QCD phase diagram and its dualities: baryon density, chiral and isospin imbalance

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IZMIRAN, IHEP

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Russian Science Foundation

БАЗИС

Фонд развития теоретической физики и математики

K.G. Klimenko, IHEP
T.G. Khunjua, University of Georgia, MSU

in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

details can be found in

JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

Acknowlegements

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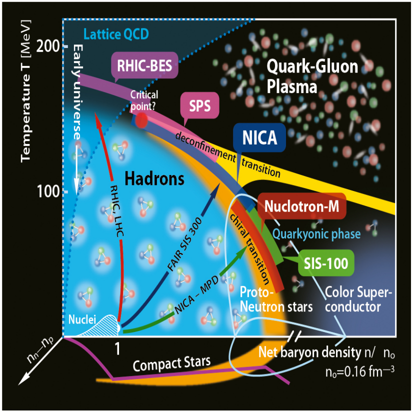
QCD at T and μ
(QCD at extreme conditions)

- neutron stars
- heavy ion collisions
- Early Universe

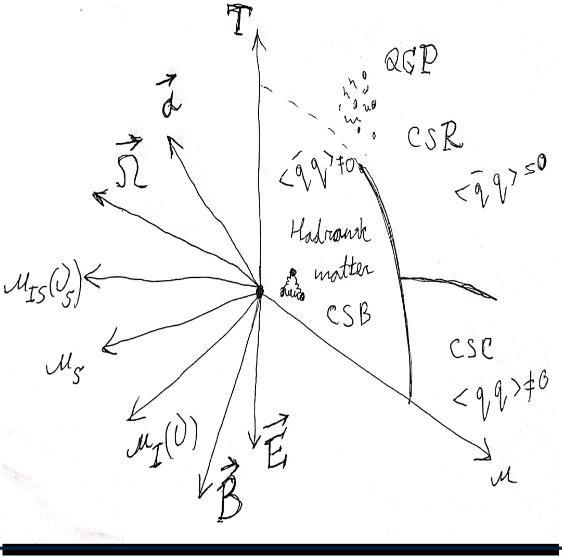
Methods of dealing with QCD

- First principle calcultion – lattice QCD
- Effective models
- DSE, FRG
- .....



More than just QCD at (μ, T)

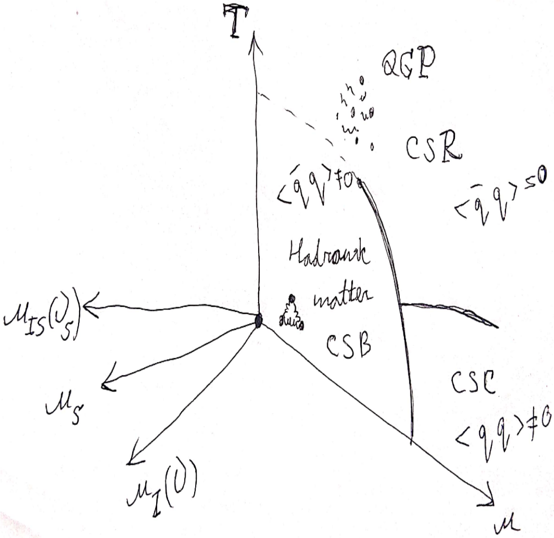
- **more chemical potentials** μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More external conditions to QCD

More than just QCD at (μ, T)

- **more chemical potentials** μ_i
- magnetic fields
- rotation of the system
- acceleration
- finite size effects (finite volume and boundary conditions)



Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

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$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad\qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

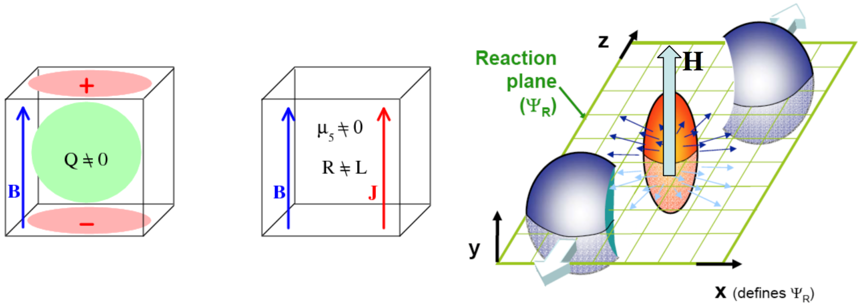
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

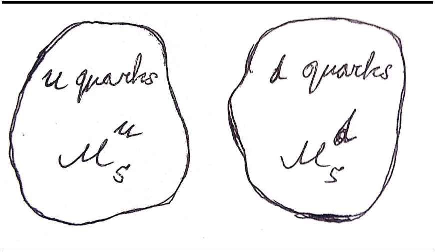
$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Chiral magnetic effect



$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

Chiral isospin imbalance n_5 and μ_5 generation

- Chiral isospin imbalance n_{I5} and hence μ_{I5} can be **generated by** $\vec{E}\,||\,\vec{B}$

  ($\textit{for } n_5 \textit{ and } \mu_5$ *M. Ruggieri, M. Chernodub, H. Warringa et al*)

- in **dense quark matter**
  - Chiral separation effect
    (Thanks for the idea to Igor Shovkovy)
  - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

$\mu, T < 600\,\mathrm{MeV}$

Parameters G, Λ, m_0

chiral limit $m_0 = 0$

in many cases chiral limit is a very good approximation

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi_a^2\Big].$$

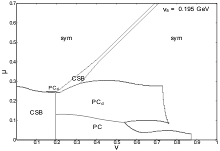
$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

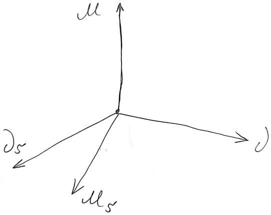
Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates

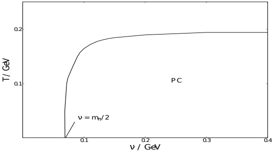
$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0.$$

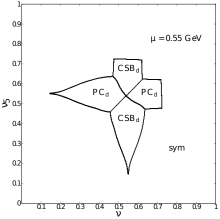
where M and π are already constant quantities.

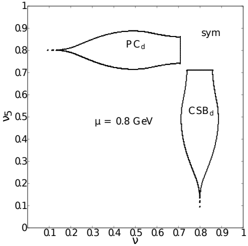
Phase diagram, lots of plots











Chiral imbalance leads to the generation of PC in dense quark matter (PC$_d$)

Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q} q \rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \pi, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

Duality in the phase portrait

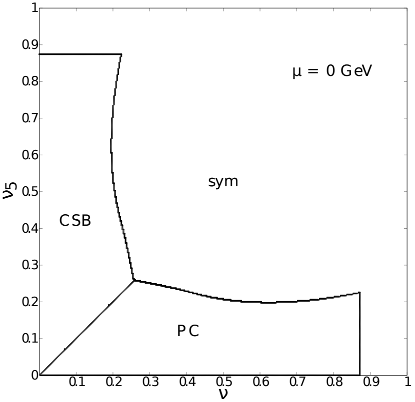


Figure: NJL model results

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$$\mathcal{D}: \ M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

Duality was found in

- In the framework of NJL model
- In the leading order of **large** N_c approximation or in **mean field**
- In the chiral limit

$$\mathcal{L}_{\rm QCD} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$

$$\mathcal{L}_{\rm NJL} = \sum_{f=u,d} \bar{q}_f \left[\mathrm{i}\gamma^\nu \partial_\nu - m_f \right] q_f + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 \vec{\tau} q)^2 \right]$$

m_f is current quark masses, $m_f:\ \frac{m_u+m_d}{2} \approx 3.5\mathrm{MeV}$

typical values of $\mu, \nu, ..., T, .. \sim 10 - 100s$ MeV, for example, 200-400 MeV

One can work in the chiral limit $m_f \to 0$

- $m_f = 0$ $\quad\to\quad$ $m_\pi = 0$
- physical m_f a few MeV $\quad\to\quad$ physical $m_\pi \sim 140$ MeV

In the chiral limit $m_f = 0$ the Duality $\mathcal{D}$ is exact

Duality between CSB and PC is **approximate** in **physical point**

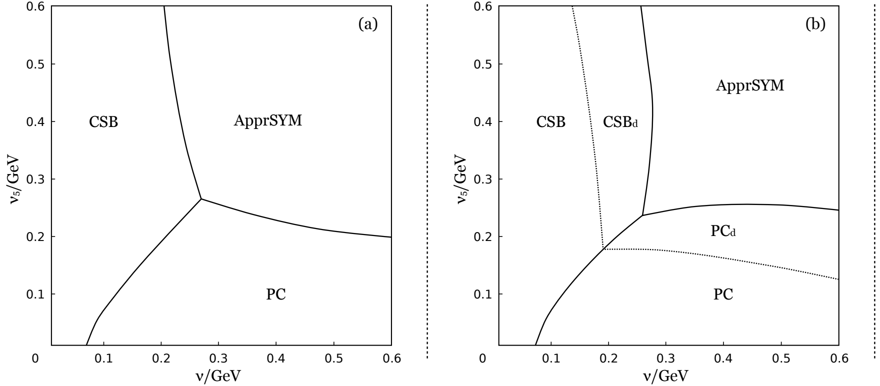


Figure: (ν, ν_5) phase diagram

Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

$\mu_B \neq 0$ impossible on lattice but if

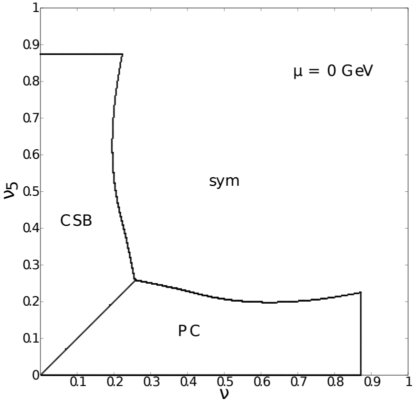
$$\mu_B = 0$$

- **QCD at** μ_5 —- (μ_5, T)

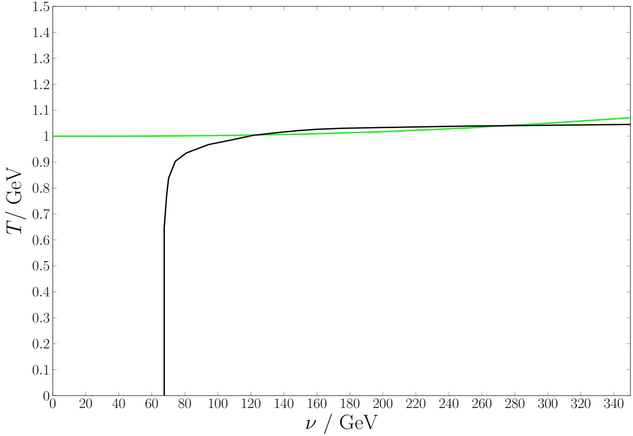
  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()



Dualities on the lattice



T_c^M as a function of μ_5 (green line) and $T_c^\pi(\nu)$ (black)

Uses of Dualities

How (if at all) it can be used

discussed in Particles 2020, 3(1), 62-79

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

- **Large $\mathbf{N}_c$ orbifold equivalences connect gauge theories with different gauge groups and matter content in the large $\mathbf{N}_c$ limit.**

M. Hanada and N. Yamamoto, JHEP 1202 (2012) 138, PoS LATTICE **2011** (2011)

Duality

$$\text{QCD at } \mu_1 \longleftrightarrow \text{QCD at } \mu_2$$

- QCD with μ_2 —- sign problem free,
- QCD with μ_1 —- sign problem (no lattice)

Investigations of (QCD with $\mu_2)_{\text{on lattice}} \Longrightarrow$ (QCD with μ_1)

Inhomogeneous phases (case)

Homogeneous case

In vacuum the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x.

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_\pm(x)\rangle = \pi, \quad \langle\pi_3(x)\rangle = 0.$$

In a dense medium $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ might depend on spatial coordinates

CDW ansatz for CSB, the single-plane-wave LOFF ansatz for PC

$$\langle\sigma(x)\rangle = M\cos(2kx^1), \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \pi\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \pi\sin(2k'x^1)$$

Duality in inhomogeneous case is shown

$$\mathcal{D}_I : \qquad M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5, \quad k \longleftrightarrow k'$$

schematic (ν_5, μ)-phase diagram

- exchange axis ν to the axis ν_5,
- rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

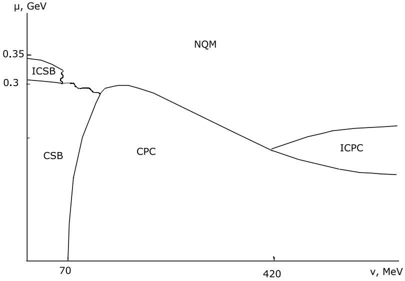


Figure: (ν, μ)-phase diagram.

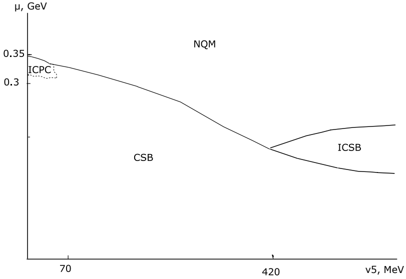


Figure: (ν_5, μ)-phase diagram

M. Buballa, S. Carignano, J. Wambach, D. Nowakovski, Lianyi He et al.

Two colour QCD case

QC$_2$D

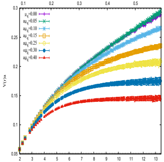
Similarity of SU(2) and SU(3)

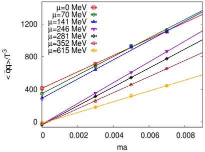
There are similar transitions:

- confinement/deconfinement
- chiral symmetry breaking/restoration

At large μ

- deconfinement
- chiral symmetry is restored





At large T

- deconfinement
- chiral symmetry is restored

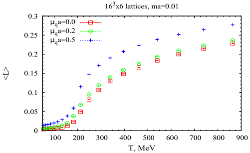


Figure 1 Polyakov loop as a function of T for three values of the baryon chemical potential.

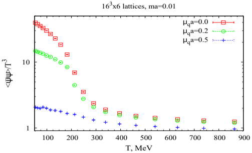


Figure 2: Chiral condensate as a function of T for three values of the baryon chemical potential. The ordinate axis is given on a logarithmic scale.

A lot of quantities coincide up to few dozens percent

SU(2)	**SU(3)**

Critical temperature

Phys. Lett. B712 (2012) 279-283, JHEP 02 (2005) 033

$T_c/\sqrt{\sigma}$=0.7092(36)	$T_c/\sqrt{\sigma}$=0.6462(30)

Topological susceptibility

Nucl.Phys.B 715 (2005) 461-482

$\chi^{\frac{1}{4}}/\sqrt{\sigma}$=0.3928(40)	$\chi^{\frac{1}{4}}/\sqrt{\sigma}$=0.4001(35)

Shear viscosity

JHEP 1509 (2015) 082, Phys.Rev. D 76 (2007) 101701

η/s=0.134(57)	η/s= 0.102(56)

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

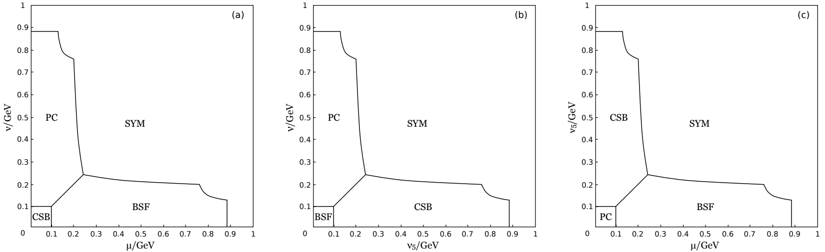
If you use Habbard-Stratanovich technique and auxiliary fileds

$$\sigma(x) = -2H(\bar{q}q), \;\; \vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q)$$
$$\Delta(x) = -2H\Big[\overline{q^c}i\gamma^5\sigma_2\tau_2 q\Big] = -2H\Big[q^T C i\gamma^5\sigma_2\tau_2 q\Big]$$
$$\Delta^*(x) = -2H\Big[\bar{q}i\gamma^5\sigma_2\tau_2 q^c\Big] = -2H\Big[\bar{q}i\gamma^5\sigma_2\tau_2 C\bar{q}^T\Big]$$

Condensates are

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi_1, \quad \langle\Delta(x)\rangle = \Delta, \quad \langle\Delta^*(x)\rangle = \Delta^*.$$

Dualities in QC_2D



$$(a) \qquad \mathcal{D}_1 : \qquad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|$$

$$(b) \qquad \mathcal{D}_3 : \qquad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1$$

$$(c) \qquad \mathcal{D}_2 : \qquad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|$$

- Based on the **duality** one can show that there is **no mixed phase**, i.e. two non-zero condensates simultaneously.

  This greatly simplifies the numeric calculations.

- Phase diagram is **highly symmetric** due to **dualities**

  The **whole phase diagram**, including diquark condensation, **in two color case** can be obtained from the results of **three color case** without any diquark condensation.

Duality $\mathcal{D}_3$ between CSB and PC and $\mathcal{D}_1$ and $\mathcal{D}_2$ were found in

- In the framework of NJL model

- In the large N_c approximation (or mean field)

$$\mathcal{D}_3: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad\qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \quad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \quad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \quad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

There is also $\mathcal{D}_1$ and $\mathcal{D}_2$

Dualities were found in

- In the framework of NJL model non-pertubartively (or beyond mean field)

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality in $SU(3)$ QCD

QCD Lagrangian is

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu} D_{\mu}\psi + \bar{\psi}\Big[\mu\gamma^{0} + \frac{\mu_I}{2}\tau_3\gamma^{0} + \frac{\mu_{I5}}{2}\tau_3\gamma^{0}\gamma^{5} + \mu_5\gamma^{0}\gamma^{5}\Big]\psi$$

$$\mathcal{D}: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^{5}\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad\qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^{C}\sigma_2\tau_2\gamma^{5}\psi \leftrightarrow i\bar{\psi}^{C}\sigma_2\tau_2\gamma^{5}\psi, \qquad \bar{\psi}^{C}\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^{C}\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \qquad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^{5}\psi, \qquad i\bar{\psi}\gamma^{5}\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^{5}\tau_3\psi$$

Duality was found in

- In the framework of NJL model non-pertubartively

  (beyond mean field or at all orders of N_c approximation)

- In QCD non-pertubartively (at the level of Lagrangian)

$$\mathcal{D} \in \quad SU_R(2) \quad \in SU_L(2) \times SU_R(2)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$M \neq 0$ breaks the chiral symmetry

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Other Dualities

They are not that strong but still...

They could still be usefull

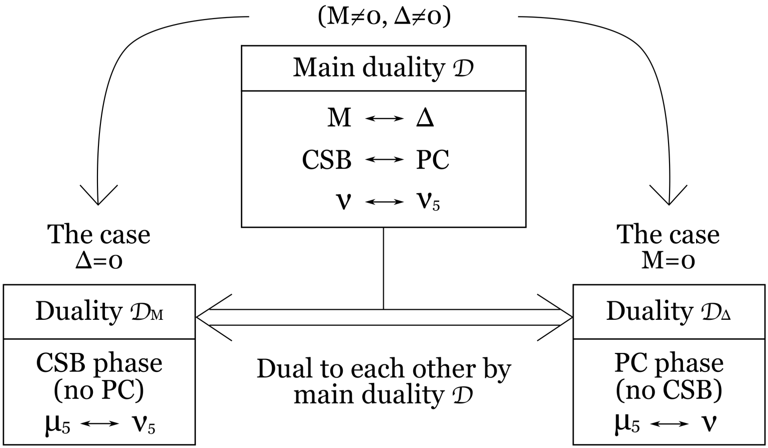


Figure: Dualities

Conclusions and Thanks for the Attention

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied
  PC *in dense matter with chiral imbalance*
  also in dense electic neutral matter in β-equilibrium
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality have been shown non-perturbarively in QCD