Present status of inflation, the simplest models and expected discoveries

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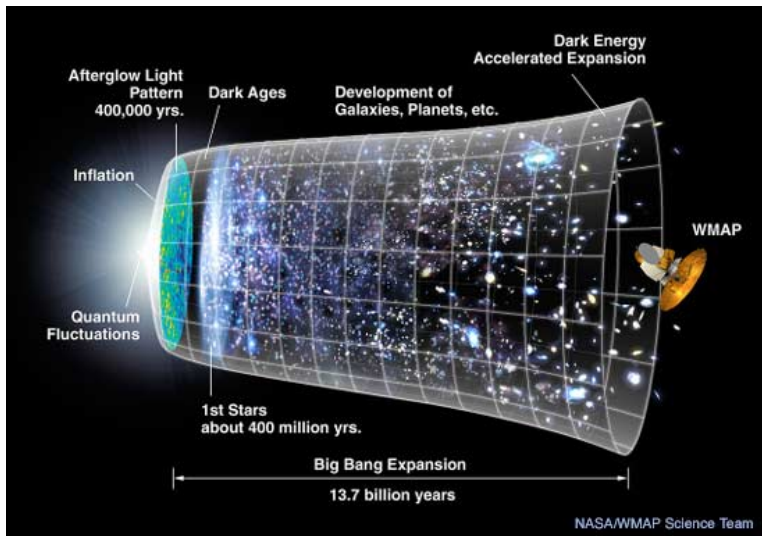
Inflation

The (minimal variant of the) inflationary scenario is based on the two cornerstone independent ideas (hypothesis):

1. Existence of inflation (or, quasi-de Sitter stage) – a stage of accelerated, close to exponential expansion of our Universe in the past preceding the hot Big Bang with decelerated, power-law expansion.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of pairs of particles - antiparticles and field fluctuations during inflation from the adiabatic vacuum (no-particle) state for Fourier modes covering all observable range of scales (and possibly somewhat beyond).

NB. This effect is the same as particle creation by black holes, but no problems with the loss of information, 'firewalls', trans-Planckian energy etc. in cosmology, as far as observational predictions are calculated.

Both assumptions can be combined into one:
Some part of the world which includes all its presently observable part was as much symmetric as possible during some period in the past - both with respect to the geometrical background and to the state of all quantum fields (no particles).

Non-universal (due to the specific initial condition) explanation of the cosmological arrow to time - entropy (in some not well defined sense) can only grow after inflation.
Still this state is an intermediate attractor for a set of pre-inflationary initial conditions with a non-zero measure.

Existing analogies in other areas of physics.
1. The present dark energy, though the required degree of metastability for the primordial dark energy is much more than is proved for the present one (more than 60 e-folds vs. ~ 3).
2. Creation of electrons and positrons in an external electric field.

Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r}) \, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \; g_{,l}^{(a)} e_m^{l(a)} = 0, \; e_{lm}^{(a)} e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$n_s(k) - 1 \equiv \frac{d \ln P_\mathcal{R}(k)}{d \ln k}, \quad r(k) \equiv \frac{P_g}{P_\mathcal{R}}$$

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\mathcal{R},\ g$). In particular:

$$\hat{\mathcal{R}}_k = \mathcal{R}_k\, i(\hat{a}_{\mathbf{k}} - \hat{a}_{\mathbf{k}}^{\dagger}) + \mathcal{O}\left((\hat{a}_{\mathbf{k}} - \hat{a}_{\mathbf{k}}^{\dagger})^2\right) + ... + \mathcal{O}(10^{-100})(\hat{a}_{\mathbf{k}} + \hat{a}_{\mathbf{k}}^{\dagger}) + , , ,$$

The last non-commutative term is time dependent, it is affected by physical decoherence and may become larger, but not as large as the second term.

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).
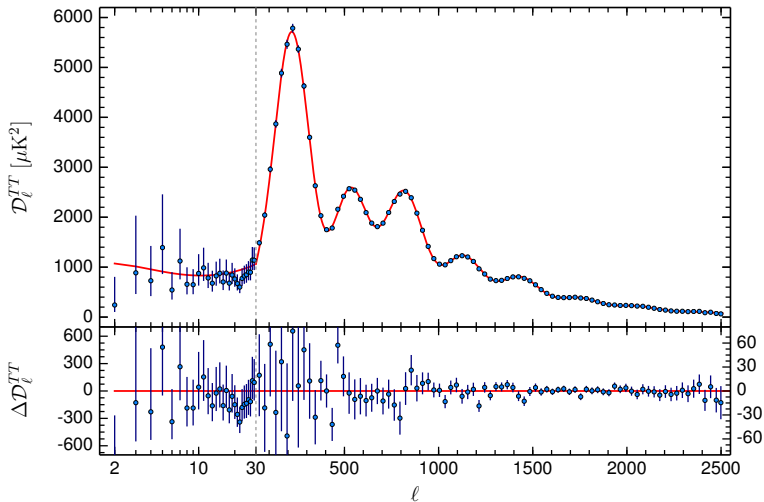
All these predictions are beyond semiclassical gravity!
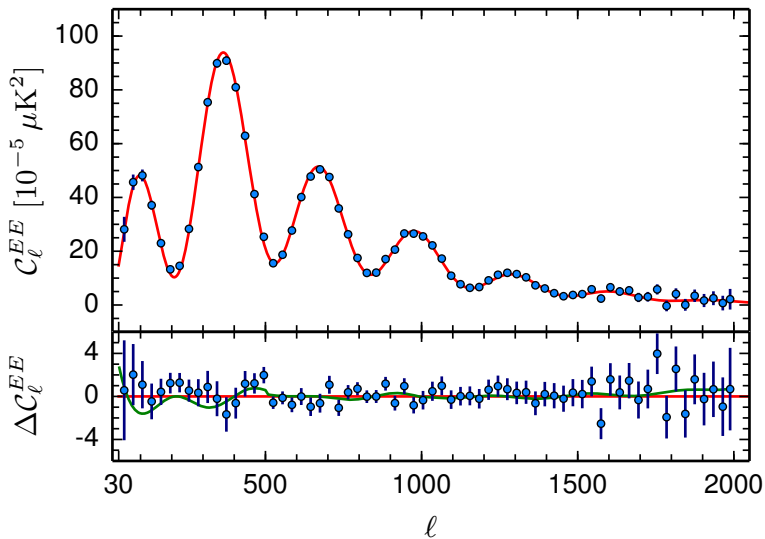
CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589

-300 -200 -100 0 100 200 300

μK_{cmb}
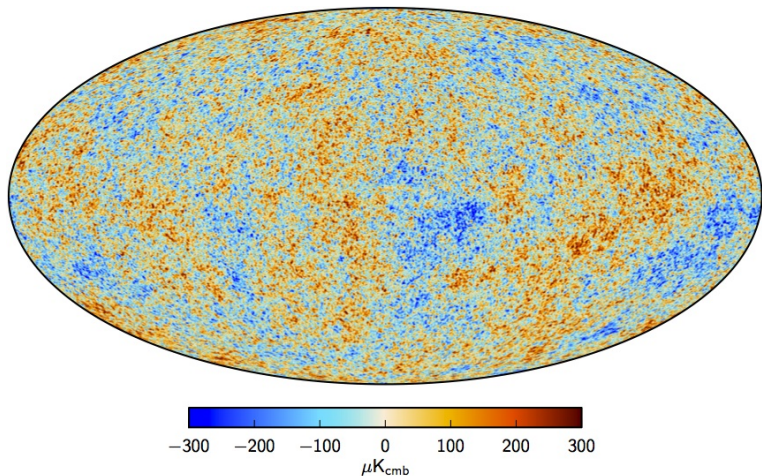
CMB temperature anisotropy multipoles

CMB E-mode polarization multipoles

New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) > = \int \frac{P_{\mathcal{R}}(k)}{k} \, dk, \ \ P_{\mathcal{R}}(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0} \right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \ n_s - 1 = -0.035 \pm 0.004, \ r < 0.065$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s) N_H \sim 2$).

Three general directions in criticizing inflation

1. "Inflation is not falsifiable and cannot predict anything". Reply. Wrong. Any concrete inflationary model is falsifiable, and most of proposed models have been already falsified. The less is the number of free parameters (to be fixed by observations only) of remaining viable models, the larger is their predictive power. The most predictive models have only one free parameter at present.

2. "Beginning of (isotropic and homogeneous) inflation requires extreme fine-tuning of pre-inflationary evolution". Reply. Wrong. Intermediate inflationary attractor is generic and inhomogeneous, it approaches (quasi-)de Sitter only locally, inside the Hubble radius. As a result, like black holes in GR, inflationary patches always form somewhere.

3. "Eternal inflation leads to multiverse in which no quantitative predictions can be made".

Reply. Not relevant (if not wrong). Irrespective of some problems with making predictions in this case (which arise due to unjustified manipulations with conditional probabilities to some extent), eternal inflation is an unfinished, still evolving process occurring outside our past (and future) light cone. Thus, it does not affect what we see in our Universe up to fantastically (exponentially) small terms which are always neglected in all quantitative considerations of eternal inflation.

The simplest models producing the observed scalar slope

1. The $R + R^2$ model (Starobinsky, 1980):

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \quad f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N} \right) M_{Pl} \approx 3.1 \times 10^{13} \, \text{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$N = \ln \frac{k_f}{k} = \ln \frac{T_\gamma}{k} - \mathcal{O}(10), \quad H_{dS}(N = 55) = 1.4 \times 10^{14} \, \text{GeV}$$

2. The same prediction from a scalar field model with $V(\phi) = \frac{\lambda \phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R \phi^2$ with $\xi < 0, \ |\xi| \gg 1$, including the Higgs inflationary model (Bezrukov and Shaposhnikov, 2008).

The simplest purely geometrical inflationary model

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_{\mathcal{R}}(k)} R^2 + (\text{small rad. corr.})$$

$$= \frac{R}{16\pi G} + 5.1 \times 10^8\, R^2 + (\text{small rad. corr.})$$

The quantum effect of creation of particles and field fluctuations works twice in this model:

a) at super-Hubble scales during inflation, to generate space-time metric fluctuations;

b) at small scales after inflation, to provide scalaron decay into pairs of matter particles and antiparticles (AS, 1980, 1981).

Weak dependence of the time t_r when the radiation dominated stage begins:

$$N(k) \approx N_H + \ln \frac{a_0 H_0}{k} - \frac{1}{3} \ln \frac{M_{\mathrm{Pl}}}{M} - \frac{1}{6} \ln(M_{\mathrm{Pl}} t_r)$$

Evolution of the $R + R^2$ model

1. During inflation ($H \gg M$):

$$H = \frac{M^2}{6}(t_f - t) + \frac{1}{6(t_f - t)} + ..., \quad |\dot{H}| \ll H^2$$

.

(for the derivation of the second term in the rhs - see
A. S. Koshelev et al., JHEP 1611 (2016) 067).

2. After inflation ($H \ll M$):

$$a(t) \propto t^{2/3} \left(1 + \frac{2}{3Mt} \sin M(t - t_1)\right)$$

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

(Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977)) can be used for simplicity, but the full integral-differential system of equations for the Bogoliubov α_k, β_k coefficients and the average EMT was in fact solved in AS (1981). Scalaron decay into graviton pairs is suppressed (A. A. Starobinsky, JETP Lett. 34, 438 (1981)).

For this channel of the scalaron decay:

$$\Gamma = \frac{24\mathcal{N}_0 M^3}{M_{\mathrm{Pl}}^2}, \ \ N(k) \approx N_H + \ln\frac{a_0 H_0}{k} - \frac{5}{6}\ln\frac{M_{\mathrm{Pl}}}{M} + \frac{1}{6}\ln\mathcal{N}_0$$

Possible microscopic origins of this phenomenological model.

1. Follow the purely geometrical approach and consider it as the specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} + (\text{small rad. corr.})$$

for which $A \gg 1$, $A \gg |B|$. Emergence of hierarchy! Approximate scale (dilaton) invariance and absence of ghosts in the curvature regime $A^{-2} \ll (RR)/M_P^4 \ll B^{-2}$.

One-loop quantum-gravitational corrections are small (their imaginary parts are just the predicted spectra of scalar and tensor perturbations), non-local and qualitatively have the same structure modulo logarithmic dependence on curvature.

2. Another, completely different way:

consider the $R + R^2$ model as an approximate description of GR + a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$) in the gravity sector::

$$\mathcal{L} = \frac{R}{16\pi G} - \frac{\xi R \phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

Geometrization of the scalar:

for a generic family of solutions during inflation, the scalar kinetic term can be neglected, so

$$\xi R \phi = -V'(\phi) + \mathcal{O}(|\xi|^{-1}) \ .$$

 No conformal transformation, we remain in the the physical (Jordan) frame!

These solutions are the same as for $f(R)$ gravity with

$$\mathcal{L} = \frac{f(R)}{16\pi G}, \; f(R) = R - \frac{\xi R \phi^2(R)}{2} - V(\phi(R)).$$

For $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$, this just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$.

The same theorem is valid for a multi-component scalar field.

More generally, R^2 inflation (with an arbitrary n_s, r) serves as an intermediate dynamical attractor for a large class of scalar-tensor gravity models.

However, this property suddenly breaks immediately after the end of inflation when R becomes negative temporarily during oscillations.

Inflation in the mixed R^2-Higgs model

M. He, A. A. Starobinsky and J. Yokoyama, JCAP **1805**, 064 (2018).

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right) - \frac{\xi R \chi^2}{2} + \frac{1}{2}\chi_{,\mu}\chi^{,\mu} - \frac{\lambda \chi^4}{4}, \ \ \xi < 0, \ \ |\xi| \gg 1$$

Can be conformally transformed to GR with two interacting scalar fields in the Einstein frame. The effective two scalar field potential for the dual model:

$$U = e^{-2\alpha\phi}\left(\frac{\lambda}{4}\chi^4 + \frac{M^2}{2\alpha^2}\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)^2\right)$$

$$\alpha = \sqrt{\frac{2}{3}}\,\kappa, \ \ R = 3M^2\left(e^{\alpha\phi} - 1 + \xi\kappa^2\chi^2\right)$$
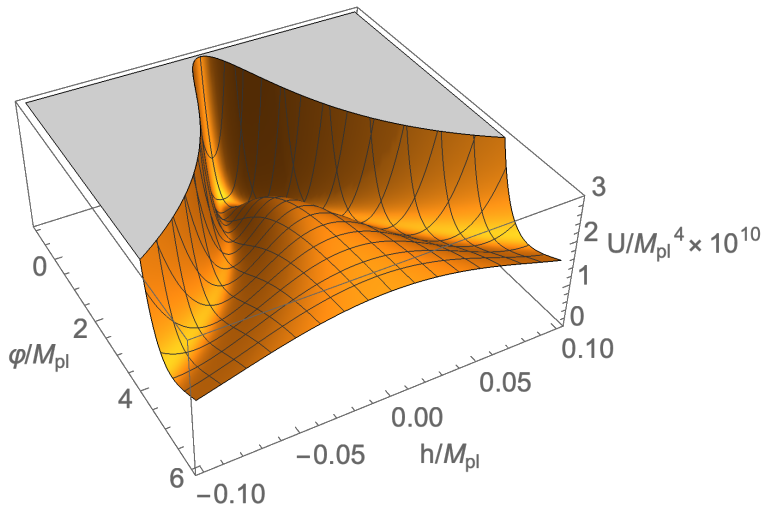
Attractiveness of the model from the quantum field theory point of view

The mixed R^2-Higgs model helps to remove some UV problems of the Higgs inflationary model and may be considered as its UV-completion up to the Planck energy if

$$\sqrt{\lambda} \lesssim \frac{|\xi| M}{M_{\mathrm{Pl}}} \lesssim 1$$

(see D. Gorbunov and A. Tokareva, Phys. Lett. B 788, 37 (2019)).

Effective potential in the Einstein frame

One-field inflation in the attractor regime

In the attractor regime during inflation:

$$\alpha\phi \gg 1, \quad \chi^2 \approx \frac{|\xi|R}{\lambda}, \quad e^{\alpha\phi} \approx \chi^2\left(|\xi|\kappa^2 + \frac{\lambda}{3|\xi|M^2}\right)$$

that directly follows from the geometrization of the Higgs boson in the physical (Jordan) frame. Thus, we return to the $f(R) = R + \frac{R^2}{6M^2}$ model with the renormalized scalaron mass $M \to \tilde{M}$:

$$\frac{1}{\tilde{M}^2} = \frac{1}{M^2} + \frac{3\xi^2\kappa^2}{\lambda}$$

Double-field inflation reduces to the single $(R + R^2)$ one for the most of trajectories in the phase space.

Viable two-parametric inflationary models in GR

The simplest inflationary models with a scalar field minimally coupled to gravity which are not excluded by observations are two-parametric. Their potential can be derived assuming the "no-scale" behaviour $P_{\mathcal{R}}(k) \propto N^\beta$ and using the reconstruction method.

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_{\mathcal{R}}(k(t(\phi))), \quad C = \frac{12\pi^2}{\kappa^6}$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^4}{12\pi^2} \int \frac{dN}{P_{\mathcal{R}}(N)}$$

$$\kappa\phi = \int dN \sqrt{\frac{d\ln V}{dN}}$$

Here, $N \gg 1$ stands both for $\ln(k_f/k)$ at the present time and the number of e-folds back in time from the end of inflation. First derived in H. M. Hodges and G. R. Blumenthal, Phys. Rev. D 42, 3329 (1990).

The best fit to the Planck data is produced by $\beta = 2$. Then:

$$V = V_0 \frac{N}{N + N_0} = V_0 \tanh^2 \frac{\kappa\phi}{2\sqrt{N_0}}$$

$$r = \frac{8N_0}{N(N + N_0)}$$

$r \sim 0.003$ for $N_0 \sim 1$. From the upper limit on r:

$$N_0 < \frac{0.07N^2}{8 - 0.07N}$$

$N_0 < 57$ for $N = 57$.

Different types of quantum corrections to the simplest model

- Logarithmic running of the free model parameter M with curvature.
- Terms with higher derivatives of R considered perturbatively (to avoid the appearance of ghosts).
- Terms arising from the conformal anomaly.

At present, no necessity to break the Lorentz invariance and to introduce additional spatial dimensions at the energy (Hubble) scale of inflation.

Logarithmic running of M with curvature

Due to the scale-invariance of the $R + R^2$ model for $R \gg M^2$, one may expect logarithmic running of the dimensionless coefficient in front of the R^2 term for large energies and curvatures. The concrete 'asymptotically safe' model with

$$f(R) = R + \frac{R^2}{6M^2 \left[1 + b \ln \left(\frac{R}{\mu^2} \right) \right]}$$

was recently considered in L.-H. Liu, T. Prokopec, A. A. Starobinsky, Phys. Rev. D **98**, 043505 (2018).

However, comparison with CMB observational data shows that b is small by modulus: $|b| \lesssim 10^{-2}$. Thus, from the observational point of view this model can be simplified to

$$f(R) = R + \frac{R^2}{6M^2} \left[1 - b \ln \left(\frac{R}{\mu^2} \right) \right],$$

for which the analytic solution exists:

$$n_s - 1 = -\frac{4b}{3} \left(e^{\frac{2bN}{3}} - 1 \right)^{-1}$$

$$r = \frac{16b^2}{3} \frac{e^{\frac{4bN}{3}}}{\left(e^{\frac{2bN}{3}} - 1 \right)^2}$$

For $|b|N \ll 1$, these expressions reduce to those for the $R + R^2$ model.

Second type: terms with higher derivatives of R

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R + \alpha R^2 + \gamma R \Box R \right], \quad \alpha = \frac{1}{6M^2}$$

An inflationary regime in this model was first considered in S. Gottlöber, H.-J. Schmidt and A. A. Starobinsky, Class. Quant. Grav. **7**, 803 (1990). But this model, if taken in full, has a scalar ghost in addition to a physical massive scalar and the massless graviton.

Its recent re-consideration avoiding ghosts: A. R. R. Castellanos, F. Sobreira, I. L. Shapiro and A. A. Starobinsky, JCAP **1812**, 007 (2018).

The idea is to treat the $\gamma R \square R$ term perturbatively with respect to the $R + R^2$ gravity, i.e., to consider only those solutions which reduce to the solutions of the $R + R^2$ gravity in the limit $\gamma - 0$. Then the second (ghost) scalar degree of freedom does not appear.

Results:

1. $|k| \lesssim 0.3$ where $k = \frac{\gamma}{6\alpha^2}$.

2. In the limit $kN \ll 1$, leading corrections $\propto kN$ to $n_s - 1$ and r vanish. The first result is in the agreement with that in a more general non-local gravity model without ghosts constructed in A. S. Koshelev, L. Modesto, L. Rachwal and A. A. Starobinsky, JHEP **1611**, 067 (2016) which contains an infinite number of R derivatives.

Third type: terms arising from the conformal (trace) anomaly

The tensor producing the $\propto \left(R_{\mu\nu} R^{\mu\nu} - \frac{R^2}{3} \right)$ term in the trace anomaly:

$$T_\mu^\nu = \frac{k_2}{2880\pi^2} \left(R_\mu^\alpha R_\alpha^\nu - \frac{2}{3} R R_\mu^\nu - \frac{1}{2} \delta_\mu^\nu R_{\alpha\beta} R^{\alpha\beta} + \frac{1}{4} \delta_\mu^\nu R^2 \right)$$

It is covariantly conserved in the isotropic case only! Can be generalized to the weakly anisotropic case by adding a term proportional to the first power of the Weyl tensor.

$$T_0^0 = \frac{3H^4}{\kappa^2 H_1^2}, \quad T = -\frac{1}{\kappa^2 H_1^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{R^2}{3} \right), \quad H_1^2 = \frac{2880\pi^2}{\kappa^2 k_2}$$

The spectrum of scalar and tensor perturbations in this case was calculated already in A. A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).

$$n_s - 1 = -2\beta \frac{e^{\beta N}}{e^{\beta N} - 1}, \quad \beta = \frac{M^2}{3H_1^2}$$

If $n_s > 0.957$ and $N = 55$, then $H_1 > 7.2M$.

Perspectives of future discoveries

- Primordial gravitational waves from inflation: r.
  $r \lesssim 8(1 - n_s) \approx 0.3$ (confirmed!) but may be much less. However, under reasonable assumptions one may expect that $r \gtrsim (n_s - 1)^2 \approx 10^{-3}$. The target prediction in the simplest (one-parametric) models is
  $r = 3(n_s - 1)^2 \approx 0.004$.

- A more precise measurement of $n_s - 1 \implies$ duration of transition from inflation to the radiation dominated stage $\implies$ information on inflaton (scalaron) couplings to known elementary particles at super-high energies $E \lesssim 10^{13}$ Gev.

- Local non-smooth features in the scalar power spectrum at cosmological scales (?).

- Local enhancement of the power spectrum at small scales leading to a significant amount of primordial black holes (?).

PBHs and small-scale GWs in two-field models of inflation

M. Braglia, D. K. Hazra, F. Fimelli, G. F. Smoot,
L. Sriramkumar, A. A. Starobinsky, JCAP **2008** (2020) 001.

The simplest one-parameter inflationary models do not predict PBHs, at least whose existing at present with $M > 10^{15}$ g. Previously known ways to obtain a large peak in the primordial power spectrum of scalar adiabatic perturbations at small scales:

1. A local feature in the inflaton potential $V(\phi)$ (a rapid change of its slope or its amplitude, an inflection point with a large $V'''(\phi)$).

2. A rapid turn of the inflaton trajectory in the field space in the case of many-field models of inflation.

3. Phase transitions leading to large isocurvature perturbations which transform to adiabatic ones afterwards.

A novel mechanism: a two-field inflation with different inflaton effective masses (that leads to two stages of inflation) and a large non-standard kinetic coupling of the heavier field.

$$S(\phi, \chi) = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - \frac{f(\phi)}{2}(\partial\chi)^2 - V(\phi, \chi) \right]$$

$$V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{m_\chi^2}{2}\chi^2$$

Large kinetic coupling: $f(\phi) = \exp(b\phi)$, $bM_{Pl} \gg 1$. The peak in the spectrum arises when the heavier field goes out of the slow-roll regime. It can lead to the formation of PBHs with a wide range of masses and to the generation of stochastic background of primordial gravitational waves produced by second order scalar perturbations.
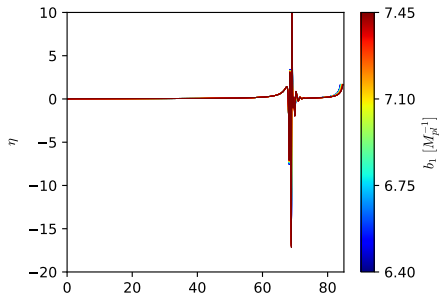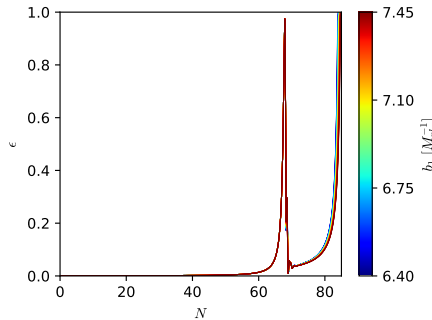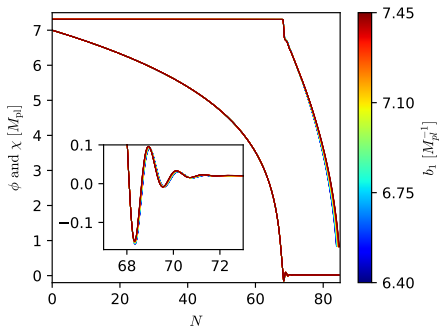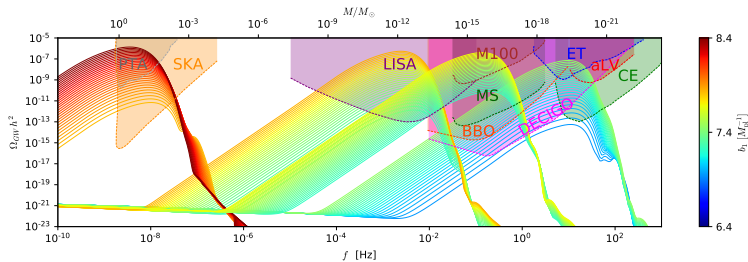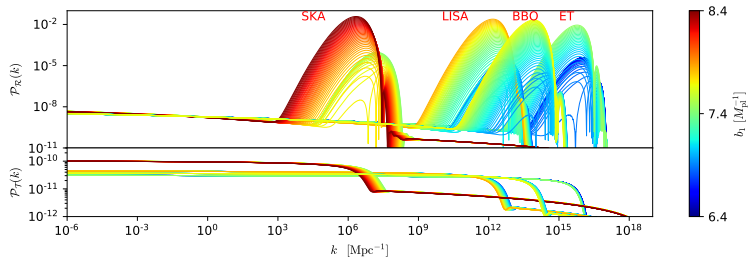
Generality of inflation

Inflation as a generic intermediate solution.

Theorem. In inflationary models in GR and $f(R)$ gravity, there exists an open set of classical solutions with a non-zero measure in the space of initial conditions at curvatures much exceeding those during inflation which have a metastable inflationary stage with a given number of e-folds.

For the GR inflationary model this follows from the generic late-time asymptotic solution for GR with a cosmological constant found in A. A. Starobinsky, JETP Lett. 37, 55 (1983). For the $R + R^2$ model, this was proved in A. A. Starobinsky and H.-J. Schmidt, Class. Quant. Grav. 4, 695 (1987). For the power-law and $f(R) = R^p, p < 2, 2 - p \ll 1$ inflation – in V. Müller, H.-J. Schmidt and A. A. Starobinsky, Class. Quant. Grav. 7, 1163 (1990).

Generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter (also called the Fefferman-Graham expansion):

$$ds^2 = dt^2 - \gamma_{ik}dx^i dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + ...$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular. b_{ik} is unambiguously defined through the 3-D Ricci tensor constructed from a_{ik}. c_{ik} contains a number of arbitrary physical functions (two - in the vacuum case, or with radiation) – tensor hair.

A similar but more complicated construction with an additional dependence of H_0 on spatial coordinates in the case of $f(R) = R^p$ inflation – scalar hair.

Consequences:

1. (Quasi-) de Sitter hair exist globally and are partially observable after the end of inflation.

2. The appearance of an inflating patch does not require that all parts of this patch should be causally connected at the beginning of inflation.

Similar property in the case of a generic curvature singularity formed at a spacelike hypersurface in GR and modified gravity. However, 'generic' does not mean 'omnipresent'.

What was before inflation?

Duration of inflation was finite inside our past light cone. In terms of e-folds, difference in its total duration in different points of space can be seen by the naked eye from a smoothed CMB temperature anisotropy map.

ΔN formalism: $\Delta \mathcal{R}(\mathbf{r}) = \Delta N_{tot}(\mathbf{r})$ where
$N_{tot} = \ln \left(\frac{a(t_{fin})}{a(t_{in})} \right) = N_{tot}(\mathbf{r})$ (AS, 1982,1985).

For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta, \phi)}{T_\gamma} = -\frac{1}{5} \Delta \mathcal{R}(r_{LSS}, \theta, \phi) = -\frac{1}{5} \Delta N_{tot}(r_{LSS}, \theta, \phi)$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\Delta N \sim 5 \times 10^{-5}$, and for $H \sim 10^{14} \, \text{GeV}$, $\Delta t \sim 5 t_{Pl}$!

Different possibilities were considered historically:

1. Creation of inflation "from nothing" (Grishchuk and Zeldovich, 1981).

One possibility among infinite number of others.

2. De Sitter "Genesis": beginning from the exact contracting full de Sitter space-time at $t \to -\infty$ (AS, 1980).

Requires adding an additional term

$$R_i^l R_l^k - \frac{2}{3} R R_i^k - \frac{1}{2} \delta_i^k R_{lm} R^{lm} + \frac{1}{4} \delta_i^k R^2$$

to the rhs of the gravitational field equations. Not generic. May not be the "ultimate" solution: a quantum system may not spend an infinite time in an unstable state.

3. Bounce due to a positive spatial curvature (AS, 1978). Generic, but probability of a bounce is small for a large initial size of a universe $W \sim 1/Ma_0$.

Formation of inflation from generic curvature singularity

In classical gravity (GR or modified $f(R)$): space-like curvature singularity is generic. Generic initial conditions near a curvature singularity in modified gravity models (the $R + R^2$ and Higgs ones): anisotropic and inhomogeneous (though quasi-homogeneous locally).

Recent analytical and numerical investigation for $f(R)$ gravity in the Bianchi I type model in D. Muller, A. Ricciardone, A. A. Starobinsky and A. V. Toporensky, Eur. Phys. J. C **78**, 311 (2018). Two types of singularities in $f(R)$ gravity with the same structure at $t \to 0$:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i} a_l^{(i)} a_m^{(i)} dx^l dx^m, \ 0 < s \leq 3/2, \ u = s(2-s)$$

where $p_i < 1$, $s = \sum_i p_i$, $u = \sum_i p_i^2$ and $a_l^{(i)}$, p_i are functions of $\mathbf{r}$. Here $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$.

Bianchi I type models with inflation in R^2 gravity

Type A. $1 \leq s \leq 3/2, \ R \propto |t|^{1-s} \to +\infty$

Type B. $0 < s < 1, \ R \to R_0 < 0, \ f'(R_0) = 0$

For $f(R) = R^2$ even an exact solution can be found.

$$ds^2 = \tanh^{2\alpha}\left(\frac{3H_0 t}{2}\right)\left(dt^2 - \sum_{i=1}^{3} a_i^2(t)dx_i^2\right)$$

$$a_i(t) = \sinh^{1/3}(3H_0 t)\tanh^{\beta_i}\left(\frac{3H_0 t}{2}\right), \ \sum_i \beta_i = 0, \ \sum_i \beta_i^2 < \frac{2}{3}$$

$$\alpha^2 = \frac{\frac{2}{3} - \sum_i \beta_i^2}{6}, \ \ \alpha > 0$$

Next step: relate arbitrary functions of spatial coordinates in the generic solution near a curvature singularity to those in the quasi-de Sitter solution. Spatial gradients may become important for some period before the beginning of inflation.

The same structure of generic singulariry for a non-minimally coupled scalar field (scalar-tensor gravity)

$$S = \int \left(f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) \right) \sqrt{-g}\, d^4x + S_m$$

$$f(\phi) = \frac{1}{2\kappa^2} - \xi\phi^2$$

Type A. $\xi < 0, |\phi| \to \infty$
Type B. $\xi > 0, |\phi| \to 1/\sqrt{2\xi}\kappa$

The asymptotic regimes and a number of exact solutions in the Bianchi type I model are presented in A. Yu. Kamenshchik, E. O. Pozdeeva, A. A. Starobinsky, A. Tronconi, G. Venturi and S. Yu. Vernov, Phys. Rev. D **97**, 023536 (2018) with some of them borrowed from A. A. Starobinsky, MS Degree thesis, Moscow State University, 1971, unpublished.

What is sufficient for beginning of inflation in classical (modified) gravity, is:

1) the existence of a sufficiently large compact expanding region of space with the Riemann curvature much exceeding that during the end of inflation ($\sim M^2$) – realized near a curvature singularity;

2) the average value $< R >$ over this region positive and much exceeding $\sim M^2$, too, – type A singularity;

3) the average spatial curvature over the region is either negative, or not too positive.

On the other hand, causal connection is certainly needed to have a "graceful exit" from inflation, i.e. to have practically the same amount of the total number of e-folds during inflation N_{tot} in some sub-domain of this inflating patch.

Conclusions

- The typical inflationary predictions that $|n_s - 1|$ is small and of the order of N_H^{-1}, and that r does not exceed $\sim 8(1 - n_s)$ are confirmed. Typical consequences following without assuming additional small parameters: $H_{55} \sim 10^{14} \, \mathrm{GeV}, \ m_{infl} \sim 10^{13} \, \mathrm{GeV}$.

- In $f(R)$ gravity, the simplest $R + R^2$ model is one-parametric and has the preferred values $n_s - 1 = -\frac{2}{N} \approx -0.035$ and $r = 3(n_s - 1)^2 \approx 0.004$. The first value produces the best fit to present observational CMB data. The same prediction follows for the Higgs and the mixed R^2-Higgs models though actual values of N are slightly different for these 3 cases.

- Inflation in $f(R)$ gravity represents a dynamical attractor for slow-rolling scalar fields strongly coupled to gravity. As a result, double field inflation in the mixed R^2-Higgs model reduces to the single R^2-like inflation for a dense set of the most interesting trajectories in the phase space

- Comparison with observational data shows that logarithmic high-curvature quantum corrections to the $R + R^2$ model in the observable part of inflation are small, no more than a few percents. The same refers to higher-derivative and conformal anomaly corrections.

- Inflation is generic in the $R + R^2$ inflationary model and close ones. Thus, its beginning does not require causal connection of all parts of an inflating patch of space-time (similar to spacelike singularities). However, graceful exit from inflation requires approximately the same number of e-folds during it for a sufficiently large compact set of geodesics. To achieve this, causal connection inside this set is necessary (though still may appear insufficient).

- Inflation can form generically and with not a small probability from generic space-like curvature singularity.