K+K− correlations in Pb–Pb collisions at √s_{NN} = 2.76 TeV by ALICE at the LHC

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Outline:

- Motivation
- Formalism
- Data and PID, purity, momentum resolution
- Fit data with different models
- Conclusion
Motivation

- First measurement of $K^+K^-$ at the LHC
- Constrain couplings and masses of $f_0(980)$ and $a_0(980)$: $K^0 S K^\pm$: strong FSI, $a_0(980)$ only [ALICE PLB774 (2017) 64]
- $K^+K^-$: s-wave strong FSI (both $a_0$ and $f_0$), p-wave FSI (ϕ meson) and Coulomb FSI [all sensitive to source size]
- $a_0$, $f_0$ couplings and masses: four model descriptions → $a_0$ couplings and mass were fixed from ALICE papers [PLB774 (2017) 64, PLB 790 (2019) 22] → are these $a_0$ values suitable for $K^+K^-$
- Cross-check of identical kaon ($K^\pm K^\mp$ and $K^0 S K^0_S$) results

**K^+K^- existing results**


- Coulomb stays above data
- Dip at $Q_{inv} \sim 50-150$ due to strong interaction
- Strong and Coulomb FSI does good job

STAR AuAu 200 GeV [WPCF 2017, J. Lidrych]

- No fit, comparison with Lednický model
- Data is described qualitatively for large source
- Phi production mechanism is not taken into account
The \( K^+K^- \) correlation function (CF) at given \( k^* \) and 3-momentum \( P \):
\[
C_{s_{FSI}}(k^*, P) = \int d^3 r^* S_\alpha(r^*, P) \sum_{\alpha'} \left| \psi_{-k^*}^{\alpha'}(r^*) \right|^2 \quad (1)
\]
Spatial separation: \( S(r^*) \sim \exp(-r^2/4R^2) \)

The s-wave scattering amplitude \( f(k^*) \):
\[
f_0(k^*) = \frac{\gamma_{f_0\rightarrow K+K-}}{m_{f_0}^2 - s - i(\gamma_{f_0\rightarrow K+K-k^*} + \gamma_{f_0\rightarrow \pi\pi\pi})} \quad \text{and} \quad f_1(k^*) = \frac{\gamma_{f_1\rightarrow K+K-}}{m_{a_0}^2 - s - i(\gamma_{a_0\rightarrow K+K-k^*} + \gamma_{a_0\rightarrow \pi\eta\pi\eta})} \quad (2)
\]

The p-wave strong interaction through \( \phi \) meson resonance [R.Lednicky Part. Nucl. Letters 8(2011)965]:
\[
C_\phi(p_1, p_2) = N^{-1}(p_1, p_2) \int d^3 r W_P(r, k) \sum_{\alpha' m'} \left| \psi_{-k}^{\alpha' m'}(r) \right|^2 \quad (3)
\]

The total correlation function:
\[
C_{FSI}(p_1, p_2) = 1 + C_{s_{FSI}}(p_1, p_2) + N_1 C_{\phi-direct}(p_1, p_2) + N_2 C_{\phi}(p_1, p_2) \quad (4)
\]
\( C_{\phi-direct}(p_1, p_2) \) is a non-relativistic Breit-Wigner function.
ALICE at the LHC

Tracking and vertex
- Time Projection Chamber (TPC)
- Inner Tracking System (ITS)

Particle Identification
- TPC and Time of Flight

Centrality determination
- V0
Dataset and particle identification

- Data: 40 million events, the same as for $K^\pm K^\pm$ in Pb-Pb at 2.76 TeV [PRC 92 (2015) 054908]
- TPC: $2\sigma \ 0.2<p<0.4\ \text{GeV/c}$, $1\sigma \ 0.4<p<0.45\ \text{GeV/c}$
  TOF+TPC(3$\sigma$): $2\sigma \ 0.45<p<0.8\ \text{GeV/c}$, $1.5\sigma \ 0.8<p<1.0\ \text{GeV/c}$, $1.0\sigma \ 1.0<p<1.5\ \text{GeV/c}$
- Bins(24 CF): same as they were in $K^\pm K^\pm$ analysis
  $k_t[8]: \ 0.2-0.3,...,1.0-1.3\ \text{GeV/c}$
  Centrality[3]: 0-10, 10-30, 30-50 %
The obtained purity decreases with increasing centrality.

The single kaon purity is higher than 96%.

PairPurity\(\left(k_T\right) = \text{SinglePurity}(p_1) \cdot \text{SinglePurity}(p_2)\)

\[k_T = \frac{|p_{T,1} + p_{T,2}|}{2}\]

The value of the pair purity is higher than 99% for \(K^+K^-\) pairs.
Momentum resolution (MR) with MC:

- The finite track momentum resolution smears the relative momentum correlation function.

- Correlation function smeared by MR:

\[
C(Q_{\text{rec}}) = \frac{\sum Q_{\text{true}} C(Q_{\text{true}}, Q_{\text{rec}}) M(Q_{\text{true}}, Q_{\text{rec}})}{\sum Q_{\text{true}} M(Q_{\text{true}}, Q_{\text{rec}})}
\]

- \(M(Q_{\text{true}}, Q_{\text{rec}})\) – HIJING [PRD 44, 3501, 1991] + GEANT

- Cross-check MR using \(\phi\) peak in \(K^+K^-\) CF

- \(\phi\) peak is fitted with the convolution of a non-relativistic Breit-Wigner peak and a Gaussian [PRC 91(2015)024609]

- The mass resolutions obtained from the two methods are in agreement
Input for fit:

- $C(q) = \text{Norm} \cdot [1 + \lambda \cdot C_{\text{FSI}}(q,R) + \lambda \cdot C_{\phi}(q,M,\sigma)]$
  
  $C_{\text{FSI}}(q,R)$ - Lednicky model
  
  $C_{\phi}(q,M,\sigma)$ - Breit-Wigner $\Gamma_{\phi} = 4.25\text{MeV}$

- $a_0$ parameters fixed from Achasov\textsuperscript{2} [ALICE PLB774 (2017) 64, PLB 790 (2019) 22]

- $f_0$ mass and coupling parameters are free

<table>
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<tr>
<th>Model</th>
<th>$m_{f_0}^2$</th>
<th>$m_{a_0}^2$</th>
<th>$Y_{f_0 \to K+K-}$</th>
<th>$Y_{f_0 \to \pi\pi}$</th>
<th>$Y_{a_0 \to K+K-}$</th>
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</table>

The masses ($a_0$ and $f_0$) and coupling parameters

K+K-@PbPb2.76TeV

Konstantin Mikhaylov ICPPA-2020
Correlation functions fitted by FSI with new $f_0$ parameters

- Fit function gives a good description for all $k_T$ (left to right) and centrality (top to bottom) bins.

K+K-@PbPb2.76TeV

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Comparison radii and $\lambda$ of $K^+K^-$ and $K^\pm K^\pm$

- Radii are in agreement within errors
- $\lambda$ of $K^+K^-$ tend to be larger than for $\lambda$ of $K^\pm K^\pm$ (possible due to FSI slightly underestimate)
- New preliminary $f_0(980)$ couplings and mass parameters:

\[
\begin{align*}
\gamma_{f_0 \to K^+K^-} &= 0.31 \pm 0.062 \pm 0.092 \text{ GeV} \\
\gamma_{f_0 \to \pi\pi} &= 0.081 \pm 0.0162 \pm 0.024 \text{ GeV} \\
M_{f_0} &= 0.972 \pm 0.003 \pm 0.005 \text{ GeV/c}^2
\end{align*}
\]
What does the PDG say? And what about ALICE measurements?

$\pi\pi$ dominant

$\gamma\gamma$ seen

Present analysis (preliminary):

• Mass is $M_{f_0} = 972 \pm 3\text{(stat)} \pm 5\text{(sys)} \text{MeV/c}^2$

• $\Gamma_{f_0} = \gamma_{f_0 \rightarrow \pi\pi} \cdot \frac{k}{m_{f_0}} = 39.7 \pm 7.94\text{(stat)} \pm 11.8\text{(sys)} \text{MeV}$

• The measured width and mass of the $f_0(980)$ is close to the PDG result
Conclusions

- The ALICE experimental data on the non-identical $K^+K^-$ correlation were presented
- The pair purity of $K^+K^-$ was better than 99 %
- The smearing matrix was used to take into account momentum resolution
- For the first time the $K^+K^-$ correlation functions were fitted with free $f_0(980)$ mass and couplings and with restriction on radii to be close to the corresponding identical $K^\pm K^\pm$
- The measured preliminary width of the $f_0(980)$ is $39.7 \pm 7.94(\text{stat}) \pm 11.8(\text{sys})$ MeV and mass is $972 \pm 3(\text{stat}) \pm 5(\text{sys})$ MeV/$c^2$ which do not contradict the PDG data

*Thank you very much for your attention !!!*
Backup slides
Femtoscopy

Correlation femtoscopy:
Measurement of space-time characteristics $R$, $c\tau$ of particle production using particle correlations due to the effects of quantum statistics (QS) and final state interactions (FSI)

Two-particle correlation function:

theory:
$$C(q) = \frac{N_2(p_1, p_2)}{N_1(p_1) \cdot N_2(p_1)}, C(\infty) = 1$$

experiment:
$$C(q) = \frac{S(q)}{B(q)}, q = p_1 - p_2$$

Parametrizations used:

1D CF:
$$C(q_{inv}) = 1 + \lambda e^{-R^2 q_{inv}^2}$$
$R$ – Gaussian radius in PRF,
$\lambda$ – correlation strength parameter

3D CF:
$$C(q_{out}, q_{side}, q_{long}) = 1 + \lambda e^{-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2}$$
$R$ and $q$ are in Longitudinally Co-Moving Frame (LCMS)
long $\parallel$ beam; out $\parallel$ transverse pair velocity $v_T$; side normal to out,long
K⁺K⁻ existing results: STAR

- K⁺K⁻ in AuAu at $\sqrt{s_{NN}}=200$ GeV
  [WPCF 2017, Jindřich Lidrych]

- CF=$(C_{F_{theor}}-1)\cdot \lambda + 1$ (no fit)
  $C_{F_{theor}} \rightarrow$ Lednický model

- Data is described qualitatively for large source

Observations:
- The model underpredicts the strength of the correlation functions in the region of resonance with decreasing $R_{inv}$
- Model fails for smaller system ($\sim$3fm and smaller)

- Does not take into account $\phi$ production mechanism
New and previous mass and couplings $f_0(980)$:

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Uncertainties for models are about 10-15% for couplings and 3-5% for masses

- New values of $f_0$ mass is close to Martin, Antonelli and Achasov results. But new couplings are smaller.
- The ratio $\gamma_{f_0 \rightarrow K^+K^-}/\gamma_{f_0 \rightarrow \pi\pi}$ is about 4-5 for all results
- ALICE $\gamma_{f_0 \rightarrow K^+K^-}/\gamma_{f_0 \rightarrow \pi\pi} \sim 4$. 

K+K-@PbPb2.76TeV

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The K+K- correlation function at given k* and 3-momentum P as

\[ C_{FSI}(k^*, P) = \int d^3r^* S_\alpha(r^*, P) \left| \psi_{-k^*}^{\alpha}(r^*) \right|^2 \]

where \( \alpha \) means K+K- and summing over intermediate channels \( \alpha' = K^+K^-, K^0\bar{K}^0 \)

Assume a spherically symmetric Gaussian distribution of the particle emitter spatial separation \( r^* \) in the PRF with size \( R \)

A stationary solution of the scattering problem at large distances has the asymptotic form of a superposition of a plane wave and an outgoing spherical wave:

\[ S(r^*) \sim \exp(-r^2/4R^2) \]

The s-wave K+ K− scattering amplitude \( f(k^*) \) is dominated by the near threshold s-wave isoscalar \( f_0(980) \) and isovector \( a_0(980) \) resonances characterized by their masses and respective couplings.

The general form of the strong FSI part (the s-wave scattering only) of the K+K- correlation function is:

\[ C(q) = 1 + \lambda \left[ \frac{1}{2} \left| \frac{f(q/2)}{R} \right|^2 + \frac{2\Re f(q/2)}{\sqrt{\pi R}} F_1(qR) - \frac{3 f(q/2)}{R} F_2(qR) \right] \]

\[ F_1(z) \equiv \int_0^z dx \frac{e^{x^2-z^2}}{z}, \quad F_2(z) \equiv \frac{1-e^{-z^2}}{z} \]
The wave function of K+K− with the Coulomb interaction may be written (see Appendix in Sov. J. Nucl. Phys. 35, 770 (1982)):

\[ \Psi_{-k^*}(r^*) = \sqrt{A_c(\eta)}e^{i\delta_c} \left[ e^{-ik^*r^*/2}F(-i\eta, 1, i\xi) + f_c(k^*)\frac{G(\rho, \eta)}{|r^*|} \right] \]

where \( A_c \) is Gamov factor

\[ A_c = 2\pi \eta [\exp(2\pi \eta - 1)]^{-1}, \eta = 1/(k^*a_c), a_c = 1/(\mu z_1 z_2 e^2) \]

Bohr radius

The corresponding effective amplitude (renormalized by Coulomb interaction):

\[ f_c(k^*) = \left[ \frac{1}{f} - ik^* A_c(\eta) - \frac{2}{a_c} h(\eta) \right]^{-1} \]

The p-wave strong interaction through \( \Phi \) meson resonance [R.Lednicky Part. Nucl. Letters 8(2011)965]:

\[ C_\Phi(p_1, p_2) = N^{-1}(p_1, p_2) \int d^3r W_P(r, k) \sum_{\alpha'\mu'} |\psi_{-k^*;\alpha}(r)|^2 \]

The emission function with possible position-momentum \((r,k)\) correlation:

\[ W_P(r, k) = \exp \left( -\frac{r^2}{4R^2} + brk \right) \quad N(p_1, p_2) = 8\pi^{3/2}R^3 \exp(bk^2R^2) \quad N(p_1, p_2) = \int d^3r W_P(r, k) \]

The height of CF at \( \Phi \) meson peak position \( \Delta CF \sim 1/R^3 \)

The total correlation function is defined as a sum:

\[ C_{FSI}(p_1, p_2) = 1 + C_{sFSI}(p_1, p_2) + N_1 C_{\Phi-direct}(p_1, p_2) + N_2 C_\Phi(p_1, p_2), \]

\( C_{\Phi-direct}(p_1, p_2) \) is a non-relativistic Breit-Wigner function.

“Direct” \( \Phi \) mesons are all phi mesons before last interaction (all stages of reaction including hadron cascade).
A coupled channel analysis of the centrally produced $K^+K^-$ and $\pi^+\pi^-$ final states

In this present paper we use the Flatté formula [6] to describe the $f_0(980)$, this is referred to as Method I. For the $\pi^+\pi^-$ channel the Breit-Wigner has the form:

$$BW(M_{\pi\pi}) = \frac{m_0 \sqrt{\Gamma_i \sqrt{\Gamma_\pi}}}{m_0^2 - m^2 - im_0(\Gamma_\pi + \Gamma_K)}$$

and in the $K^+K^-$ channel the Breit-Wigner has the form:

$$BW(M_{KK}) = \frac{m_0 \sqrt{\Gamma_i \sqrt{\Gamma_K}}}{m_0^2 - m^2 - im_0(\Gamma_\pi + \Gamma_K)}$$

where $\Gamma_i$ is absorbed into the intensity of the resonance. $\Gamma_\pi$ and $\Gamma_K$ describe the partial widths of the resonance to decay to $\pi\pi$ and $KK$ and are given by

$$\Gamma_\pi = g_\pi (m^2/4 - m_\pi^2)^{1/2}$$

$$\Gamma_K = g_K/2 \left[ (m^2/4 - m_{K^*}^2)^{1/2} + (m^2/4 - m_{K^0}^2)^{1/2} \right]$$

The $f_0(980)$ couplings: $g_\pi = 0.19 \pm 0.03 \pm 0.04$ $g_K = 0.40 \pm 0.04 \pm 0.04$

These values best relate to Martin’s model[NPB121(1977)514]: $g_\pi = 0.199 \pm 0.014$ $g_K = 0.792 \pm 0.099$
Position-momentum correlations

\[ W_p(r,k) \sim \exp\left[ -r^2 / 4r_0^2 + bk r \right]; \]  
\( r \)-\( k \) correlation may result from the collective flows and resonance decays. CF suppressed by a factor \( W_p(0,k) \sim \exp[-b^2 r_0^2 k^2] \)

One can estimate \( b \) from experimental phi-meson peak in K+K- correlation function. The peak height decreases with the increasing centrality in qualitative agreement with the inverse volume(\( r_0^{-3} \)) dependence.

Does the \( r \)-\( k \) correlation help to understand the ALICE data?

\( \Phi \) peak value (\( \Delta R = CF(m_\phi)-1 \)) according to \( W_p(r,k) \) space distribution with \( r_0 = 5 \)fm. Simple Gaussian emission function(\( b=0 \)) leads to overestimates the peak value. Factor \( b \approx 0.12 \) is sufficient to describe NA49 data on K+K- in PbPb at 158GeV.