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Abstract

We study the gravitational lensing by a black hole in the framework of the Poincare gauge theory of gravity. By using a recent black hole solution in this setup, we derive the deflection angle for light rays grazing the black hole and numerically solve the resulting integral. Numerical result show that the effects of torsion generally increases the deflection angle.

Introduction

One of the consequences of Einstein's general theory of relativity is bending of light as it passes through a gravitational field. Examining the path of light in a very strong gravitational field of a black hole can provide a huge amount of information about the geometry and characteristics of the surrounding space. On the other hand, the path of light rays, extent, and shape of gravitational lensing, are directly related to the type of background geometry in which light is emitted. Since the theory of general relativity in very high energies and very strong gravitational fields is expected to be corrected, researchers have been looking at the phenomenon of gravitational lensing in the context of alternative theories for general relativity to find out the needed corrections for the results of general relativity and these corrections are likely to be more significant in a very strong gravitational field of a black hole.

Among the various theories that have been proposed for correcting the gravity in high energies, gauge theories of gravity have great importance. One of the important results of these theories is changing the geometry for the background in general relativity, Riemannian space-time, to a non-Riemannian geometry in which, in addition to curvature, there is also torsion. In these theories, the presence of torsion coupled to spin of a matter can affect the path of light rays and correct the results of gravitational lensing.

In this work, we want to study the effects of non-Riemannian geometry on the gravitational lensing of a black hole, and in particular the effects of torsion and spin in this context.

Gravitational lensing with torsion

➤ Gravitational lensing by a black hole in general relativity

First we review gravitational lensing by a black hole in general relativity. Generic static spherically symmetric metric can be put in the form

$$ds^2 = A(r) dt^2 - B(r) dr^2 - C(r) (d\theta^2 + \sin^2\theta d\phi^2).$$

Suppose that a photon comes from infinite distance, grazes the black hole at a minimum distance r_m and goes away to infinity. The deflection angle, defined as the angle between the asymptotic incoming and outgoing trajectories, can be easily derived by the analysis of the geodesics equations as¹

$$\alpha = -\pi + 2 \int_{r_m}^{\infty} u \sqrt{\frac{B(r)}{C(r) [C(r)/A(r) - u^2]}} dr,$$

Where u is the impact parameter.

➤ Black hole solution with torsion

Poincare gauge theory of gravity (PGT) is a gravitational theory derived by applying the gauge procedure (replacing global symmetries by local ones) to the Poincare symmetry. The result is a gravitational theory in which the gravitational interactions are characterized by both curvature and torsion². In PGT, torsion couples with spin of the matter, as a result we can naturally introduce not only energy-momentum but also the spin of matter fields into gravitational dynamics. The Lagrangian of the PGT is assumed to be quadratic in terms of curvature and torsion tensors. We assume the action of the PGT to be in the form

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\mathcal{L}_m - R - \frac{1}{2} (2c_1 + c_2) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\lambda\rho} + c_1 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} + c_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} + d_1 \tilde{R}_{\mu\nu} (\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}) \right].$$

Where $\tilde{R}_{\lambda\rho\mu\nu}$ is the curvature tensor constructed by the full connection. Recently a new exact static and spherically symmetric vacuum solution has been found in [3]. This solution is given by the line element in the form

$$ds^2 = \Psi(r) dt^2 - \frac{dr^2}{\Psi(r)} - r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

Where the function $\Psi(r)$ is found to be³

$$\Psi(r) = 1 - \frac{2m}{r} + \frac{s}{r^2}. \quad (\text{I})$$

where s is a parameter related to the torsion of the space time. This solution describes a Reissner-Nordstrom type geometry, supported only by the metric and torsion fields rather than an electric or magnetic source.

➤ Gravitational lensing by a black hole in Poincaré gauge theory of gravity

Gravitation lensing by a black hole with torsion has been studied in the framework of Einstein-Cartan-Sciama-Kibble theory⁴. This theory is a special case of the PGT where the Lagrangian is assumed to be the same as in general relativity. Here we focus on the gravitational lensing by a black hole in PGT. We begin by the spherically symmetric line element in the form

$$ds^2 = -e^{\nu(r)} dt^2 + e^{-\nu(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

$$e^{\nu(r)} = \left(1 - \frac{2m}{r} + \frac{s}{r^2} \right),$$

As the field is isotropic, we assume the motion of light to be in equatorial plane, that is $\theta = \frac{\pi}{2}$. Using this, we have

$$e^{\nu(r)} \frac{dt}{dp} = \frac{dr}{dp},$$

The geodesic equations are

$$\frac{d^2 t}{dp^2} + 2 \left(\frac{m}{r^2} - \frac{s}{r^3} \right) \left(\frac{dt}{dp} \right)^2 - \frac{r}{2} \left(\frac{d\phi}{dp} \right)^2 = 0,$$

$$\frac{d^2 r}{dp^2} - \frac{3}{2} r \left(1 - \frac{2m}{r} + \frac{s}{r^2} \right) \left(\frac{d\phi}{dp} \right)^2 = 0,$$

$$\frac{d^2 \phi}{dp^2} - \left(\frac{2}{r} \right) \frac{dr}{dp} \frac{d\phi}{dp} = 0.$$

Using the last geodesic equation, we find

$$\frac{d}{dp} \left\{ \ln \frac{d\phi}{dp} + \ln r^2 \right\} = 0 \rightarrow r^2 \frac{d\phi}{dp} = j \text{ (Const)},$$

where j plays the role of an angular momentum per unit mass. Having $dr^2 = E dp^2$ from geodesic equation, applying $\theta = \frac{\pi}{2}$ and $j^2 = r^4 (d\phi/dp)^2$ helping us to reach

$$\frac{d^2 r}{dp^2} + \left(\frac{j}{r} \right)^2 \left(2 \left(\frac{m}{r^2} - \frac{s}{r^3} \right) - \frac{1}{2r} \left(1 - \frac{2m}{r} + \frac{s}{r^2} \right) \right) = 0.$$

Eliminating dp , our differential equation reads as follows

$$\frac{d^2 r}{d\phi^2} - \frac{2}{r} \left(\frac{dr}{d\phi} \right)^2 + 2r^2 \left(\frac{m}{r^2} - \frac{s}{r^3} \right) - \frac{r}{2} \left(1 - \frac{2m}{r} + \frac{s}{r^2} \right) = 0,$$

which leads us to the deflection angle integral

$$\phi = \pm \int \frac{2}{\sqrt{4r^4 c_1 - 2r^2 + 8mr - 5s}} dr - C_2. \quad (\text{II})$$

Choosing $C_2 = \pi$, we have the minimum deflection angle equal to π when the integral is zero.

Results

The integral in equation (II) is of the elliptic type which can be solved numerically. We perform the numerical analysis using 4th order Runge-Kutta method to find the deflection angle for a black hole when the exterior of the black hole is given by equation (I). We also used Mathematica's NIntegrate package to cross-check our results.

Figure 1 shows the deflection angle with respect to r_m for fixed values of $C_1 = m = s = 1$. As can be seen from the figure, the deflection angle decreases when r_m increases as expected. α is equal to $\phi + \pi$.

Figure 2 shows the deflection angle with respect to the torsion parameter s for fixed values of $C_1 = m = r_m = 1$. one can see that the deflection angle increases by increasing s , This means that the effects of torsion enhances lensing by a black hole in non-Riemannian spacetimes.

Fig. 1

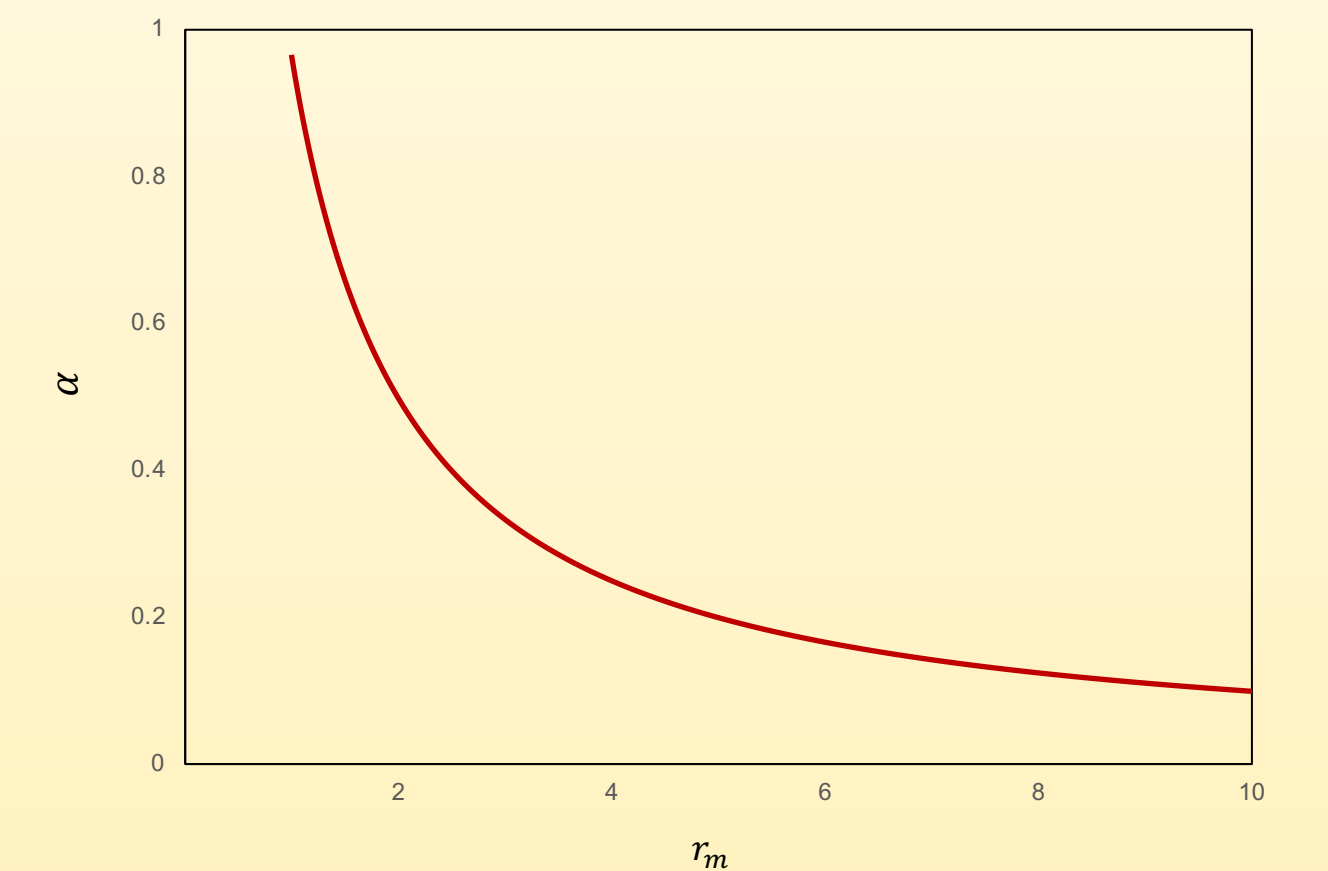
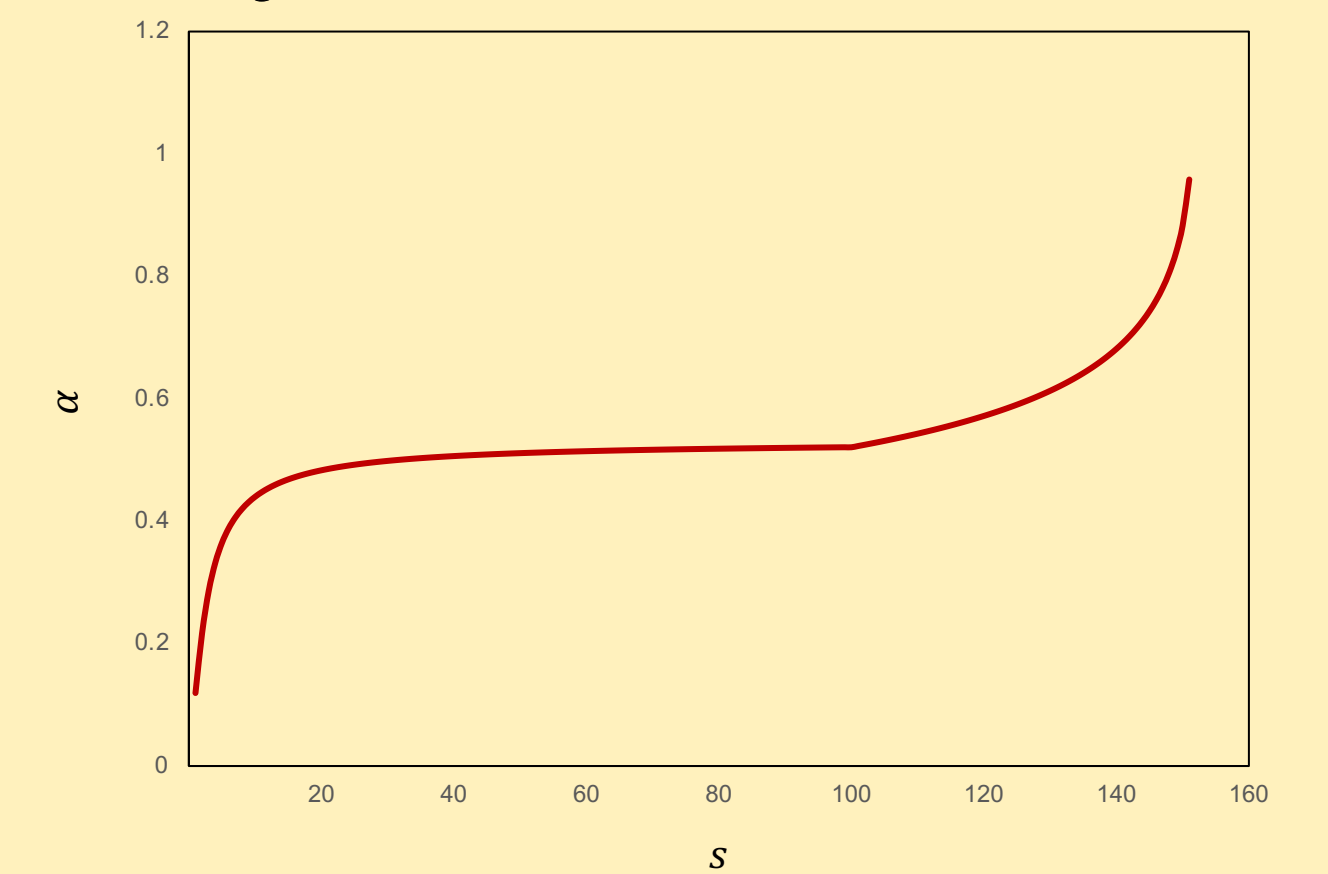


Fig. 2



Conclusion

We studied gravitational lensing by a black hole in the framework of the Poincare gauge theory of gravity. Using the recent solution for static spherically symmetric line element in this theory, we derived an integral for the deflection angle for light rays grazing a black hole. By numerically solving this integral, we show that the effects of the spin and torsion will increase the deflection angle compared to when the torsion tensor is zero. Moreover, the deflection angle tends to vanish when the minimum distance r_m goes to infinity.

References

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