Shear viscosity of nucleons and pions in A+A collisions at NICA energies

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Based on publications:

   (proc. of Quark Matter’19, to be published in NPA)
Motivation

The diagram illustrates the phase space of high-energy physics, focusing on the transition region between hadrons and quark-gluon plasma. Key points include:

- **Lattice QCD**
- **RHIC-BES**
- **SPS**
- **NICA**
- **Nuclotron-M**
- **SIS-100**

The diagram highlights the critical point of phase transition and the transition from hadron to quark-gluon plasma. Various experiments like RHIC, LHC, and NICA are shown along with the net baryon density n/ n_o = 0.16 fm^-3.

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\( \eta \) of \( N \) and \( \pi \) in A+A at NICA; ICPPA 2020, Moscow
Motivation

- L.Csernai, J.Kapusta, L.McLerran. PRL 97, 152303 (2006)
- S.Plumari et al. PRC 86, 054902 (2012)
- ALICE collaboration, CERN COURIER (14.10.2016)
- J.Rose et al. PRC 97, 055204 (2018)
- ...

courtesy of R. Lacey and A. Taranenko
Comparison of several calculations for the hadron gas $\eta/s$ at $\mu_B = 0$
[from J. Rose et al. PRC97 (2018) 055204]
Green-Kubo: shear viscosity $\eta$ may be defined as:

$$\eta(t_0) = \frac{1}{\hbar T} \int_{t_0}^{\infty} dt \langle \pi(t) \pi(t_0) \rangle_t = \frac{\tau}{\hbar T} \langle \pi(t_0) \pi(t_0) \rangle,$$

where

$$\langle \pi(t) \pi(t_0) \rangle_t = \frac{1}{3} \sum_{i,j=1}^{3} \lim_{t_{\text{max}} \to \infty} \frac{1}{t_{\text{max}} - t_0} \int_{t_0}^{t_{\text{max}}} dt' \pi^{ij}(t + t') \pi^{ij}(t')$$

$$= \langle \pi(t_0) \pi(t_0) \rangle \exp \left( -\frac{t - t_0}{\tau} \right)$$

with

$$\pi^{ij}(t) = \frac{1}{V} \sum_{\text{particles}} \frac{p^i(t) p^j(t)}{E(t)}$$

$t_0$: initial cut-off time to start with
Model setup: cell calculations

- UrQMD calculations, central Au+Au collisions at energies $E \in [5, 10, 20, 30, 40]$ AGeV of the projectile, 51200 events per each

- central cell $5 \times 5 \times 5$ fm$^3$ $\Rightarrow$ \{\(\varepsilon, \rho_B, \rho_S\}\} at times $t_{\text{cell}} = 1 \div 20$ fm/c

- statistical model (SM): \{\(\varepsilon, \rho_B, \rho_S\}\} $\Rightarrow$ \{\(T, s, \mu_B, \mu_S\)\}
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Equilibration in the Central Cell

Kinetic equilibrium:
Isotropy of velocity distributions
Isotropy of pressure

Thermal equilibrium:
Energy spectra of particles are described by Boltzmann distribution

\[
\beta_{cm} \quad \quad 2R/\gamma_{cm}
\]
\[
\Delta z
\]
\[
t^{\text{cross}} = \frac{2R}{(\gamma_{cm} \beta_{cm})}
\]
\[
t^{\text{eq}} \geq t^{\text{cross}} + \Delta z/(2\beta_{cm})
\]


\[
\frac{dN_i}{4\pi pEdE} = \frac{V g_i}{(2\pi \hbar)^3} \exp \left( \frac{\mu_i}{T} \right) \exp \left( -\frac{E_i}{T} \right)
\]

Chemical equilibrium:
Particle yields are reproduced by SM with the same values of \((T, \mu_B, \mu_S)\):

\[
N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp \left( \frac{\mu_i}{T} \right) \exp \left( -\frac{E_i}{T} \right)
\]
Statistical model of ideal hadron gas

\[ \varepsilon^{\text{mic}} = \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S), \]
\[ \rho_B^{\text{mic}} = \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S), \]
\[ \rho_S^{\text{mic}} = \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S). \]

Multiplicity

Energy

Pressure

Entropy density

\[ N_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp, \]
\[ E_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} f(p, m_i) dp \]
\[ P^{\text{SM}} = \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}} f(p, m_i) dp \]
\[ s^{\text{SM}} = -\sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f(p, m_i) \left[ \ln f(p, m_i) - 1 \right] p^2 dp \]
UrQMD box calculations at $\{\varepsilon, \rho_B, \rho_S\}$ for every energy and cell time $t_{cell}$ from cell calculations, 100 points in total, 12800 events per each

- $\rho_B$ is included as $N_p : N_n = 1 : 1$
- $\rho_S$ is included via kaons $K^-$
- box size: $10 \times 10 \times 10$ fm$^3$
- box boundaries: transparent

- $\pi^{\text{ij}}(t)$ data extraction: $t = 1 \div 1000$ fm/c in box time, all types of hadrons are taken into account
Model setup: box calculations

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Box with periodic boundary conditions

M. Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species
(N, \( \Delta \), hyperons and their resonances with \( m \leq 2.25 \text{ GeV}/c^2 \))
32 different meson species
(including resonances with \( m \leq 2 \text{ GeV}/c^2 \)) and their respective antistates.
For higher mass excitations a string mechanism is invoked.

Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra
Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)

M. Belkacem et al., PRC 58, 1727 (1998)

L. Bravina et al., PRC 62, 064906 (2000)
Dependence of $\varepsilon, \rho_B, \rho_S$ (from cell) and of $T, \mu_B, \mu_S$ (from SM) on $t_{cell}$.
Results: $\langle \pi(t) \pi(t_0) \rangle_t$ at $E \in [10, 20, 30, 40]$ AGeV

Time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$

$t_0 = 300$ fm/c ; $t_{\text{cell}} \in \{1 \div 20\}$ fm/c
Results: viscosity $\eta(t_0)$

Dependence of $\eta$ on $t_0$
Results: $\eta/s_{SM}$

Dynamics of $\eta/s_{SM}$ in cell as function of time, $T$, $\mu_B$, $\mu_S$.
Entropy density $s$ is defined as:

$$s = \sum_i \frac{g_i}{(2\pi\hbar)^3} \int_0^\infty f_i(p, m_i) [\ln f_i(p, m_i) - 1] \, d^3p$$

where microscopic distribution function reads

$$f_i^{\text{mic}}(p) = \frac{(2\pi\hbar)^3}{V g_i} \frac{dN_i}{d^3p}$$
Results: $\eta/s_{\text{noneq}}$.

Dynamics of $\eta/s_{\text{noneq}}$ in cell
$\eta/s$ drops with time for $t_{\text{cell}} \leq 6$ fm/c. Then it increases for all five energies
Pronounced minima for all reactions
Partial viscosities of nucleons and pions

Green-Kubo formula:

$$\eta(t_0) = \frac{V}{T} \int_{t_0}^{\infty} dt \langle \pi(t) \pi(t_0) \rangle_t = \frac{V}{T} \langle \pi(t_0) \pi(t_0) \rangle_T,$$

where

$$\langle \pi(t) \pi(t_0) \rangle_t = \frac{1}{3} \sum_{i,j=1}^{3} \lim_{t_{\text{max}} \to \infty} \frac{1}{t_{\text{max}} - t_0} \int_{t_0}^{t_{\text{max}}} dt' \pi^{ij}(t + t') \pi^{ij}(t')$$

$$= \langle \pi(t_0) \pi(t_0) \rangle \exp \left(-\frac{t - t_0}{\tau}\right)$$

with

$$\pi^{ij}(t) = \frac{1}{V} \sum_{\text{particles}} \frac{p^i(t) p^j(t)}{E(t)}$$

Now "particles" means (1) nucleons ($NN$) and (2) pions ($\pi\pi$) only.
Results: $\eta$ of nucleons and pions

Partial $\eta$ from $N-N$ and $\pi-\pi$ correlators

What makes the main contribution to shear viscosity?
Conclusions

- Data from central cell of UrQMD calculations are used as input for SM to calculate temperature $T$ and entropy density $s$, and for UrQMD box calculations to estimate shear viscosity $\eta$.
- Box data are taken within the range $200 \leq t_0 \leq 800$ fm/c because:
  - Values at $t_0 < 200$ fm/c are distorted by the initial fluctuation in the box.
  - Values at $t_0 > 800$ fm/c may be disturbed by the analog of Brownian motion.
- It is shown that for all tested energies $\eta$ and $s$ in the cell drop with time.
- Ratios $\eta/s$ reach minima about $0.3(0.5)$ at $t \approx 5$ fm/c for all energies. Then, the ratios rise to $1.0 \div 1.2$ ($1.3 \div 1.6$) at $t = 20$ fm/c.
- Partial contributions of nucleons and pions to shear viscosity seem to be low. $\pi N$? Resonances?
Back-up slides
• in high energy collisions hadrons can be excited into **strings**

• a **color flux-tube** is formed by pulling one valence quark away from the remaining ones in the hadron

• if the color-field increases beyond a critical value (defined by the **string-tension**), spontaneous quark-antiquark creation from the **Dirac sea** occurs (**Schwinger mechanism**)

• newly created (anti-)quarks require a **formation time** to form hadrons

• **leading hadrons** interact with reduced **cross sections** during their formation time

• newly created hadrons have **zero cross section** during their formation time
Pre-equilibrium: Homogeneity of baryon matter

L. Bravina et al., PRC 60 (1999) 024904

The local equilibrium in the central zone is quite possible.
Dynamics of Boltzmann entropy density $s$ and of $s/\rho_B$ in cell
Results: $\langle \pi(t) \pi(t_0) \rangle_t$ at fixed $t_{\text{cell}}$

![Graph showing time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$](image)

**Time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$**

Subplot: the same but at linear scale

$t_0 = 300$ fm/c

$t_{\text{cell}} = 7$ fm/c
Results: $\tau(t_0)$

Dependence of $\tau_{int}$ on $t_0$

(a) 10 AGeV
- $t_{cell} = 1$ fm/c
- $t_{cell} = 3$ fm/c

(b) 20 AGeV
- $t_{cell} = 6$ fm/c
- $t_{cell} = 9$ fm/c

(c) 30 AGeV
- $t_{cell} = 12$ fm/c
- $t_{cell} = 15$ fm/c

(d) 40 AGeV
- $t_{cell} = 18$ fm/c
- $t_{cell} = 20$ fm/c
Results: $\tau$ from the fit

Dependence of $\tau_{fit}$ on $t_0$
Results: Comparison of $\tau_{\text{int}}$ and $\tau_{\text{fit}}$

$t_0 \in [200, 800]$
Results: viscosity $\eta(t_{\text{cell}})$

$\eta(t_{\text{cell}})$

$t_0 \in [200, 800]$

Dynamics of $\eta$ in cell

All curves sit on the top of each other for $t_{\text{cell}} \geq 7 \text{ fm/c}$