# The QCD equation of state at finite density, from the known to the unknown

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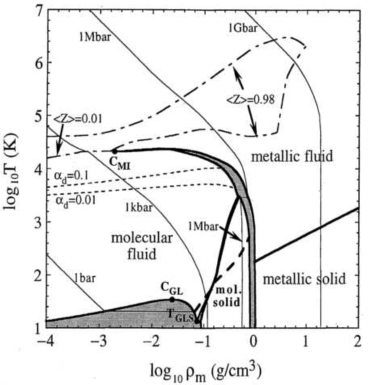
Thanks to:
V. Vovchenko, A. Motornenko, A. Mukherjee, S. Schramm, M. Hanauske, L. Rezzolla and H. Stöcker

## The legacy of high energy nuclear physics?

Can we eventually draw a diagram like this for the textbooks?(Hydrogen)




Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

# Motivation

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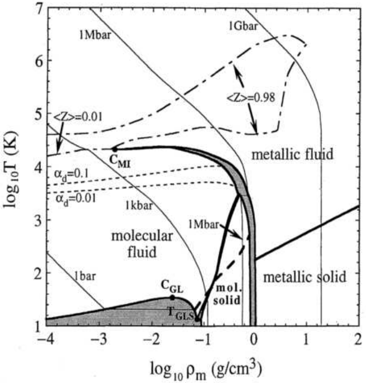


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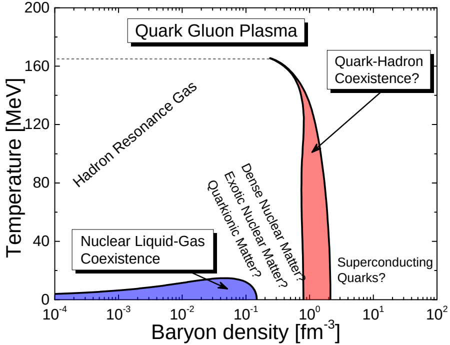
QCD



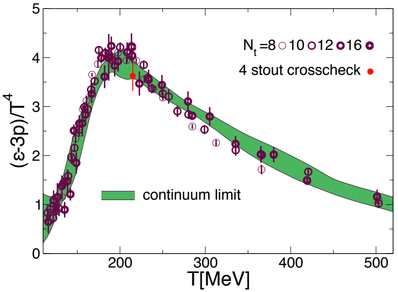

**JS** , V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm and H. Stoecker, Phys. Rev. C **81**, 044913 (2010)

## Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV.

## Constraints from lQCD:

- The Interaction measure, thermodynamics at $\mu_B = 0$
- Derivatives of the pressure wrt $\mu_B$. Expansion into finite real $\mu_B$.
- Calculations at imaginary $\mu$.




S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B **730**, 99 (2014)

# Getting the most out of lattice QCD $\rightarrow$ the CEM model

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- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$

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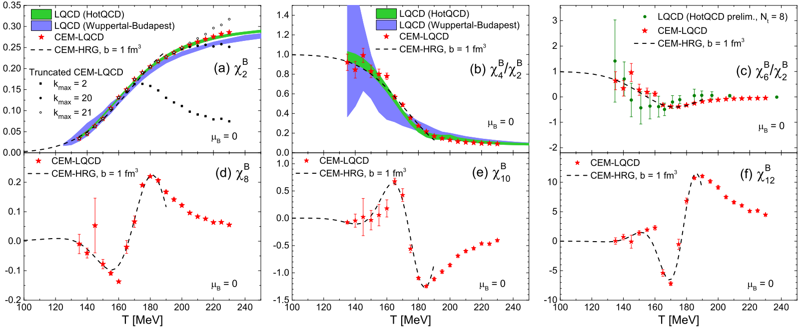


## Results on the applicability

Radius of convergence: $\mu_B/T < \pi$

V. Vovchenko, **JS** , O. Philipsen and H. Stoecker, Phys. Rev. D **97**, no.11, 114030 (2018)

# Taylor expansion in real $\mu_B$

**Instead of expanding in imaginary $\mu$, do a Taylor expansion in real $\mu_B$**

- Write the expansion of the pressure using susceptibilities:

$$P = P_0 + T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi^{i,j,k}_{B,Q,S} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k , \tag{1}$$

A. Bazavov et. al., Phys. Rev. D 95, 054504 (2017)

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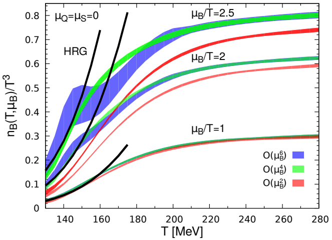


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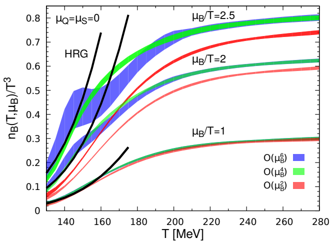


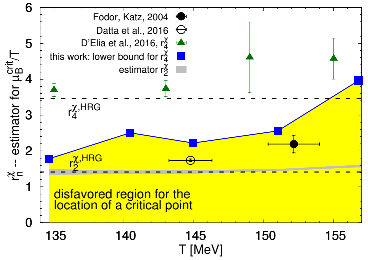


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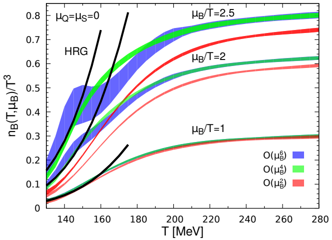


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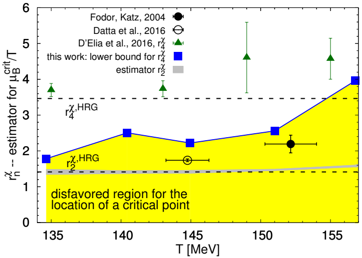
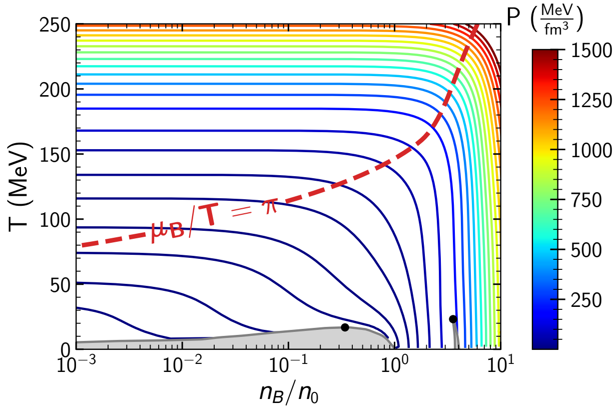



- Artifacts appear around $\mu_B/T > 2.5$
- Radius of convergence $\mu_B/T < 3$
- High $T$ rule out quark-repulsion.

**JS** and S. Schramm, Phys. Lett. B **736**, 241-245 (2014)

# Why the breakdown at $\mu_B/T \approx 3$?




### Why do the methods break down?

- Sudden change of isobaric lines at this point.
- From Boson (mesons/gluons) dominated matter to fermionic matter (nucleons/quarks).

A. Motornenko, **JS**, V. Vovchenko, S. Schramm and H. Stöcker, (Quark Matter 2019), Wuhan, China, November 3-9 2019

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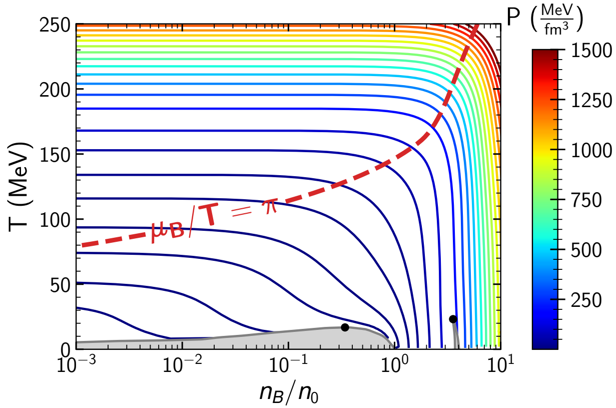


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- First principle calculations seem to fail for fermionic matter.

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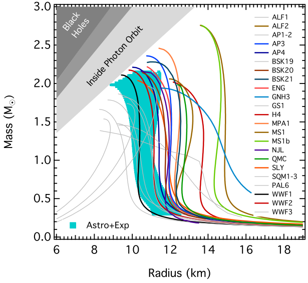


F. Özel and P. Freire, Ann. Rev. Astron. Astrophys.

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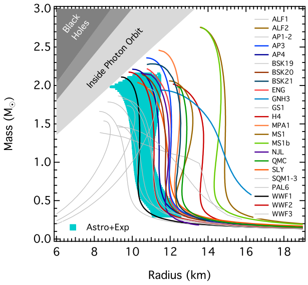


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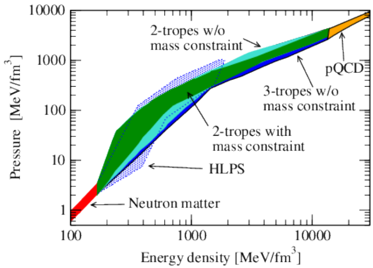


A. Kurkela, E. S. Fraga, J. Schaffner-Bielich and A. Vuorinen, Astrophys. J. **789**, 127 (2014)

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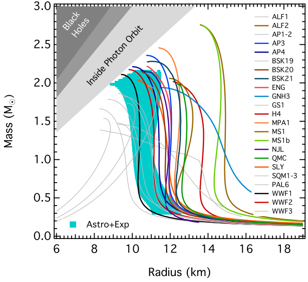


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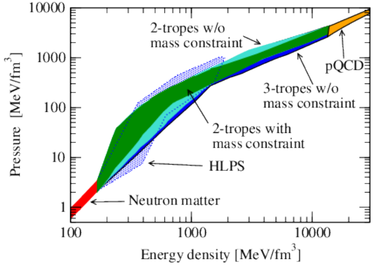


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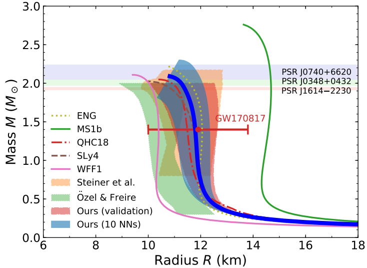
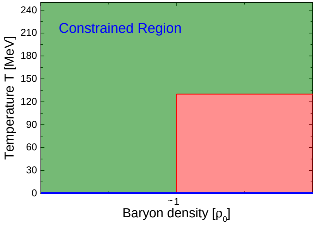



Y. Fujimoto, K. Fukushima and K. Murase, Phys.

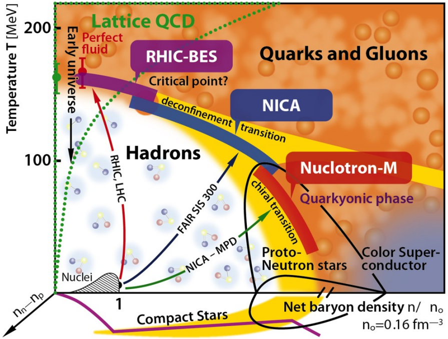
# Constraints at $T = 0$

- Here we have guidance from measured neutron star masses
- Without Radii no real constraints!
- Add constraints from PQCD.
- Still missing the important region. Extension to finite temperature $\rightarrow$ New degrees of freedom.



# The QCD EoS in Heavy Ion collisions

The NICA-JINR phase diagram in T and $\rho_B$.

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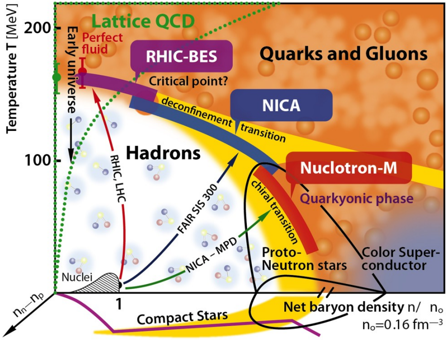


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Let's estimate the densities expected for central collisions.

- Geometrical Overlap Model:
  - $\rho = 2\gamma_{cm}\rho_0$
  - $\epsilon = 2\gamma_{cm}^2\epsilon_0$
- UrQMD with and without nuclear potentials.
  - Average densities in a box with $-0.5 < z < 0.5$ fm, $-3 < x, y < 3$ fm.
- Should give a good estimate on expected maximum compression.

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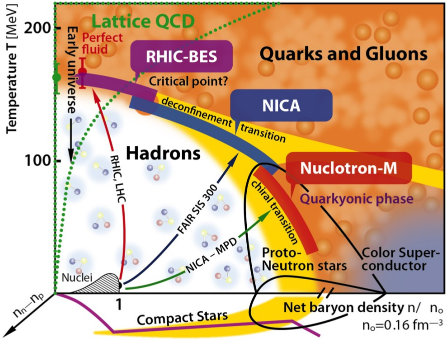


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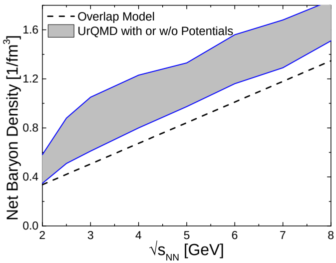
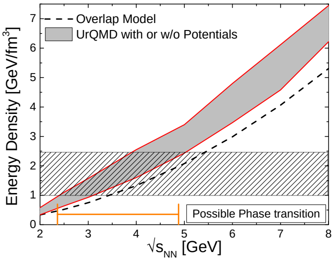


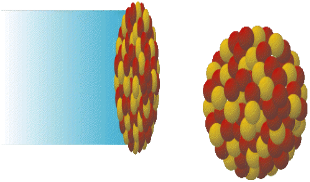
# How to study the equation of state using hadrons

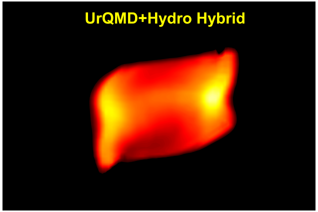
Much of we today think about hadronic observables is motivated by the fluid dynamic picture of HIC:

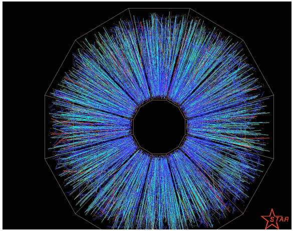
Pre-equilibrium phase

Equilibrated? phase

Final stage and particle freeze-out






Non-equilibrium initial state

Fluid dynamic evolution

Freeze-out: chemical and thermal

H. Petersen, **JS**, G. Burau, M. Bleicher and H. Stöcker, Phys. Rev. C **78** (2008) 044901

# An example: The $v_1$ story

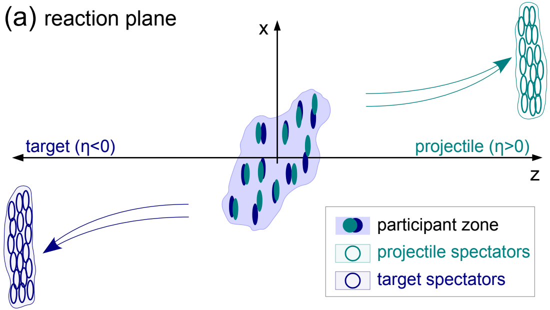
## Maybe:

- Early studies proposed the directed flow as a signal of the phase transition
- They where done using only 1 or 2 fluid dynamics.

## What is directed flow?

Deflection of matter in the reaction plane:

$v_1 = \langle p_x/p_T \rangle (y)$




B. Abelev et al. [ALICE Collaboration], Phys. Rev. Lett. 111, no. 23, 232302 (2013)

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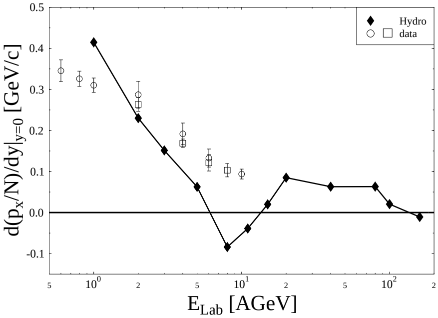


K. Paech, M. Reiter, A. Dumitru, H. Stoecker and W. Greiner, Nucl. Phys. A 681, 41 (2001)

## What is directed flow?

One is interested in the slope of $v_1 = \langle p_x/p_T \rangle (y)$ w.r.t the rapidity.

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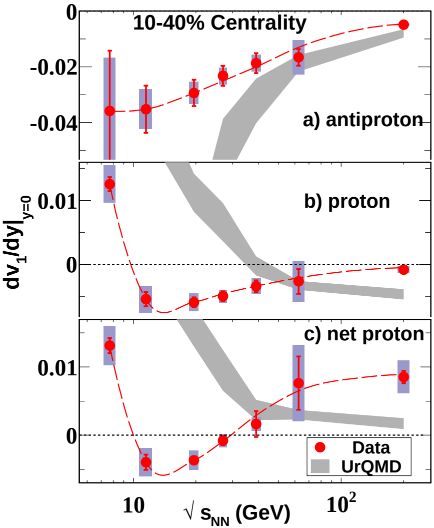
- Resent STAR measurements show a negative slope of net proton $v_1$.
- Is it the phase transition?

## STAR data

Data on the net proton $v_1$ slope show the predicted behavior.




L. Adamczyk et al. [STAR Collaboration], Phys. Rev. Lett. 112, 162301 (2014)

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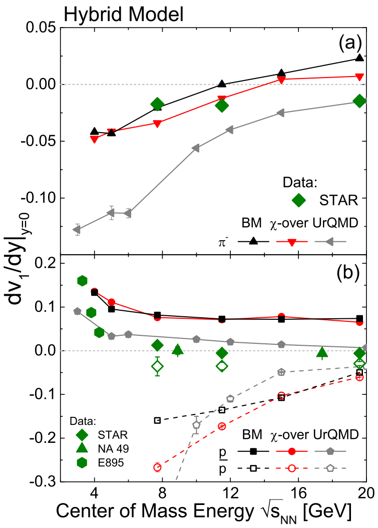
## Hybrid Model

However, when checked with state of the art hydro, no signal is found.

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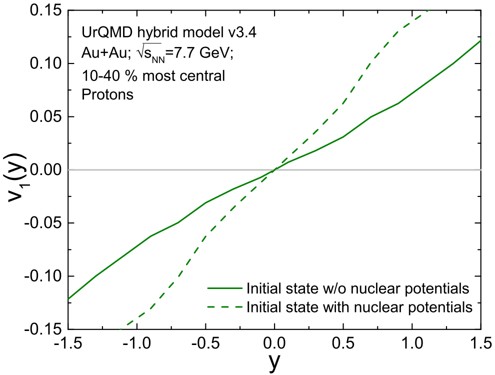
## Hybrid Model

However, when the stiffness of the initial state is changed one observes a sensitivity!




So $v_1$ might be sensitive to the 'softness' of the initial state...

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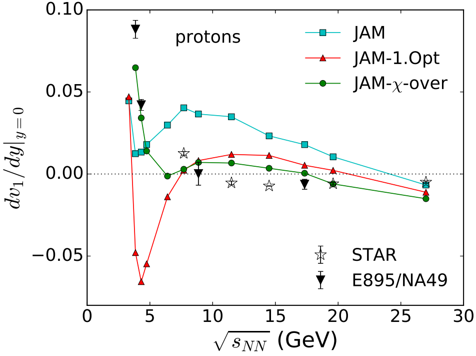
## Microscopic Transport with EoS

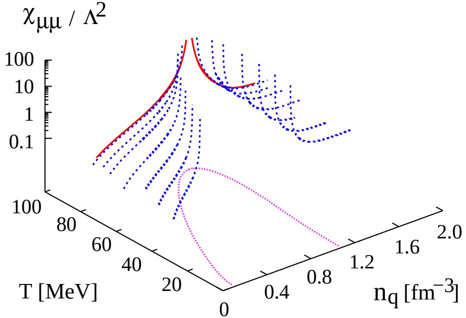
If a fully microscopic transport simulation with EoS (JAM) is used the effect persists.




Y. Nara, H. Niemi, **JS** and H. Stöcker, Phys. Lett. B **769**, 543-548 (2017)

# A Phase Transition in Fluid Dynamics

- In a dynamical scenario, locally the system may not be in phase eq.
- Phase separation occurs.
- At the critical point: divergence of correlation length.




C.Sasaki, B.Friman and K.Redlich, Phys. Rev. Lett. 99, 232301 (2007)

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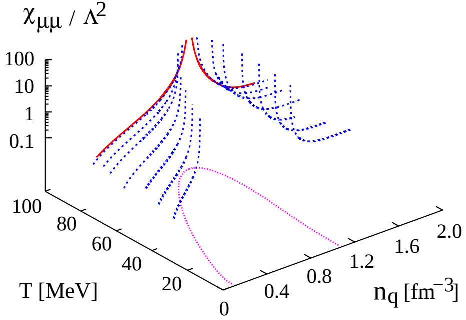
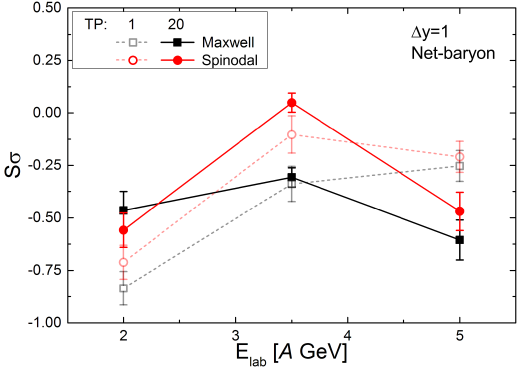



C.Sasaki, B.Friman and K.Redlich, Phys. Rev. Lett. 99, 232301 (2007)

- Susceptibilities diverge due to mechanically unstable phase.
- Separation of the two phases: Spinodal Instabilities.
- It's not the amplitude of the density fluctuation which diverges!

# What is the data situation?

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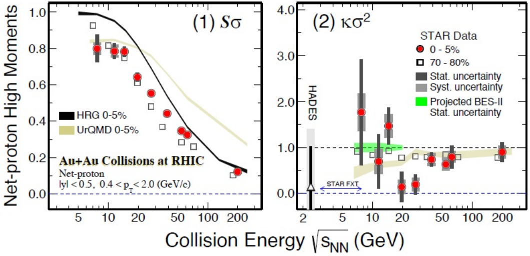


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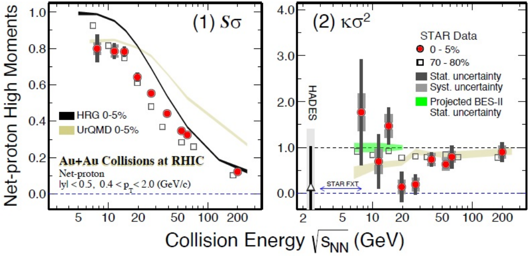


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**In short: what is really measured are fluctuations and correlations in momentum space.**

- The downsides of hadrons: freeze-out and rescattering wash out signals
- Implementation of EoS for the fully dynamical description from pre-equilibrum to freeze-out necessary

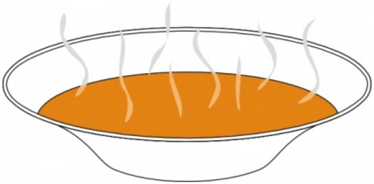
# Electromagnetic probes

Electromagnetic probes offer a chance to probe the whole time evolution of the fireball.

In particular di-lepton pairs created by the decay of hadrons or quark annihilation.

- $\rho \rightarrow e^+ + e^-$
- $q + \overline{q} \rightarrow e^+ + e^-$

Process sensitive to the medium in which it takes place ($T$ and $\rho_B$).



# Electromagnetic probes

Electromagnetic probes offer a chance to probe the whole time evolution of the fireball.

In particular di-lepton pairs created by the decay of hadrons or quark annihilation.

- $\rho \rightarrow e^{+} + e^{-}$
- $q + \overline{q} \rightarrow e^{+} + e^{-}$

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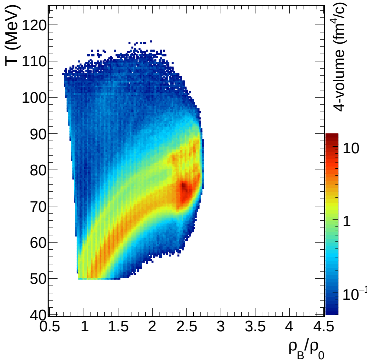
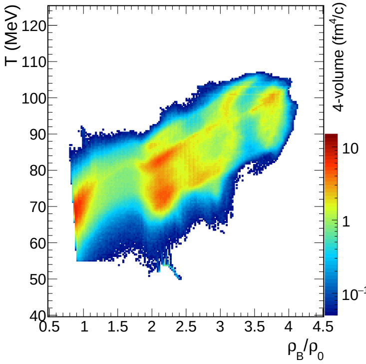
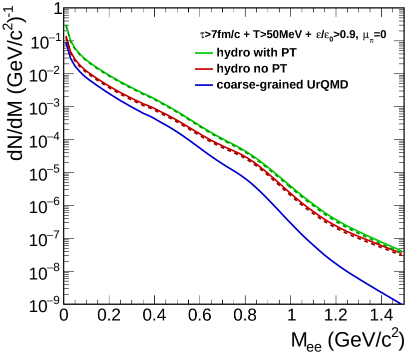


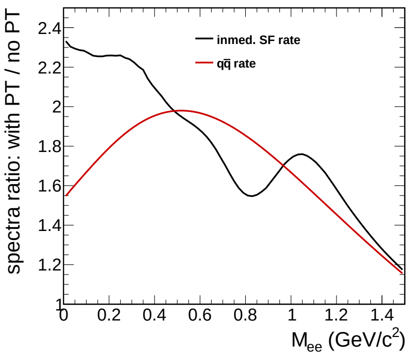




Distinct differences with or without a phase transition

## Indeed di-lepton emission shows a significant effect

- A simulation for Au+Au at the current SIS18 beam energy.
- A factor 2 enhancement of di-lepton emission due to extended 'cooking'.



# The strategy

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- Employ these models for heavy ion collisions as well neutron star mergers.
- Find a consistent description
- Possibly new analysis methods that combine many observables and statistical / machine learning methods.

# One example: Effective model for this - the CMF

## Effective SU$(3)_f$ chiral mean field model based on:

- Chiral symmetry for hadrons via nucleon parity partners: Describes nuclear matter and lattice phenomenology on masses.
- Deconfined quarks and gluons via effective Polyakov Loop potential and removal of hadrons via excluded volume.

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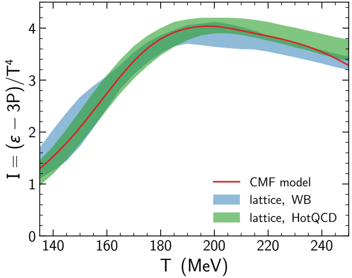


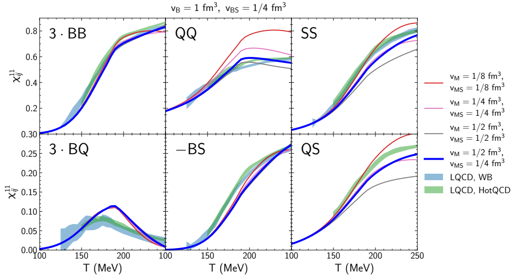

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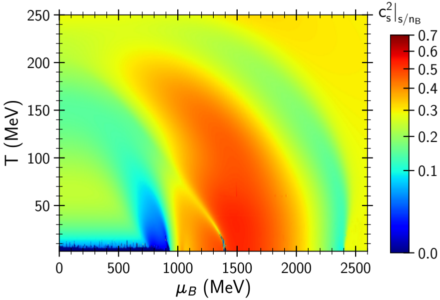

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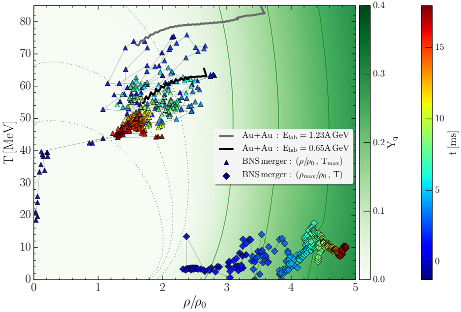


M. Hanauske, **JS** *et al.*, J. Phys. Conf. Ser. **878**, no. 1, 012031 (2017).

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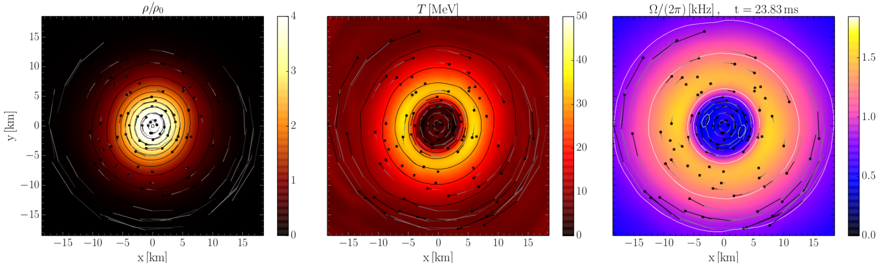
- A dense and cold core with a hot hadronic corona.

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# Summary

- Lattice QCD seem to be only useful up to $\mu_B/T \approx 3$ ,after that fermions become the dominant d.o.f.
- Neutron star properties constrain $T = 0$.
- No sign of a critical point or phase transition yet.
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- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 3$ A GeV) probe complementary region in the phase diagram.
- Treat both on the same footing $\rightarrow$ Combining QCD thermodynamics, relativistic fluid dynamics and GR.
- Use statistical/ML methods to combine the wealth of data for a consistent picture of the QCD phase diagram.