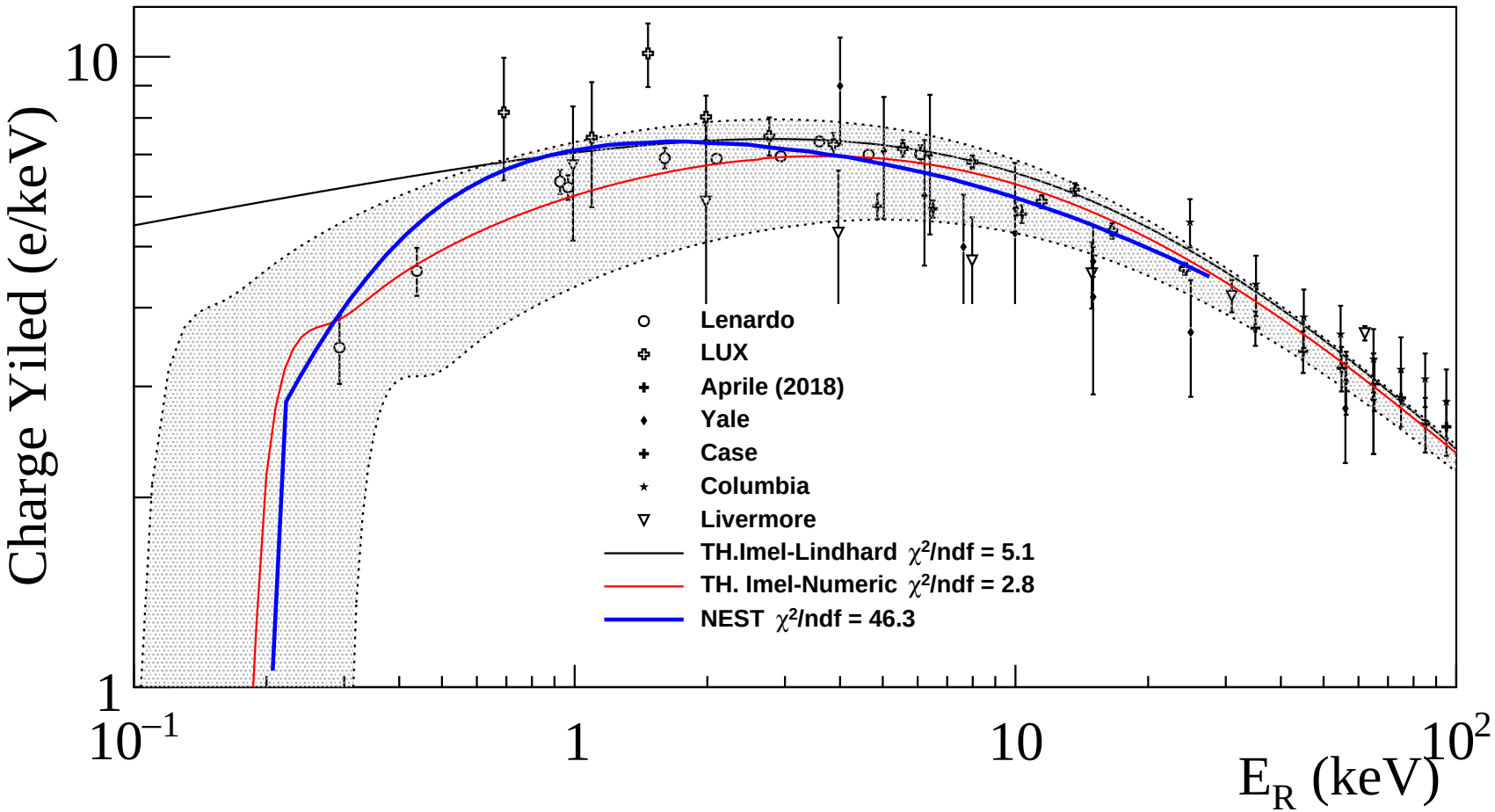

f_n
10^{-1}
10^{-2}
10^{-1}
1
10
10^2
E (keV)


U (keV)
k
$\times 10^3$
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
186.70 188.37 192.82 194.90 199.82 205.60 212.25 219.57
340.89 356.46 379.54 387.25 403.01 419.05 435.20 451.60
184.40 186.57 192.32 194.95 201.08 208.25 216.41 225.35
117.01 114.14 113.76 114.61 117.59 122.06 127.84 134.76
47.13 40.03 34.85 34.43 35.24 38.01 42.46 48.38
13.17 5.78 1.95 2.35 5.26 10.63 18.11 27.39
6.90 2.47 4.48 7.09 14.73 25.21 38.05 52.90
4.88 12.78 36.66 47.21 71.40 99.07 129.47 162.04
0.1 0.12 0.14 0.16 0.18 0.2
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05

Xenon


Light Yield (ph/keV)
10
1
10^{-1}
10^{-1}
1
10
10^2
E_R (keV)
Akimov
LUX
Arneodo
Aprile (2005)
Manzur
Aprile (2009)
Plante
TH.Imel-Lindhard χ^2/ndf = 8.5
TH.Imel-Numeric, χ^2/ndf = 2.7
NEST χ^2/ndf = 2.5

Xenon


Charge Yiled (e/keV)
10
1
10^{-1}
1
10
10^2
E_R (keV)
Lenardo
LUX
Aprile (2018)
Yale
Case
Columbia
Livermore
TH.Imel-Lindhard χ^2/ndf = 5.1
TH. Imel-Numeric χ^2/ndf = 2.8
NEST χ^2/ndf = 46.3

Introduction

Lindhard parametrization was deduced based in the following:

i Neglect atomic movement from electrons, since is negligible at low energies.

ii **Neglect the binding energy $U = 0$.**

iii Energy transferred to ionized electrons is small compared to that transferred to recoiling ions.

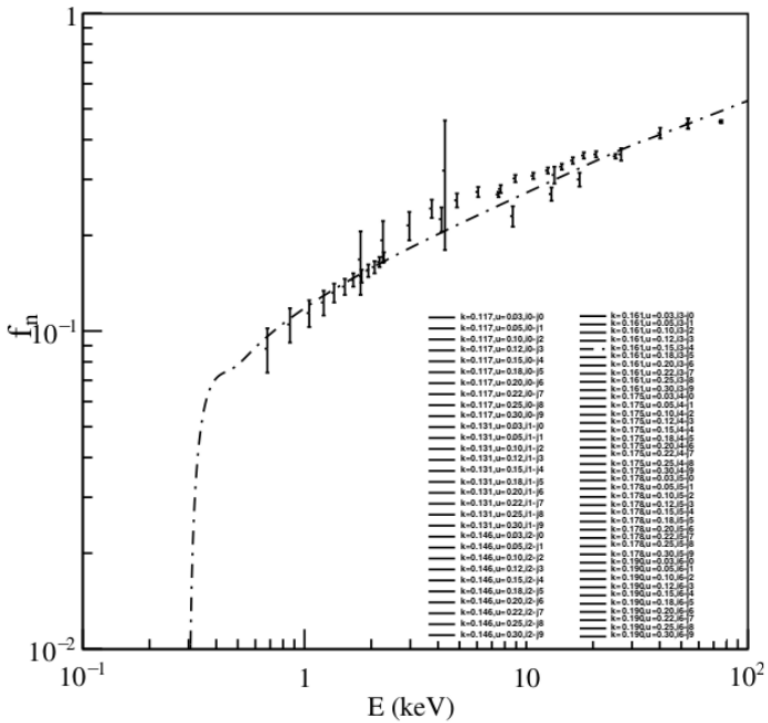
iv Effects of electronic and atomic collisions can be treated separately.

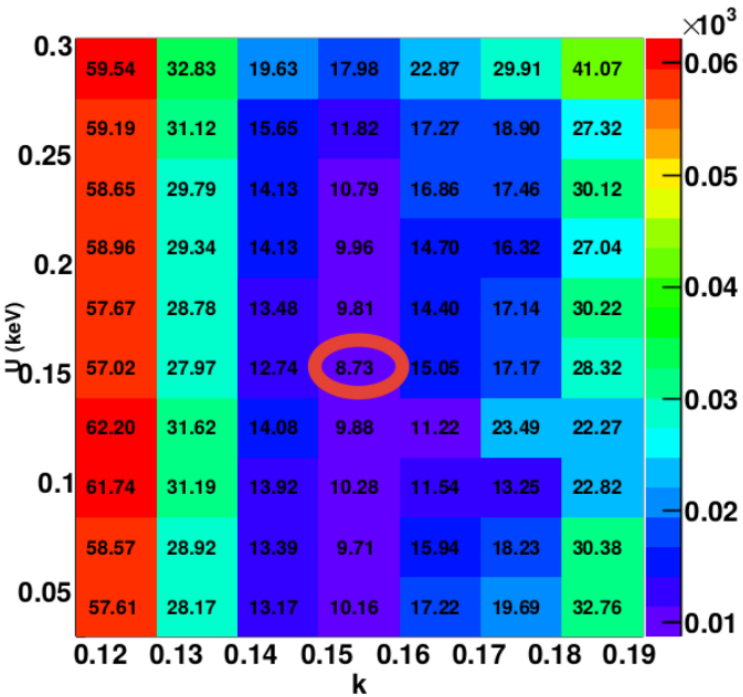
v T_n is also small compared to the energy E.

The main achievement of this work is to incorporate in the physical model the binding energy U.

Fits to data

We set a grid of 7x7 points in U and k region, in an acceptable ranges, and compute the χ^2/ndf of each (U,k) point to determine the optimal value, we do this for Si and Ge.





Lindhard's model

1. Using dimensionless units ($\varepsilon = 11.5E(\mathrm{keV})/\mathrm{Z}^{7/3}$,), the quenching factor is defined:

$$quenching = \frac{\text{total ionization energy}}{\text{total deposited energy}} = f_n = \frac{\bar{\eta}}{\varepsilon_R}$$

where $\bar{\eta}$ and ε_R are the ionization energy and the total recoil energy in adimensional units.

2. We concern about determining the fraction of ε_R which is given to electrons, $\bar{\eta}$, and that which is given to atomic motion,$\bar{\nu}$, assuming $\bar{\varepsilon}_R = \bar{\eta} + \bar{\nu}$. Hence; $\bar{\eta} = \varepsilon_R - \bar{\nu}$.