

# **PRODUCTION OF DIJETS WITH LARGE RAPIDITY SEPARATION AT COLLIDERS**

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# MOTIVATION

## ENERGY FLOW AND JET VETO

- Many signals of new physics is observed via hadronic jets
- Multiple soft gluon emission can transfer the energy or transverse momentum away from hard jets (ENERGY FLOW)
- It is not possible to forbid very soft real QCD emission as well as virtual QCD corrections experimentally
- Various Veto conditions are introduced experimentally (JET VETO)
- Our aim is to calculate the effect of JET VETO

# INTERJET VETO

## SEARCH FOR THE BALITSKY-FADIN-KURAEV-LIPATOV (BFKL) EFFECTS IN THE ATLAS EXPERIMENT

G.Aad et al. (ATLAS Collaboration) J. High Energy Phys. 09(2011)053

$$\mathcal{R}(\Delta y, \overline{p}_T) = \frac{d\sigma^{dijet\ veto} / d\Delta y d^2\overline{p}_T}{d\sigma^{dijet} / d\Delta y d^2\overline{p}_T}$$

- pp at 7 TeV

- two selection types:

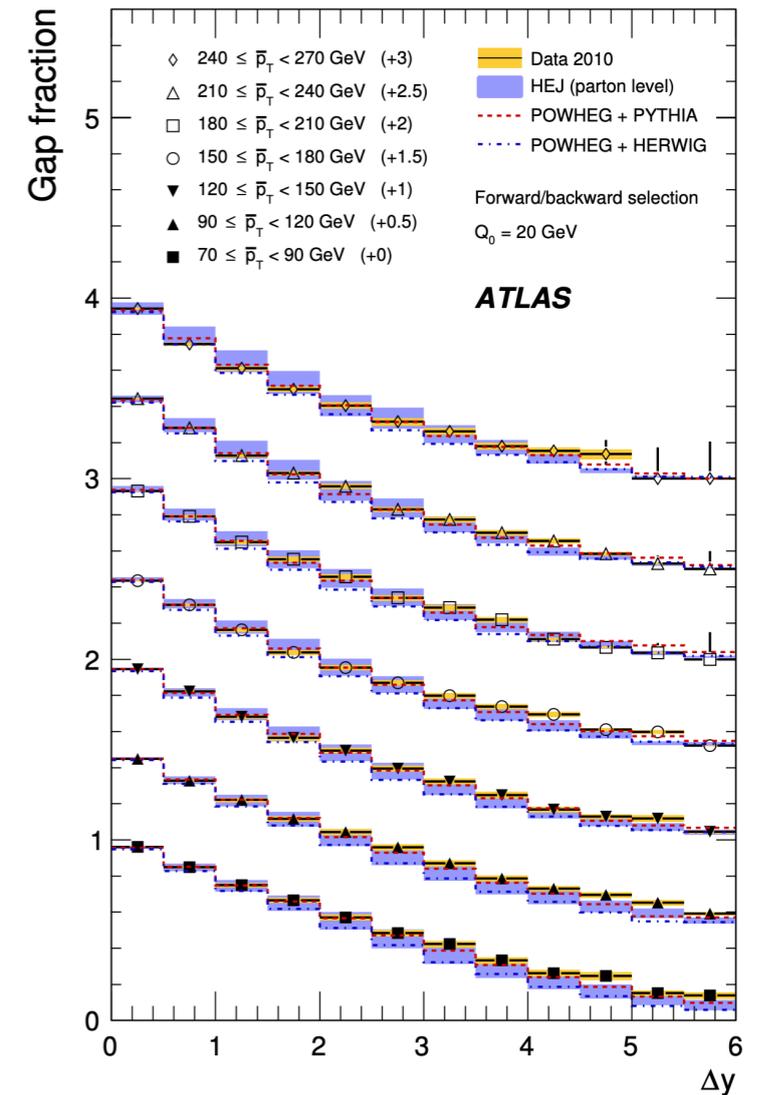
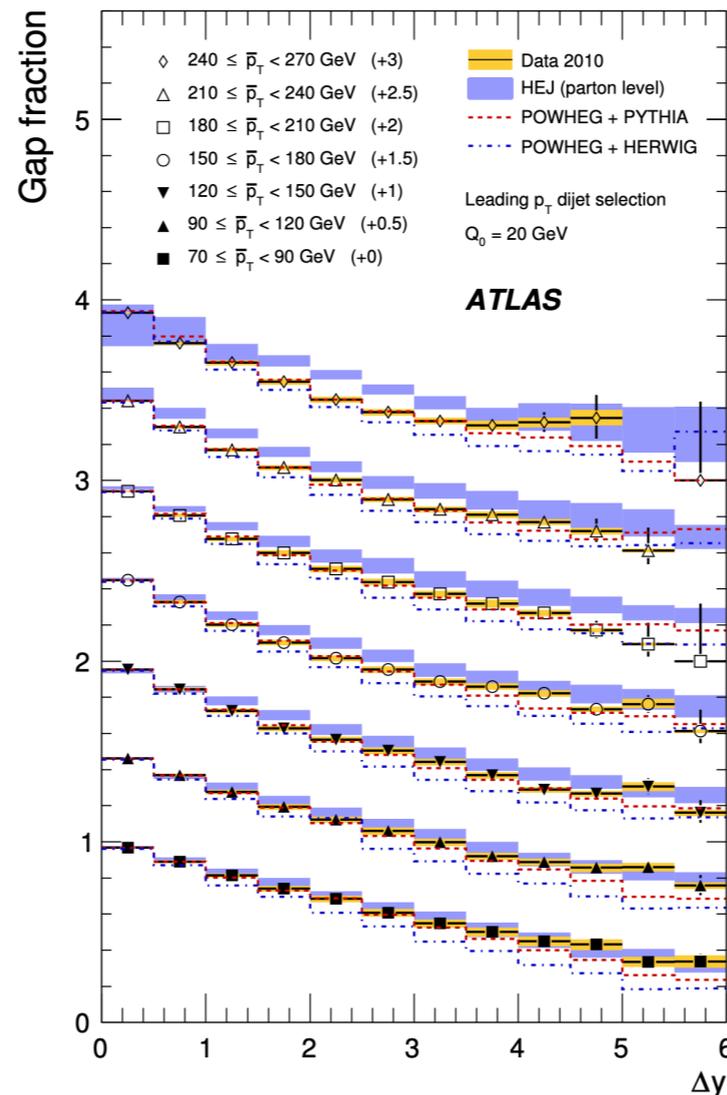
Leading  $p_T$  dijet

Forward/Backward dijet

-  $70 < \overline{p}_T < 270$  GeV

-  $\Delta y = |y_1 - y_2| < 6$

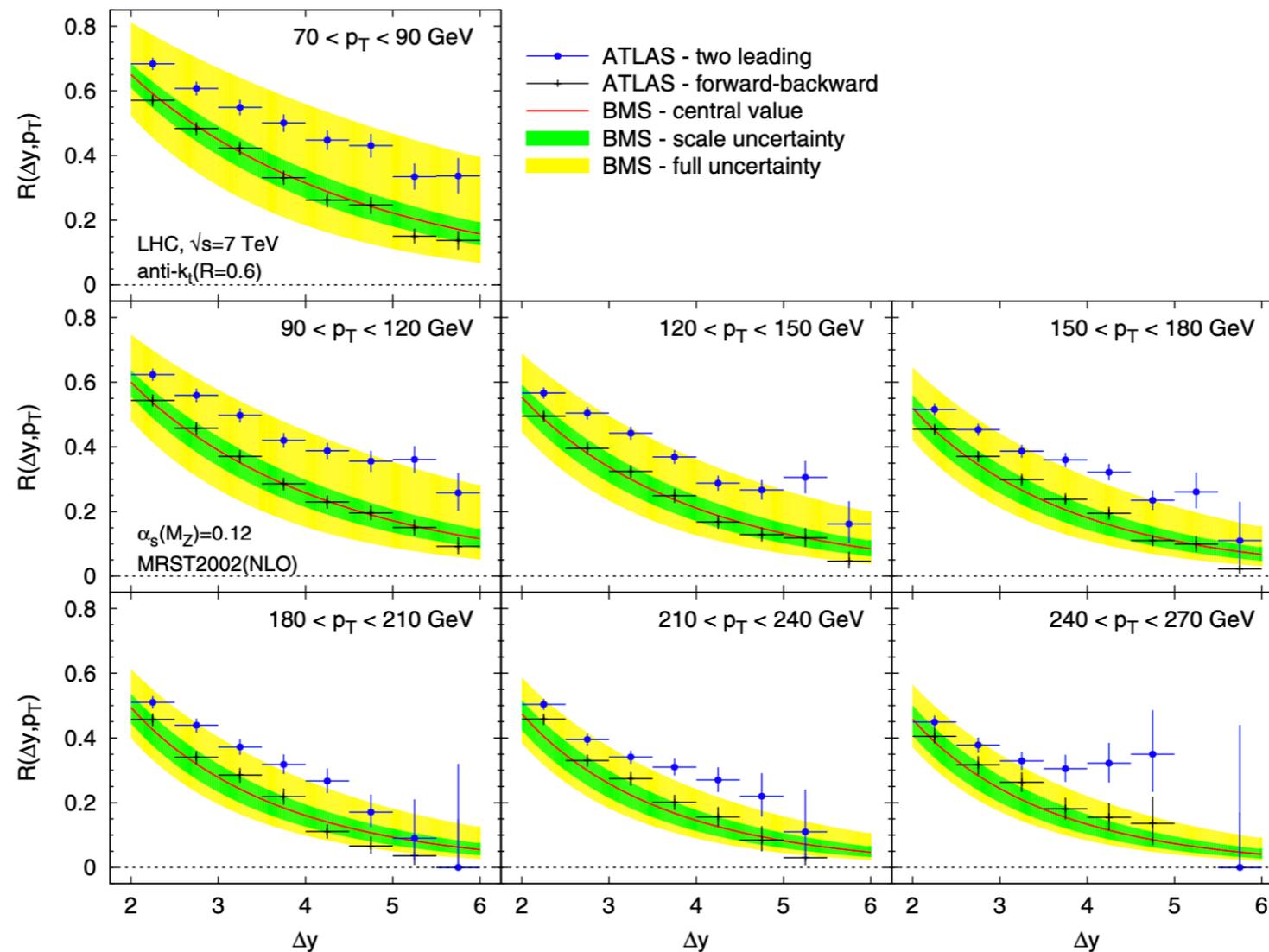
- **Veto  $Q = 20$  GeV between jets**



# APPROACH OF BMS

BANFI-MARCHESINI-SMYE (BMS) *JHEP* 08(2002)006

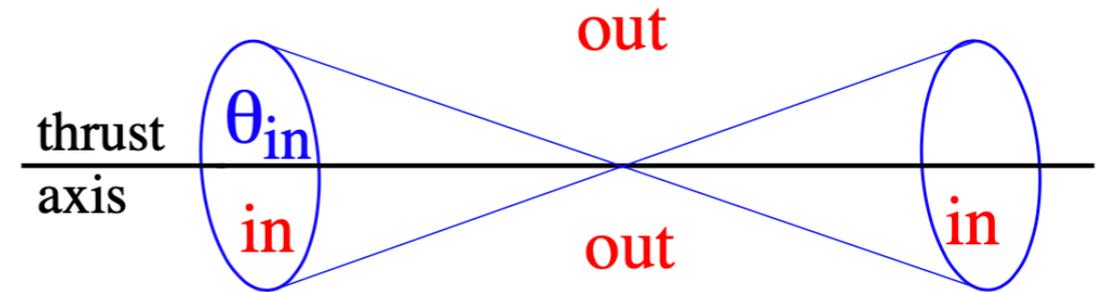
Y.Hatta et al. *Phys. Rev. D* 87, 054016 (2013)



# BMS EQUATION

BANFI-MARCHESINI-SMYE (BMS) *JHEP* 08 (2002) 006

$$\Sigma_{out}(Q, E_{out}) = \sum_n \int \frac{d\sigma_n}{\sigma_T} \Theta(E_{out} - \sum_{out} q_{ti})$$



Soft gluon approximation, large  $N_c$ , energy ordering;

Large angle emission, Sudakov and non-global logarithms;

$$\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab}) \Sigma_{ab} + \int_{in} \frac{d\Omega_q}{4\pi} w_{ab}(q) [\Sigma_{aq}(\tau) \Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)]$$

$$\tau = \int_{Q_0}^Q \frac{dq_t}{q_t} \bar{\alpha}_s(q_t); \quad Q_0 = E_{out}; \quad \bar{\alpha}_s = \frac{N_c \alpha_s}{\pi} = \frac{C_A \alpha_s}{\pi}$$

$$R_{ab}(\tau) = \int_{E_{out}}^Q \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \int_{out} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab}; \quad w_{ab}(q) = \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{aq})(1 - \cos \theta_{qb})}$$

# RATIOS OF X-SECTIONS

## VETO TO INCLUSIVE

$$\mathcal{R}(\Delta y, p_T) = \frac{d\sigma^{\text{veto}}/d\Delta y d^2p_T}{d\sigma^{\text{incl}}/d\Delta y d^2p_T}$$

## inclusive x-section:

$$\frac{d\sigma^{\text{incl}}}{d\Delta y d^2p_T} = \sum_{ij}^{q, \bar{q}, g} \int_{Y_{\min}(p_T, \Delta y)}^{Y_{\max}(p_T, \Delta y)} dY x_1 f_i(x_1, p_T) x_2 f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

## qq' scattering with one gluon exchange

$$\frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} h^A(\hat{s}, \hat{t}, \hat{u}) \quad h^A(s, t, u) = g^4 \frac{C_F}{N_c} \left( \frac{s^2 + u^2}{t^2} \right)$$

if consider only  $\Delta y > 0$

$$\frac{d\hat{\sigma}_{qq'}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} (h^A(\hat{s}, \hat{t}, \hat{u}) + h^A(\hat{s}, \hat{u}, \hat{t}))$$

# INTERJET VETO

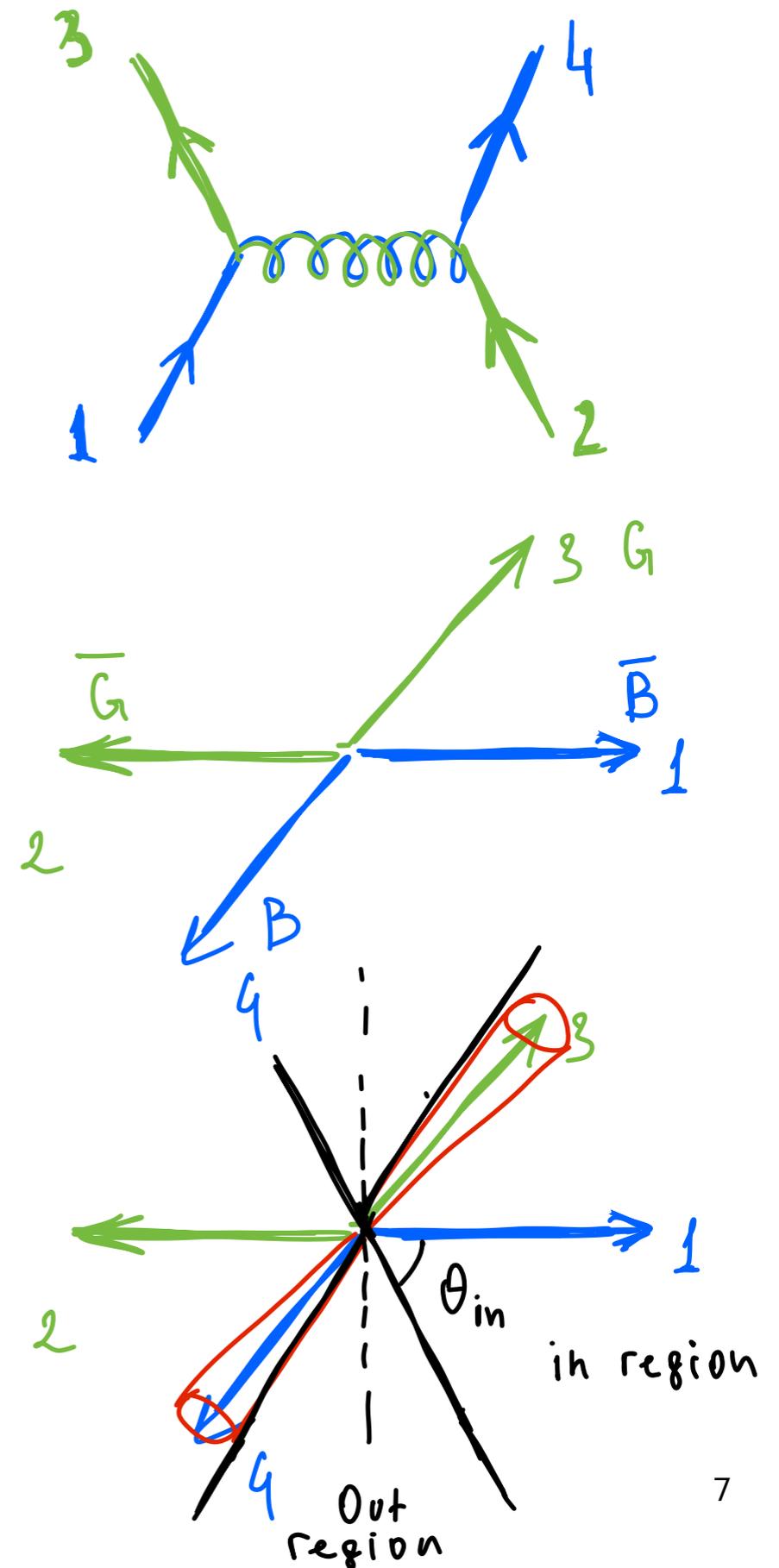
$$\frac{d\sigma^{\text{veto}}}{d\Delta y d^2p_T} = \sum_{ij}^{q, \bar{q}, g} \int_{Y_{\min}(p_T, \Delta y)}^{Y_{\max}(p_T, \Delta y)} dY x_1 f_i(x_1, p_T) x_2 f_j(x_2, p_T) \frac{1}{\pi} \frac{d\hat{\sigma}_{ij}^{\text{veto}}}{d\hat{t}}$$

at large  $N_c$  limit, and in one-gluon exchange, color flows as  $1 \rightarrow 4$  and  $2 \rightarrow 3$

$$\frac{d\hat{\sigma}_{qq'}^{\text{veto}}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} (h^A(\hat{s}, \hat{t}, \hat{u}) \Sigma_{14}\Sigma_{23} + h^A(\hat{s}, \hat{u}, \hat{t}) \Sigma_{13}\Sigma_{24})$$

$$y_{in} = \frac{\Delta y}{2} - R_{jet}$$

$$y_{in} = -\log \tan\left(\frac{\theta_{in}}{2}\right)$$



# JET VETO

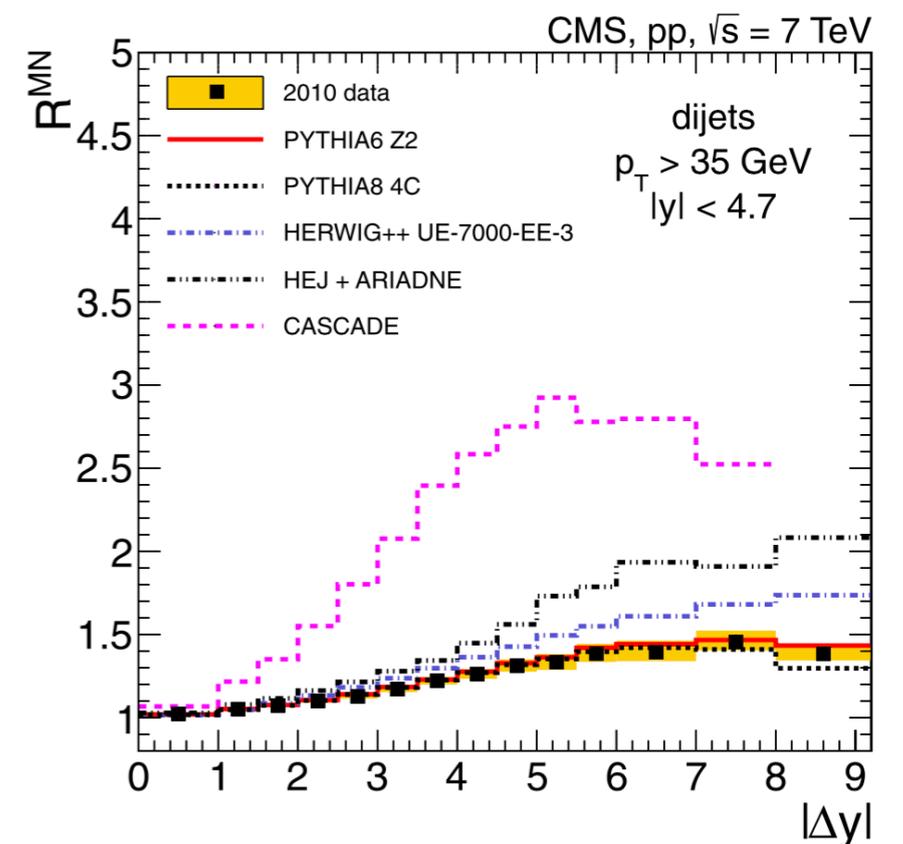
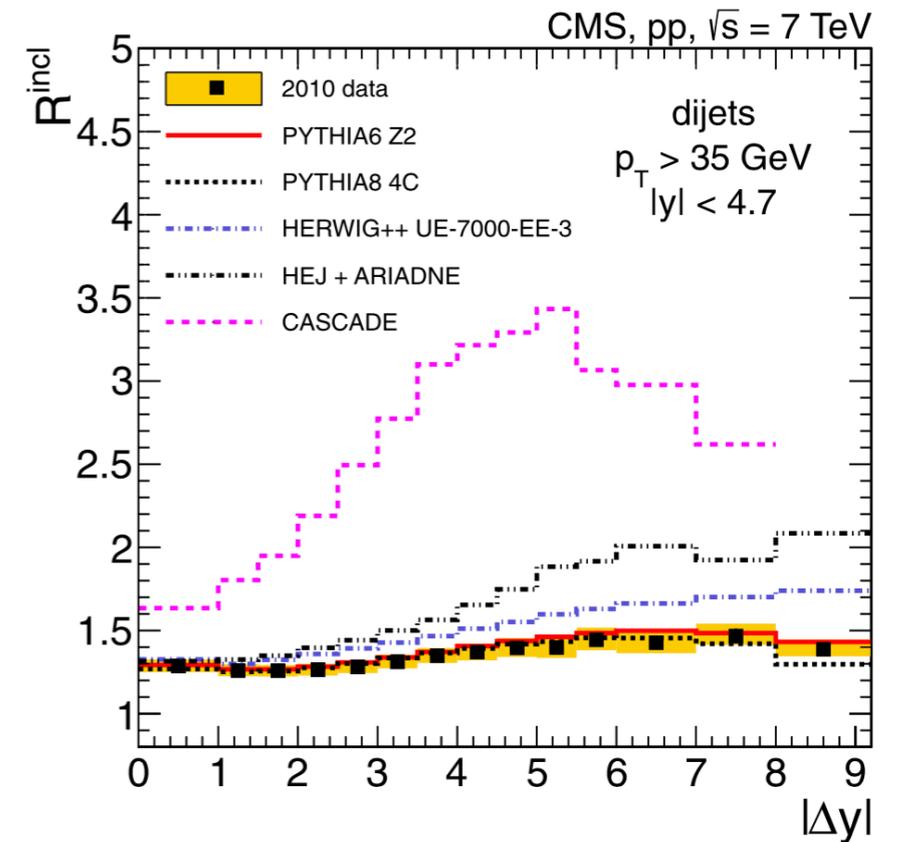
CMS SEARCH OF BFKL EUR. PHYS. J. C (2012) 72:2216

- pp at 7 TeV
- two selection types:

Inclusive dijets

Mueller-Navelet dijets

- $p_T > 35$  GeV
- $\Delta y = |y_1 - y_2| < 9.4$
- **Veto**  $p_{T\ veto} = 35$  GeV



# JET VETO

$$y_{bound} = 4.7$$

$$\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab})\Sigma_{ab} + \int_{C_{in}} \frac{d\Omega_q}{4\pi} w_{ab}(q) [\Sigma_{aq}(\tau)\Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)]$$

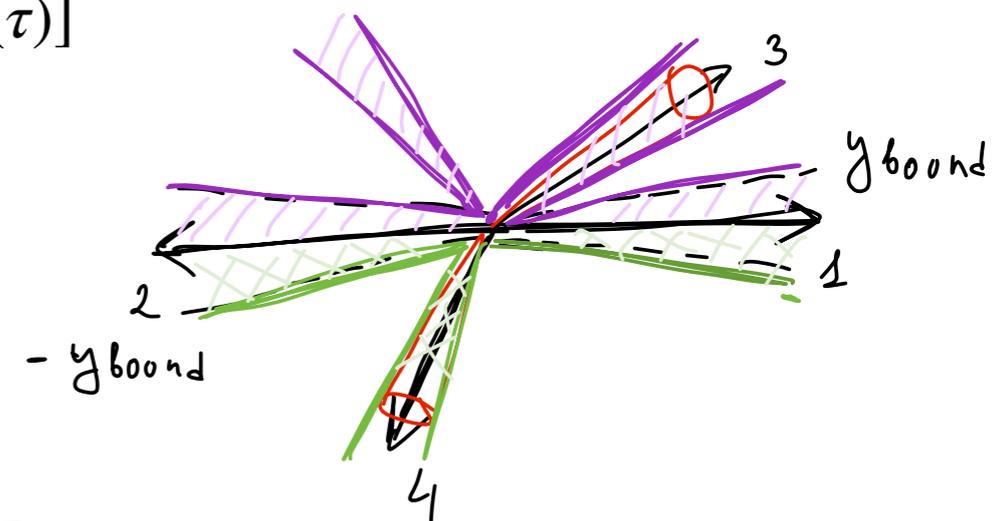
$$R_{ab}(\tau) = \int_{E_{out}}^Q \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \int_{C_{out}} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab} C_{out}$$

Before  $C_{out}$  is controlled by  $\Delta y$  (100 Systems)

Now  $C_{out}$  is a function of dipole type as well as:

$$y_{bound}, Y, \Delta y$$

(100x100x6 = 60k Systems)



$$\Sigma_{24} \quad C_{out} \in [-y_{bound}, y_4 - R] \\ [y_4 + R, y_{bound}]$$

$$\Sigma_{34} \quad C_{out} \in [-y_{bound}, y_4 - R] \\ [y_4 + R, y_3 - R] \\ [y_3 + R, y_{bound}]$$

# LARGE ANGLE SOFT MC

MARCHESINI 2006 [ARXIV:HEP-PH/0601068](https://arxiv.org/abs/hep-ph/0601068)

**Generating functional**

$$\Sigma_{ab}[Q, u] = \sum_n \frac{1}{n!} \int \frac{d\sigma_{ab}^{(n)}}{\sigma_{abT}} \prod_{i=1}^n u(q_i)$$

**In soft and planar limit**

$$Q\partial_Q \Sigma_{ab}[Q, u] = \int \frac{d\Omega_q}{4\pi} \bar{\alpha}_s w_{ab}(q) \{ u(q) \Sigma_{aq}[Q, u] \Sigma_{qb}[Q, u] - \Sigma_{ab}[Q, u] \}$$

**Sudakov form factor**

$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(q_{abt}) w_{ab}(q) \theta(q_{abt} - Q_0)$$

$$q_{abt}^2 = \frac{2\omega_q^2}{w_{ab}(q)} \quad w_{ab}(q) = \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{aq})(1 - \cos \theta_{qb})}$$

$$\Sigma_{ab}[Q, u] = S_{ab}(Q, Q_0) + \int_{Q_0}^Q dP_{ab}(q) u(q) \Sigma_{aq}[\omega_q, u] \Sigma_{qb}[\omega_q, u]$$

# MC ALGORITHM

MARCHESINI 2006 [ARXIV:HEP-PH/0601068](https://arxiv.org/abs/hep-ph/0601068)

Splitting probability  $dP_{ab}(q) = d\left(\frac{S_{ab}(Q, Q_0)}{S_{ab}(\omega_q, Q_0)}\right) dR_{ab}(\Omega_q)$

Angle distribution  $\frac{dR_{ab}(\Omega_q)}{d\Omega_q} = \frac{\bar{\alpha}_s(q_{abt})w_{ab}(q)}{N_{ab}(\omega_q)}\theta(q_{abt} - Q_0)$

Algorithm: start with ab-dipole.

use r random number to generate energy of branch:

if  $r < S(Q, Q_0)$  - dipole does not branch (with  $Q_0$  cutoff).

if  $r > S(Q, Q_0)$  - dipole split with  $\omega_q$  such that  $S_{ab}(\omega_q, Q_0) \cdot r = S_{ab}(Q, Q_0)$

generate angle with  $dR_{ab}(\Omega_q)$

repeat with aq and qb dipoles until  $Q_0$

$$q_{abt}^2 = \frac{2\omega_q^2}{w_{ab}(q)}$$

**MC ALGORITHM DOES NOT REPRODUCE BMS EQUATION**

**MC**

$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(q_{abt}) w_{ab}(q) \theta(q_{abt} - Q_0)$$

**BMS**

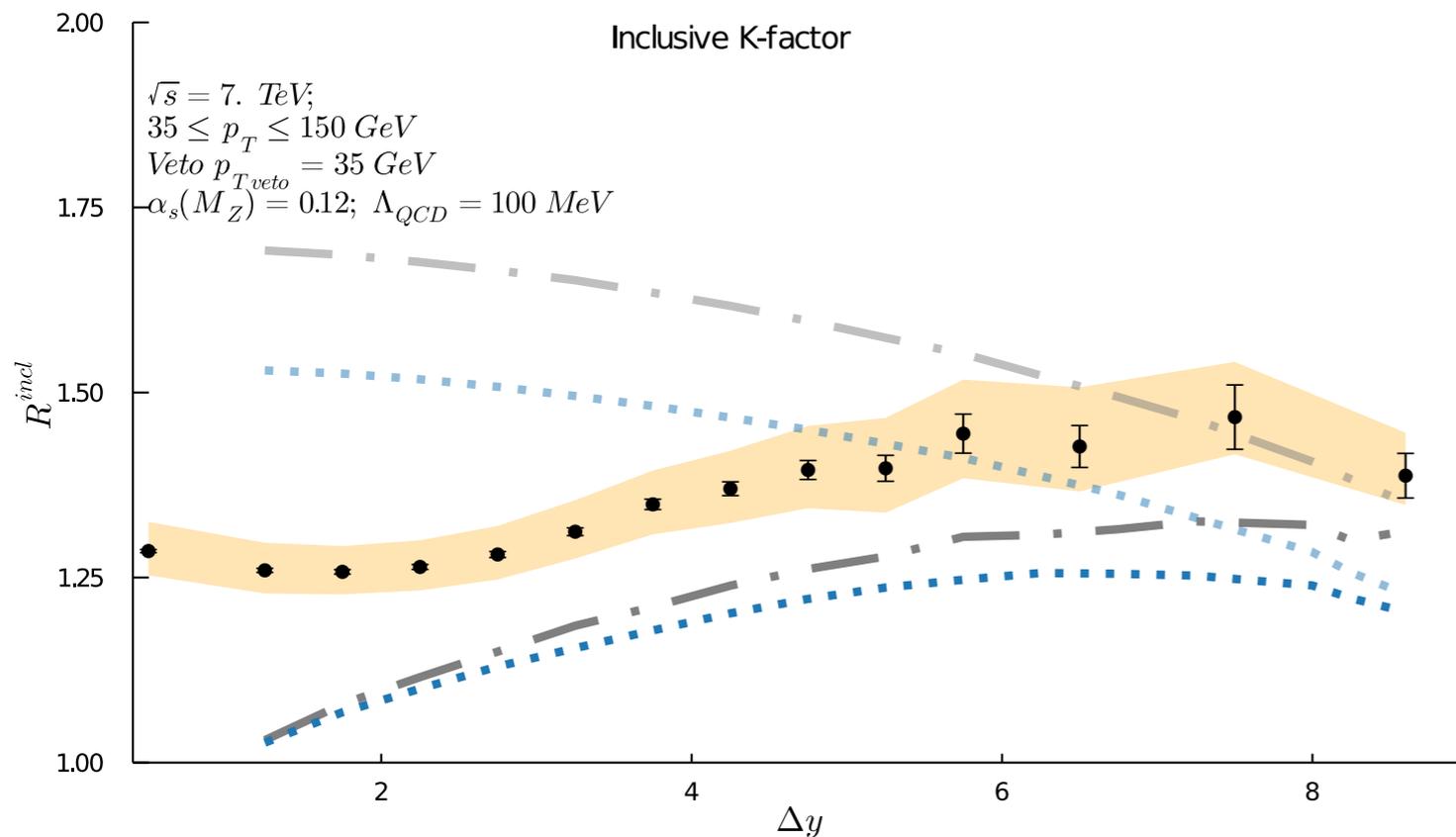
$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(\omega_q) w_{ab}(q)$$



$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(\omega_q) w_{ab}(q) \Theta(\cos(\theta)_{\max} - \cos(\theta))$$

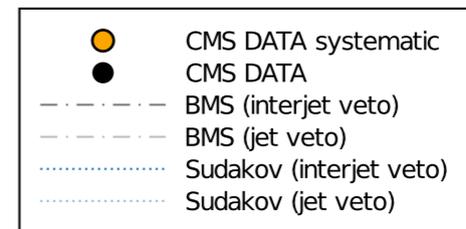
# RESULTS

## SUDAKOV AND BMS



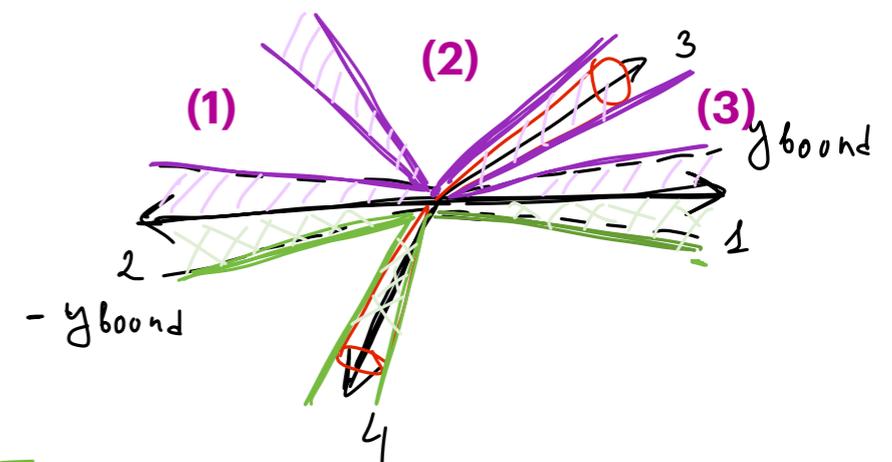
- **Sudakov:**  $\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab})\Sigma_{ab}$

$$R_{ab}(\tau) = \int_{E_{out}}^Q \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \int_{C_{out}} \frac{d\Omega_q}{4\pi} w_{ab}(q) \approx \tau f_{ab} C_{out}$$



- **BMS**

$$\Sigma_{34}(\tau) = \Sigma_{34}(\tau)^{(1)} \Sigma_{34}(\tau)^{(2)} \Sigma_{34}(\tau)^{(3)}$$

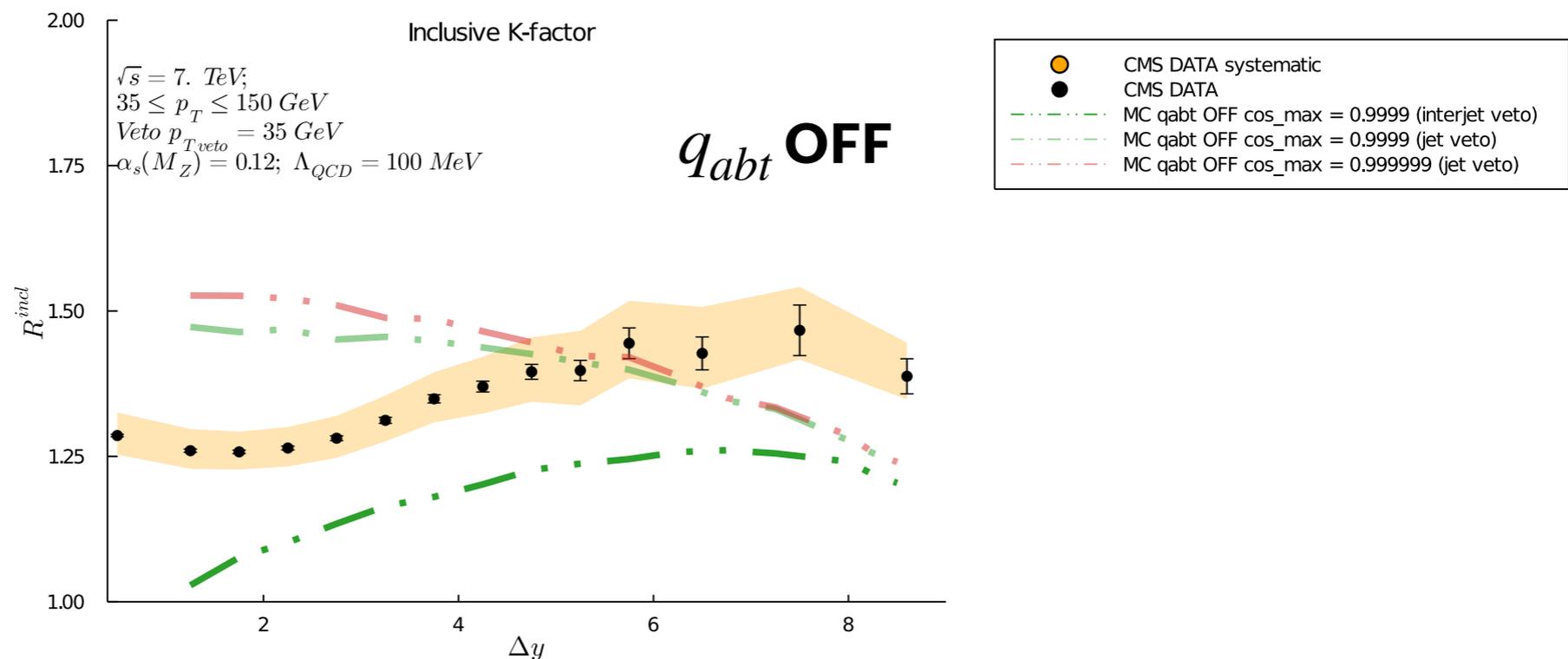
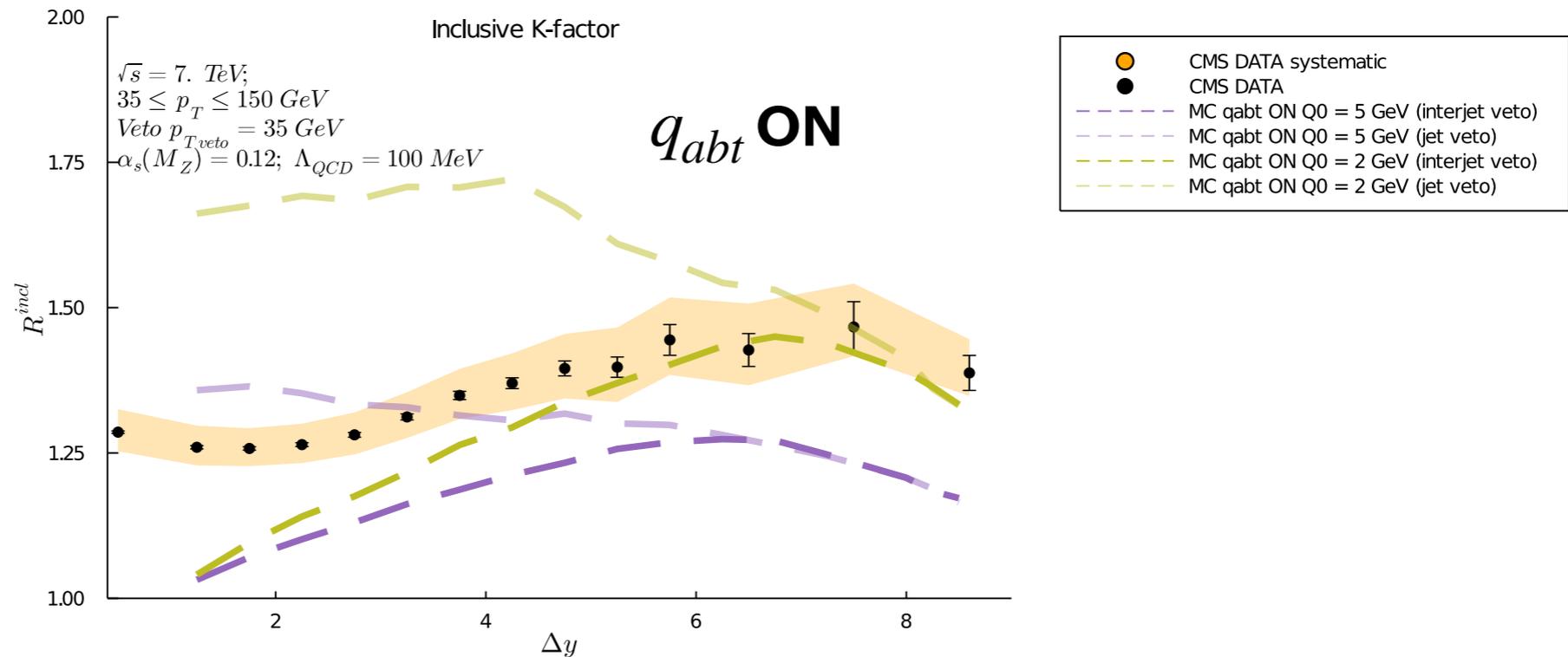


$$\Sigma_{24} C_{out} \in [-y_{bound}, y_4 - R] \\ [y_4 + R, y_{bound}]$$

$$\Sigma_{34} C_{out} \in [-y_{bound}, y_4 - R] \\ [y_4 + R, y_3 - R] \\ [y_3 + R, y_{bound}]$$

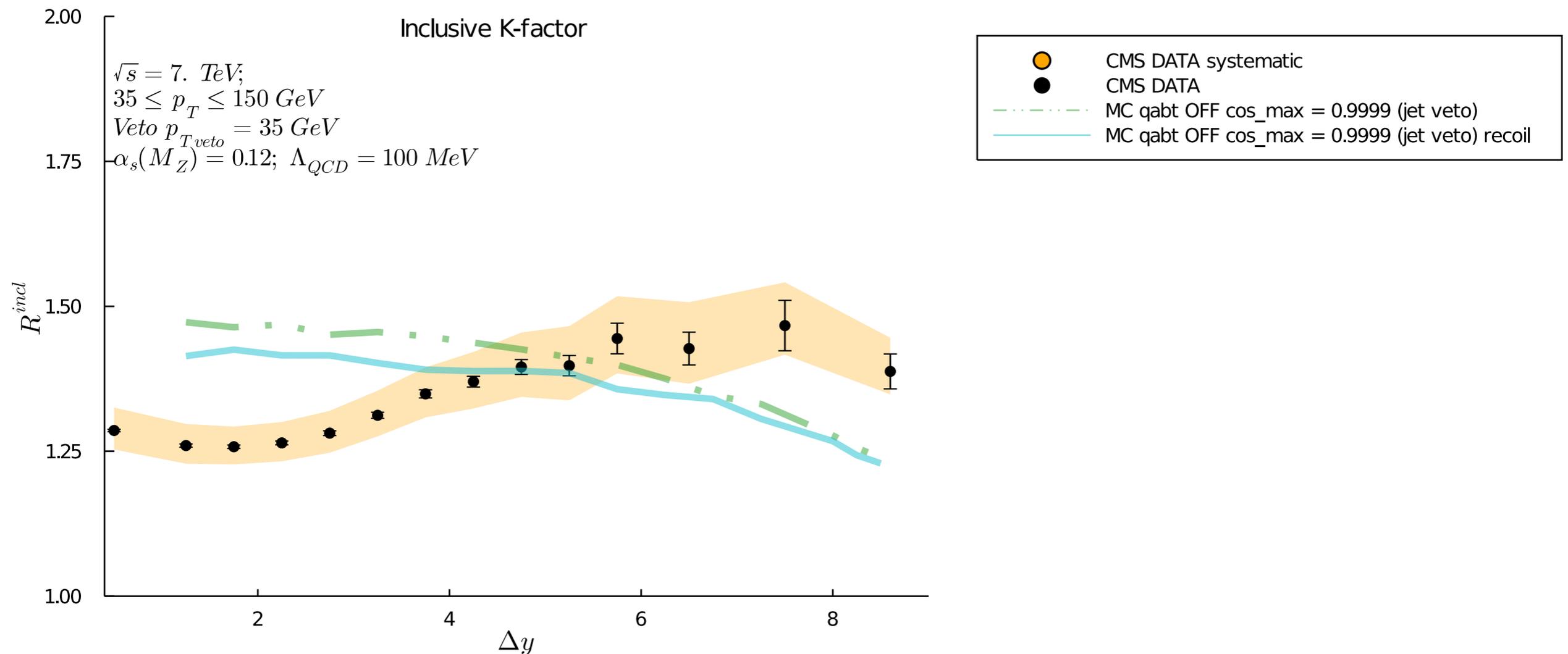
# RESULTS

MC



# IMPROVEMENTS

ATTEMPT TO TAKE RECOIL INTO ACCOUNT (ENERGY-MOMENTUM CONSERVATION)



# CONCLUSIONS

- **Soft gluon approximation is suitable tool to study the ENERGY FLOW and JET VETO**
- **MC algorithm gives solution much faster**
- **MC algorithm is more suitable for jet veto**
- **Improvements are needed (such as inclusion the energy-momentum conservation and recoil)**
- **This tool works better when Veto region is far from jets**

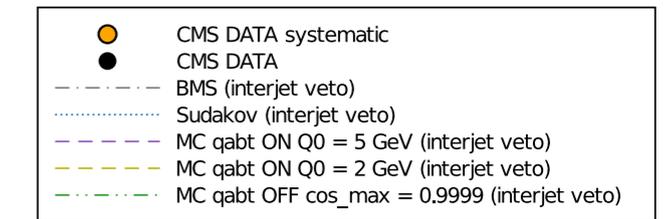
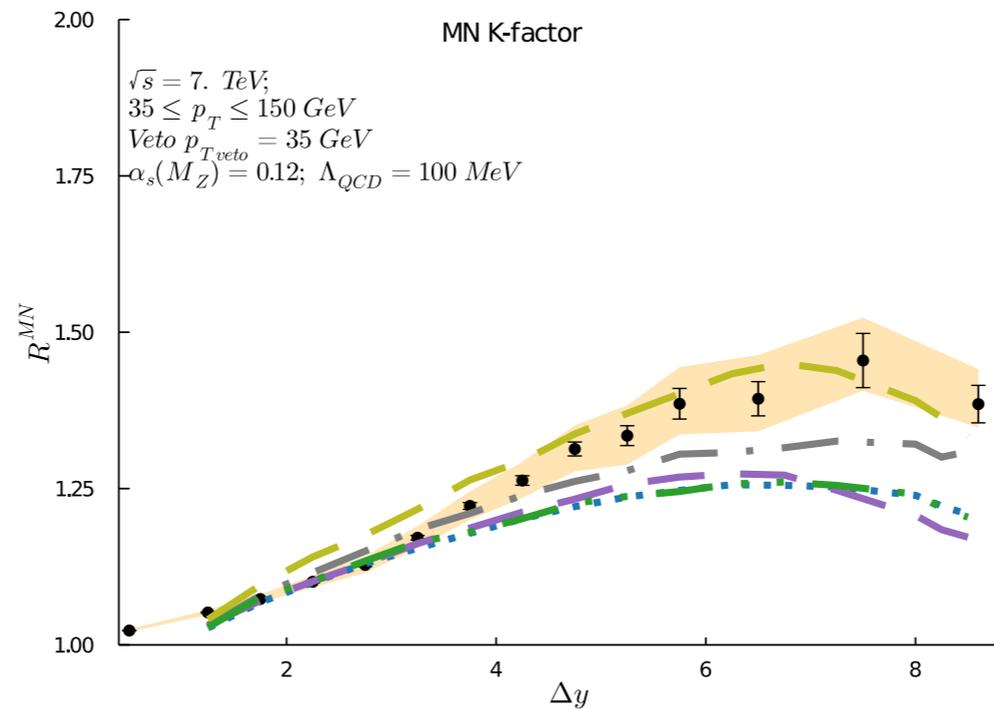
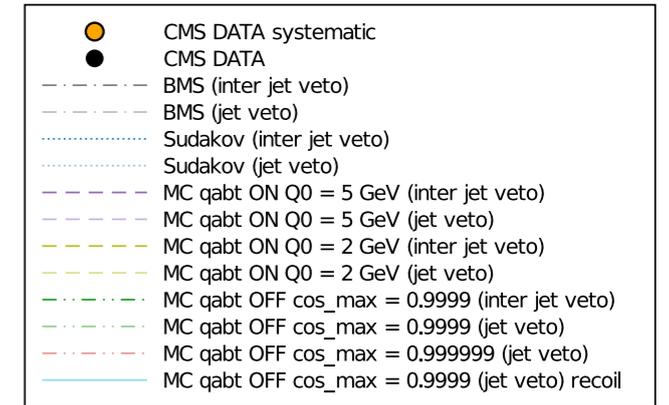
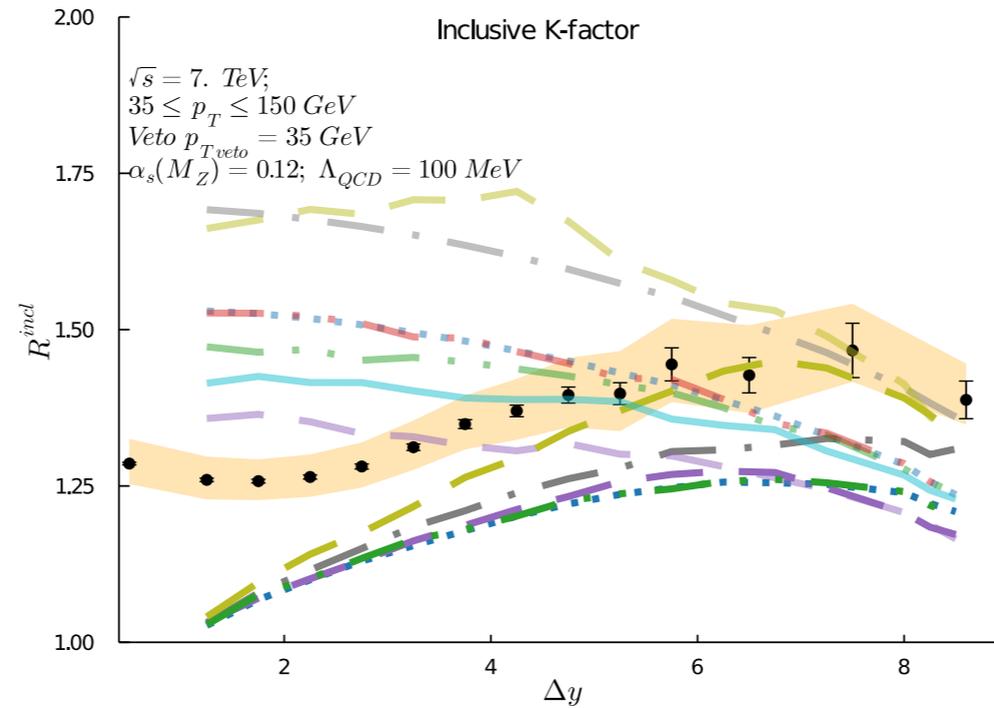
**THANK YOU**



**BACKUP**

# RESULTS

ALL IN ONE



# COLOR FLOW

PDG REVIEW  $gg \rightarrow gg$  x-section

$$\frac{d\sigma_{gg \rightarrow gg}}{d\Omega} = \frac{9\alpha_s^2}{8s} \left( 3 - \frac{ut}{s^2} - \frac{su}{t^2} - \frac{st}{u^2} \right)$$

BMS JHEP 08 (2002) 006

$$\frac{d\sigma_{gg \rightarrow gg}}{dt} = \frac{1}{16\pi s} (h^D(s, t, u) + h^D(s, u, t) + h^D(u, t, s))$$

$$h^D(s, t, u) = 2g^4 \frac{N_c^2}{N_c^2 - 1} \left( 1 - \frac{tu}{s^2} - \frac{su}{t^2} + \frac{u^2}{st} \right)$$

VETO

$$\frac{d\sigma_{gg \rightarrow gg}^{veto}}{dt} = \frac{1}{16\pi s} (h^D(s, t, u) \Sigma_{12} \Sigma_{13} \Sigma_{24} \Sigma_{34} + h^D(s, u, t) \Sigma_{12} \Sigma_{14} \Sigma_{23} \Sigma_{34} + h^D(u, t, s) \Sigma_{14} \Sigma_{24} \Sigma_{13} \Sigma_{23})$$

# SOLUTION OF BMS

$$\partial_\tau \Sigma_{ab}(\tau) = -(\partial_\tau R_{ab})\Sigma_{ab} + \int_{in} \frac{d\Omega_q}{4\pi} w_{ab}(q) [\Sigma_{aq}(\tau)\Sigma_{qb}(\tau) - \Sigma_{ab}(\tau)]$$

in region  $C_{in}$

$$\phi_q \in [0, 2\pi]; \quad \theta_q \in [0, \theta_{in}] \cup [\pi - \theta_{in}, \pi]$$

$\tau$  range

$$\tau = \int_{Q_0}^Q \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) \quad \tau_{max} = \int_{p_{T \text{ veto}}}^{\sqrt{s}/2} \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) = \int_{20}^{7000} \frac{dq_t}{q_t} \bar{\alpha}_s(q_t) = 0.651$$

$$\Sigma_{ab}(\tau) = \Sigma(\theta_a, \phi_a, \theta_b, \phi_b, \tau, \theta_{in}) = \Sigma(\theta_a, \theta_b, \Delta\phi, \tau, \theta_{in})$$

we have to solve the equation for each  $\theta_{in}(\Delta y)$

# SOLUTION OF BMS

## NUMERICAL SOLUTION

Make transform  $\Sigma_{ab}(\tau) = e^{-\tau f_{ab}} g_{ab}(\tau)$

$$\partial_{\tau} g_{ab}(\tau) = \int_{C_{in}} \frac{d\Omega_q}{4\pi} w_{ab}(q) U_{abq} [g_{aq}(\tau) g_{qb}(\tau) - g_{ab}(\tau)]$$

$$U_{abq} = e^{-\tau(f_{aq} + f_{qb} - f_{ab})}$$

Calculate  $\theta_{in}$  in 100 points from 0 to  $\pi/2$

Divide in region  $\theta_q \in [0, \theta_{in}] \cup [\pi - \theta_{in}, \pi]$  in 81 points

Divide  $\Delta\phi \in [0, \pi]$  in 21 points

$$\partial_{\tau} g_{ij}(\tau) = \sum_{k \neq i, k \neq j, k \in C_{in}} \frac{\Delta\Omega_k}{4\pi} w_{ij}(k) U_{ijk} [g_{ik}(\tau) g_{kj}(\tau) - g_{ij}(\tau)]$$

# SOLUTION OF BMS

## NUMERICAL SOLUTION

System of  $81 \times 81 \times 21 \sim 140k$  ODE for each  $\theta_{in}$

$$\partial_{\tau} g_{ij}(\tau) = \sum_{k \neq i, k \neq j, k \in C_{in}} \frac{\Delta \Omega_k}{4\pi} w_{ij}(k) U_{ijk} [g_{ik}(\tau) g_{kj}(\tau) - g_{ij}(\tau)]$$

$$g_{ij}(0) = 1 \quad \tau \in [0, 0.8]$$

Runge-Kutta 4 with tau step  $\Delta\tau = 0.01$

use cpython

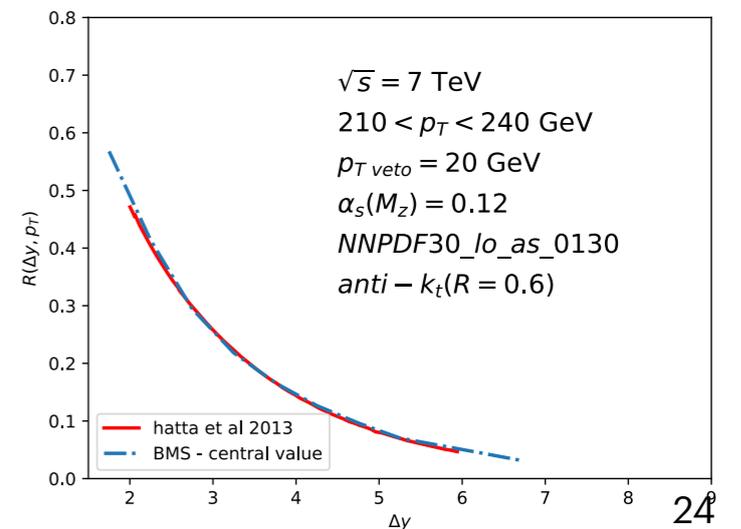
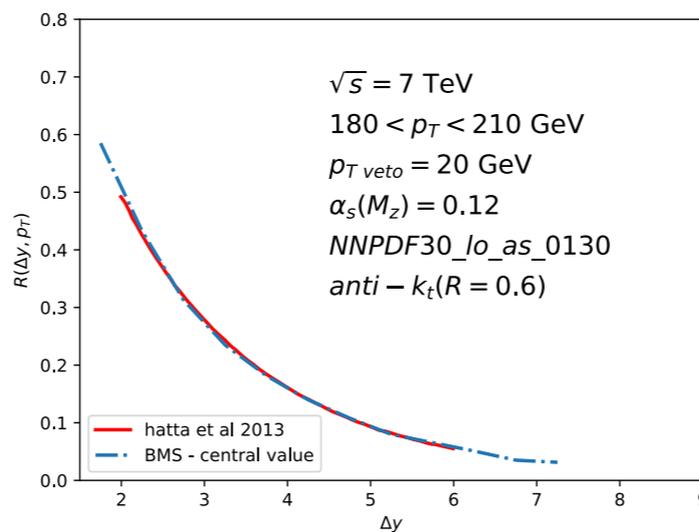
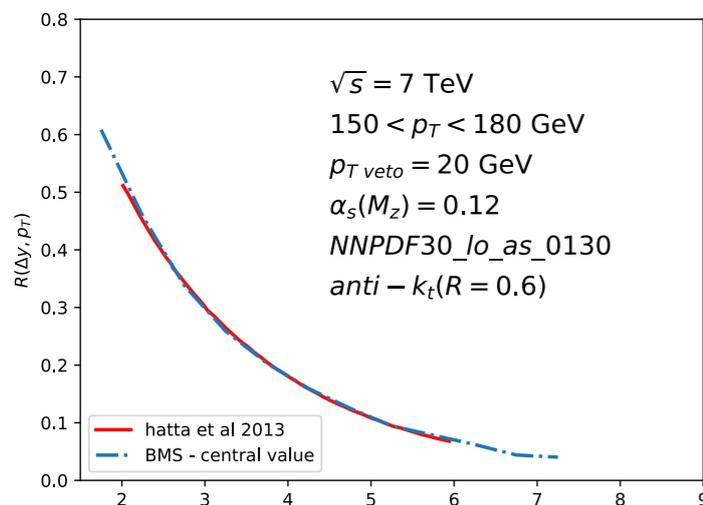
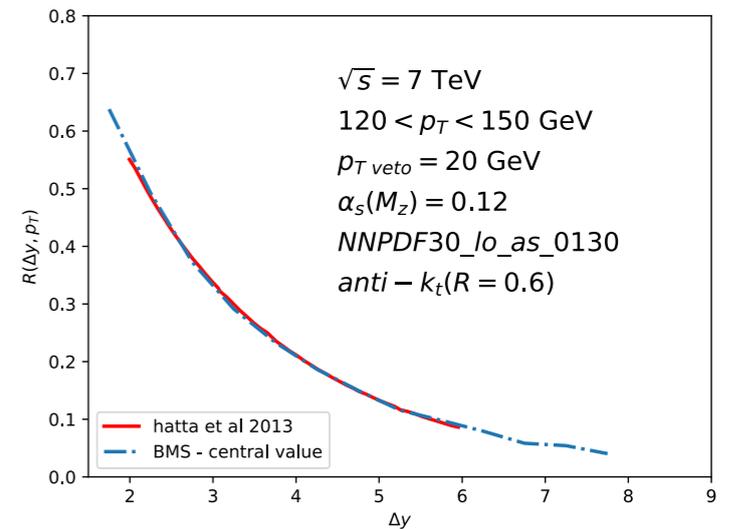
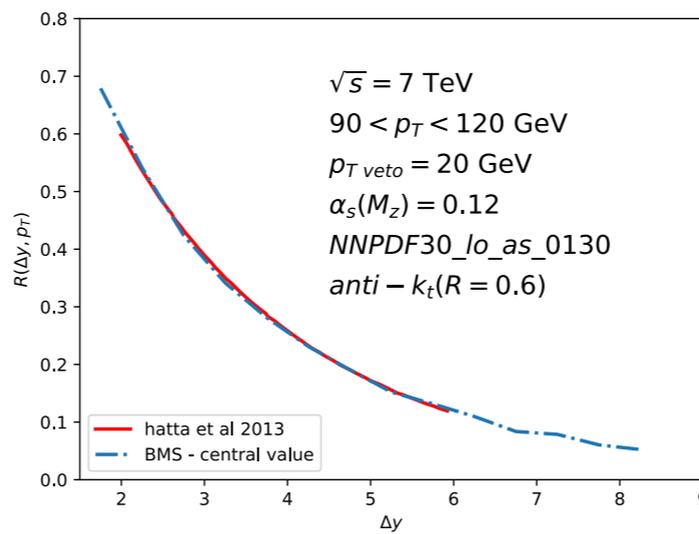
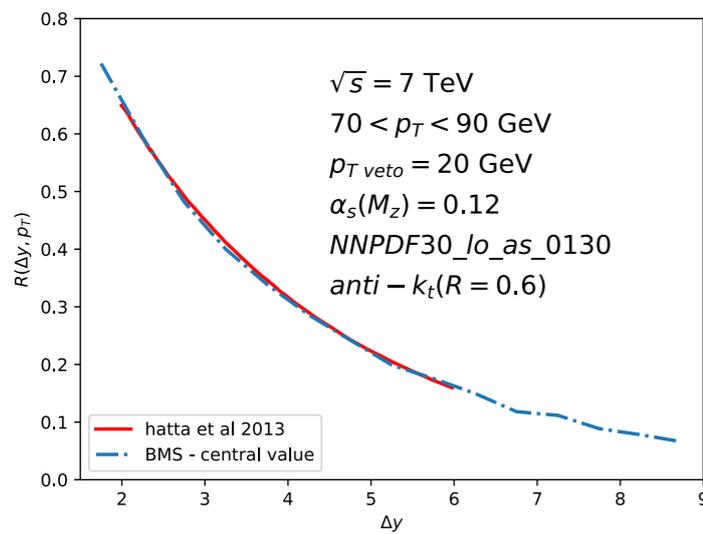
for each  $\theta_{in}$  can be solved within 10h in Supercomputing Data Center PIK (SDC PIK)

Solution is  $\sim 8$  Gb .npz (numpy) files

# REPRODUCE RESULTS

HATTA AT AL. PHYS. REV. D 87, 054016 (2013)

ATLAS MEASUREMENT at 7 TEV  $\mathcal{R}(\Delta y, p_T) = \frac{d\sigma^{\text{veto}} / d\Delta y d^2 p_T}{d\sigma^{\text{incl}} / d\Delta y d^2 p_T}$



# SHORTCOMINGS

- Solution known only in knots - difficult to use with MC integration
- equal spacing in  $\theta_{in}$  - rough at large rapidities

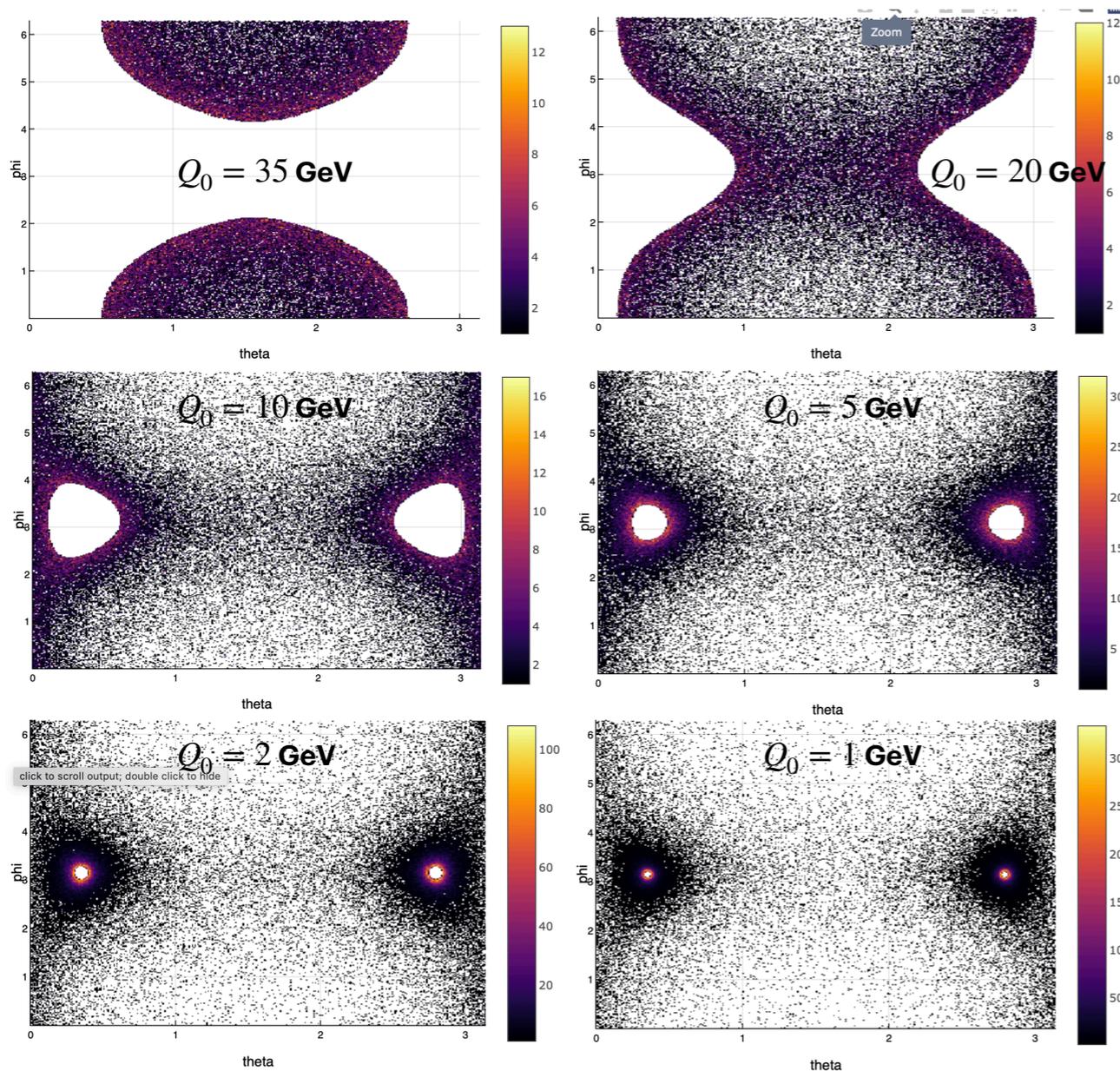
$$\theta_{in} = \{0^\circ, 1^\circ, 2^\circ, 3^\circ, 4^\circ, 5^\circ\} \implies \eta = \{\infty, 4.05, 3.35, 2.94, 2.66, 2.43\}$$

- difficult to apply to jet veto (**not inter jet veto**)

# REALISATION MC

$$S_{ab}(\omega_q, Q_0) \cdot r = S_{ab}(Q, Q_0)$$

$$\ln S_{ab}(Q, Q_0) = - \int_{Q_0}^Q \frac{d\omega_q}{\omega_q} \frac{d\Omega_q}{4\pi} \bar{\alpha}_s(q_{abt}) w_{ab}(q) \theta(q_{abt} - Q_0)$$



$$\frac{dR_{ab}(\Omega_q)}{\Omega_q} = \frac{\bar{\alpha}_s(q_{abt}) w_{ab}(q)}{N_{ab}(\omega_q)} \theta(q_{abt} - Q_0)$$

