Impact of Weak Annihilation Contribution on Rare Semileptonic $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ Decay

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5th International Conference on Particle Physics and Astrophysics (ICPPA-2020) MEPHi, Moscow, Russia, 7 October 2020

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Rare Decays Induced by $b \rightarrow s$ and $b \rightarrow d$ FCNCs

- Rare semileptonic decays of $B$-mesons and $\Lambda_b$-baryons due to $b \rightarrow s$ and $b \rightarrow d$ transitions, where $b$, $s$, and $d$ are quarks with $Q = -1/3$, may be sensitive to “New Physics”
- At present, the proton-proton collider LHC and $B$-factory SuperKEKB are the only sources of experimental data on these decays
- Branching fractions of semileptonic $B$-meson decays due to $b \rightarrow s$ transition, like $B^\pm \rightarrow K^{(*)\pm} \mu^+ \mu^-$, $B^0 \rightarrow K^{(*)0} \mu^+ \mu^-$, $B^0_s \rightarrow \phi \mu^+ \mu^-$, lepton-pair invariant mass distributions in them, and coefficients in angular distributions, are experimentally measured quite precisely
- As for exclusive decays originated by the $b \rightarrow d$ neutral current, $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay only was observed by the LHCb Collab. in 2012
\(\mu^-\mu^+\)-distributions in semileptonic \(B\)-meson decays

\(B^+ \rightarrow \pi^+\mu^-\mu^+ \) \cite{JHEP 10 (2015) 034}

\(B^+ \rightarrow K^+\mu^-\mu^+ \) \cite{JHEP 06 (2014) 133}

\(B^0 \rightarrow K^{*0}\mu^-\mu^+ \) \cite{JHEP 11 (2016) 047}

\(B_s^0 \rightarrow \phi\mu^-\mu^+ \) \cite{JHEP 09 (2015) 179}
Effective Hamiltonian for $b \rightarrow s \,(d)$ transitions

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left[ V_{tb}^* V_{tp} \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + V_{ub}^* V_{up} \sum_{i=1}^{2} C_i(\mu) (O_i(\mu) - O_i^{(u)}(\mu)) \right]$$

- $p = s, \, d$ is the light-quark flavor
- $V_{q_1 q_2}$ are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements
- $C_i$ are Wilson coefficients
- $O_i$ are local operators responsible for $b \rightarrow s \,(d)$ transitions

$$O_1 = (\bar{p}_L \gamma_\mu T^A c_L) (\bar{c}_L \gamma^\mu T^A b_L), \quad O_2 = (\bar{p}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L),$$

$$O_7 = \frac{e m_b}{g_{\text{st}}^2} (\bar{p}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \quad O_8 = \frac{m_b}{g_{\text{st}}} (\bar{p}_L \sigma^{\mu\nu} T^A b_R) G_{\mu\nu}^A,$$

$$O_9 = \frac{e^2}{g_{\text{st}}^2} (\bar{p}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell} \gamma_\mu \gamma_5 \ell), \quad O_{10} = \frac{e^2}{g_{\text{st}}^2} (\bar{p}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell} \gamma_\mu \gamma_5 \ell)$$
Matrix elements of $B \to P$ transition

$$
\langle P(k)|\bar{p}\gamma^\mu b|B(p_B)\rangle = f_+(q^2) \left[ p_B^\mu + k^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_P^2}{q^2} q^\mu
$$

$$
\langle P(k)|\bar{p}\gamma^\mu \gamma_5 b|B(p_B)\rangle = 0
$$

$$
\langle P(k)|\bar{p}\sigma^{\mu\nu} q_\nu b|B(p_B)\rangle = i \left[ (p_B^\mu + k^\mu) q^2 - q^\mu (m_B^2 - m_P^2) \right] \frac{f_T(q^2)}{m_B + m_P}
$$

$$
\langle P(k)|\bar{p}\sigma^{\mu\nu} \gamma_5 q_\nu b|B(p_B)\rangle = 0
$$

- $q^\mu = p_B^\mu - k^\mu$ is momentum transferred
- $f_+(q^2), f_0(q^2), f_T(q^2)$ are the transition form factors
$B^+ \rightarrow \pi^+ \ell^+ \ell^- \text{ decay}$

Operators $\mathcal{O}_7$, $\mathcal{O}_9$, and $\mathcal{O}_{10}$ give tree-level contributions to the decay amplitude ($N = \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \frac{e^2}{g_{st}^2}$)

\[
\mathcal{M}_9 = N C_9 \left\langle \pi(P_\pi) | \bar{d}_L \gamma^\mu b_L \right| B(P_B) \right\rangle \left[ \bar{U}(q_1) \gamma_\mu U(-q_2) \right]
\]

\[
\mathcal{M}_{10} = N C_{10} \left\langle \pi(P_\pi) | \bar{d}_L \gamma^\mu b_L \right| B(P_B) \right\rangle \left[ \bar{U}(q_1) \gamma_\mu \gamma_5 U(-q_2) \right]
\]

\[
\mathcal{M}_7 = -i N \frac{2m_b}{q^2} C_7 \left\langle \pi(P_\pi) | \bar{d}_L \sigma^{\mu\nu} q_\nu b_R \right| B(P_B) \right\rangle \left[ \bar{U}(q_1) \gamma_\mu U(-q_2) \right]
\]
$B \rightarrow P \ell^+ \ell^-$ differential branching fraction

Depends on the dilepton invariant-mass $q^2$

$$
\frac{d\text{Br} (B \rightarrow P \ell^+ \ell^-)}{dq^2} = S_P \frac{2G_F^2\alpha_{em}^2\tau_B}{3(4\pi)^5 m_B^3} |V_{tb}V_{tp}^*|^{2} \beta_{\ell} \lambda^{3/2}(q^2) F^{BP}(q^2),
$$

$$
F^{BP}(q^2) = F_{97}^{BP}(q^2) + F_{10}^{BP}(q^2)
$$

$$
F_{97}^{BP}(q^2) = \left(1 + \frac{2m^2_{\ell}}{q^2}\right) \left| C_{9}^{\text{eff}}(q^2) f_{+}^{BP}(q^2) + \right. \left. \frac{2m_b}{m_B + m_P} C_{7}^{\text{eff}}(q^2) f_{T}^{BP}(q^2) + L_{A}^{BP}(q^2) \right|^{2}
$$

$$
F_{10}^{BP}(q^2) = \left(1 - \frac{4m^2_{\ell}}{q^2}\right) \left| C_{10}^{\text{eff}} f_{+}^{BP}(q^2) \right|^{2} + \frac{6m^2_{\ell}}{q^2} \frac{(m^2_{B} - m^2_{P})^2}{\lambda(q^2)} \left| C_{10}^{\text{eff}} f_{0}^{BP}(q^2) \right|^{2}
$$

$S_P$ is the final-meson flavor factor ($S_{\pi^\pm} = 1$ and $S_{\pi^0} = 1/2$)

$p = s, d$ is flavor in $b \rightarrow p$ transition

$$
\beta_{\ell} = \sqrt{1 - 4m^2_{\ell}/q^2}, \quad \lambda(q^2) = (m^2_{B} + m^2_{\pi} - q^2)^2 - 4m^2_{B} m^2_{\pi}
$$
Weak annihilation contribution in $B^+ \rightarrow \pi^+ \ell^+ \ell^-$ decay

Can be calculated within the LEET [Eur.Phys.J.C41:173-188 2005]

\[
L^B_{A\pi(t)}(q^2) = Q_q \frac{\pi^2}{3} \frac{4f_B f_\pi}{m_b} \lambda_{B,-}^{-1}(q^2) C_{34}
\]

\[
L^B_{A\pi(u)}(q^2) = -Q_q \frac{\pi^2}{3} \frac{4f_B f_\pi}{m_b} \lambda_{B,-}^{-1}(q^2) C_{12}
\]

- $Q_q$ is relative charge of spectator quark
- $f_B$ and $f_\pi$ are $B$- and $\pi$-meson decay constants
- $C_{34} = C_3 + \frac{4}{3}(C_4 + 12C_5 + 16C_6)$; $C_{12} = 3C_2$ are combinations of Wilson coefficients

First inverse moment of $B$-meson LCDA enters these contributions

\[
\lambda_{B,-}^{-1}(q^2) = \frac{e^{-q^2/(m_B \omega_0)}}{\omega_0} \left[ i\pi - Ei(q^2/(m_B \omega_0)) \right]
\]

\[
Ei(z) = \int_{z}^{\infty} dt \frac{e^t}{t} \text{ is the Exponential integral}
\]
Effective Wilson Coefficients

- Branching fractions of $B$-meson decays induced by $b \rightarrow s (d) \ell^- \ell^+$ transitions are expressed through $C_{7}^{\text{eff}}$, $C_{9}^{\text{eff}}$, and $C_{10}^{\text{eff}}$

- In NNLO, effective Wilson coefficients are given as [H. Asatrian et al., PRD 69 (2004) 074007]

\[
C_{7}^{\text{eff}} = \left[ 1 + \frac{\alpha_s}{\pi} \omega_7(\hat{s}) \right] A_7 - \frac{\alpha_s}{4\pi} \left[ C_{1}^{(0)} F_{1,c}^{(7)} + C_{2}^{(0)} F_{2,c}^{(7)} + \sum_{k=3}^{6} C_{k}^{(0)} F_{k}^{(7)} + A_{8}^{(0)} F_{8}^{(7)} \right] - \frac{\alpha_s}{4\pi} \xi(q) \left\{ C_{1}^{(0)} \left[ F_{1,c}^{(7)} - F_{1,u}^{(7)} \right] + C_{2}^{(0)} \left[ F_{2,c}^{(7)} - F_{2,u}^{(7)} \right] \right\} \\
C_{9}^{\text{eff}} = \left[ 1 + \frac{\alpha_s}{\pi} \omega_9(\hat{s}) \right] \left\{ A_9 + T_9 h(\hat{m}_c^2, \hat{s}) + U_9 h(1, \hat{s}) + W_9 h(0, \hat{s}) + \xi(s) T_{9a} \times \right. \\
\left. \times \left[ h(\hat{m}_c^2, \hat{s}) - h(0, \hat{s}) \right] \right\} - \frac{\alpha_s}{4\pi} \left[ C_{1}^{(0)} F_{1,c}^{(9)} + C_{2}^{(0)} F_{2,c}^{(9)} + \sum_{k=3}^{6} C_{k}^{(0)} F_{k}^{(9)} + A_{8}^{(0)} F_{8}^{(9)} \right] - \frac{\alpha_s}{4\pi} \xi(q) \left\{ C_{1}^{(0)} \left[ F_{1,c}^{(9)} - F_{1,u}^{(9)} \right] + C_{2}^{(0)} \left[ F_{2,c}^{(9)} - F_{2,u}^{(9)} \right] \right\} \\
C_{10}^{\text{eff}} = \left[ 1 + \frac{\alpha_s}{\pi} \omega_{10}(\hat{s}) \right] A_{10}
\]

- $\hat{m}_c = m_c/m_b$; $\xi(q) = \lambda_{\mu}^{(q)}/\lambda_{\tau}^{(q)}$ ($q = d, s$)

- $\hat{s} = q^2/m_b^2$ is scaled lepton-pair momentum squared
Form Factor parameterizations

- Ball-Zwicky (BZ) parameterization \( (i = +, T) \)

\[
f_i(q^2) = f_i(0) - \frac{r_2^{(i)}}{1 - q^2/m_B^2} + \frac{r_2^{(i)}}{1 - q^2/m_{fit}^{(i)}^2}
\]

\[
f_0(q^2) = \frac{f_0(0)}{1 - q^2/m_{fit}^{(0)}^2}
\]

- Boyd-Grinstein-Lebed (BGL) parameterization
- Bourrely-Caprini-Lellouch (BCL) parameterization
Form Factor parameterizations

- Ball-Zwicky (BZ) parameterization
- Boyd-Grinstein-Lebed (BGL) parameterization \((i = +, 0, T)\)

\[
f_i(q^2) = \frac{1}{P_i(q^2)\phi_i(q^2, q_0^2)} \sum_{k=0}^{N} a_k^{(i)} \left[z(q^2, q_0^2)\right]^k,
\]

\[
z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}},
\]

\[
m_+ = m_B + m_\pi, \quad q_0^2 = 0.65(m_B - m_\pi)^2
\]

- Blaschke factor: \(P_{i=+, T}(q^2) = z(q^2, m_{B^*}^2)\) and \(P_0(q^2) = 1\)
- \(\phi_i(q^2, q_0^2)\) is outer function depending on isospin factor and three parameters \(K_i, \alpha_i,\) and \(\beta_i\)
- Bourrely-Caprini-Lellouch (BCL) parameterization
Form Factor parametrizations

- Ball-Zwicky (BZ) parameterization
- Boyd-Grinstein-Lebed (BGL) parameterization
- Bourrely-Caprini-Lellouch (BCL) parameterization ($i = +, T$)

\[
f_i(q^2) = \frac{1}{1 - q^2/m_B^*} \sum_{k=0}^{N-1} b_k^{(i)} \left( \left[ z(q^2, q_0^2) \right]^k - (-1)^{k-N} \frac{k}{N} \left[ z(q^2, q_0^2) \right]^N \right)
\]

\[
f_0(q^2) = \sum_{k=0}^{N-1} b_k^{(0)} z(q^2, q_0^2)^k
\]

\[m_+ = m_B + m_\pi, \quad q_0^2 = m_+ (\sqrt{m_B} - \sqrt{m_\pi})^2\]

Form factors are considered as truncated series at $N = 4$
Dilepton invariant-mass distribution for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in large-recoil approximation

**Without WA:**

**With WA:**

**FF par.:**
- Ball, Zwicky: PRD 71 (2005) 014015
- Bailey et all: PRD 92 (2015) 014024
Dilepton invariant-mass distribution for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the entire kinematically allowed region.

Experimental data are taken from [LHCb Collab., JHEP 10 (2015) 034]
Differential branching fraction of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in bins of dilepton invariant mass squared

### BCL

![Graph showing BCL data](image)

### BZ

![Graph showing BZ data](image)

<table>
<thead>
<tr>
<th></th>
<th>BCL</th>
<th></th>
<th>BZ</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td>$[4m_\mu^2, 8 \text{ GeV}^2]$</td>
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<td>$[4m_\mu^2, 8 \text{ GeV}^2]$</td>
</tr>
<tr>
<td>$Br_{th} \times 10^8$</td>
<td>$1.78_{-0.48}^{+0.63}$</td>
<td>$0.65_{-0.33}^{+0.47}$</td>
<td>$2.20_{-0.44}^{+0.58}$</td>
<td>$0.78_{-0.18}^{+0.23}$</td>
</tr>
<tr>
<td>$Br_{WA} \times 10^8$</td>
<td>$1.86_{-0.51}^{+0.66}$</td>
<td>$0.72_{-0.36}^{+0.49}$</td>
<td>$2.28_{-0.45}^{+0.59}$</td>
<td>$0.86_{-0.19}^{+0.24}$</td>
</tr>
<tr>
<td>$Br_{exp} \times 10^8$</td>
<td></td>
<td></td>
<td>Total: $1.83 \pm 0.29$</td>
<td></td>
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</tbody>
</table>
Summary and outlook

• For $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ branching fraction, there is a good agreement between the theory and experiment within uncertainties, taking into account the contribution of annihilation diagrams.

• In addition, accounting the weak annihilation contribution makes it possible to obtain a better agreement with the experimental data on the dimuon invariant-mass distribution in the lowest $q^2$-part of the entire range.

• Explaining experimental threshold enhancement, the present analysis should be supplemented by an inclusion of long distance contributions from light neutral vector mesons.

• Present data on $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay correspond to an integrated luminosity of 3.0 fb$^{-1}$, collected by the LHCb experiment at centre-of-mass energies of 7 and 8 TeV, so it is interesting to wait an update from the LHCb, based on statistics at 13 TeV, and data from Belle-II at SuperKEKB.
Backup Slides
Wilson Coefficients

- At the matching scale $\mu_W$, Wilson coefficients can be calculated as a perturbative expansion
  \[
  C_j(\mu_W) = \sum_{k=0}^{\infty} \left[ \frac{\alpha_s(\mu_W)}{4\pi} \right]^k C_j^{(k)}(\mu_W)
  \]

- Mixed under the operator renormalization
- Evolved to the $b$-quark scale by RGE
- Hierarchy in the Wilson coefficients exists; in the naive dimensional regularization scheme at the next-to-leading logarithmic (NLL) order

| $C_1(m_b)$  | 1.080 |
| $C_2(m_b)$  | −0.177 |
| $C_{7\gamma}(m_b)$ | −0.317 |
| $C_{8g}(m_b)$ | 0.149 |
| $C_3(m_b)$  | 0.011 |
| $C_4(m_b)$  | −0.033 |
| $C_5(m_b)$  | 0.010 |
| $C_6(m_b)$  | −0.040 |
| $C_7(m_b)$  | $4.9 \times 10^{-4}$ |
| $C_8(m_b)$  | $4.6 \times 10^{-4}$ |
| $C_9(m_b)$  | $-9.8 \times 10^{-3}$ |
| $C_{10}(m_b)$ | $1.9 \times 10^{-3}$ |
Partial branching ratios for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay

Partial branching ratios for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay, integrated over the indicated ranges

<table>
<thead>
<tr>
<th>$[q^2_{min}, q^2_{max}]$</th>
<th>$10^8 \Delta B_{th}$</th>
<th>$10^8 \Delta B_{WA}$</th>
<th>$10^8 \Delta B_{exp}$</th>
<th>$10^8 \Delta B_{th}$</th>
<th>$10^8 \Delta B_{WA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.1, 2.0]</td>
<td>$0.16^{+0.14}_{-0.10}$</td>
<td>$0.21^{+0.16}_{-0.11}$</td>
<td>$0.36^{+0.10}_{-0.09}$</td>
<td>$0.18^{+0.05}_{-0.04}$</td>
<td>$0.24^{+0.06}_{-0.05}$</td>
</tr>
<tr>
<td>[2.0, 4.0]</td>
<td>$0.16^{+0.13}_{-0.09}$</td>
<td>$0.18^{+0.13}_{-0.10}$</td>
<td>$0.12^{+0.08}_{-0.07}$</td>
<td>$0.20^{+0.06}_{-0.05}$</td>
<td>$0.21^{+0.06}_{-0.05}$</td>
</tr>
<tr>
<td>[4.0, 6.0]</td>
<td>$0.16^{+0.11}_{-0.08}$</td>
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<td>$0.17^{+0.07}_{-0.06}$</td>
<td>$0.20^{+0.06}_{-0.05}$</td>
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</tr>
<tr>
<td>[6.0, 8.0]</td>
<td>$0.16^{+0.08}_{-0.07}$</td>
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<td>$0.13^{+0.06}_{-0.05}$</td>
<td>$0.20^{+0.06}_{-0.05}$</td>
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</tr>
<tr>
<td>[11.0, 12.5]</td>
<td>$0.12^{+0.04}_{-0.03}$</td>
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<td>$0.13^{+0.05}_{-0.04}$</td>
<td>$0.15^{+0.04}_{-0.03}$</td>
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</tr>
<tr>
<td>[15.0, 17.0]</td>
<td>$0.14^{+0.03}_{-0.02}$</td>
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<tr>
<td>[17.0, 19.0]</td>
<td>$0.13^{+0.02}_{-0.01}$</td>
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<tr>
<td>[19.0, 22.0]</td>
<td>$0.17^{+0.03}_{-0.01}$</td>
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<tr>
<td>[22.0, 25.0]</td>
<td>$0.10^{+0.01}_{-0.01}$</td>
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<tr>
<td>Total</td>
<td>$1.78^{+0.63}_{-0.48}$</td>
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Maximum limits of uncertainty arising from errors in determining form factors for BCL

BCL parameterization works fine in the $q^2 \geq 11 GeV^2$