Shear viscosity of nucleons and pions in A+A collisions at NICA energies

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in collaboration with
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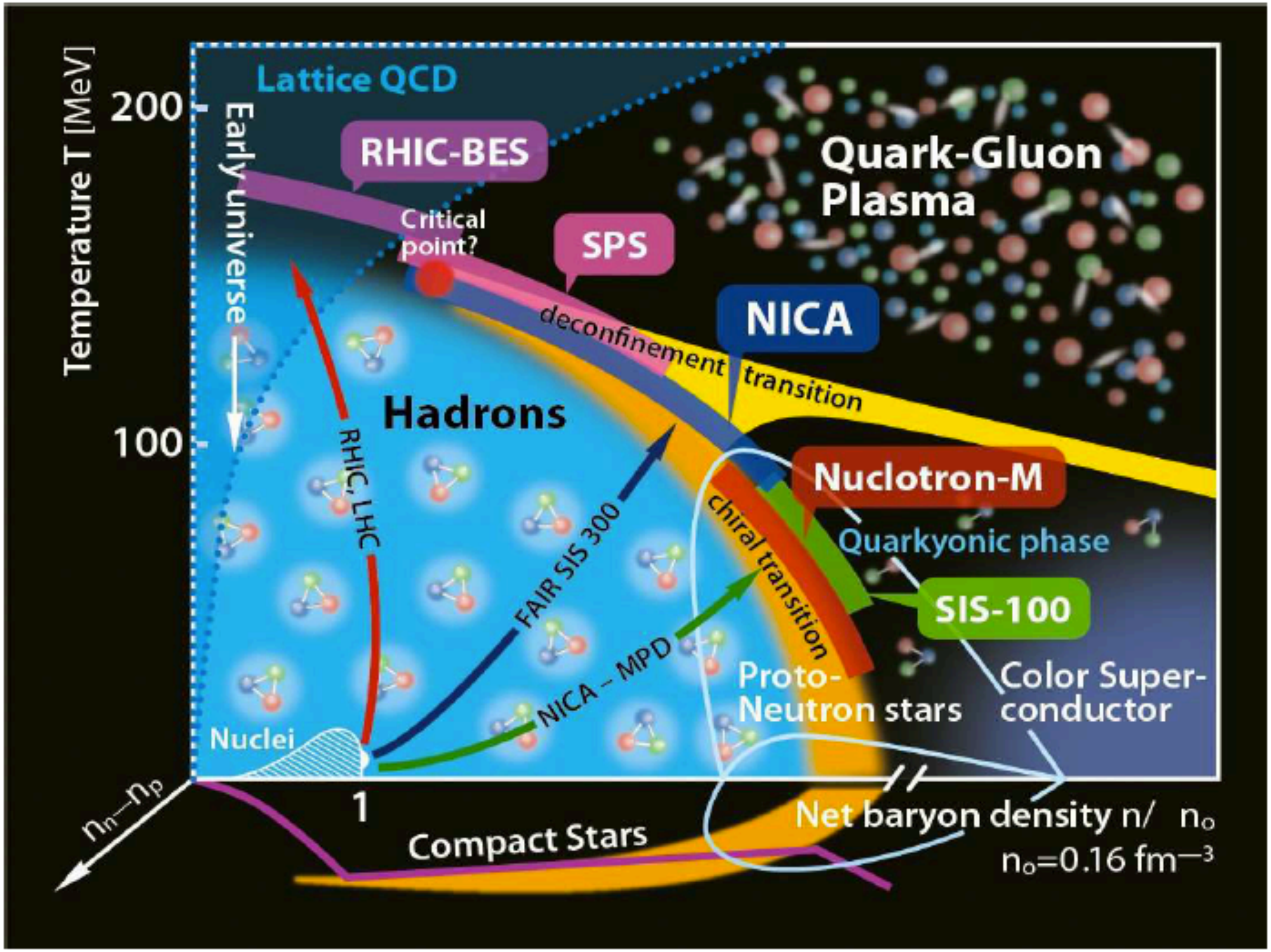
MEPhI, Moscow, 5-9.10.2020

Based on publications:

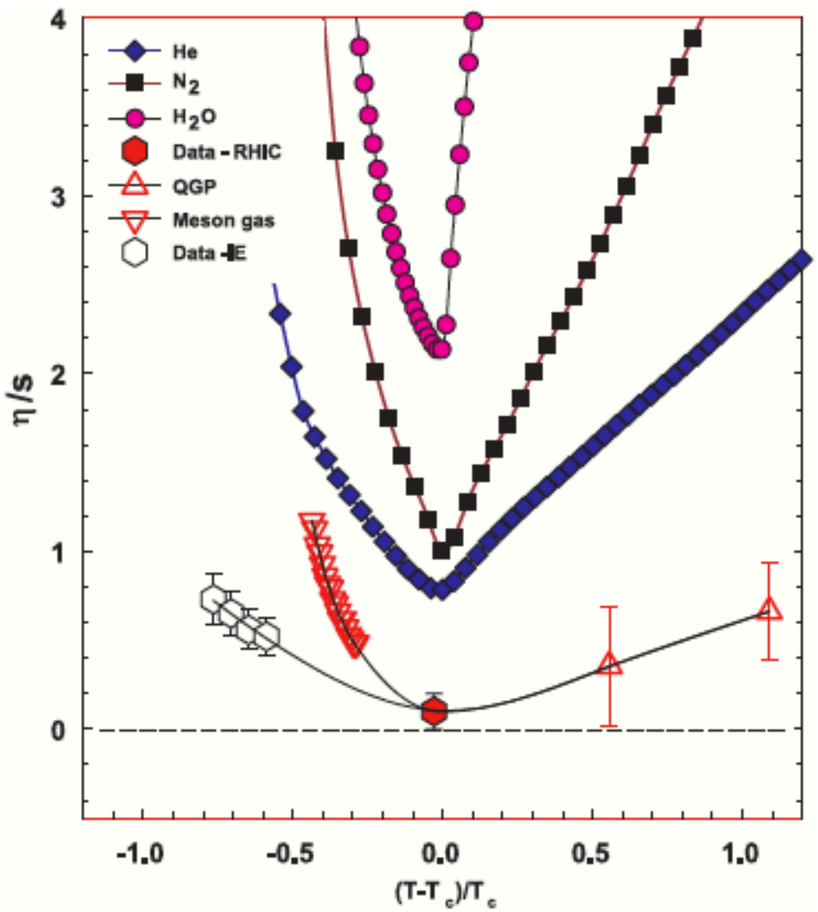
1. M.Teslyk, L.Bravina, O.Panova, O.Vitiuk, E.Z., PRC 101 (2020) 014904
2. E.Z., M.Teslyk, O.Vitiuk, L.Bravina, Phys. Scripta 95 (2020) 7, 074009
3. E.Z., L.Bravina, M.Teslyk, O.Vitiuk, arXiv: 2002.05181 [nucl-th] (proc. of Quark Matter'19, to be published in NPA)

Motivation



Temperature T [MeV]
200
100
Lattice QCD
Early universe
RHIC-BES
Critical point?
SPS
NICA
Quark-Gluon Plasma
deconfinement transition
Hadrons
RHIC, LHC
FAIR SIS 300
NICA – MPD
Nuclotron-M
chiral transition
Quarkyonic phase
SIS-100
Proto-Neutron stars
Color Super-conductor
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o=0.16 fm^{-3}

Motivation



courtesy of R. Lacey and A. Taranenko

- P.Kovtun, D.T.Son, O.Starinets. PRL 94, 111601 (2005)
- A.Muronga. PRC 69, 044901 (2004)
- L.Csernai, J.Kapusta, L.McLerran. PRL 97, 152303 (2006)
- P.Romatschke, U.Romatschke. PRL 99, 172301 (2007)
- S.Plumari et al. PRC 86, 054902 (2012)
- ALICE collaboration, CERNCOURIER (14.10.2016)
- J.Rose et al. PRC 97, 055204 (2018)
- ...

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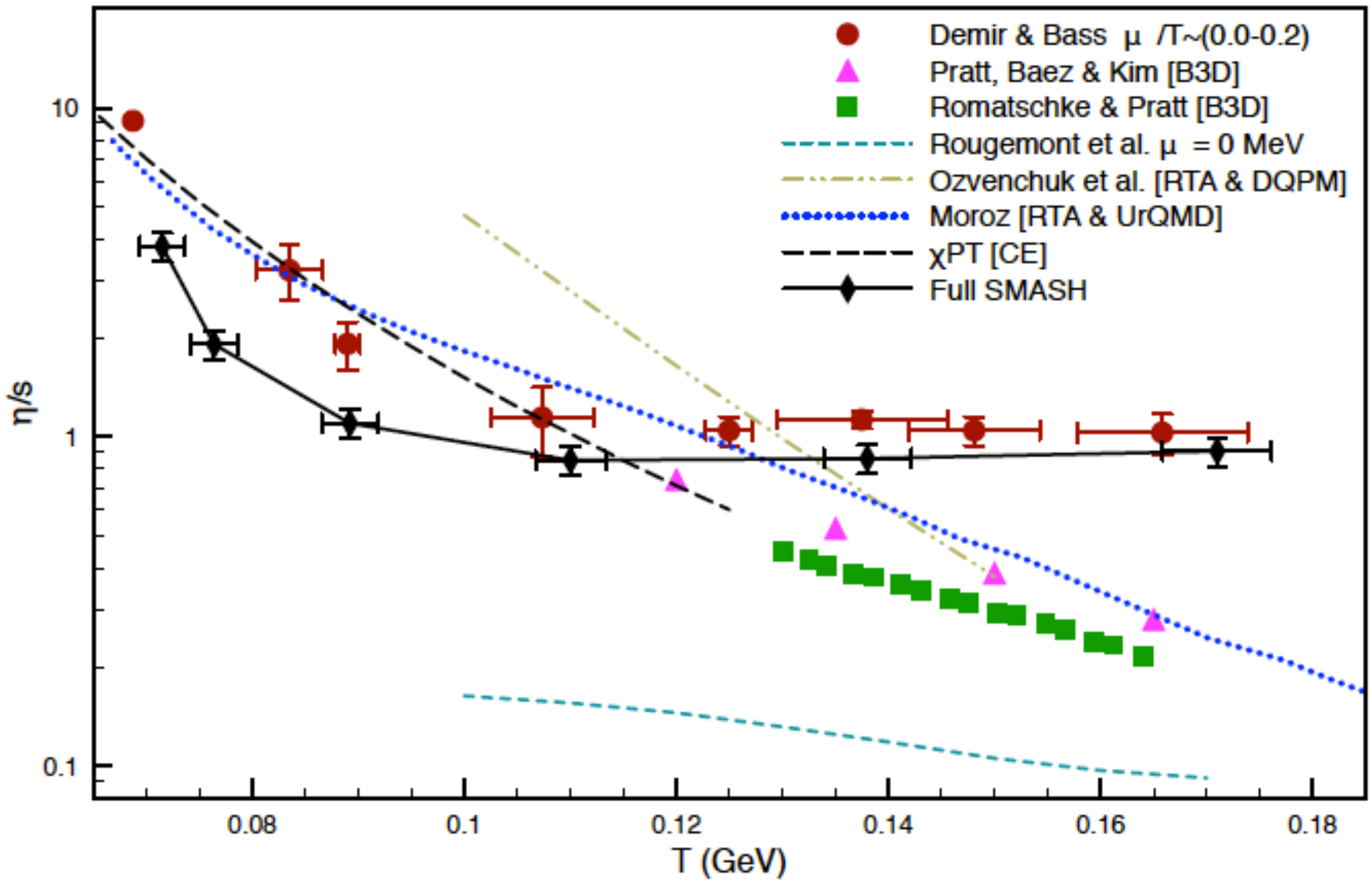

Comparison of several calculations for the hadron gas η/s at $\mu_B = 0$ [*from J.Rose et al. PRC*97 (2018) 055204]

Theory

Green-Kubo: shear viscosity η may be defined as:

$$\eta\left(t_0\right)=\frac{1}{\hbar}\frac{V}{T}\int_{t_0}^{\infty}\mathrm{d}t\langle\pi\left(t\right)\pi\left(t_0\right)\rangle_t=\frac{\tau}{\hbar}\frac{V}{T}\langle\pi\left(t_0\right)\pi\left(t_0\right)\rangle,$$

where

$$\begin{aligned}\langle\pi\left(t\right)\pi\left(t_0\right)\rangle_t &= \frac{1}{3}\sum_{\substack{i,j=1\\ i\neq j}}^{3}\lim_{t_{\max}\to\infty}\frac{1}{t_{\max}-t_0}\int_{t_0}^{t_{\max}}\mathrm{d}t'\pi^{ij}\left(t+t'\right)\pi^{ij}\left(t'\right)\\ &= \langle\pi\left(t_0\right)\pi\left(t_0\right)\rangle\exp\left(-\frac{t-t_0}{\tau}\right)\end{aligned}$$

with

$$\pi^{ij}\left(t\right)=\frac{1}{V}\sum_{\text{particles}}\frac{p^i\left(t\right)p^j\left(t\right)}{E\left(t\right)}$$

t_0: initial cut-off time to start with

Model setup: cell calculations

- UrQMD calculations, central Au+Au collisions at energies $E \in [5, 10, 20, 30, 40]$ AGeV of the projectile, 51200 events per each
- central cell $5 \times 5 \times 5$ fm^3 $\Rightarrow$ $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ at times $t_{\rm cell} = 1 \div 20$ fm/c
- statistical model (SM): $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ $\Rightarrow$ $\{T, s, \mu_{\rm B}, \mu_{\rm S}\}$

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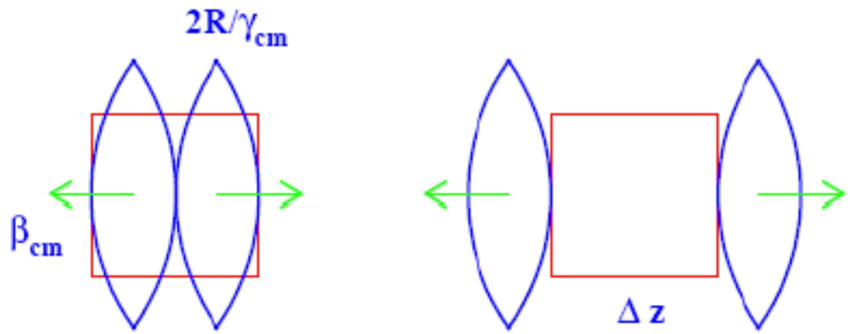
Equilibration in the Central Cell



$t^{cross} = 2R/(\gamma_{cm}\,\beta_{cm})$

$t^{eq} \geq t^{cross} + \Delta z/(2\beta_{cm})$

Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equilibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values — output values

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

$$N_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

Energy

$$E_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2\sqrt{p^2+m_i^2}\,f(p,m_i)dp$$

Pressure

$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2\frac{p^2}{3(p^2+m_i^2)^{1/2}}\,f(p,m_i)dp$$

Entropy density

$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,\left[\ln f(p,m_i)-1\right]\,p^2dp$$

Model setup: box calculations

- UrQMD box calculations at $\{\varepsilon, \rho_{\rm B}, \rho_{\rm S}\}$ for every energy and cell time $t_{\rm cell}$ from cell calculations, 100 points in total, 12800 events per each
  $\rho_{\rm B}$ is included as $N_p : N_n = 1:1$
  $\rho_{\rm S}$ is included via kaons K^-
  box size: $10 \times 10 \times 10$ fm^3
  box boundaries: transparent
- $\pi^{ij}(t)$ data extraction: $t = 1 \div 1000$ fm/c in box time, all types of hadrons are taken into account

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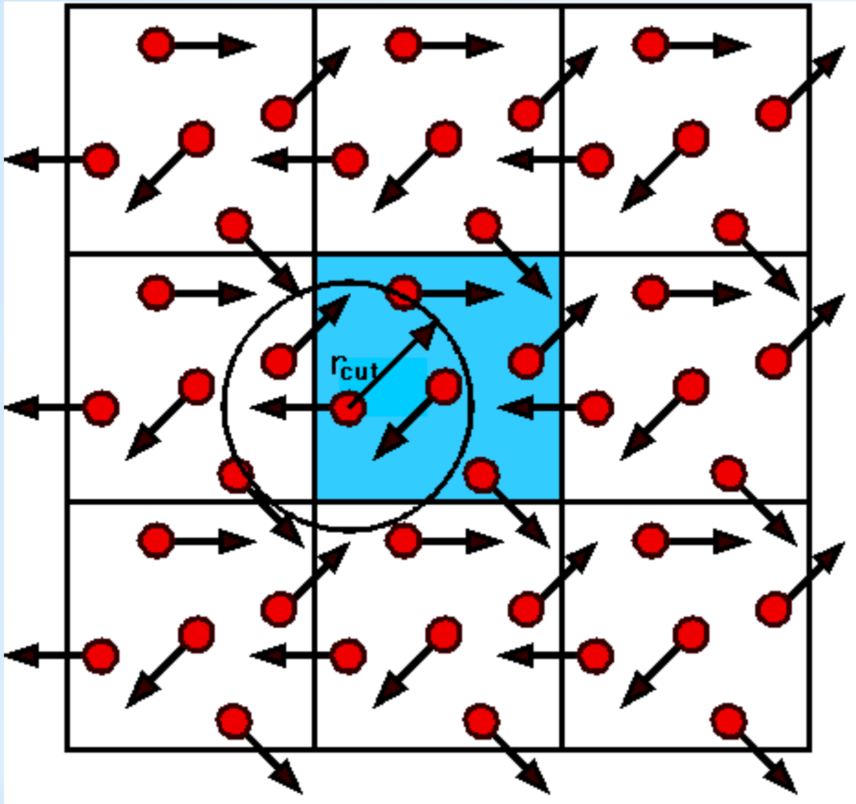
Box with periodic boundary conditions



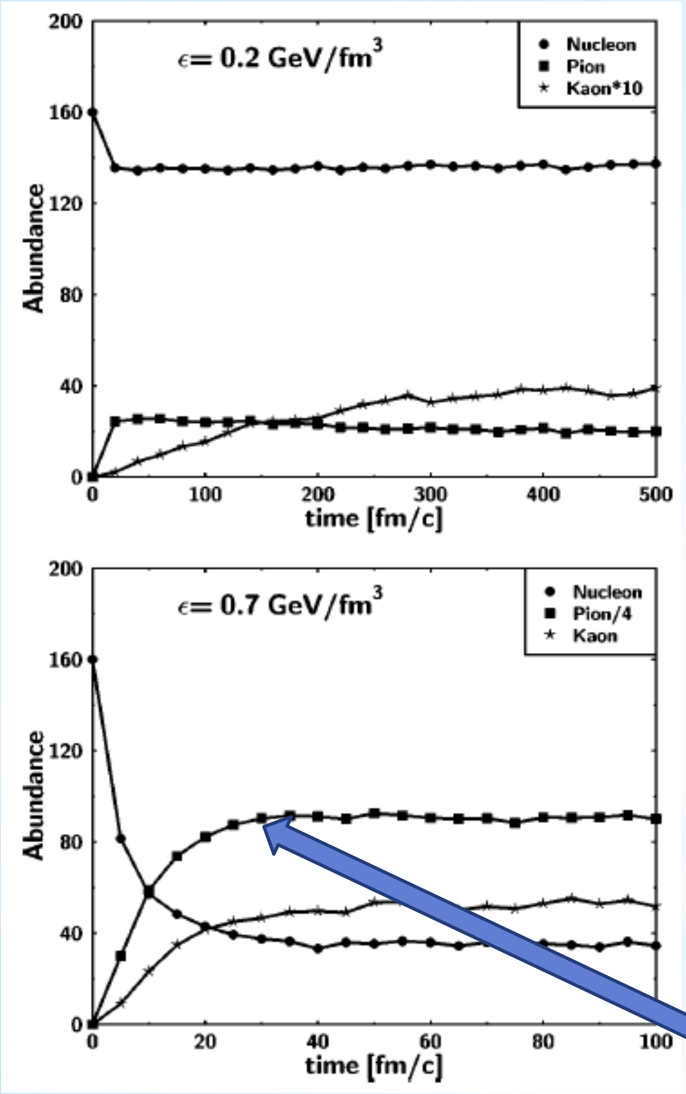
M.Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species
(N, Δ, hyperons and their
resonances with
$m \leq 2.25$ GeV/c^2)
32 different meson species
(including resonances with
$m \leq 2$ GeV/c^2) and their
respective antistates.
For higher mass excitations
a string mechanism is invoked.

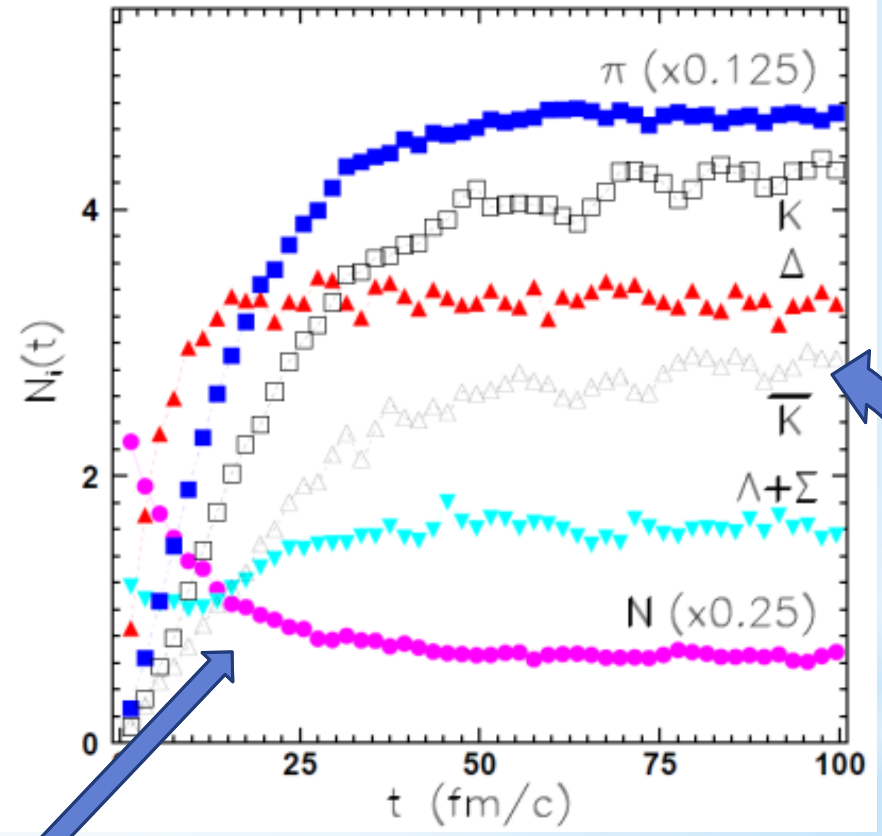
Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

Box: particle abundances



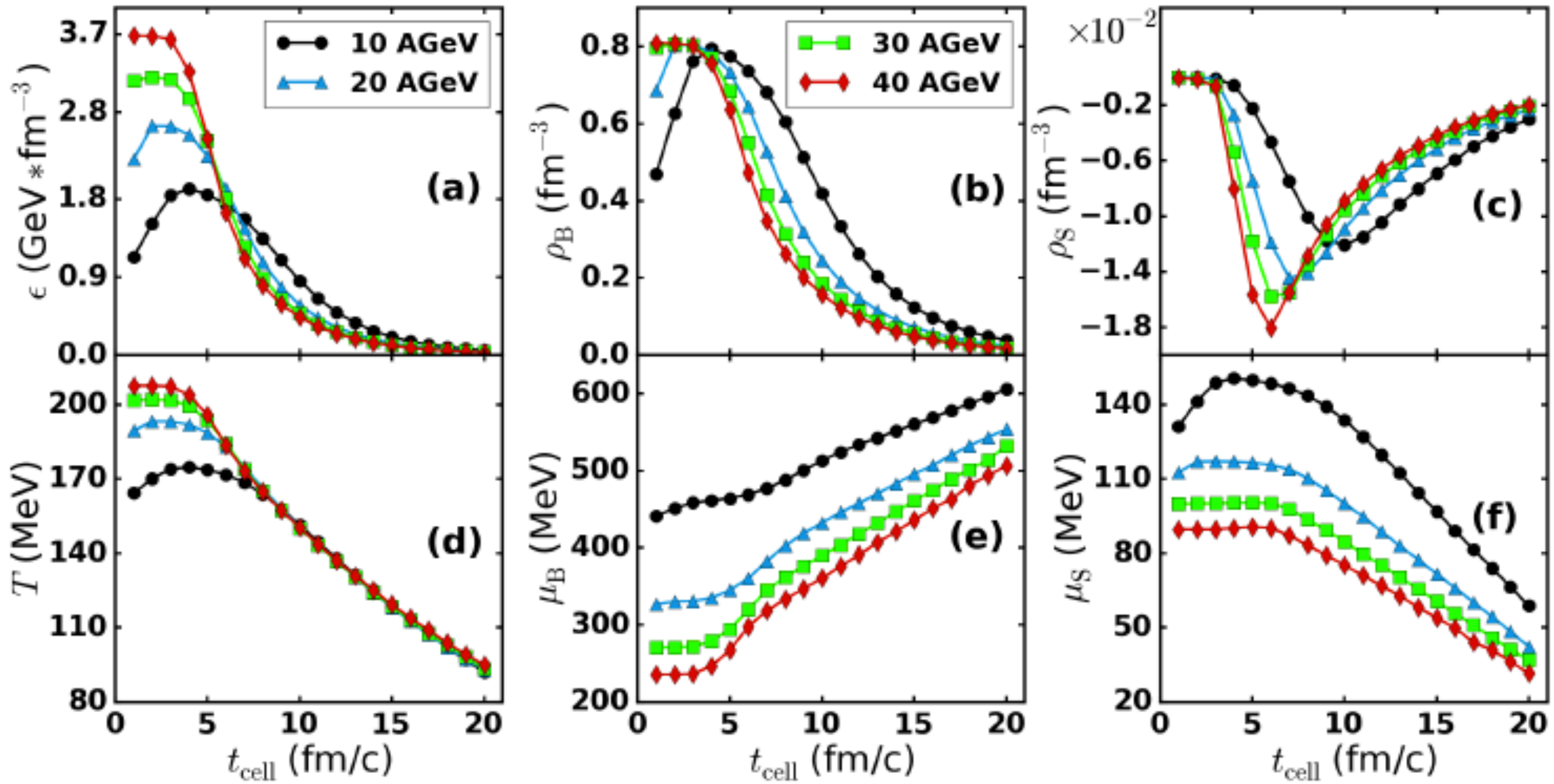
M.Belkacem et al., PRC 58, 1727 (1998)



L.Bravina et al., PRC 62, 064906 (2000)

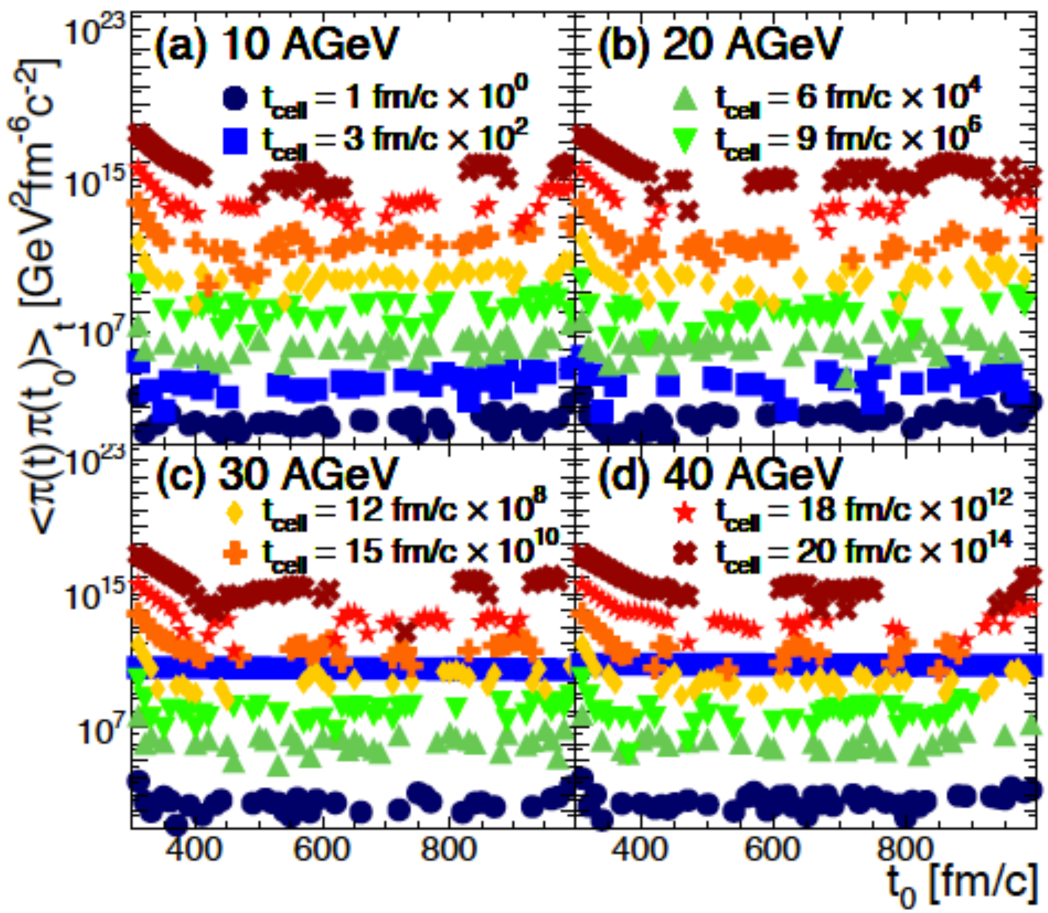
Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)

Cell + SM



Dependence of $\varepsilon, \rho_{\rm B}, \rho_{\rm S}$ (from cell) and of $T, \mu_{\rm B}, \mu_{\rm S}$ (from SM) on $t_{\rm cell}$

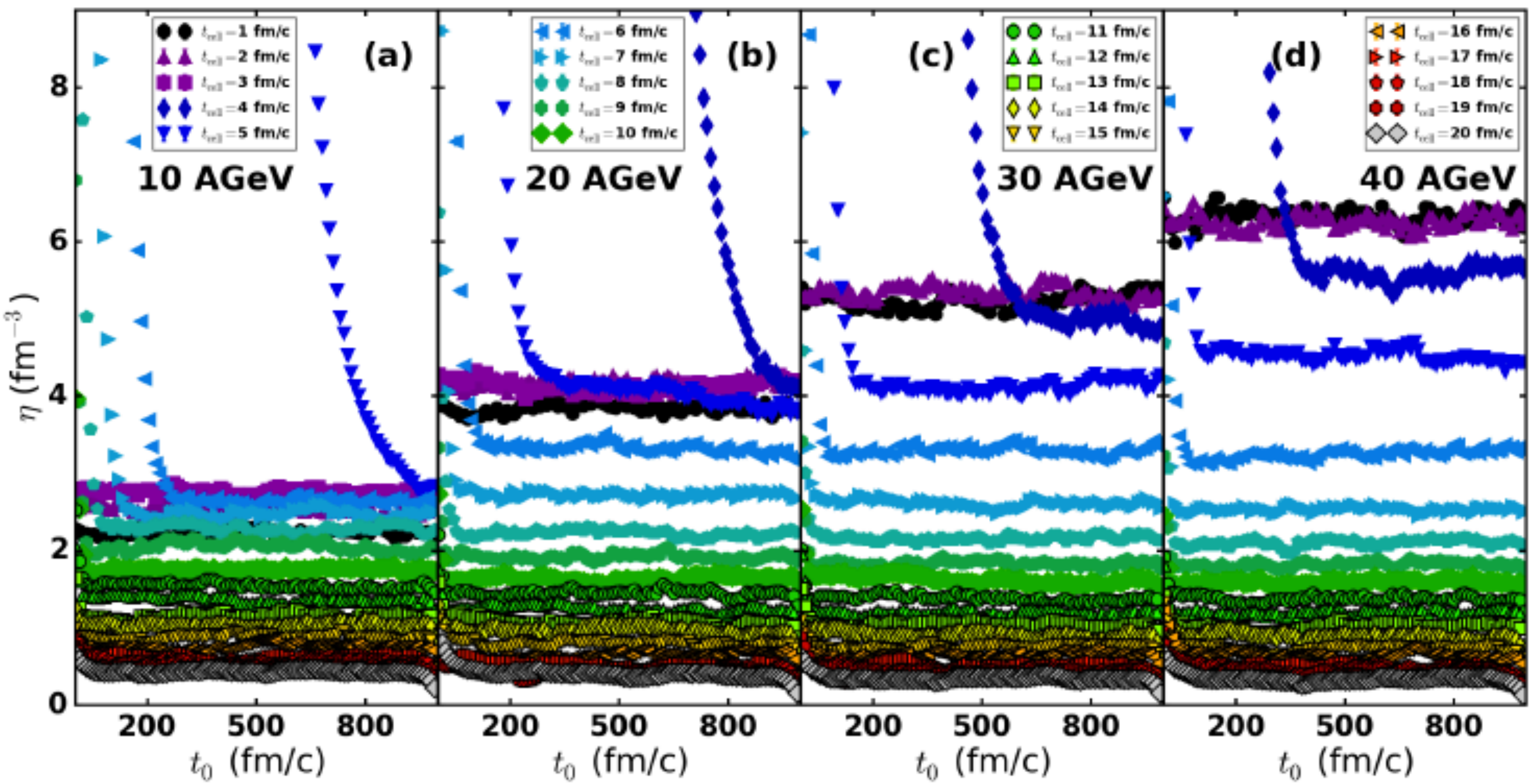
Results: $\langle \pi(t)\,\pi(t_0)\rangle_t$ at $E \in [10, 20, 30, 40]$ AGeV



Time dependence of correlators $\langle \pi(t)\,\pi(t_0)\rangle_t$

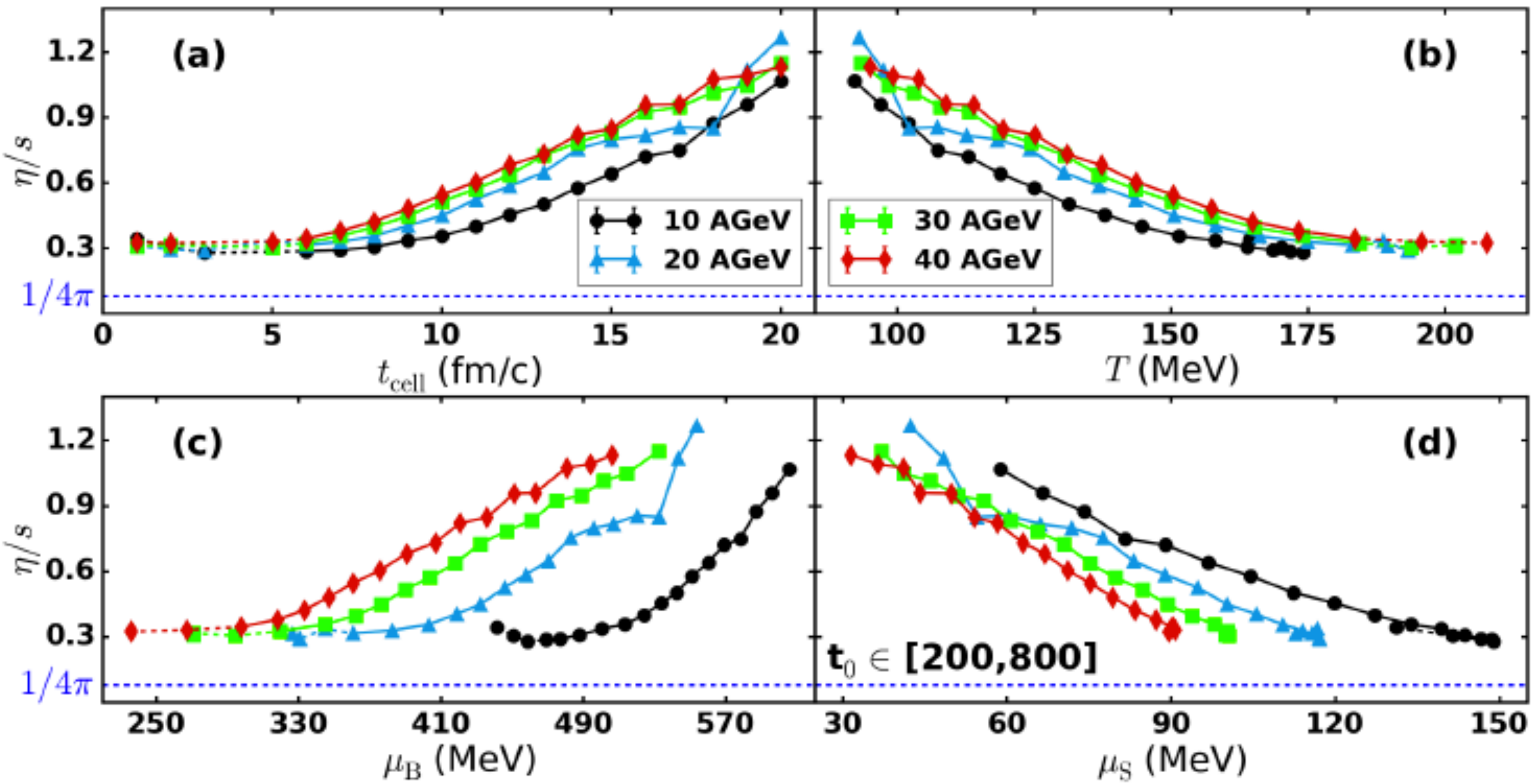
$t_0 = 300$ fm/c ; $t_{\rm cell} \in \{1 \div 20\}$ fm/c

Results: viscosity $\eta\,(t_0)$



Dependence of η on t_0

Results: η/s_{SM}



Dynamics of η/s_{SM} in cell
as function of time, T, $\mu_{\rm B}$, $\mu_{\rm S}$

Entropy density of nonequilibrium state

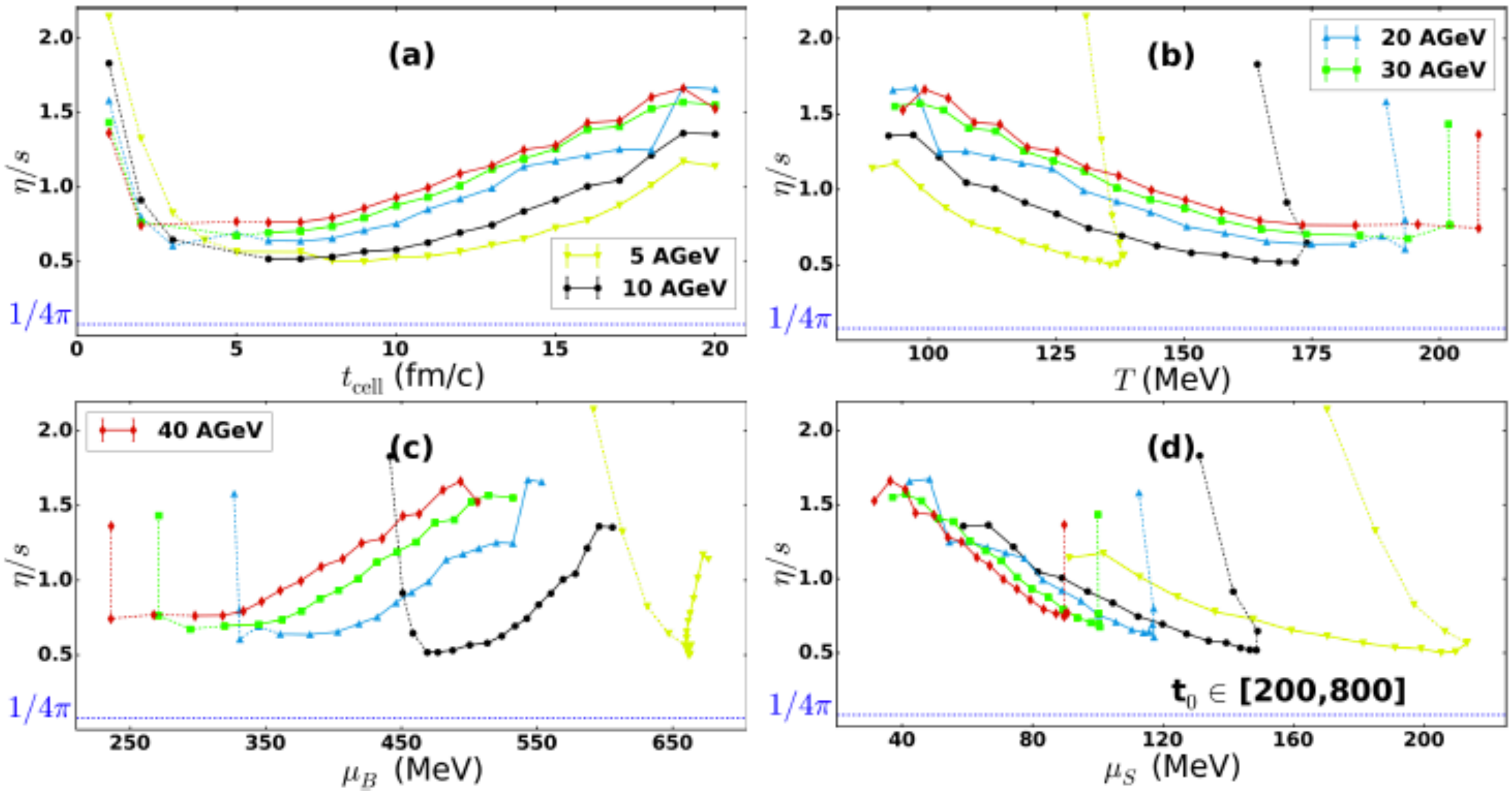
Entropy density s is defined as:

$$s = \sum_i \frac{g_i}{(2\pi\hbar)^3} \int_0^\infty f_i\,(p, m_i)\,[\ln f_i\,(p, m_i) - 1]\,\mathrm{d}^3 p$$

where microscopic distribution function reads

$$f_i^{mic}(p) = \frac{(2\pi\hbar)^3}{V g_i} \frac{dN_i}{d^3 p}$$

Results: $\eta/s_{noneq.}$



Dynamics of $\eta/s_{noneq.}$ in cell

η/s drops with time for $t_{\rm cell} \leq 6$ fm/c. Then it increases for all five energies

Pronounced minima for all reactions

Partial viscosities of nucleons and pions

Green-Kubo formula:

$$\eta(t_0) = \frac{V}{T}\int_{t_0}^{\infty} \mathrm{d}t \langle \pi(t)\,\pi(t_0)\rangle_t = \frac{V}{T}\langle \pi(t_0)\,\pi(t_0)\rangle,$$

where

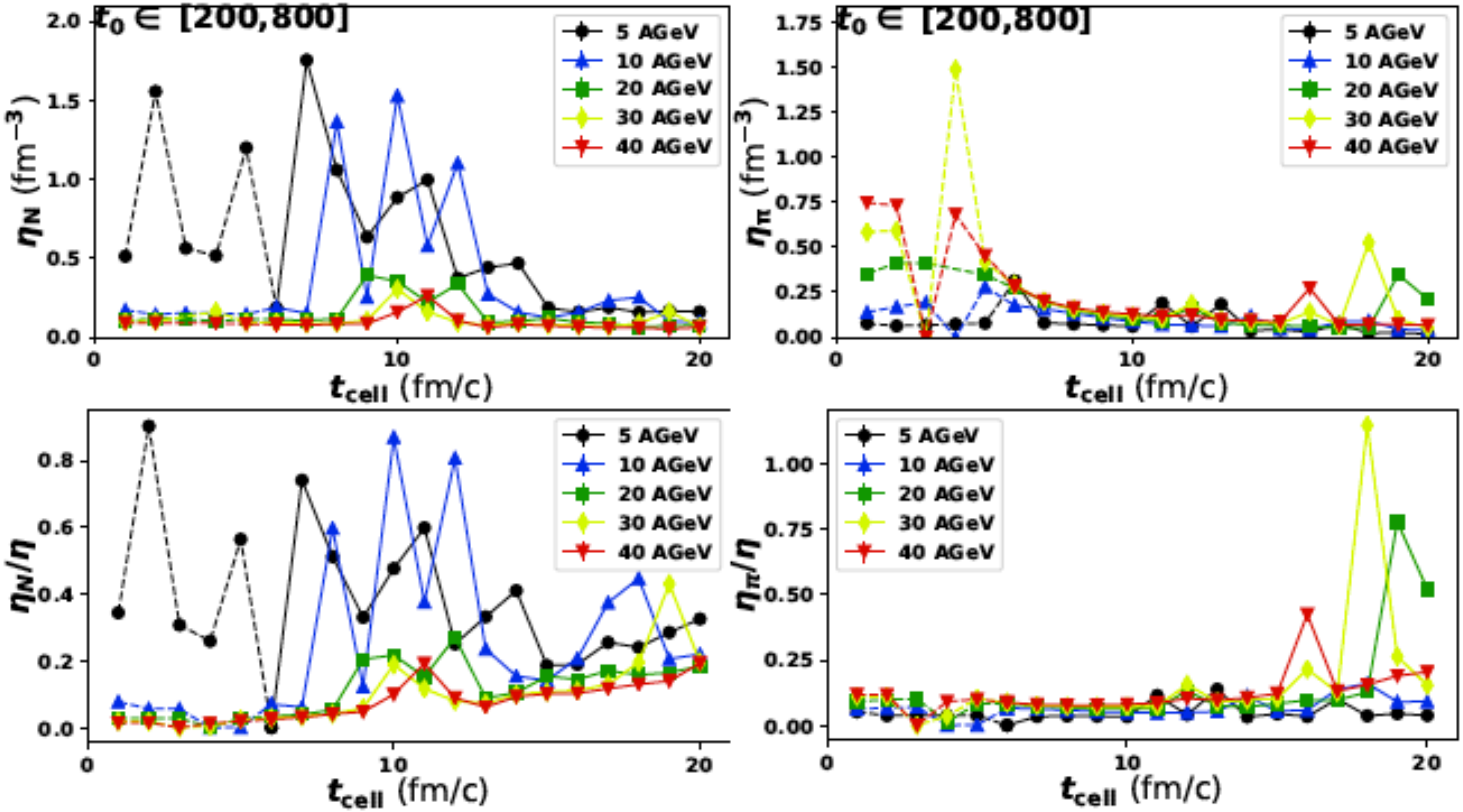
$$\begin{aligned}\langle \pi(t)\,\pi(t_0)\rangle_t &= \frac{1}{3}\sum_{\substack{i,j=1\\ i\neq j}}^{3} \lim_{t_{\max}\to\infty} \frac{1}{t_{\max}-t_0}\int_{t_0}^{t_{\max}} \mathrm{d}t'\pi^{ij}(t+t')\,\pi^{ij}(t') \\ &= \langle \pi(t_0)\,\pi(t_0)\rangle \exp\left(-\frac{t-t_0}{\tau}\right)\end{aligned}$$

with

$$\pi^{ij}(t) = \frac{1}{V}\sum_{\mathrm{particles}} \frac{p^i(t)\,p^j(t)}{E(t)}$$

Now "particles" means (1) nucleons (NN) and (2) pions ($\pi\pi$) only

Results: η of nucleons and pions



Partial η from N-N and π-π correlators
What makes the main contribution to shear viscosity?

Conclusions

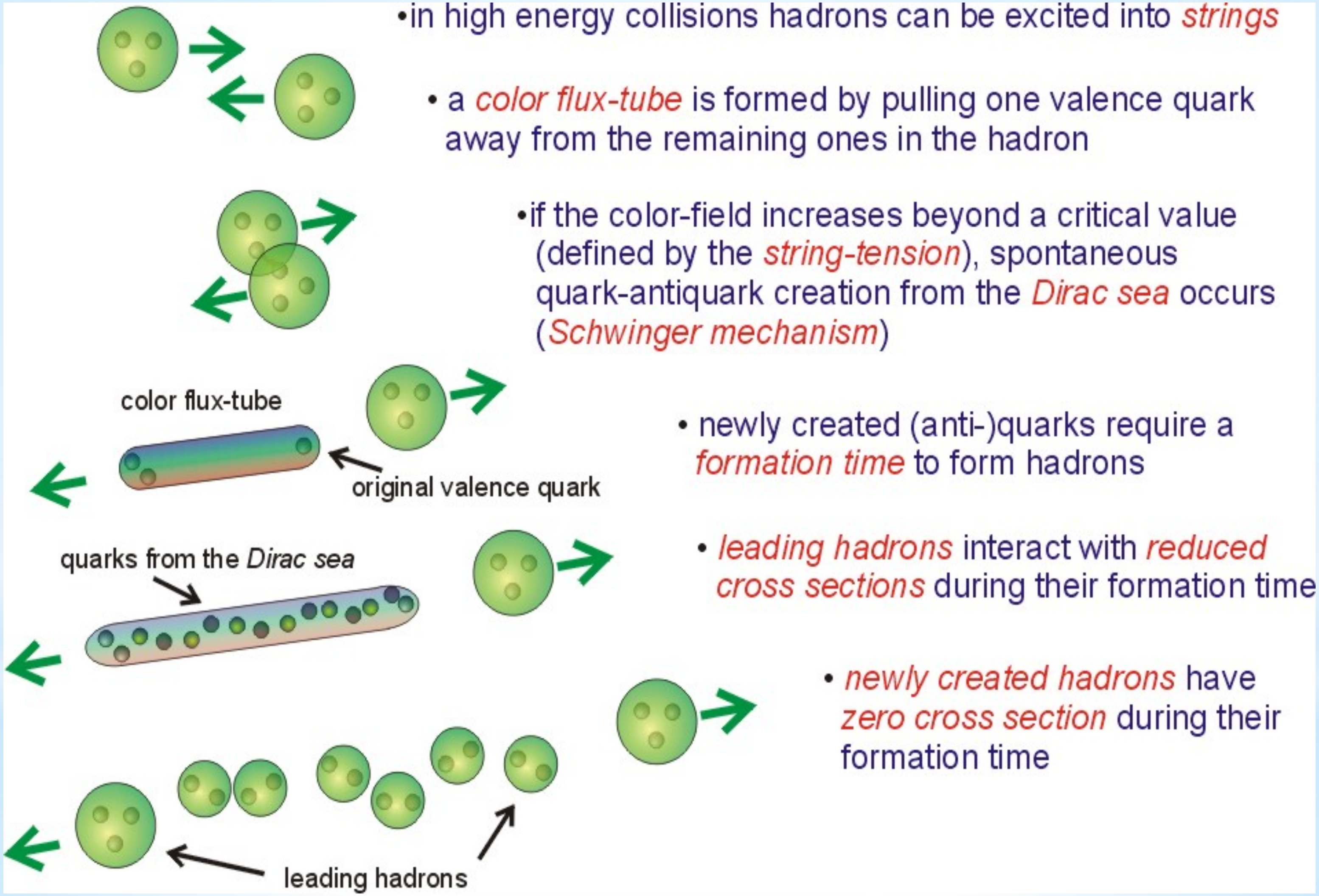
- data from central cell of UrQMD calculations are used as input for SM to calculate temperature T and entropy density s, and for UrQMD box calculations to estimate shear viscosity η
- box data are taken within the range $200 \leq t_0 \leq 800$ fm/c because:

  - values at $t_0 < 200$ fm/c are distorted by the initial fluctuation in the box

  - values at $t_0 > 800$ fm/c may be disturbed by the analog of Brownian motion
- it is shown that for all tested energies η and s in the cell drop with time
- ratios η/s reach minima about 0.3(0.5) at $t \approx 5$ fm/c for all energies. Then, the ratios rise to $1.0 \div 1.2$ $(1.3 \div 1.6)$ at $t = 20$ fm/c
- partial contributions of nucleons and pions to shear viscosity seem to be low. πN? Resonances?

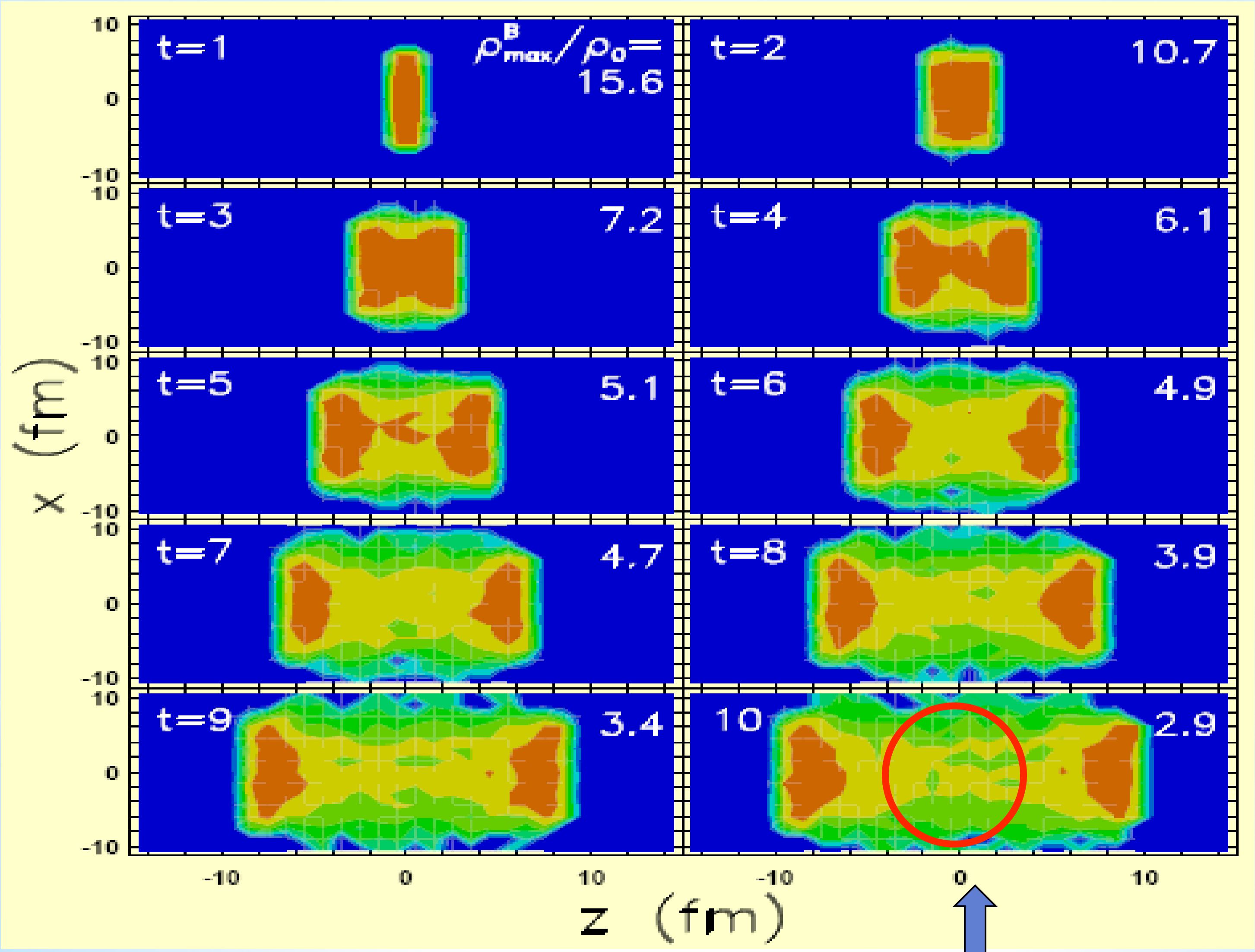
Back-up slides

Initial Particle Production in UrQMD



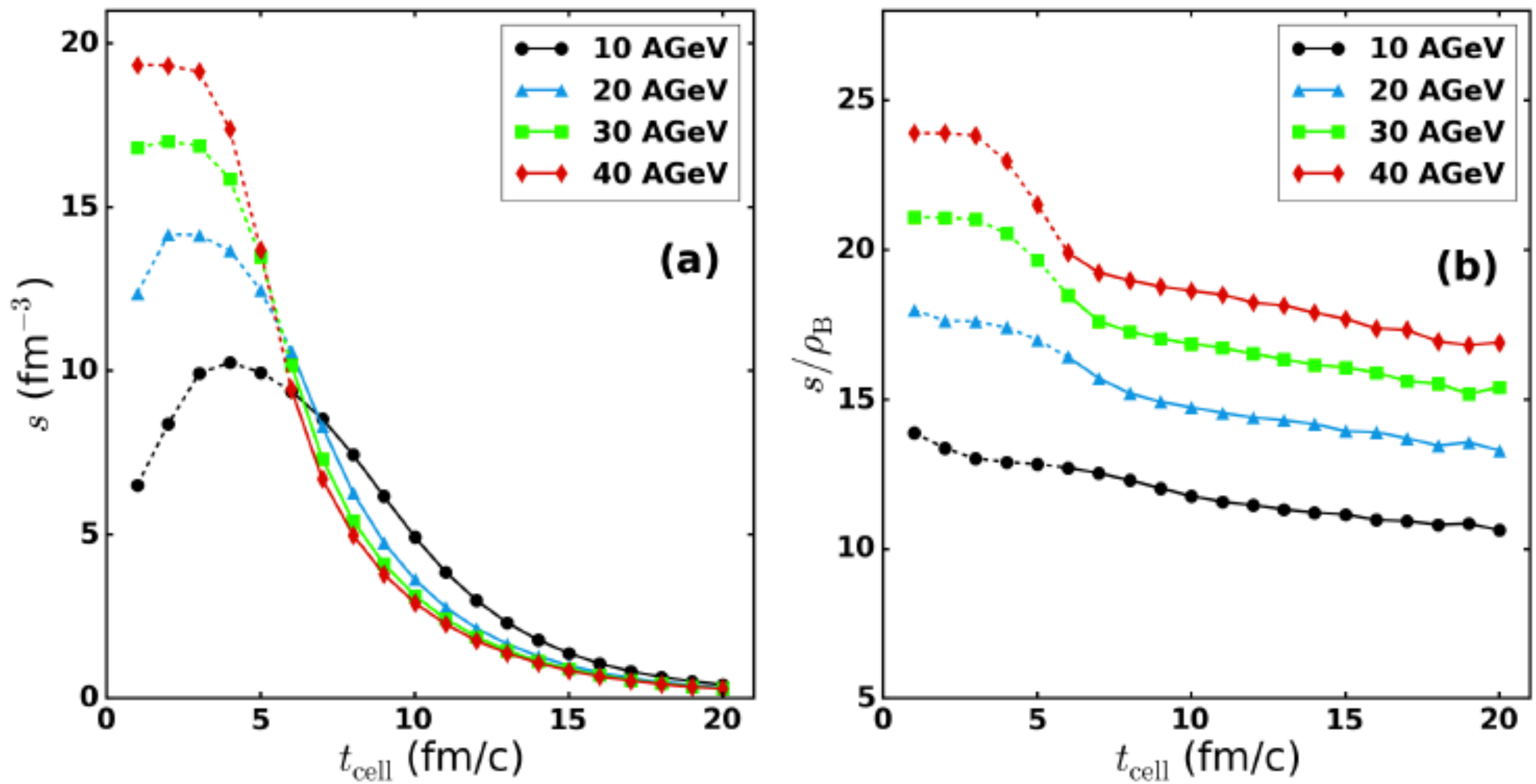
Pre-equilibrium: Homogeneity of baryon matter

L.Bravina et al., PRC 60 (1999) 024904



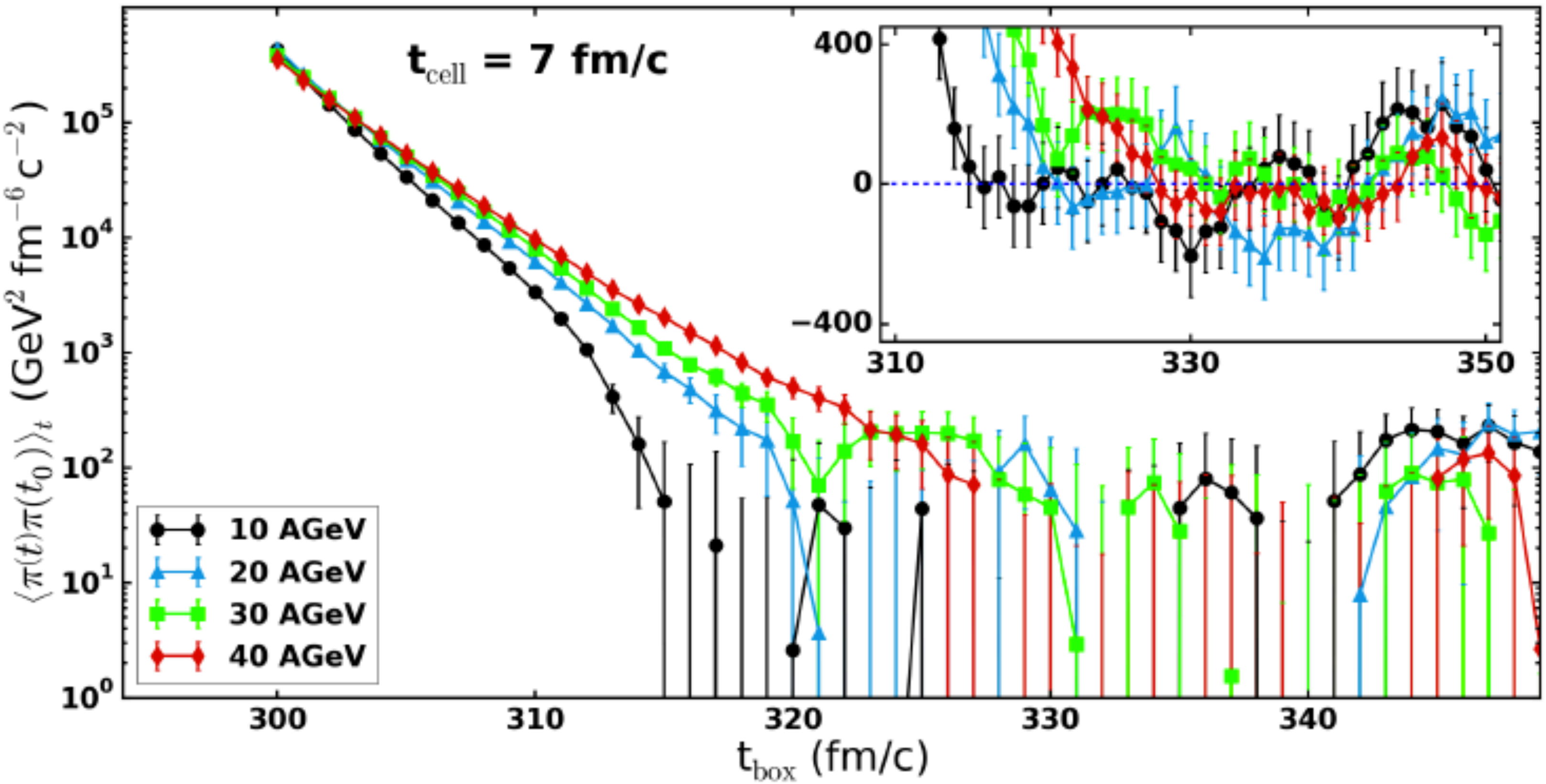
The local equilibrium in the central zone is quite possible

SM, Boltzmann entropy s



Dynamics of Boltzmann entropy density s and of $s/\rho_{\rm B}$ in cell

Results: $\langle\pi(t)\,\pi(t_0)\rangle_t$ at fixed $t_{\rm cell}$



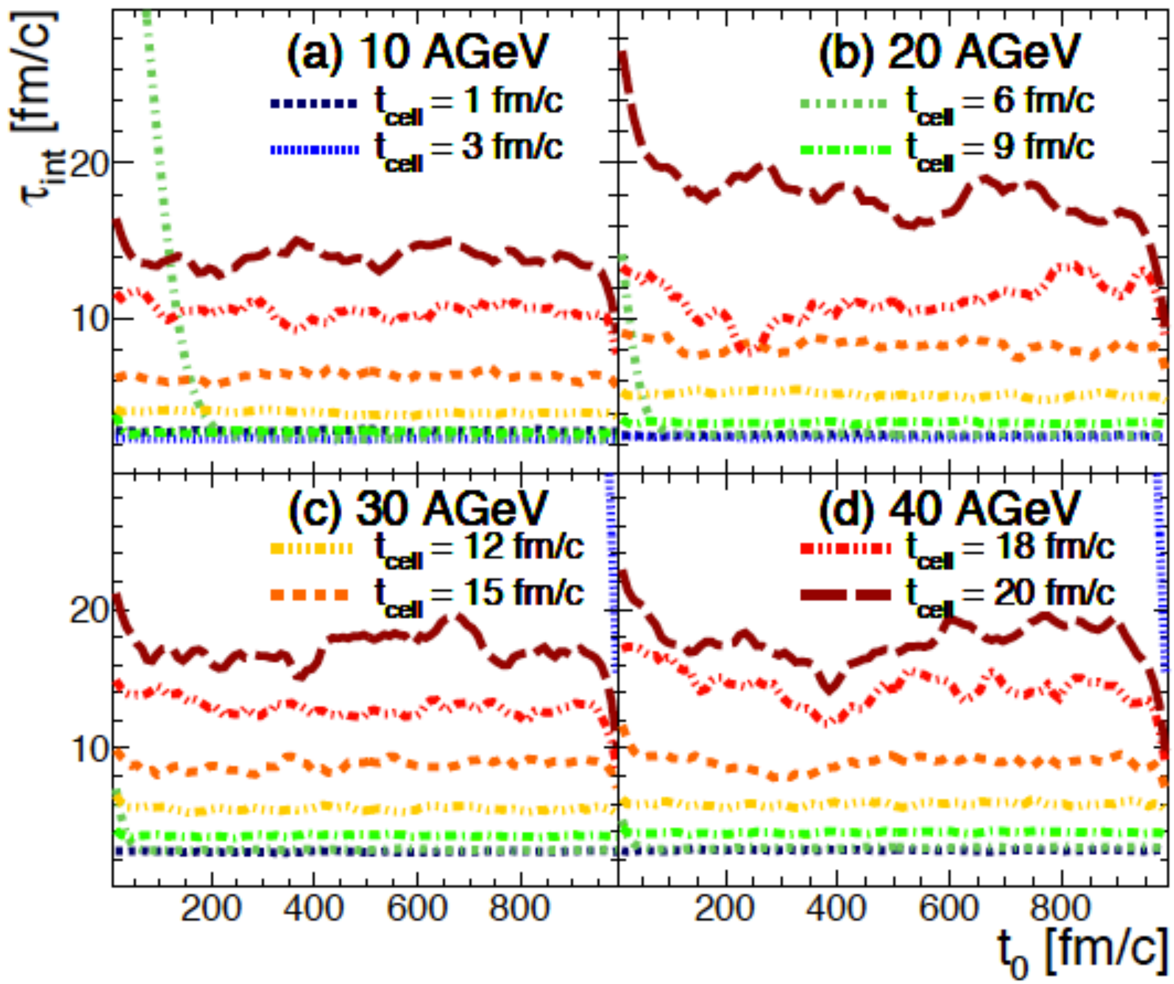
Time dependence of correlators $\langle\pi(t)\,\pi(t_0)\rangle_t$

Subplot: the same but at linear scale

$t_0 = 300$ fm/c

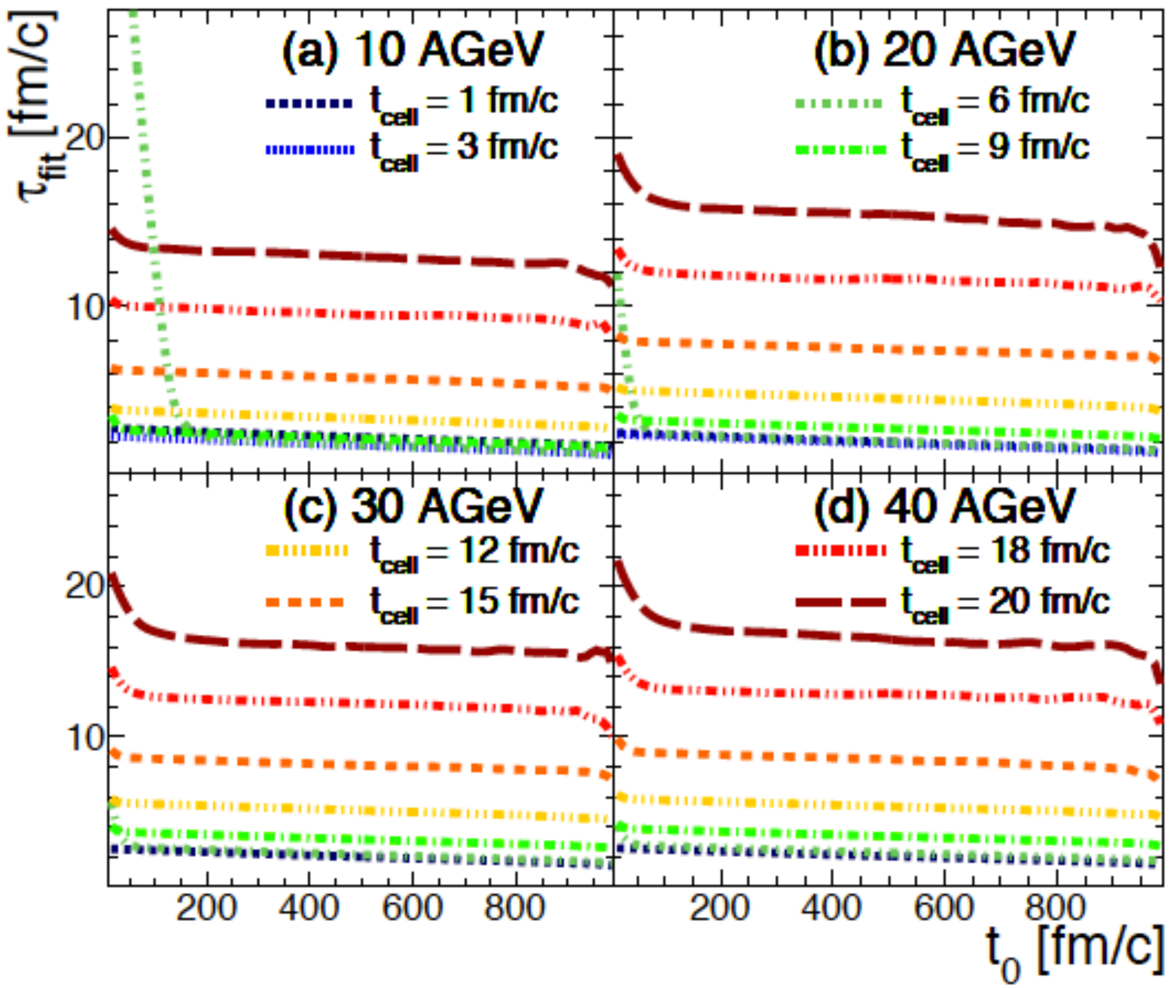
$t_{\rm cell} = 7$ fm/c

Results: $\tau\,(t_0)$



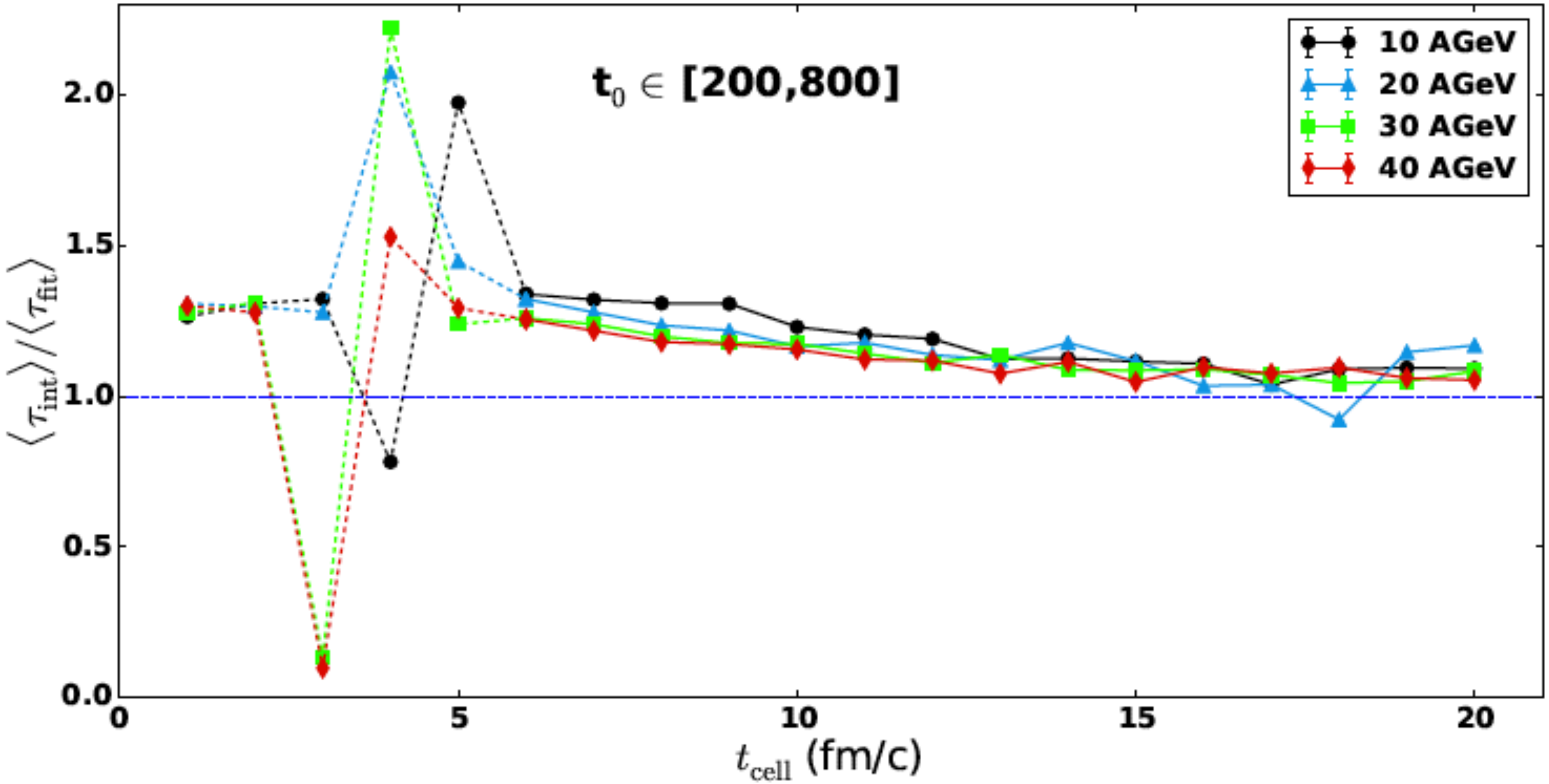
Dependence of τ_{int} on t_0

Results: τ from the fit



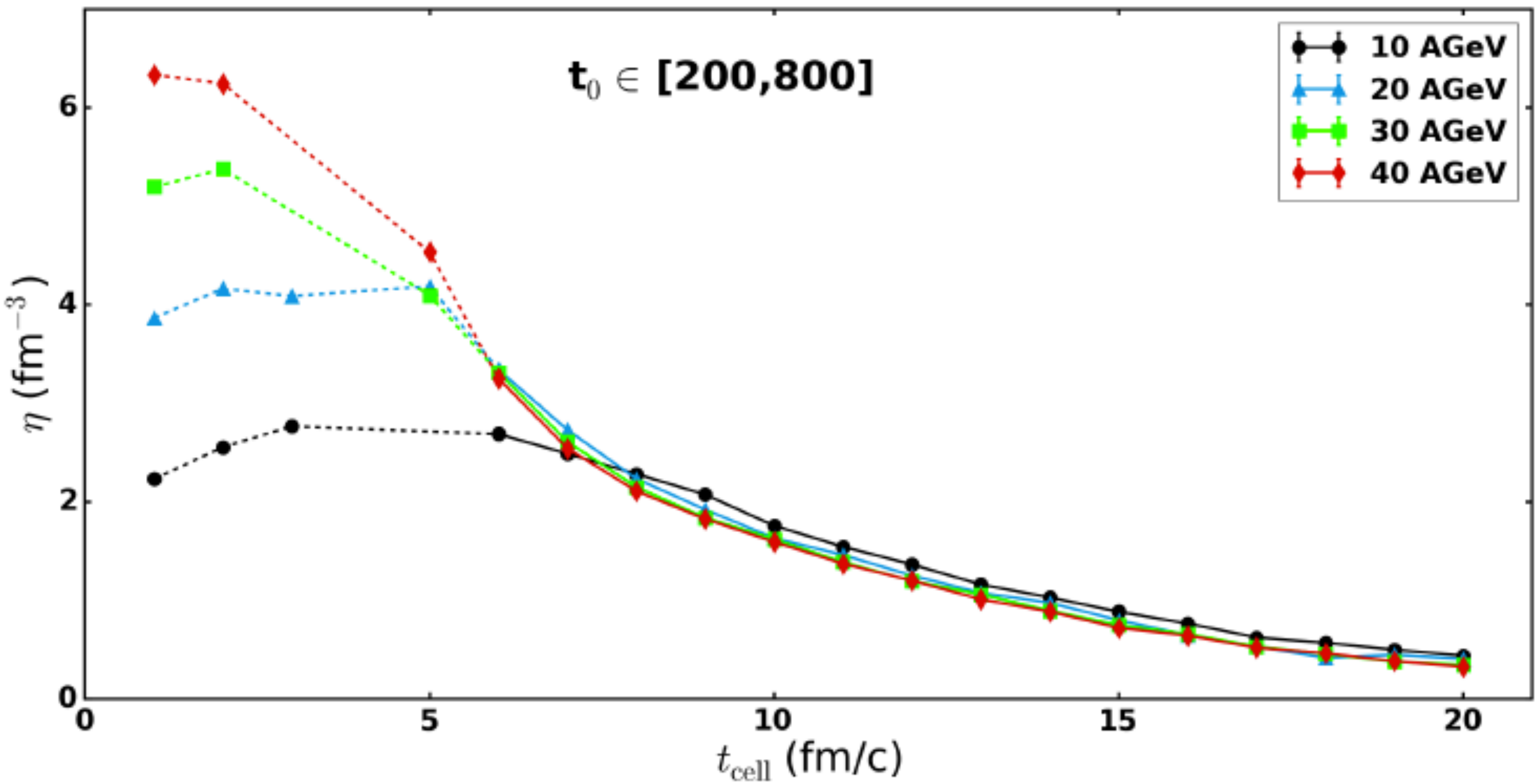
Dependence of τ_{fit} on t_0

Results: Comparison of τ_{int} and τ_{fit}



Ratio of τ_{int}/τ_{fit}

Results: viscosity $\eta\,(t_{\text{cell}})$



Dynamics of η in cell
All curves sit on the top of each other for $t_{\text{cell}} \geq 7$ fm/c