The QCD equation of state at finite density, from the known to the unknown

Jan Steinheimer

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Thanks to:
V. Vovchenko, A. Motornenko, A. Mukherjee, S. Schramm, M. Hanauske, L. Rezzolla and H. Stöcker
Motivation

The legacy of high energy nuclear physics?

Can we eventually draw a diagram like this for the textbooks? (Hydrogen)


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Robust constraints on the Equation of state from:

- Lattice QCD, for \( T \geq 130 \text{ MeV} \).

Constraints from lQCD:

- The Interaction measure, thermodynamics at \( \mu_B = 0 \)
- Derivatives of the pressure wrt \( \mu_B \).
- Expansion into finite real \( \mu_B \).
- Calculations at imaginary \( \mu \).

Using only the Fourier coefficients $b_k$ from imaginary $\mu_B$ simulations as input:

\begin{align*}
\rho_{B,T}^3 &= \partial_p \left( \frac{p}{T^4} \right) \partial_{\mu_B/T} = \sum_{k=1}^{\infty} b_k \left( T \right) \sinh \left( \frac{k\mu_B}{T} \right)
\end{align*}

Assuming the proper SB limit and using only the first two coefficients one can exactly predict finite $\mu_B$ thermodynamics

\begin{align*}
b_k \left( T \right) &= \alpha_k \left[ b_2 \left( T \right) \right]_k - 1 \left[ b_1 \left( T \right) \right]_k - 2
\end{align*}

Results on the applicability

Radius of convergence: $\mu_B/T < \pi$

Using only the Fourier coefficients $b_k$ from imaginary $\mu_B$ simulations as input:

- One can write the density of QCD as a cluster expansion:

$$ \rho_B T^3 = \frac{\partial (p/T^4)}{\partial (\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh \left( \frac{k \mu_B}{T} \right) $$

- Assuming the proper SB limit and using only the first two coefficients one can exactly predict finite $\mu_B$ thermodynamics

$$ b_k(T) = \alpha_k \left[ b_2(T) \right]^{k-1} - \left[ b_1(T) \right]^{k-2} $$

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Getting the most out of lattice QCD → the CEM model

Using only the Fourier coefficients $b_k$ from imaginary $\mu_B$ simulations as input:

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  \[ \frac{\rho_B}{T^3} = \frac{\partial (p/T^4)}{\partial (\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh \left( \frac{k \mu_B}{T} \right) \]

- Assuming the proper SB limit and using only the first two coefficients one can exactly predict finite $\mu_B$ thermodynamics

- $b_k(T) = \alpha_k \left[ \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}} \right]$. 


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Using only the Fourier coefficients \( b_k \) from imaginary \( \mu_B \) simulations as input:

- One can write the density of QCD as a cluster expansion:
  \[
  \rho_B \frac{T^3}{3} = \frac{\partial}{\partial (\mu_B/T)} \left( \frac{\rho}{T^4} \right) = \sum_{k=1}^{\infty} b_k(T) \sinh \left( \frac{k \mu_B}{T} \right)
  \]

- Assuming the proper SB limit and using only the first two coefficients on can exactly predict finite \( \mu_B \) thermodynamics

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  b_k(T) = \alpha_k \left[ \frac{b_2(T)}{b_1(T)} \right]^{k-1}
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Results on the applicability

Radius of convergence: \( \mu_B/T < \pi \)

Taylor expansion in real $\mu_B$

Instead of expanding in imaginary $\mu$, do a Taylor expansion in real $\mu_B$

- Write the expansion of the pressure using susceptibilities:

$$P = P_0 + T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{i,j,k,B,Q,S} \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k,$$

(1)

Artifacts appear around $\mu_B/T > 2.5$.

Radius of convergence $\mu_B/T < 3$.

High $T$ rule out quark-repulsion.

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Why the breakdown at $\mu_B/T \approx 3$?

Why do the methods break down?
- Sudden change of isobaric lines at this point.
- From Boson (mesons/gluons) dominated matter to fermionic matter (nucleons/quarks).

A. Motornenko, JS, V. Vovchenko, S. Schramm and H. Stöcker, (Quark Matter 2019), Wuhan, China, November 3-9 2019
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- First principle calculations seem to fail for fermionic matter.

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Constraints at $T = 0$

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Y. Fujimoto, K. Fukushima and K. Murase, Phys.
Constraints at $T = 0$

- Here we have guidance from measured neutron star masses.
- Without Radii no real constraints!
- Add constraints from PQCD.
- Still missing the important region. Extension to finite temperature $\rightarrow$ New degrees of freedom.

![Diagram](image-url)
The QCD EoS in Heavy Ion collisions

The NICA-JINR phase diagram in $T$ and $\rho_B$.

Details depend on the experiment.
This one is from: https://nica.jinr.ru/physics.php
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Let’s estimate the densities expected for central collisions.

- Geometrical Overlap Model:
  - $\rho = 2\gamma_{cm}\rho_0$
  - $\epsilon = 2\gamma_{cm}^2\epsilon_0$

- UrQMD with and without nuclear potentials.
  - Average densities in a box with $-0.5 < z < 0.5$ fm, $-3 < x, y < 3$ fm.

- Should give a good estimate on expected maximum compression.
The QCD EoS in Heavy Ion collisions

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How to study the equation of state using hadrons

Much of we today think about hadronic observables is motivated by the fluid dynamic picture of HIC:

- Pre-equilibrium phase
- Equilibrated? phase
- Final stage and particle freeze-out

Non-equilibrium initial state
- Fluid dynamic evolution
- Freeze-out: chemical and thermal

An example: The $v_1$ story

Maybe:
- Early studies proposed the directed flow as a signal of the phase transition
- They where done using only 1 or 2 fluid dynamics.

What is directed flow?
Deflection of matter in the reaction plane:
$v_1 = \langle \frac{p_x}{p_T} \rangle (y)$

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What is directed flow?

One is interested in the slope of $v_1 = \langle p_x/p_T \rangle (y)$ w.r.t the rapidity.

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- Recent STAR measurements show a negative slope of net proton $v_1$.
- Is it the phase transition?

STAR data

Data on the net proton $v_1$ slope show the predicted behavior.

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Hybrid Model

However, when checked with state of the art hydro, no signal is found.

**References**

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- But changing the initial EoS changes the slope.

**Hybrid Model**

However, when the stiffness of the initial state is changed one observes a sensitivity!

So $v_1$ might be sensitive to the 'softness' of the initial state...
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**Microscopic Transport with EoS**
If a fully microscopic transport simulation with EoS (JAM) is used the effect persists.

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Phase separation occurs.
At the critical point: divergence of correlation length.

\[ \chi_{\mu\mu} / \Lambda^2 \]

A Phase Transition in Fluid Dynamics

- In a dynamical scenario, locally the system may not be in phase eq.
- Phase separation occurs.
- At the critical point: divergence of correlation length.

\[ \chi_{\mu\mu} / \Lambda^2 \]

- Susceptibilities diverge due to mechanically unstable phase.
- Separation of the two phases: Spinodal Instabilities.
- It’s not the amplitude of the density fluctuation which diverges!

What is the data situation?

- Full model simulations for fluctuations are scarce
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- STAR data, recently corrected, shows no clear signal.
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- Separation of the two phases: Spinodal Instabilities.
- STAR data, recently corrected, shows no clear signal.

In short: what is really measured are fluctuations and correlations in momentum space.

- The downsides of hadrons: freeze-out and rescattering wash out signals
- Implementation of EoS for the fully dynamical description from pre-equilibrium to freeze-out necessary
Electromagnetic probes

Electromagnetic probes offer a chance to probe the whole time evolution of the fireball.

In particular di-lepton pairs created by the decay of hadrons or quark annihilation.

\[ \rho \rightarrow e^+ + e^- \]
\[ q + \bar{q} \rightarrow e^+ + e^- \]

Process sensitive to the medium in which it takes place ($T$ and $\rho_B$).
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Process sensitive to the medium in which it takes place ($T$ and $\rho_B$).

Distinct differences with or without a phase transition
Electromagnetic probes

Indeed di-lepton emission shows a significant effect

- A simulation for Au+Au at the current SIS18 beam energy.
- A factor 2 enhancement of di-lepton emission due to extended 'cooking'.
What can be done to study the EoS at high density?

- Design effective models that match lattice QCD at low $\mu_B$ and neutron stars at high density.
The strategy

<table>
<thead>
<tr>
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The strategy

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- Design effective models that match lattice QCD at low $\mu_B$ and neutron stars at high density.
- Employ these models for heavy ion collisions as well neutron star mergers.
- Find a consistent description
- Possibly new analysis methods that combine many observables and statistical / machine learning methods.
Effective $SU(3)_f$ chiral mean field model based on:

- Chiral symmetry for hadrons via nucleon parity partners: Describes nuclear matter and lattice phenomenology on masses.
- Deconfined quarks and gluons via effective Polyakov Loop potential and removal of hadrons via excluded volume.

One example: Effective model for this - the CMF

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- Deconfined quarks and gluons via effective Polyakov Loop potential and removal of hadrons via excluded volume.
- Free parameter fitted to lattice QCD thermodynamics As well as Susceptibilities from lattice.
- Phase diagram seems reasonable

This EoS enables us to treat heavy ion collisions and NS mergers on the same footing.

What area of the phase diagram are tested and what is the overlap?
The CMF and neutron star mergers

- This EoS enables us to treat heavy ion collisions and NS mergers on the same footing
- What area of the phase diagram are tested and what is the overlap?
- Low beam energy HIC compared to NS merger simulations.
- Disclaimer: Not the same EoS used yet.

This EoS enables us to treat heavy ion collisions and NS mergers on the same footing. What area of the phase diagram are tested and what is the overlap? Low beam energy HIC compared to NS merger simulations. Disclaimer: Not the same EoS used yet. A dense and cold core with a hot hadronic corona.

Lattice QCD seem to be only useful up to $\mu_B/T \approx 3$, after that fermions become the dominant d.o.f.

Neutron star properties constrain $T = 0$.

No sign of a critical point or phase transition yet.

Combined/Complex models are necessary to describe the matter in low energy HIC and neutron star mergers.

We have to take all constraints seriously.

Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe complementary region in the phase diagram.
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- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe complementary region in the phase diagram.
- Treat both on the same footing $\rightarrow$ Combining QCD thermodynamics, relativistic fluid dynamics and GR.
- Use statistical/ML methods to combine the wealth of data for a consistent picture of the QCD phase diagram.