$J/\psi$ mass shift using D meson loop effect

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Outline

- Aim and motivation
- Methodology
- Results
- Summary
Need to study the in medium $J/\psi$?

- To understand the possible outcomes of future experiments
- $J/\psi$ Suppression
  - CBM (Compressed Baryonic Matter)
  - PANDA (anti-Proton ANnihilations at DArmstadt)

Under the FAIR (Facility of Antiproton and Ion Research) project at GSI, Germany.
Hadron gas:
- Moderate temperatures and densities
- Quarks and gluons are confined
- Chiral symmetry is spontaneously broken

Quark-gluon plasma:
- Very high temperatures and densities
- Deconfined quarks and gluons
- Chiral symmetry is restored

Color superconductor:
- $T < 100$ MeV and very high densities
- Quarks form bosonic pairs in analogy to BCS theory
SU(4) extension of light-flavor chiral-symmetric Lagrangians of pseudoscalar meson that is extended to the charm sector.

SU(4) flavor symmetry is strongly broken in nature, and in order to stay as close as possible to phenomenology, we use experimental values for the charmed mesons masses and use the empirically known meson coupling constants.
We are interested in the difference of the in-medium mass and the vacuum mass.

\[ \Delta m = m^*_\psi - m_\psi, \]

With the bare mass \( m^0_\psi \) calculated as,

\[ m^2_\psi = (m^0_\psi)^2 + \Sigma (k^2 = m^2_\psi). \]

Where \( \Sigma (k^2) \) is the total J/Ψ self-energy obtained from the sum of the contributions from the DD pairs.
We take the averaged, equal masses for the neutral and charged $D$ mesons, i.e. $m_{D^0} = m_{D^\pm}$. Averaging over the three polarizations of $J/\Psi$, one can calculate the $D\bar{D}$-loop contribution to the $J/\Psi$ self-energy $\Sigma_{D\bar{D}}$ as

\[ \Sigma_I(m_{\psi}^2) = -\frac{g_{\psi I}^2}{3\pi^2} \int_0^\infty dq^2 F_I(q^2) K_I(q^2), \]

where $F_I(q^2)$ is the product of vertex form-factors

\[ K_{D\bar{D}}(\bar{q}^2) = \frac{1}{\omega_D} \left( \frac{\bar{q}^2}{\omega_D^2 - m_{\psi}^2/4} - \xi \right), \]

where $\omega_D = (\bar{q}^2 + m_D^2)^{1/2}$, $\xi = 0$.
\[ u_D(q^2) = \left( \frac{\Lambda_D^2 + m^2}{\Lambda_D^2 + 4\omega_D^2(q)} \right)^2, \quad F_{DD}(q^2) = u_D^2(q^2), \]

where \( \Lambda_D \) is a cutoff mass.
CHIRAL SU(3) MODEL

• **Chiral symmetry**
  Limit of massless quarks i.e \( m_u = m_d = m_s = 0 \)
  Invariance of QCD Lagrangian under \( SU(3)_L \times SU(3)_R \) transformation.

• **Broken scale invariance**, which leads to non zero trace of energy momentum tensor

\[
\theta^\mu_\mu = \left\langle \frac{\beta_{QCD}}{2g} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle
\]

• **Mean-field approximation**
Lagrangian Density is given as

\[ \mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB} \]

- **Kinetic term**
- **Interaction of Vector mesons**
- **Explicit symmetry breaking**
- **Baryon meson interactions**
- **Meson–meson interactions**
Coupled equations of motion of scalar fields derived from effective Lagrangian using mean field approximation:

For sigma field:
\[
k_0 \chi^2 \sigma - 4k_1 (\sigma^2 + \zeta^2) \sigma - 2k_2 (\sigma^3) - 2k_3 \chi \sigma \zeta - \frac{d}{3} \chi^4 \left(\frac{2}{\sigma}\right) + \left(\frac{\chi}{\chi_0}\right)^2 m_{\pi f_{\pi}}^2 - \sum g_{\sigma i} \rho_i^s = 0
\]

For zeta field:
\[
k_0 \chi^2 \zeta - 4k_1 (\sigma^2 + \zeta^2) \zeta - 4k_2 (\zeta^3) - k_3 \chi \sigma^2 - \frac{d}{3} \chi^4 - \left(\frac{\chi}{\chi}\right)^2 \left[\sqrt{2} m_{k f_k}^2 - \frac{1}{\sqrt{2}} m_{\pi f_{\pi}}^2\right] - \sum g_{\zeta i} \rho_i^s = 0
\]

For dilaton field:
\[
k_0 \chi (\sigma^2 + \zeta^2) - k_3 \sigma^2 \zeta + \chi^3 \left[1 + \ln \left(\frac{\chi^4}{\chi_0^4}\right)\right]
+ (4k_4 - d) \chi^3 - \frac{4}{3} d \chi^3 \ln \left\{\left[\frac{(\sigma^2 \zeta)}{\sigma_0^2 \zeta_0}\right] \left(\frac{\chi^4}{\chi_0^4}\right)\right\}
+ 2 \frac{\chi}{\chi_0} \left[m_{\pi f_{\pi}}^2 \sigma + \left(\sqrt{2} m_{k f_k}^2 - \frac{1}{\sqrt{2}} m_{\pi f_{\pi}}^2\right) \zeta\right] = 0
\]
On solving these coupled equations we can calculate the density and temperature dependence of scalar fields $\sigma, \zeta$ and dilaton field $\chi$. Strangeness fraction is defined as $f_s = \frac{\sum_i |s_i| \rho_i}{\rho_B}$. Where $s_i$ is the number of strange quarks and $\rho_i$ is the Number density of ith baryon.
Variation of quark, gluon and strange condensates with temperature and strangeness fraction.
In QCD Sum rule approach we start with the two point correlation function.

\[ \Pi_{\mu\nu} (q) = i \int d^4 x \ e^{iq \cdot x} \langle T \{ J_\mu (x) J_\nu^\dagger (0) \} \rangle_{\rho_B,T} \]

Where \( J_\mu (x) \) denotes the isospin averaged current \( \mathbf{c} = \chi^\mu = (\chi^0, \chi) \) is the four coordinate and \( q = q^\mu = (q^0, q) \) is the four momentum, and \( T \) denotes the time ordered operation on the product of quantities in the brackets.

Vector meson isospin averaged average currents are given by the expression

\[ J_\mu (x) = J_\mu^+ (x) = \frac{\bar{c} (x) \gamma_\mu q (x) + \bar{q} (x) \gamma_\mu c (x)}{2}, \]

Here \( q \) denotes light quark and \( c \) denotes heavy charm quark.
The two point correlation function can be decomposed into the vacuum part, static one-nucleon part and pion bath contribution.

\[
\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}^0(q) + \frac{\rho_B}{2M_N} T_{\mu\nu}^N(q) + \Pi_{\mu\nu}^{P,B.}(q),
\]

Where,

\[
T_{\mu\nu}^N(\omega, q) = i \int d^4xe^{iq\cdot x} \langle N(p)|T \left\{ J_\mu(x)J_\nu^\dagger(0) \right\}|N(p)\rangle.
\]

Here \(|N(p)\rangle\) denotes the isospin and spin averaged static nucleon part. With the four momentum \(p = (M_N, 0)\). Third term in the equation denotes the contribution from pion bath at finite temperature. In our present work we take the effect of finite temperature on the vector D and B meson through the temperature dependence of fields \(\sigma\), \(\varsigma\) and \(\chi\).

Shift in mass can be defined by the relation

\[
\delta M_{D^*} = \sqrt{m_{D^*}^2 + \Delta m_{D^*}^2} - m_{D^*}
\]
Borel transformation equation can be written as
\[
a \left\{ \frac{1}{M^2} e^{-\frac{M^2}{M^2_D^*}} - \frac{s_0}{M^4_{D^*}} e^{-\frac{s_0}{M^2}} \right\} + b \left\{ e^{-\frac{M^2}{M^2_D^*}} - \frac{s_0}{M^2_{D^*}} e^{-\frac{s_0}{M^2}} \right\}
\]
\[
+ B \left[ \frac{1}{(M_H + M_N)^2 - M^2_{D^*}} - \frac{1}{M^2} \right] e^{-\frac{M^2}{M^2_D^*}} - \frac{B}{(M_H + M_N)^2 - M^2_{D^*}} e^{-\frac{(M_H + M_N)^2}{M^2}}
\]
\[
= \left\{ -\frac{m_c \langle \bar{q}q \rangle_N}{2} - \frac{2 \langle q^\dagger iD_0 q \rangle_N}{3} + \frac{m_c^2 \langle q^\dagger iD_0 q \rangle_N}{M^2} \right\} e^{-\frac{m_c^2}{M^2}} + \frac{m_c \langle q g_s \sigma G q \rangle_N}{3 M^2} e^{-\frac{m_c^2}{M^2}}
\]
\[
+ \left\{ \frac{8 m_c \langle \bar{q} iD_0 iD_0 q \rangle_N}{3 M^2} - \frac{m_c^3 \langle \bar{q} iD_0 iD_0 q \rangle_N}{M^4} \right\} e^{-\frac{m_c^2}{M^2}}
\]
\[
- \frac{1}{24} \frac{\langle \alpha_s G G \rangle_N}{\pi} \int_0^1 dx \left( 1 + \frac{\tilde{m}_c^2}{2 M^2} \right) e^{-\frac{\tilde{m}_c^2}{M^2}}
\]
\[
+ \frac{1}{48 M^2} \frac{\langle \alpha_s G G \rangle_N}{\pi} \int_0^1 dx \frac{1 - x}{x} \left( \frac{\tilde{m}_c^2 - \tilde{m}_c^4}{M^4} \right) e^{-\frac{\tilde{m}_c^2}{M^2}}.
\]

This eqn. has two unknowns \( a \) and \( b \). To solve this we differentiate this eqn w.r.t \( \frac{1}{M^2} \) to get two equations with two unknowns. Solving these two coupled equations we can calculate \( a \) and \( b \). Where

\[
B = \frac{2 f^2_{D^*} M^2_{D^*} M_N (M_H + M_N) g^2_{D^* NH}}{(M_H + M_N)^2 - M^2_{D^*}}
\]

Within Chiral SU(3) model we can calculate the values of \( \mathcal{O}_{\rho_B} \) at finite density of the nuclear medium, hence can find the \( \mathcal{O}_N \)

\[
\mathcal{O}_N = [\mathcal{O}_{\rho_B} - \mathcal{O}_{\text{vacuum}}] \frac{2M_N}{\rho_B}.
\]

Here \( \mathcal{O}_N \) is nucleon expectation value of the operator at finite baryonic density, \( \mathcal{O}_{\text{vacuum}} \) is vacuum expectation value of operator, \( \mathcal{O}_{\rho_B} \) is the expectation value of operator at finite baryonic density.

In our present investigation of hadrons properties, we are interested in light quark condensates \( \bar{u}u \) and \( \bar{d}d \), which are proportional to the non-strange scalar field \( \sigma \), within chiral SU(3) model. Considering equal mass of light quark, \( u = d = 0.006 \text{ GeV} \). We write,

\[
\langle \bar{q}q \rangle_{\rho_B} = \frac{1}{2m_q} \left( \frac{\chi}{\chi_0} \right)^2 \left[ m_\pi^2 f_\pi \sigma \right].
\]

Quark condensate, \( \langle \bar{q}q \rangle_{\rho_B} \) is calculated from the chiral SU(3) model.
\[ \langle \bar{q}q \rangle_{\rho_B} \] Can be used to calculate the other condensates, which are

\[ \langle \bar{q}g_8 \sigma Gq \rangle_{\rho_B} = \lambda^2 \langle \bar{q}q \rangle_{\rho_B} + 3.0 \text{GeV}^2 \rho_B. \]

\[ \langle \bar{q}iD_0^iD_0q \rangle_{\rho_B} + \frac{1}{8} \langle \bar{q}g_8 \sigma Gq \rangle_{\rho_B} = 0.3 \text{GeV}^2 \rho_B. \]

Value of the condensate \( \langle q^\dagger iD_0 q \rangle \) is not calculated from the chiral SU(3) model. It’s value is approximated as \(0.18 \text{GeV}^2 \rho_B\).

Qualitative Results and Discussion

D Meson Mass Shift:

Temperature: decreases the magnitude of shift in mass.

Strange medium $f_s$: Increases the magnitude of mass shift.

Density $\rho_B$: Increases the magnitude of mass shift.
Shift in mass of J/ψ meson by using DD loop effect, but mass of D meson was calculated through QMC model.

Only in nuclear matter, and observed shift 3 MeV,

In our work, 3-4 MeV.

Temperature of the medium causes a drop in the shift.

Presence of strange hadrons (along with nucleons) cause enhancement in the magnitude of the shift.
Summary

- We have used effective Lagrangian approach to calculate in mass of $J/\psi$ meson in hot and dense symmetric hadronic medium, by considering pseudoscalar D meson loop effect.

- In medium mass of D meson was calculated through chiral model and QCD sum rule approach to observe the effect of strange medium, temperature and baryonic density on the shift in masses and decay constants of D meson.

- Strange medium cause a drop in the mass of $J/\psi$ meson.

- Finite temperature causes an increase in the mass.

- With increase in density of the medium increases the magnitude of the shift.
Thank you
Parameters used

Nuclear saturation density used in the present investigation is 0.15 $\text{fm}^{-3}$. Coupling constant used in this case is $g_{D^*N\Lambda_c} \approx g_{D^*N\Sigma_c} \approx g_{B^*N\Lambda_b} \approx g_{B^*N\Sigma_b} \approx 3.86$. The masses of mesons $m_{D^*}$, $m_{B^*}$, $m_{D_{s^*}}$ and $m_{B_{s^*}}$ are 2.01, 5.325, 2.112 and 5.415 MeV respectively. Values of decay constants of $f_{D^*}$, $f_{B^*}$, $f_{D_{s^*}}$ and $f_{B_{s^*}}$ are 0.270, 0.195, 1.16*, $f_{D^*}$ and 1.16* $f_{B^*}$ respectively. Masses of quarks namely (up)u, (down)d, strange(s), (charm)c and (bottom)b are taken as 0.006, 0.006, 0.095, 1.35 and 4.7 GeV respectively. Values of threshold parameter $s_0$ used in the present investigations for $D^*$, $B^*$, $D_{s^*}$ and $B_{s^*}$ mesons are 6.5, 35, 7.5 and 38 GeV$^2$ respectively. To represent the exact mass and decay shift we chose a suitable Borel window within which there is almost no variation in the mass and decay constant the detailed information regarding Borel window chosen and parameters used can be found from the paper [ ]