Impact of Weak Annihilation Contribution on Rare Semileptonic $B^+ \to \pi^+\ell^+\ell^-$ Decay

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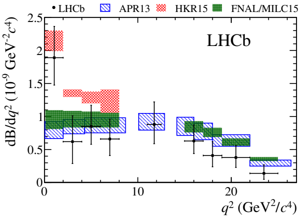
Rare Decays Induced by $b \to s$ and $b \to d$ FCNCs

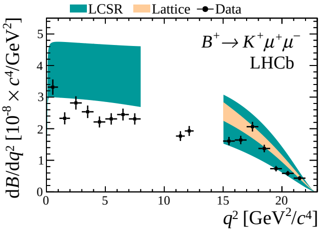
- Rare semileptonic decays of B-mesons and Λ_b-baryons due to $b \to s$ and $b \to d$ transitions, where b, s, and d are quarks with $Q = -1/3$, may be sensitive to "New Physics"
- At present, the proton-proton collider LHC and B-factory SuperKEKB are the only sources of experimental data on these decays
- Branching fractions of semileptonic B-meson decays due to $b \to s$ transition, like $B^\pm \to K^{(*)\pm}\mu^+\mu^-$, $B^0 \to K^{(*)0}\mu^+\mu^-$, $B_s^0 \to \phi\mu^+\mu^-$, lepton-pair invariant mass distributions in them, and coefficients in angular distributions, are experimentally measured quite precisely
- As for exclusive decays originated by the $b \to d$ neutral current, $B^+ \to \pi^+\mu^+\mu^-$ decay only was observed by the LHCb Collab. in 2012

$\mu^-\mu^+$-distributions in semileptonic B-meson decays

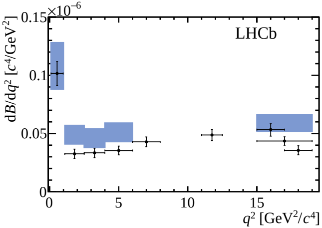
$B^+ \to \pi^+\mu^-\mu^+$ [JHEP 10 (2015) 034]



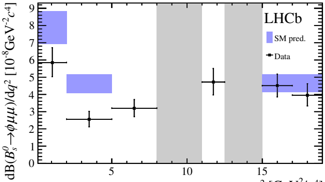
$B^+ \to K^+\mu^-\mu^+$ [JHEP 06 (2014) 133]



$B^0 \to K^{*0}\mu^-\mu^+$ [JHEP 11 (2016) 047]



$B_s^0 \to \phi\mu^-\mu^+$ [JHEP 09 (2015) 179]



Effective Hamiltonian for $b \to s\,(d)$ transitions

$$H_{eff} = -\frac{4G_F}{\sqrt{2}}\left[V_{tb}^* V_{tp} \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + V_{ub}^* V_{up} \sum_{i=1}^{2} C_i(\mu)\Big(O_i(\mu) - O_i^{(u)}(\mu)\Big)\right]$$

- $p = s,\ d$ is the light-quark flavor
- $V_{q_1 q_2}$ are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements
- C_i are Wilson coefficients
- O_i are local operators responsible for $b \to s\,(d)$ transitions

$$O_1 = \left(\bar{p}_L \gamma_\mu T^A c_L\right)\left(\bar{c}_L \gamma^\mu T^A b_L\right), \quad O_2 = \left(\bar{p}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right),$$

$$O_7 = \frac{e\, m_b}{g_{\rm st}^2}\left(\bar{p}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}, \quad O_8 = \frac{m_b}{g_{\rm st}}\left(\bar{p}_L \sigma^{\mu\nu} T^A b_R\right) G_{\mu\nu}^A$$

$$O_9 = \frac{e^2}{g_{\rm st}^2}\left(\bar{p}_L \gamma^\mu b_L\right) \sum_\ell \left(\bar{\ell} \gamma_\mu \ell\right), \quad O_{10} = \frac{e^2}{g_{\rm st}^2}\left(\bar{p}_L \gamma^\mu b_L\right) \sum_\ell \left(\bar{\ell} \gamma_\mu \gamma_5 \ell\right)$$

Matrix elements of $B \to P$ transition

$$\langle P(k)|\bar{p}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + k^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

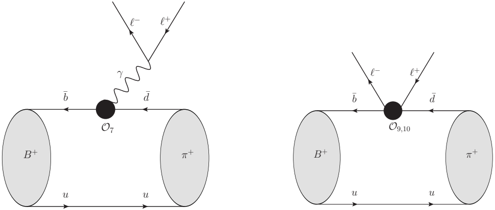
$$\langle P(k)|\bar{p}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 0$$

$$\langle P(k)|\bar{p}\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = i\left[(p_B^\mu + k^\mu)q^2 - q^\mu\left(m_B^2 - m_P^2\right)\right]\frac{f_T(q^2)}{m_B + m_P}$$

$$\langle P(k)|\bar{p}\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 0$$

- $q^\mu = p_B^\mu - k^\mu$ is momentum transfered
- $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$ are the transition form factors

$B^+ \to \pi^+\ell^+\ell^-$ decay



Operators $\mathcal{O}_7$, $\mathcal{O}_9$, and $\mathcal{O}_{10}$ give tree-level contributions to the decay amplitude ($N = \frac{G_F}{\sqrt{2}}\ V_{tb}V_{td}^*\ \frac{e^2}{g_{st}^2}$)

$$\mathcal{M}_9 = N\, C_9 \left\langle \pi(P_\pi)|\bar{d}_L\gamma^\mu b_L|B(P_B)\right\rangle \left[\bar{U}(q_1)\gamma_\mu U(-q_2)\right]$$

$$\mathcal{M}_{10} = N\, C_{10} \left\langle \pi(P_\pi)|\bar{d}_L\gamma^\mu b_L|B(P_B)\right\rangle \left[\bar{U}(q_1)\gamma_\mu\gamma_5 U(-q_2)\right]$$

$$\mathcal{M}_7 = -i\, N\, \frac{2m_b}{q^2}\, C_7 \left\langle \pi(P_\pi)|\bar{d}_L\sigma^{\mu\nu}q_\nu b_R|B(P_B)\right\rangle \left[\bar{U}(q_1)\gamma_\mu U(-q_2)\right]$$

$B \to P\ell^+\ell^-$ differential branching fraction

Depends on the dilepton invariant-mass q^2

$$\frac{d\mathrm{Br}\left(B \to P\ell^+\ell^-\right)}{dq^2} = S_P \frac{2G_F^2\alpha_{\rm em}^2\tau_B}{3(4\pi)^5 m_B^3}|V_{tb}V_{tp}^*|^2\beta_\ell\lambda^{3/2}(q^2)F^{BP}(q^2),$$

$$F^{BP}(q^2) = F_{97}^{BP}(q^2) + F_{10}^{BP}(q^2)$$

$$F_{97}^{BP}(q^2) = \left(1 + \frac{2m_\ell^2}{q^2}\right)\Bigg|C_9^{\rm eff}(q^2)\, f_+^{BP}(q^2) + $$

$$+\frac{2m_b}{m_B + m_P}\, C_7^{\rm eff}(q^2)\, f_T^{BP}(q^2) + L_A^{BP}(q^2)\Bigg|^2$$

$$F_{10}^{BP}(q^2) = \left(1 - \frac{4m_\ell^2}{q^2}\right)\left|C_{10}^{\rm eff}\, f_+^{BP}(q^2)\right|^2 + \frac{6m_\ell^2}{q^2}\,\frac{\left(m_B^2 - m_P^2\right)^2}{\lambda(q^2)}\left|C_{10}^{\rm eff}\, f_0^{BP}(q^2)\right|^2$$

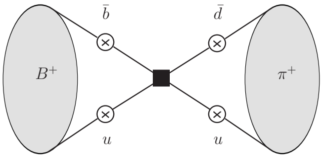
S_P is the final-meson flavor factor ($S_{\pi^\pm} = 1$ and $S_{\pi^0} = 1/2$)

$p = s, d$ is flavor in $b \to p$ transition

$\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2}$, $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$

Weak annihilation contribution in $B^+ \to \pi^+ \ell^+ \ell^-$ decay

Can be calculated within the LEET [Eur.Phys.J.C41:173-188 2005]



$$L_A^{B\pi(t)}(q^2) = Q_q \, \frac{\pi^2}{3} \frac{4 f_B f_\pi}{m_b} \, \lambda_{B,-}^{-1}(q^2) \, C_{34}$$

$$L_A^{B\pi(u)}(q^2) = -Q_q \, \frac{\pi^2}{3} \frac{4 f_B f_\pi}{m_b} \, \lambda_{B,-}^{-1}(q^2) \, C_{12}$$

- Q_q is relative charge of spectator quark
- f_B and f_π are B- and π-meson decay constants
- $C_{34} = C_3 + \frac{4}{3}(C_4 + 12 C_5 + 16 C_6)$; $C_{12} = 3 C_2$ are combinations of Wilson coefficients

  First inverse moment of B-meson LCDA enters these contributions

$$\lambda_{B,-}^{-1}(q^2) = \frac{e^{-q^2/(m_B \omega_0)}}{\omega_0} \left[i\pi - Ei(q^2/(m_B \omega_0)) \right]$$

$Ei(z) = \int\limits_{z}^{-\infty} dt \, e^t/t$ is the Exponential integral

Effective Wilson Coefficients

- Branching fractions of B-meson decays induced by $b \to s\,(d)\,\ell^-\ell^+$ transitions are expressed through $C_7^{\rm eff}$, $C_9^{\rm eff}$, and $C_{10}^{\rm eff}$
- In NNLO, effective Wilson coefficients are given as [H. Asatrian et al., PRD 69 (2004) 074007]

$$
\begin{aligned}
C_7^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_7(\hat{s})\right] A_7 - \frac{\alpha_s}{4\pi}\left[C_1^{(0)}\,F_{1,c}^{(7)} + C_2^{(0)}\,F_{2,c}^{(7)} + \sum_{k=3}^{6} C_k^{(0)}\,F_k^{(7)} + A_8^{(0)} F_8^{(7)}\right] - \\
&- \frac{\alpha_s}{4\pi}\,\xi^{(q)}\left\{C_1^{(0)}\left[F_{1,c}^{(7)} - F_{1,u}^{(7)}\right] + C_2^{(0)}\left[F_{2,c}^{(7)} - F_{2,u}^{(7)}\right]\right\} \\
C_9^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_9(\hat{s})\right]\Big\{A_9 + T_9\,h(\hat{m}_c^2,\hat{s}) + U_9\,h(1,\hat{s}) + W_9\,h(0,\hat{s}) + \xi^{(s)}\,T_{9a}\times \\
&\times\,\left[h(\hat{m}_c^2,\hat{s}) - h(0,\hat{s})\right]\Big\} - \frac{\alpha_s}{4\pi}\left[C_1^{(0)}\,F_{1,c}^{(9)} + C_2^{(0)}\,F_{2,c}^{(9)} + \sum_{k=3}^{6} C_k^{(0)}\,F_k^{(9)} + A_8^{(0)} F_8^{(9)}\right] - \\
&- \frac{\alpha_s}{4\pi}\,\xi^{(q)}\left\{C_1^{(0)}\left[F_{1,c}^{(9)} - F_{1,u}^{(9)}\right] + C_2^{(0)}\left[F_{2,c}^{(9)} - F_{2,u}^{(9)}\right]\right\} \\
C_{10}^{\rm eff} &= \left[1 + \frac{\alpha_s}{\pi}\,\omega_{10}(\hat{s})\right] A_{10}
\end{aligned}
$$

- $\hat{m}_c = m_c/m_b; \quad \xi^{(q)} = \lambda_u^{(q)}/\lambda_t^{(q)} \quad (q = d,\, s)$
- $\hat{s} = q^2/m_b^2$ is scaled lepton-pair momentum squared

Form Factor parameterizations

- Ball-Zwicky (BZ) parameterization ($i = +,\ T$)

$$f_i(q^2) = \frac{f_i(0) - r_2^{(i)}}{1 - q^2/m_{B^*}^2} + \frac{r_2^{(i)}}{1 - q^2/m_{fit}^{(i)2}}$$

$$f_0(q^2) = \frac{f_0(0)}{1 - q^2/m_{fit}^{(0)2}}$$

- Boyd-Grinstein-Lebed (BGL) parameterization
- Bourrely-Caprini-Lellouch (BCL) parameterization

Form Factor parameterizations

- Ball-Zwicky (BZ) parameterization
- Boyd-Grinstein-Lebed (BGL) parameterization ($i = +,\ 0,\ T$)

$$f_i(q^2) = \frac{1}{P_i(q^2)\phi_i(q^2, q_0^2)} \sum_{k=0}^{N} a_k^{(i)} \left[z(q^2, q_0^2)\right]^k ,$$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}},$$

$$m_+ = m_B + m_\pi, \quad q_0^2 = 0.65(m_B - m_\pi)^2$$

  - Blaschke factor: $P_{i=+,T}(q^2) = z(q^2, m_{B^*}^2)$ and $P_0(q^2) = 1$
  - $\phi_i(q^2, q_0^2)$ is outer function depending on isospin factor and three parameters K_i, α_i, and β_i
- Bourrely-Caprini-Lellouch (BCL) parameterization

Form Factor parametrizations

- Ball-Zwicky (BZ) parameterization
- Boyd-Grinstein-Lebed (BGL) parameterization
- Bourrely-Caprini-Lellouch (BCL) parameterization ($i = +,\ T$)

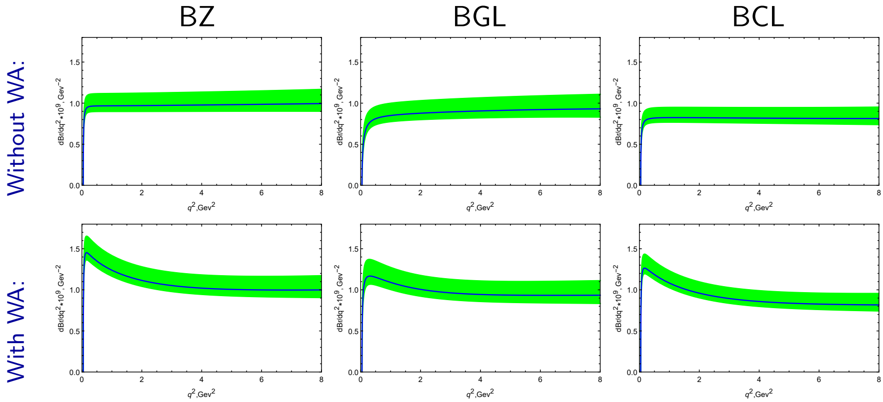
$$f_i(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{N-1} b_k^{(i)} \left(\left[z(q^2, q_0^2)\right]^k - (-1)^{k-N} \frac{k}{N} \left[z(q^2, q_0^2)\right]^N \right)$$

$$f_0(q^2) = \sum_{k=0}^{N-1} b_k^{(0)} z(q^2, q_0^2)^k$$

$$m_+ = m_B + m_\pi, \quad q_0^2 = m_+(\sqrt{m_B} - \sqrt{m_\pi})^2$$

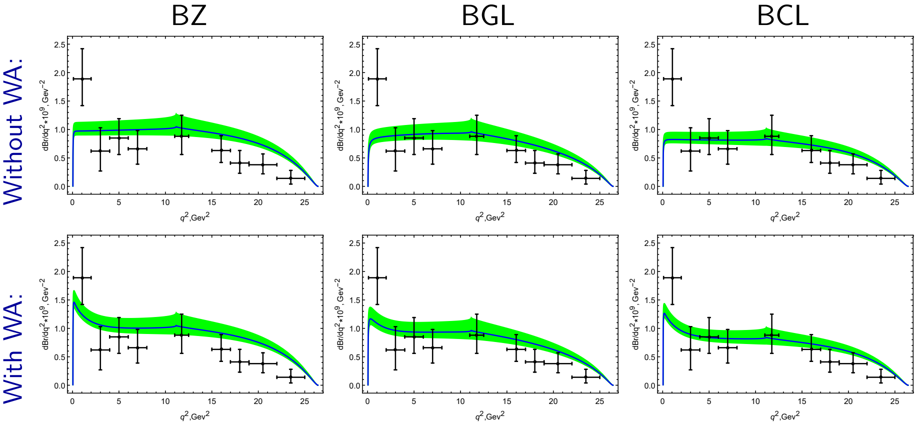
Form factors are considered as truncated series at $N = 4$

Dilepton invariant-mass distribution for $B^+ \to \pi^+\mu^+\mu^-$ in large-recoil approximation



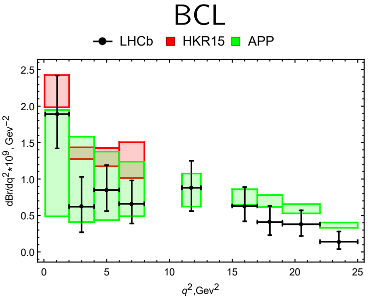
FF par.:	Ball, Zwicky	Ali, Parkhomenko, Rusov	Bailey et all.
	PRD 71 (2005) 014015	PRD 89 (2014) 094021	PRD 92 (2015) 014024

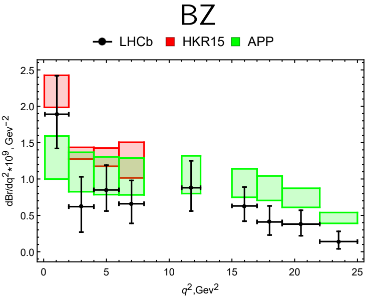
Dilepton invariant-mass distribution for $B^+ \to \pi^+\mu^+\mu^-$ in the entire kinematically allowed region



Experimental data are taken from [LHCb Collab., JHEP 10 (2015) 034]

Differential branching fraction of $B^+ \to \pi^+\mu^+\mu^-$ in bins of dilepton invariant mass squared





Branching fraction for $B^+ \to \pi^+\mu^+\mu^-$

	BCL		BZ	
	Total	$[4m_\mu^2, 8\,\mathrm{GeV}^2]$	Total	$[4m_\mu^2, 8\,\mathrm{GeV}^2]$
$\mathrm{Br_{th}} \times 10^8$	$1.78^{+0.63}_{-0.48}$	$0.65^{+0.47}_{-0.33}$	$2.20^{+0.58}_{-0.44}$	$0.78^{+0.23}_{-0.18}$
$\mathrm{Br_{th}^{WA}} \times 10^8$	$1.86^{+0.66}_{-0.51}$	$0.72^{+0.49}_{-0.36}$	$2.28^{+0.59}_{-0.45}$	$0.86^{+0.24}_{-0.19}$
$\mathrm{Br_{exp}} \times 10^8$	Total: 1.83 ± 0.29			

Summary and outlook

- For $B^+ \to \pi^+\mu^+\mu^-$ branching fraction, there is a good agreement between the theory and experiment within uncertainties, taking into account the contribution of annihilation diagrams
- In addition, accounting the weak annihilation contribution makes it possible to obtain a better agreement with the experimental data on the dimuon invariant-mass distribution in the lowest q^2-part of the entire range
- Explaining experimental threshold enhancement, the present analysis should be supplemented by an inclusion of long distance contributions from light neutral vector mesons
- Present data on $B^+ \to \pi^+\mu^+\mu^-$ decay correspond to an integrated luminosity of 3.0 fb^{-1}, collected by the LHCb experiment at centre-of-mass energies of 7 and 8 TeV, so it is interesting to wait an update from the LHCb, based on statistics at 13 TeV, and data from Belle-II at SuperKEKB

Backup Slides

Wilson Coefficients

- At the matching scale μ_W, Wilson coefficients can be calculated as a perturbative expansion

$$C_j(\mu_W) = \sum_{k=0}^{\infty} \left[\frac{\alpha_s(\mu_W)}{4\pi} \right]^k C_j^{(k)}(\mu_W)$$

- Mixed under the operator renormalization
- Evolved to the b-quark scale by RGE
- Hierarchy in the Wilson coefficients exists; in the naive dimensional regularization scheme at the next-to-leading logarithmic (NLL) order

$C_1(m_b)$	1.080	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	-0.177	$C_4(m_b)$	-0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	-0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	-0.040	$C_{10}(m_b)$	1.9×10^{-3}

Partial branching ratios for $B^+ \to \pi^+\mu^+\mu^-$ decay

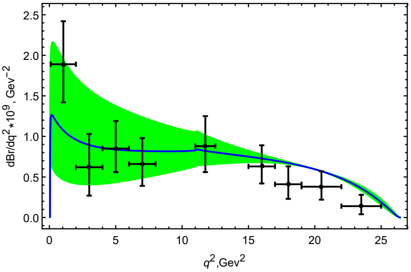
Partial branching ratios for $B^+ \to \pi^+\mu^+\mu^-$ decay, integrated over the indicated ranges

$[q^2_{min}, q^2_{max}]$	$10^8 \times \Delta\mathcal{B}_{th}$	$10^8 \times \Delta\mathcal{B}^{WA}_{th}$	$10^8 \times \Delta\mathcal{B}_{exp}$	$10^8 \times \Delta\mathcal{B}_{th}$	$10^8 \times \Delta\mathcal{B}^{WA}_{th}$
$[0.1, 2.0]$	$0.16^{+0.14}_{-0.10}$	$0.21^{+0.16}_{-0.11}$	$0.36^{+0.10}_{-0.09}$	$0.18^{+0.05}_{-0.04}$	$0.24^{+0.06}_{-0.05}$
$[2.0, 4.0]$	$0.16^{+0.13}_{-0.09}$	$0.18^{+0.13}_{-0.10}$	$0.12^{+0.08}_{-0.07}$	$0.20^{+0.06}_{-0.05}$	$0.21^{+0.06}_{-0.05}$
$[4.0, 6.0]$	$0.16^{+0.11}_{-0.08}$	$0.17^{+0.11}_{-0.08}$	$0.17^{+0.07}_{-0.06}$	$0.20^{+0.06}_{-0.05}$	$0.20^{+0.06}_{-0.05}$
$[6.0, 8.0]$	$0.16^{+0.08}_{-0.07}$	$0.16^{+0.08}_{-0.07}$	$0.13^{+0.06}_{-0.05}$	$0.20^{+0.06}_{-0.04}$	$0.20^{+0.06}_{-0.04}$
$[11.0, 12.5]$	$0.12^{+0.04}_{-0.03}$	$0.12^{+0.04}_{-0.03}$	$0.13^{+0.06}_{-0.05}$	$0.15^{+0.04}_{-0.03}$	$0.15^{+0.04}_{-0.03}$
$[15.0, 17.0]$	$0.14^{+0.03}_{-0.02}$	$0.14^{+0.03}_{-0.02}$	$0.13^{+0.05}_{-0.04}$	$0.18^{+0.05}_{-0.03}$	$0.18^{+0.05}_{-0.03}$
$[17.0, 19.0]$	$0.13^{+0.02}_{-0.01}$	$0.13^{+0.02}_{-0.01}$	$0.08^{+0.04}_{-0.04}$	$0.17^{+0.04}_{-0.03}$	$0.17^{+0.04}_{-0.03}$
$[19.0, 22.0]$	$0.17^{+0.03}_{-0.01}$	$0.17^{+0.03}_{-0.01}$	$0.11^{+0.06}_{-0.05}$	$0.21^{+0.05}_{-0.03}$	$0.21^{+0.05}_{-0.03}$
$[22.0, 25.0]$	$0.10^{+0.01}_{-0.01}$	$0.10^{+0.01}_{-0.01}$	$0.04^{+0.04}_{-0.03}$	$0.13^{+0.03}_{-0.02}$	$0.13^{+0.03}_{-0.02}$
Total	$1.78^{+0.63}_{-0.48}$	$1.86^{+0.66}_{-0.51}$	$1.83^{+0.29}_{-0.29}$	$2.20^{+0.58}_{-0.44}$	$2.28^{+0.59}_{-0.45}$

BCL (columns 2–3) *BZ* (columns 5–6)

Limits of uncertainty arising from errors

Maximum limits of uncertainty arising from errors in determining form factors for BCL



BCL parameterization works fine in the $q^2 \geq 11 GeV^2$