PRODUCTION OF DIJETS WITH LARGE RAPIDITY SEPARATION AT COLLIDERS

ANATOLII EGOROV (1, 2), VICTOR KIM (1, 2)

1. Petersburg Nuclear Physics Institute NRC Kurchatov Institute, Gatchina
2. Peter the Great St. Petersburg Polytechnic University
MOTIVATION

ENERGY FLOW AND JET VETO

- Many signals of new physics is observed via hadronic jets

- Multiple soft gluon emission can transfer the energy or transverse momentum away from hard jets (ENERGY FLOW)

- It is not possible to forbid very soft real QCD emission as well as virtual QCD corrections experimentally

- Various Veto conditions are introduced experimentally (JET VETO)

- Our aim is to calculate the effect of JET VETO
INTERJET VETO

SEARCH FOR THE BALITSKY-FADIN-KURAEV-LIPATOV (BFKL) EFFECTS IN THE ATLAS EXPERIMENT


\[ \mathcal{R}(\Delta y, p_T) = \frac{d\sigma^{\text{dijet veto}}/d\Delta y d^2p_T}{d\sigma^{\text{dijet}}/d\Delta y d^2p_T} \]

- pp at 7 TeV

- two selection types:
  
  Leading \( p_T \) dijet

  Forward/Backward dijet

- \( 70 < p_T < 270 \) GeV

- \( \Delta y = |y_1 - y_2| < 6 \)

- **Veto** \( Q = 20 \) GeV between jets

ANATOLII EGOROV 07.10.2020
APPROACH OF BMS

BANFI-MARCHESINI-SMYE (BMS) JHEP 08(2002)006

Soft gluon approximation, large $N_c$, energy ordering;

Large angle emission, Sudakov and non-global logarithms;

\[
\tau = \int_{Q_0}^{Q} \frac{dq_t}{q_t} \tilde{\alpha}_s(q_t); \quad Q_0 = E_{out}; \quad \tilde{\alpha}_s = \frac{N_c \alpha_s}{\pi} = \frac{C_A \alpha_s}{\pi}
\]

\[
R_{ab}(\tau) = \int_{E_{out}}^{Q} \frac{dq_t}{q_t} \tilde{\alpha}_s(q_t) \int_{out}^{Q} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab}; \quad w_{ab}(q) = \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{aq})(1 - \cos \theta_{qb})}
\]
RATIOS OF X-SECTIONS

VETO TO INCLUSIVE

\[ \mathcal{R}(\Delta y, p_T) = \frac{d\sigma^{\text{veto}}/d \Delta y d^2 p_T}{d\sigma^{\text{incl}}/d \Delta y d^2 p_T} \]

inclusive x-section:

\[ \frac{d\sigma^{\text{incl}}}{d \Delta y d^2 p_T} = \sum_{ij}^{q, \bar{q}, g} \int_{Y_{\text{min}}(p_T, \Delta y)}^{Y_{\text{max}}(p_T, \Delta y)} dY_{\Delta y} x_1 f_i(x_1, p_T) x_2 f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}_{ij}}{d\hat{t}} \]

qq’ scattering with one gluon exchange

\[ \frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} h^A(\hat{s}, \hat{t}, \hat{u}) \]

\[ h^A(s, t, u) = g^A \frac{C_F}{N_c} \left( \frac{s^2 + u^2}{t^2} \right) \]

if consider only \( \Delta y > 0 \)

\[ \frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} (h^A(\hat{s}, \hat{t}, \hat{u}) + h^A(\hat{s}, \hat{u}, \hat{t})) \]
INTERJET VETO

\[ \frac{d\sigma^{\text{veto}}}{d\Delta y d^2 p_T} = \sum_{ij} \frac{Y_{\text{max}}(p_T, \Delta y)}{Y_{\text{min}}(p_T, \Delta y)} dYx_1f_i(x_1, p_T)x_2f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}^{\text{veto}}_{ij}}{d\hat{t}} \]

at large \( N_c \) limit, and in one-gluon exchange, color flows as \( 1 \to 4 \) and \( 2 \to 3 \)

\[ \frac{d\hat{\sigma}^{\text{veto}}_{qq'}}{d\hat{t}} = \frac{1}{16\pi s^2} (h^A(\hat{s}, \hat{t}, \hat{u}) \Sigma_{14} \Sigma_{23} + h^A(\hat{s}, \hat{u}, \hat{t}) \Sigma_{13} \Sigma_{24}) \]

\[ y_{\text{in}} = \frac{\Delta y}{2} - R_{\text{jet}} \quad y_{\text{in}} = -\log \tan\left(\frac{\theta_{\text{in}}}{2}\right) \]
JET VETO


- pp at 7 TeV

- two selection types:
  
  Inclusive dijets

  Mueller-Navelet dijets

- $p_T > 35$ GeV

- $\Delta y = |y_1 - y_2| < 9.4$

- Veto $p_{T,veto} = 35$ GeV
JET VETO

\( y_{\text{bound}} = 4.7 \)

\[ \partial_\tau \Sigma_{ab}(\tau) = - (\partial_\tau R_{ab}) \Sigma_{ab} + \int_{C_{\text{in}}} \frac{d\Omega_q}{4\pi} w_{ab}(q) [\Sigma_{aq}(\tau) \Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)] \]

\[ R_{ab}(\tau) = \int_{\bar{q}_t}^{Q} \frac{dq_t}{q_t} \tilde{\alpha}(q_t) \int_{C_{\text{out}}} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab} C_{\text{out}} \]

Before \( C_{\text{out}} \) is controlled by \( \Delta y \) (100 Systems)

Now \( C_{\text{out}} \) is a function of dipole type as well as:

\( y_{\text{bound}}, Y, \Delta y \)

(100x100x6 = 60k Systems)
Generating functional
\[ \Sigma_{ab}[Q, u] = \sum_n \frac{1}{n!} \int \frac{d\sigma^{(n)}_{ab}}{\sigma_{abT}} \prod_{i=1}^{n} u(q_i) \]

In soft and planar limit
\[ Q \partial_Q \Sigma_{ab}[Q, u] = \int \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \left\{ u(q) \Sigma_{aq}[Q, u] \Sigma_{qb}[Q, u] - \Sigma_{ab}[Q, u] \right\} \]

Sudakov form factor
\[ \ln S_{ab}(Q, Q_0) = -\int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(q_{ab}) w_{ab}(q) \theta(q_{ab} - Q_0) \]
\[ q_{ab}^2 = \frac{2\omega_q^2}{w_{ab}(q)} \quad w_{ab}(q) = \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{aq})(1 - \cos \theta_{qb})} \]
\[ \Sigma_{ab}[Q, u] = S_{ab}(Q, Q_0) + \int_{Q_0}^{Q} dP_{ab}(q) u(q) \Sigma_{aq}[\omega_q, u] \Sigma_{qb}[\omega_q, u] \]
MC ALGORITHM

MARCHESINI 2006 ARXIV:HEP-PH/0601068

Splitting probability

\[ dP_{ab}(q) = d\left( \frac{S_{ab}(Q, Q_0)}{S_{ab}(\omega_q, Q_0)} \right) dR_{ab}(\Omega_q) \]

Angle distribution

\[ \frac{dR_{ab}(\Omega_q)}{d\Omega_q} = \frac{\tilde{\alpha}_s(q_{abt}) w_{ab}(q)}{N_{ab}(\omega_q)} \theta(q_{abt} - Q_0) \]

Algorithm: start with ab-dipole.

use \( r \) random number to generate energy of branch:

if \( r < S(Q, Q_0) \) - dipole does not branch (with \( Q_0 \) cutoff).

if \( r > S(Q, Q_0) \) - dipole split with \( \omega_q \) such that \( S_{ab}(\omega_q, Q_0) \cdot r = S_{ab}(Q, Q_0) \)

generate angle with \( dR_{ab}(\Omega_q) \)

repeat with aq and qb dipoles until \( Q_0 \)
\[ q_{abt}^2 = \frac{2\omega_q^2}{w_{ab}(q)} \]

**MC ALGORITHM DOES NOT REPRODUCE BMS EQUATION**

\[
\ln S_{ab}(Q, Q_0) = -\int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \tilde{\alpha}_s(q_{abt}) w_{ab}(q) \theta(q_{abt} - Q_0)
\]

**BMS**

\[
\ln S_{ab}(Q, Q_0) = -\int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \tilde{\alpha}_s(\omega_q) w_{ab}(q)
\]

\[
\ln S_{ab}(Q, Q_0) = -\int_{Q_0}^{Q} \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \tilde{\alpha}_s(\omega_q) w_{ab}(q) \Theta(\cos(\theta)_{\text{max}} - \cos(\theta))
\]
RESULTS

SUDAKOV AND BMS

- Sudakov: \( \partial_\tau \Sigma_{ab}(\tau) = - (\partial_\tau R_{ab}) \Sigma_{ab} \)

\[
R_{ab}(\tau) = \int_{E_{out}}^{Q} \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \int_{Q}^{} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab} C_{out}
\]

- BMS

\[
\Sigma_{34}(\tau) = \Sigma_{34}(\tau)^{(1)} \Sigma_{34}(\tau)^{(2)} \Sigma_{34}(\tau)^{(3)}
\]
RESULTS

MC

Inclusive K-factor

\( \sqrt{s} = 7 \text{ TeV} \)
35 \( \leq p_T \leq 150 \text{ GeV} \)
Veto \( p_T^{\text{jet}} = 35 \text{ GeV} \)
\( \alpha_s(M_Z) = 0.12; \Lambda_{\text{QCD}} = 100 \text{ MeV} \)

\( q_{abt} \text{ ON} \)

\( q_{abt} \text{ OFF} \)

Inclusive K-factor

\( \sqrt{s} = 7 \text{ TeV} \)
35 \( \leq p_T \leq 150 \text{ GeV} \)
Veto \( p_T^{\text{jet}} = 35 \text{ GeV} \)
\( \alpha_s(M_Z) = 0.12; \Lambda_{\text{QCD}} = 100 \text{ MeV} \)

ANATOLII EGOROV 07.10.2020
IMPROVEMENTS

ATTEMPT TO TAKE RECOIL INTO ACCOUNT (ENERGY-MOMENTUM CONSERVATION)

\[ \sqrt{s} = 7 \text{ TeV}; \]
\[ 35 \leq p_T \leq 150 \text{ GeV}; \]
\[ V_{\text{eto}} p_{T,\text{veto}} = 35 \text{ GeV}; \]
\[ \alpha_s(M_Z) = 0.12; \quad \Lambda_{QCD} = 100 \text{ MeV}. \]
CONCLUSIONS

- Soft gluon approximation is suitable tool to study the ENERGY FLOW and JET VETO

- MC algorithm gives solution mach faster

- MC algorithm is more suitable for jet veto

- Improvements are needed (such as inclusion the energy-momentum conservation and recoil)

- This tool works better when Veto region is far from jets
THANK YOU
BACKUP
RESULTS

ALL IN ONE
COLOR FLOW

PDG REVIEW $gg \rightarrow gg$ x-section

$$\frac{d\sigma_{gg\rightarrow gg}}{d\Omega} = \frac{9\alpha_s^2}{8s} \left( 3 - \frac{ut}{s^2} - \frac{su}{t^2} - \frac{st}{u^2} \right)$$

BMS JHEP 08 (2002) 006

$$\frac{d\sigma_{gg\rightarrow gg}}{dt} = \frac{1}{16\pi s} (h^D(s, t, u) + h^D(s, u, t) + h^D(u, t, s))$$

$$h^D(s, t, u) = 2g^4 \frac{N_c^2}{N_c^2 - 1} \left( 1 - \frac{tu}{s^2} - \frac{su}{t^2} + \frac{u^2}{st} \right)$$

VETO

$$\frac{d\sigma_{gg\rightarrow gg}^{\text{veto}}}{dt} = \frac{1}{16\pi s} (h^D(s, t, u)\Sigma_{12}\Sigma_{13}\Sigma_{24}\Sigma_{34} + h^D(s, u, t)\Sigma_{12}\Sigma_{14}\Sigma_{23}\Sigma_{34} + h^D(u, t, s)\Sigma_{14}\Sigma_{24}\Sigma_{13}\Sigma_{23})$$
SOLUTION OF BMS

\[\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab})\Sigma_{ab} + \int_{in} \frac{d\Omega_q}{4\pi} w_{ab}(q)[\Sigma_{aq}(\tau)\Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)]\]

in region \(C_{in}\)

\(\phi_q \in [0,2\pi] ; \quad \theta_q \in [0,\theta_{in}] \cup [\pi - \theta_{in}, \pi]\)

\(\tau\) range

\[\tau = \int_{Q_0}^{Q} \frac{dq_t}{q_t} \tilde{\alpha}_s(q_t) \quad \tau_{max} = \int_{p_{T \text{ veto}}}^{\sqrt{s}/2} \frac{dq_t}{q_t} \tilde{\alpha}_s(q_t) = \int_{20}^{7000} \frac{dq_t}{q_t} \tilde{\alpha}_s(q_t) = 0.651\]

\[\Sigma_{ab}(\tau) = \Sigma(\theta_a, \phi_a, \theta_b, \phi_b, \tau, \theta_{in}) = \Sigma(\theta_a, \theta_b, \Delta \phi, \tau, \theta_{in})\]

we have to solve the equation for each \(\theta_{in}(\Delta y)\)
SOLUTION OF BMS

NUMERICAL SOLUTION

Make transform \( \Sigma_{ab}(\tau) = e^{-\tau f_{ab}} g_{ab}(\tau) \)

\[
\partial_\tau g_{ab}(\tau) = \int_{C_{in}} \frac{d\Omega_q}{4\pi} w_{ab}(q) U_{abq} [g_{aq}(\tau)g_{qb}(\tau) - g_{ab}(\tau)] \\
U_{abq} = e^{-\tau (f_{aq} + f_{qb} - f_{ab})}
\]

Calculate \( \theta_{in} \) in 100 points from 0 to \( \pi/2 \)

Divide in region \( \theta_q \in [0, \theta_{in}] \cup [\pi - \theta_{in}, \pi] \) in 81 points

Divide \( \Delta \phi \in [0, \pi] \) in 21 points

\[
\partial_\tau g_{ij}(\tau) = \sum_{k \neq i, k \neq j, k \in C_{in}} \frac{\Delta \Omega_k}{4\pi} w_{ij}(k) U_{ijk} [g_{ik}(\tau)g_{kj}(\tau) - g_{ij}(\tau)]
\]
**SOLUTION OF BMS**

**NUMERICAL SOLUTION**

System of $81 \times 81 \times 21 \sim 140k$ ODE for each $\theta_{in}$

$$\partial_{\tau} g_{ij}(\tau) = \sum_{k \neq i, k \neq j, k \in C_{in}} \frac{\Delta \Omega}{4\pi} w_{ij}(k) U_{ijk} [g_{ik}(\tau) g_{kj}(\tau) - g_{ij}(\tau)]$$

$$g_{ij}(0) = 1 \quad \tau \in [0,0.8]$$

Runge-Kutta 4 with tau step $\Delta \tau = 0.01$

use cpython

for each $\theta_{in}$ can be solved within 10h in Supercomputing Data Center PIK (SDC PIK)

Solution is $\sim 8$ Gb .npz (numpy) files
REPRODUCE RESULTS

HATTA AT AL. PHYS. REV. D 87, 054016 (2013)

ATLAS MEASUREMENT at 7 TEV

\[ \mathcal{R}(\Delta y, p_T) = \frac{d\sigma^{\text{veto}}/d\Delta y d^2 p_T}{d\sigma^{\text{incl}}/d\Delta y d^2 p_T} \]

\[ \sqrt{s} = 7 \text{ TeV} \]
\[ 70 < p_T < 90 \text{ GeV} \]
\[ p_T^{\text{veto}} = 20 \text{ GeV} \]
\[ \alpha_s(M_Z) = 0.12 \]
\[ \text{NNPDF30} \_\text{lo} \_\text{as} \_0130 \]
\[ \text{anti} - k_t(R = 0.6) \]

\[ \sqrt{s} = 7 \text{ TeV} \]
\[ 90 < p_T < 120 \text{ GeV} \]
\[ p_T^{\text{veto}} = 20 \text{ GeV} \]
\[ \alpha_s(M_Z) = 0.12 \]
\[ \text{NNPDF30} \_\text{lo} \_\text{as} \_0130 \]
\[ \text{anti} - k_t(R = 0.6) \]

\[ \sqrt{s} = 7 \text{ TeV} \]
\[ 120 < p_T < 150 \text{ GeV} \]
\[ p_T^{\text{veto}} = 20 \text{ GeV} \]
\[ \alpha_s(M_Z) = 0.12 \]
\[ \text{NNPDF30} \_\text{lo} \_\text{as} \_0130 \]
\[ \text{anti} - k_t(R = 0.6) \]
SHORTCOMINGS

• Solution known only in knots - difficult to use with MC integration

• equal spacing in $\theta_{\text{in}}$ - rough at large rapidities

$\theta_{\text{in}} = \{0^\circ, 1^\circ, 2^\circ, 3^\circ, 4^\circ, 5^\circ\} \implies \eta = \{\infty, 4.05, 3.35, 2.94, 2.66, 2.43\}$

• difficult to apply to jet veto (not inter jet veto)
\[ S_{ab}(\omega_q, Q_0) \cdot r = S_{ab}(Q, Q_0) \]

\[ \ln S_{ab}(Q, Q_0) = -\int_0^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \tilde{\alpha}_s(q_{ab}) w_{ab}(q) \theta(q_{ab} - Q_0) \]

\[ \frac{dR_{ab}(\Omega_q)}{\Omega_q} = \frac{\tilde{\alpha}_s(q_{ab}) w_{ab}(q)}{N_{ab}(\omega_q)} \theta(q_{ab} - Q_0) \]