Study of the anomalous gauge couplings

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Introduction

Ways for searching for new physics:

- Direct — search for new particles, e.g. by peaks in invariant mass distribution.
- Indirect — search for deviations from the SM in the interactions of already known particles.

Search for anomalous couplings refers to indirect, model-independent way.

There are to formalisms for anomalous couplings: effective field theory (EFT) and vertex function (VF).

EFT and VF formalisms

EFT — parameterization of the Lagrangian with the operators of higher dimensions. The operators are constructed so that the gauge symmetries are respected. Usually each operator describes different vertices.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \ldots, \quad \mathcal{L}^{(d)} = \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}.$$

VF approach — parameterization of the vertex function or, equivalently, of the Lagrangian. Parameterization structures (operators) are not required to respect the gauge symmetries. Parameterization is made separately for each specific vertex.

Example for general three-boson vertex $V_1 V_2 V_3$:

$$\Gamma^{\alpha\beta\mu}_{V_1 V_2 V_3} = \Gamma^{\alpha\beta\mu}_{V_1 V_2 V_3,\mathrm{SM}} + g_{V_1 V_2 V_3} \sum_i h_i \Gamma^{\alpha\beta\mu}_{V_1 V_2 V_3,i}$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + g_{V_1 V_2 V_3} \sum_i h_i \mathcal{O}_i$$

$\Gamma^{\alpha\beta\mu}_{V_1 V_2 V_3,i}$ is the Feynman rule for the vertex described by $\mathcal{O}_i$. Simplified explanation: it is $\mathcal{O}_i$ in the momentum space.

Anomalous quartic gauge couplings (aQGCs)

AQGCs are studied only in EFT formalism.
Operators are dimension-eight, since dimension-six operators contain triple gauge couplings counterpart, and therefore are not valid for studying genuine aQGCs.

$$\mathcal{O}_{\mathbf{S0}} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right]\left[(D^\mu\Phi)^\dagger D^\nu\Phi\right],$$
$$\mathcal{O}_{\mathbf{S1}} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right]\left[(D_\nu\Phi)^\dagger D^\nu\Phi\right].$$

$$\mathcal{O}_{\mathbf{M0}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right],$$
$$\mathcal{O}_{\mathbf{M1}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right],$$
$$\mathcal{O}_{\mathbf{M2}} = \left[B_{\mu\nu}B^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right],$$
$$\mathcal{O}_{\mathbf{M3}} = \left[B_{\mu\nu}B^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right],$$
$$\mathcal{O}_{\mathbf{M4}} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}D^\mu\Phi\right]B^{\beta\nu},$$
$$\mathcal{O}_{\mathbf{M5}} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}D^\nu\Phi\right]B^{\beta\mu} + \mathbf{h.c.},$$
$$\mathcal{O}_{\mathbf{M7}} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right].$$

$$\mathcal{O}_{\mathbf{T0}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\mathbf{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right],$$
$$\mathcal{O}_{\mathbf{T1}} = \mathbf{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\mathbf{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right],$$
$$\mathcal{O}_{\mathbf{T2}} = \mathbf{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\mathbf{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right],$$
$$\mathcal{O}_{\mathbf{T3}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}_{\alpha\beta}\right]\mathbf{Tr}\left[\hat{W}^{\alpha\nu}\hat{W}^{\mu\beta}\right],$$
$$\mathcal{O}_{\mathbf{T4}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}_{\alpha\beta}\right]\left[B^{\alpha\nu}B^{\mu\beta}\right],$$
$$\mathcal{O}_{\mathbf{T5}} = \mathbf{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\left[B_{\alpha\beta}B^{\alpha\beta}\right],$$
$$\mathcal{O}_{\mathbf{T6}} = \mathbf{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\left[B_{\mu\beta}B^{\alpha\nu}\right],$$
$$\mathcal{O}_{\mathbf{T7}} = \mathbf{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\left[B_{\beta\nu}B^{\nu\alpha}\right],$$
$$\mathcal{O}_{\mathbf{T8}} = \left[B_{\mu\nu}B^{\mu\nu}\right]\left[B_{\alpha\beta}B^{\alpha\beta}\right],$$
$$\mathcal{O}_{\mathbf{T9}} = \left[B_{\alpha\mu}B^{\mu\beta}\right]\left[B_{\beta\nu}B^{\nu\alpha}\right].$$

Corresponding coefficients: $f_{\mathbf{S0}}/\Lambda^{\mathbf{4}}$, $f_{\mathbf{S1}}/\Lambda^{\mathbf{4}}$, $f_{\mathbf{M0}}/\Lambda^{\mathbf{4}}$, etc.
Previously limits on 7 coefficients were set as an interpretation of $Z(\nu\bar{\nu})\gamma jj$ ATLAS Run II analysis.

Anomalous neutral triple gauge couplings (nTGCs)

NTGCs — triple couplings between Z and γ. They are zero in the SM and studied in EFT and VF formalisms. EFT operators are dimension-eight, since dimension-six operators do not describe nTGCs.

$$\mathcal{O}_{\tilde{B}W} = i\Phi^\dagger \tilde{B}_{\mu\nu}\hat{W}^{\mu\rho}\{D_\rho, D^\nu\}\Phi + \text{h.c.}, \qquad \mathcal{O}_{BW} = i\Phi^\dagger B_{\mu\nu}\hat{W}^{\mu\rho}\{D_\rho, D^\nu\}\Phi + \text{h.c.},$$

$$\mathcal{O}_{BB} = i\Phi^\dagger B_{\mu\nu}B^{\mu\rho}\{D_\rho, D^\nu\}\Phi + \text{h.c.}, \qquad \mathcal{O}_{WW} = i\Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\rho}\{D_\rho, D^\nu\}\Phi + \text{h.c.},$$

$$\mathcal{O}_{G\pm} = \frac{2}{g}\tilde{B}_{\mu\nu}\text{Tr}\left[\hat{W}^{\mu\rho}\left(D_\rho D_\lambda \hat{W}^{\nu\lambda} \pm D^\nu D^\lambda \hat{W}^{\lambda\rho}\right)\right].$$

Coefficients: C_{BB}/Λ^4, C_{WW}/Λ^4, C_{BW}/Λ^4 (CP-violating) and $C_{\tilde{B}W}/\Lambda^4$, C_{G+}/Λ^4, C_{G-}/Λ^4 (CP-conserving).

VF approach:

$$\Gamma^{\alpha\beta\mu(8)}_{Z\gamma V^*}(q_1, q_2, q_3) = \frac{e(q_3^2 - M_V^2)}{M_Z^2}\left[\left(h_3^V + h_5^V\frac{q_3^2}{M_Z^2}\right)q_{2\nu}\epsilon^{\alpha\beta\mu\nu} + \frac{h_4^V}{M_Z^2}q_2^\alpha q_{3\nu}q_{2\sigma}\epsilon^{\beta\mu\nu\sigma}\right],$$

$$\mathcal{L} = \frac{e}{M_Z^2}\Bigg[-\left[h_3^\gamma(\partial_\sigma F^{\sigma\rho}) + h_3^Z(\partial_\sigma Z^{\sigma\rho}) + \frac{h_5^\gamma}{M_Z^2}(\partial^2\partial_\sigma F^{\rho\sigma}) + \frac{h_5^Z}{M_Z^2}(\partial^2\partial_\sigma Z^{\rho\sigma})\right]Z^\alpha\tilde{F}_{\rho\alpha}$$
$$+\left\{\frac{h_4^\gamma}{2M_Z^2}[\Box\partial^\sigma F^{\rho\alpha}] + \frac{h_4^Z}{2M_Z^2}[(\Box + M_Z^2)\partial^\sigma Z^{\rho\alpha}]\right\}Z_\sigma\tilde{F}_{\rho\alpha}\Bigg].$$

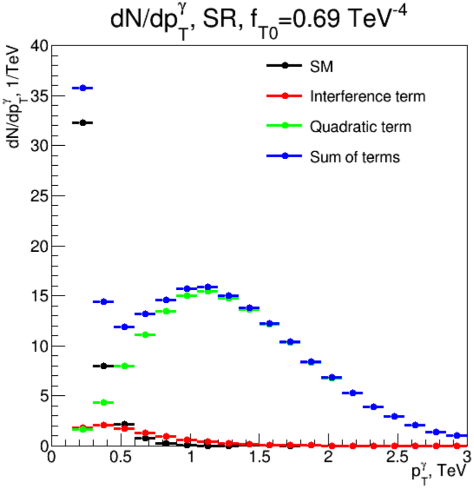
Coefficients: h_1^V, h_2^V (CP-violating) and h_3^V, h_4^V, h_5^V (CP-conserving).

The last paper 2308.16887 introduces 3 EFT, 2 VF new coefficients. Moreover, new VF formalism for off-shell Z is suggested.

Amplitude decomposition

Parameterization by a single operator:
$\mathcal{L} = \mathcal{L}_{SM} + f\mathcal{O}$.

Squared amplitude:
$|\mathcal{A}|^2 = |\mathcal{A}_{SM} + f\mathcal{A}_{BSM}|^2 =$
$= |\mathcal{A}_{SM}|^2 + f2\mathrm{Re}(\mathcal{A}_{SM}^{\dagger}\mathcal{A}_{BSM}) + f^2|\mathcal{A}_{BSM}|^2$



UFO models for nTGCs

In order to generate events in `MadGraph5` with decomposition, one needs an universal feynrules output (UFO) model.
Previous models:
1. [EFT] The first model created by Celine Degrande. In contains 4 operators and does not support direct generation of cross terms.
2. [EFT] Model created by authors of operators $\mathcal{O}_{G\pm}$. In contains 3 CP-even operators and supports direct generation of cross terms.
3. [VF] Model contains coefficients f_4^V, f_5^V, h_1^V, h_2^V, h_3^V, h_4^V and does not support direct generation of cross terms.

New model for EFT was needed for conveniency (all operators in a single model) and direct generation of any cross term. It was created using FeynRules, validated and agreed with the authors. Moreover, a small incostistency between previous models and operators from the papers was removed. New model was uploaded to the ATLAS model database and is used for the sample request for current $Z(\nu\bar{\nu})\gamma$ and $ZZ \to \ell\ell\nu\nu$ analyses.

New model for VF was also created and validated. Coefficients h_5^V are added, and direct generation of cross terms became possible. It is planned to use it for the sample request for $Z(\nu\bar{\nu})\gamma$ analysis.

Methods for increasing the sensitivity

1. Luminosity increasing.

2. Usage of sensitive variables.

The most prospective way for setting the limits is to base it on at least to variables: the first one is sensitive to quadratic term and the second one is sensitive to interference term.

Quadratic term: sensitive variables are correlated with bosonic $\sqrt{\hat{s}}$. Examples: E_{T}^{γ} (for $Z(\nu\bar{\nu})\gamma$ analysis), $p_{\mathrm{T}}^{\ell\ell}$ (for $ZZ \to \ell\ell\nu\nu$ analysis), etc.

Interference term: sensitive variables are not trivial. Examples: requirement of 1 jet with high p_{T} for reducing the suppression of interference (W^+W^- analysis), difficult angular variables ($W\gamma$ analysis, usually leptons or both bosons reconstructed are needed), matrix-based optimal observables (Higgs WG), ML (currently do not widely used for EFT).

We try to probe optimal observables and simple variables, including CP-sensitive variables. Issue: CP-sensitive variables are usually P-sensitive; however our CP-odd operators are P-even...

3. Accounting of EFT impact on backgrounds.

Optimal observables

Parameterization by one operator:

$$\mathcal{M}^2 = |\mathcal{M}_{\mathrm{SM}} + C\mathcal{M}_{\mathrm{BSM}}|^2 = |\mathcal{M}_{\mathrm{SM}}|^2 + C \cdot 2\mathrm{Re}(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}}) + C^2|\mathcal{M}_{\mathrm{BSM}}|^2 =$$

$$= |\mathcal{M}_{\mathrm{SM}}|^2 \left(1 + C\frac{2\mathrm{Re}(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}})}{|\mathcal{M}_{\mathrm{SM}}|^2} + C^2\frac{|\mathcal{M}_{\mathrm{BSM}}|^2}{|\mathcal{M}_{\mathrm{SM}}|^2}\right)$$

Optimal observables (for operator $\mathcal{O}_i$):

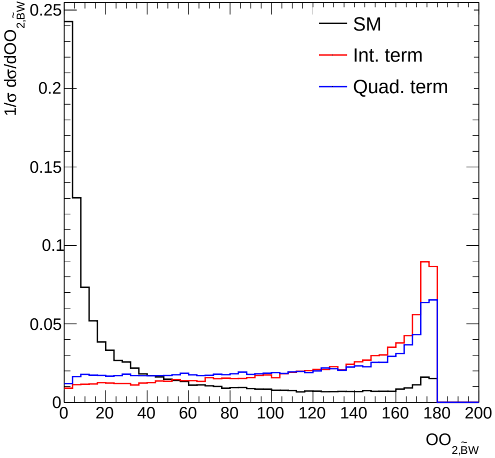
$$OO_{1,i} = 1\ \mathrm{TeV}^{-4} \cdot \frac{2\mathrm{Re}(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}})}{|\mathcal{M}_{\mathrm{SM}}|^2}; \qquad OO_{2,i} = 1\ \mathrm{TeV}^{-8} \cdot \frac{|\mathcal{M}_{\mathrm{BSM}}|^2}{|\mathcal{M}_{\mathrm{SM}}|^2}$$

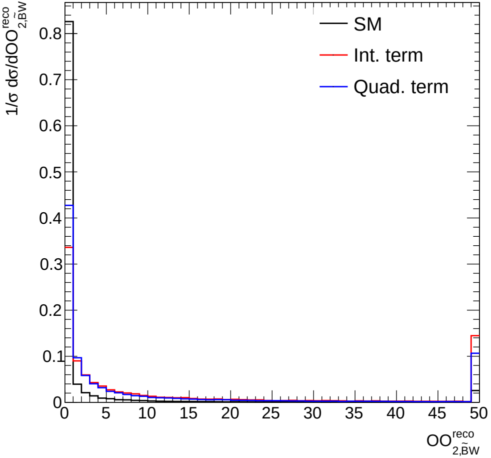
- OOs are different for different operators.
- Reconstruction of OOs for $pp \to \nu\bar{\nu}\gamma$ is impossible. Proposal of reco-level OOs: process $pp \to Z\gamma$ and

$$OO^{\mathrm{reco}}_{1,i} = 1\ \mathrm{TeV}^{-4} \cdot \frac{2\mathrm{Re}(\mathcal{M}^\dagger_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}})}{|\mathcal{M}_{\mathrm{SM}}|^2}(p^Z_z = 0); \qquad OO^{\mathrm{reco}}_{2,i} = 1\ \mathrm{TeV}^{-8} \cdot \frac{|\mathcal{M}_{\mathrm{BSM}}|^2}{|\mathcal{M}_{\mathrm{SM}}|^2}(p^Z_z = 0)$$

$OO_{2,\tilde{B}W}$

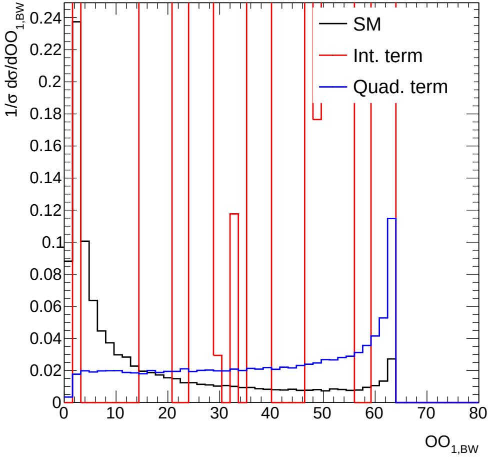
Simple example for subprocess $u\bar{u} \to Z\gamma$.

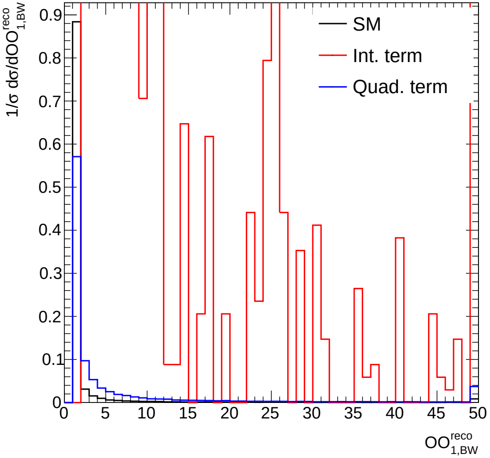




$OO_{1,BW}$

Simple example for subprocess $u\bar{u} \to Z\gamma$.





EFT impact on backgrounds: aQGC

Z(νν)γjj

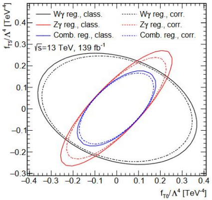
Coefficient	Class. lim. [TeV^{-4}]	Corr. lim. [TeV^{-4}]	Improvement
	Integrated luminosity of 139 fb^{-1}		
f_{T0}/Λ^4	[−0.125; 0.119]	[−0.124; 0.118]	1.3%
f_{T5}/Λ^4	[−0.125; 0.132]	[−0.122; 0.128]	2.7%
f_{M0}/Λ^4	[−6.04; 6.05]	[−5.57; 5.57]	7.8%
f_{M2}/Λ^4	[−2.42; 2.42]	[−2.20; 2.20]	9.1%

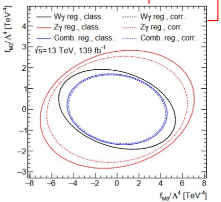
W(lν)γjj

Coefficient	Class. lim. [TeV^{-4}]	Corr. lim. [TeV^{-4}]	Improvement
	Integrated luminosity of 139 fb^{-1}		
f_{T0}/Λ^4	[−0.286; 0.292]	[−0.274; 0.278]	4.4%
f_{T5}/Λ^4	[−0.204; 0.206]	[−0.196; 0.198]	4.1%
f_{M0}/Λ^4	[−4.30; 4.28]	[−4.29; 4.27]	0.3%
f_{M2}/Λ^4	[−1.54; 1.54]	[−1.53; 1.53]	0.4%

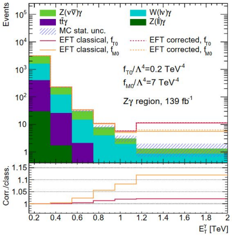
Combination

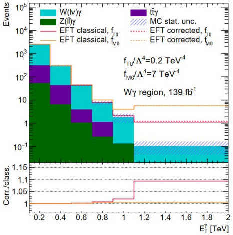
Coefficient	Class. lim. [TeV^{-4}]	Corr. lim. [TeV^{-4}]	Improvement
	Integrated luminosity of 139 fb^{-1}		
f_{T0}/Λ^4	[−0.120; 0.115]	[−0.118; 0.113]	1.6%
f_{T5}/Λ^4	[−0.114; 0.119]	[−0.110; 0.116]	3.0%
f_{M0}/Λ^4	[−3.86; 3.85]	[−3.76; 3.75]	2.7%
f_{M2}/Λ^4	[−1.43; 1.42]	[−1.39; 1.39]	2.6%





Operators basis: 1604.03555
Simulation: Madgraph5+Pythia8+Delphes3
Conditions: ATLAS Run II
Considered channels: Z(νν)γjj, W(lν)γjj and their combination
Selection is based on papers 1705.01966 and 2008.10521
Total flat systematic of 10% is applied
Limits from Z(νν)γjj are corrected using W(lν)γjj background
Limits from W(lν)γjj are corrected using Z(ll)γjj background



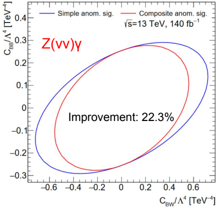


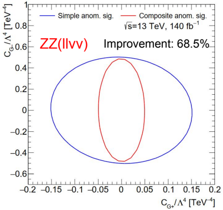
Run III integrated luminosity was also considered,
Improvement of the limits is similar
Improvement for 2D limits: up to 17.2%
Paper: 2209.07906

EFT impact on backgrounds: nTGC

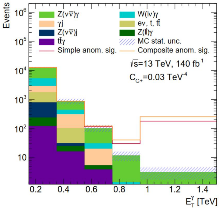
Coefficient	$Z\gamma$ anom. signal	$Z\gamma + W\gamma$ anom. signal	Improvement
C_{G+}/Λ^4	[-5.29; 5.30]$\times 10^{-3}$	[-4.43; 4.45]$\times 10^{-3}$	16.2%
C_{G-}/Λ^4	[-0.272; 0.286]	[-0.272; 0.286]	0
$C_{\tilde{B}W}/\Lambda^4$	[-0.306; 0.300]	[-0.244; 0.233]	21.4%
C_{BW}/Λ^4	[-0.549; 0.551]	[-0.447; 0.450]	18.4%
C_{BB}/Λ^4	[-0.223; 0.222]	[-0.223; 0.222]	0
C_{WW}/Λ^4	[-1.11; 1.12]	[-1.11; 1.12]	0

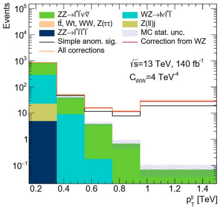
Coefficient	ZZ anom. signal	$ZZ + WZ$ anom. signal	Improvement
C_{G+}/Λ^4	[-0.124; 0.123]	[-0.041; 0.041]	67.1%
C_{G-}/Λ^4	[-0.412; 0.415]	[-0.399; 0.403]	3.1%
$C_{\tilde{B}W}/\Lambda^4$	[-0.663; 0.671]	[-0.626; 0.639]	5.2%
C_{BW}/Λ^4	[-1.53; 1.51]	[-1.42; 1.42]	6.4%
C_{BB}/Λ^4	[-0.815; 0.819]	[-0.815; 0.819]	0
C_{WW}/Λ^4	[-1.27; 1.25]	[-1.05; 1.04]	17.4%





Operators basis: 1308.6323 and 2008.04298
Simulation: Madgraph5+Pythia8+Delphes3
Conditions: ATLAS Run II
Considered channels: Z(νν)γ and ZZ(llνν)
Selection is based on papers 1810.04995 and 1905.07163
Total flat systematic of 10% is applied
Limits from Z(νν)γ are corrected using W(lν)γ background
Limits from ZZ(llνν) are corrected using WZ(lνll) background





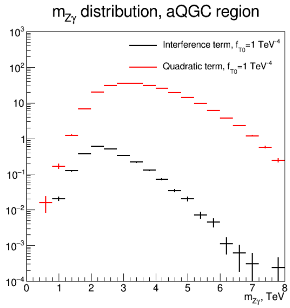
In ZZ case impact on other backgrounds was also considered. Background Z(ee/μμ)+jets provides additional improvement of 3-13%.

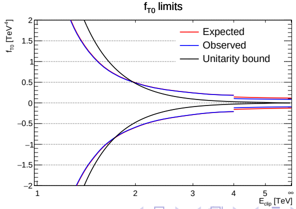
Unitarization of the limits

In order to make the limits unitarized, the clipping method was used in aQGC interpretation of $Z(\nu\bar{\nu})\gamma jj$ analysis. It is based on setting the anomalous contributions to zero if $\sqrt{\hat{s}} > E_c$.
Unitarity bounds are calculated analytically (2004.05174) basing on partial wave unitarity conditions for VBS $V_1 V_2 \to V_3 V_4$.
As a result, for some E_c the limits become unitarized.

For nTGC it is possible to make the same unitarization. Unitarity bounds are calculated for process $f\bar{f} \to Z\gamma$ in 2308.16887.

Example: $C_{\tilde{B}W}/\Lambda^4 < \dfrac{24\sqrt{2}\pi}{s^2}$





Reinterpretation of the limits

General way for re-interpretation: to write a Lagrangian with new particles interacting with the bosons, to integrate them out from the partition function. The result will be the effective Lagrangian, and it can be matched with the operators.

It was done for aQGC in 1908.09845 (basing on similar but another operators basis).

	scalar	fermion	vector
$c_1^{B^4}$	$\frac{7}{32} g_1^4 Q^4$	$\frac{1}{2} g_1^4 Q^4$	$\frac{261}{32} g_1^4 Q^4$
$c_2^{B^4}$	$\frac{1}{32} g_1^4 Q^4$	$\frac{7}{8} g_1^4 Q^4$	$\frac{243}{32} g_1^4 Q^4$
$c_1^{W^4}$	$g_2^4\left[\frac{7}{32}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{1}{2}\Lambda(\mathbf{R}_2)+\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{32}\Lambda(\mathbf{R}_2)-\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_2^{W^4}$	$g_2^4\left[\frac{1}{32}\Lambda(\mathbf{R}_2)+\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{8}\Lambda(\mathbf{R}_2)+\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{32}\Lambda(\mathbf{R}_2)-\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_3^{W^4}$	$g_2^4\left[\frac{7}{16}\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\Lambda(\mathbf{R}_2)-\frac{1}{48}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{261}{16}\Lambda(\mathbf{R}_2)+\frac{3}{16}I_2(\mathbf{R}_2)\right]$
$c_4^{W^4}$	$g_2^4\left[\frac{1}{16}\Lambda(\mathbf{R}_2)-\frac{1}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{7}{4}\Lambda(\mathbf{R}_2)-\frac{19}{336}I_2(\mathbf{R}_2)\right]$	$g_2^4\left[\frac{243}{16}\Lambda(\mathbf{R}_2)+\frac{27}{112}I_2(\mathbf{R}_2)\right]$
$c_1^{G^4}$	$g_3^4\left[\frac{7}{32}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{1}{2}\Lambda(\mathbf{R}_3)+\frac{1}{96}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{32}\Lambda(\mathbf{R}_3)-\frac{3}{32}I_2(\mathbf{R}_3)\right]$
$c_2^{G^4}$	$g_3^4\left[\frac{1}{32}\Lambda(\mathbf{R}_3)+\frac{1}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{8}\Lambda(\mathbf{R}_3)+\frac{19}{672}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{32}\Lambda(\mathbf{R}_3)-\frac{27}{224}I_2(\mathbf{R}_3)\right]$
$c_3^{G^4}$	$g_3^4\left[\frac{7}{16}\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\Lambda(\mathbf{R}_3)-\frac{1}{48}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{261}{16}\Lambda(\mathbf{R}_3)+\frac{3}{16}I_2(\mathbf{R}_3)\right]$
$c_4^{G^4}$	$g_3^4\left[\frac{1}{16}\Lambda(\mathbf{R}_3)-\frac{1}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{7}{4}\Lambda(\mathbf{R}_3)-\frac{19}{336}I_2(\mathbf{R}_3)\right]$	$g_3^4\left[\frac{243}{16}\Lambda(\mathbf{R}_3)+\frac{27}{112}I_2(\mathbf{R}_3)\right]$
$c_5^{G^4}$	$\frac{1}{32} g_3^4 I_2(\mathbf{R}_3)$	$\frac{1}{32} g_3^4 I_2(\mathbf{R}_3)$	$-\frac{9}{32} g_3^4 I_2(\mathbf{R}_3)$
$c_6^{G^4}$	$\frac{1}{224} g_3^4 I_2(\mathbf{R}_3)$	$\frac{19}{224} g_3^4 I_2(\mathbf{R}_3)$	$-\frac{81}{224} g_3^4 I_2(\mathbf{R}_3)$
$c_1^{B^2W^2}$	$\frac{7}{16} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{261}{16} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$
$c_2^{B^2W^2}$	$\frac{1}{16} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{7}{4} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{243}{16} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$
$c_3^{B^2W^2}$	$\frac{7}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$2 g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{261}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$
$c_4^{B^2W^2}$	$\frac{1}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{7}{2} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$	$\frac{243}{8} g_1^2 g_2^2 Q^2 I_2(\mathbf{R}_2)$
$c_1^{B^2G^2}$	$\frac{7}{16} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$	$g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$	$\frac{261}{16} g_1^2 g_3^2 Q^2 I_2(\mathbf{R}_3)$