The screening Horndeski cosmologies

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Horndeski theory

In 1974, Horndeski derived the action of the most general scalar-tensor theories with second-order equations of motion
[G.Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, IJTP **10**, 363 (1974)]

Horndeski Lagrangian:

$$L_{\rm H} = \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X,\Phi)\,,\\
\mathcal{L}_3 &= G_3(X,\Phi)\,\Box\Phi\,,\\
\mathcal{L}_4 &= G_4(X,\Phi)\,R + \partial_X G_4(X,\Phi)\,\delta^{\mu\nu}_{\alpha\beta}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\,,\\
\mathcal{L}_5 &= G_5(X,\Phi)\,G_{\mu\nu}\nabla^{\mu\nu}\Phi - \tfrac{1}{6}\,\partial_X G_5(X,\Phi)\,\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\,\nabla^\alpha_\mu\Phi\nabla^\beta_\nu\Phi\nabla^\gamma_\rho\Phi\,,
\end{aligned}$$

where $X = -\frac{1}{2}(\nabla\phi)^2$, and $G_k(X,\Phi)$ are arbitrary functions,
and $\delta^{\lambda\rho}_{\nu\alpha} = 2!\,\delta^\lambda_{[\nu}\delta^\rho_{\alpha]}$, $\delta^{\lambda\rho\sigma}_{\nu\alpha\beta} = 3!\,\delta^\lambda_{[\nu}\delta^\rho_\alpha\delta^\sigma_{\beta]}$

Fab Four subclass of the Horndeski theory

There is a special subclass of the theory, sometimes called Fab Four (F4), for which the coefficients are chosen such that the Lagrangian becomes

$$L_{\rm F4} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

with

$$\begin{aligned}
\mathcal{L}_J &= V_J(\Phi)\, G_{\mu\nu} \nabla^\mu \Phi \nabla^\nu \Phi\,, \\
\mathcal{L}_P &= V_P(\Phi)\, P_{\mu\nu\rho\sigma} \nabla^\mu \Phi \nabla^\rho \Phi \nabla^{\nu\sigma} \Phi\,, \\
\mathcal{L}_G &= V_G(\Phi)\, R\,, \\
\mathcal{L}_R &= V_R(\Phi)\, (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2).
\end{aligned}$$

Here the double dual of the Riemann tensor is

$$P^{\mu\nu}{}_{\alpha\beta} = -\frac{1}{4}\,\delta^{\mu\nu\gamma\delta}_{\sigma\lambda\alpha\beta}\, R^{\sigma\lambda}{}_{\gamma\delta} = -R^{\mu\nu}{}_{\alpha\beta} + 2R^{\mu}_{[\alpha}\delta^{\nu}_{\beta]} - 2R^{\nu}_{[\alpha}\delta^{\mu}_{\beta]} - R\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta]}\,,$$

whose contraction is the Einstein tensor, $P^{\mu\alpha}{}_{\nu\alpha} = G^{\mu}{}_{\nu}$.

Fab Four subclass of the Horndeski theory

Fab Four Lagrangian:

$$L_{\rm F4} = \sqrt{-g}\,(\mathcal{L}_J + \mathcal{L}_P + \mathcal{L}_G + \mathcal{L}_R - 2\Lambda)$$

- The Fab Four model is distinguished by the *screening property* – it is the most general subclass of the Horndeski theory in which flat space is a solution, despite the presence of the cosmological term Λ.
- This property suggests that Λ is actually irrelevant and hence there is no need to explain its value.
- Indeed, however large Λ is, Minkowski space is always a solution and so one may hope that a slowly accelerating universe will be a solution as well.

The model with nonminimal kinetic coupling

Action of the theory with nonminimal kinetic coupling:

$$S = \frac{1}{2}\int \left(M_{\rm Pl}^2 R - (\alpha\, G_{\mu\nu} + \varepsilon\, g_{\mu\nu})\nabla^\mu\Phi\nabla^\nu\Phi - 2\Lambda\right)\sqrt{-g}\, d^4x + S_{\rm m}$$

The gravitational equations:

$$M_{\rm Pl}^2 G_{\mu\nu} + \Lambda g_{\mu\nu} = \alpha\,\mathcal{T}_{\mu\nu} + \varepsilon\, T^{(\Phi)}_{\mu\nu} + T^{(\rm m)}_{\mu\nu},$$

with

$$\mathcal{T}_{\mu\nu} = P_{\alpha\mu\nu\beta}\nabla^\alpha\Phi\nabla^\beta\Phi + \frac{1}{2}\, g_{\mu\lambda}\,\delta^{\lambda\rho\sigma}_{\nu\alpha\beta}\,\nabla^\alpha_\rho\Phi\nabla^\beta_\sigma\Phi - X G_{\mu\nu}\,,$$
$$T^{(\Phi)}_{\mu\nu} = \nabla_\mu\Phi\nabla_\nu\Phi + X g_{\mu\nu}\,,$$
$$T^{(\rm m)}_{\mu\nu} = (\rho + p)U_\mu U_\mu + p g_{\mu\nu}\,,$$

The scalar equation

$$\nabla_\mu((\alpha G^{\mu\nu} + \varepsilon g^{\mu\nu})\nabla_\nu\Phi) = 0\,.$$

Cosmological models

The FLRW ansatz for the metric:

$$ds^2 = -dt^2 + \mathrm{a}^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)\right],$$

$\mathrm{a}(t)$ *cosmological factor*, $H = \dot{\mathrm{a}}/\mathrm{a}$ *Hubble parameter*

Gravitational equations:

$$-3M_{\mathrm{Pl}}^2\left(H^2 + \frac{K}{\mathrm{a}^2}\right) + \frac{1}{2}\varepsilon\,\psi^2 - \frac{3}{2}\alpha\,\psi^2\left(3H^2 + \frac{K}{\mathrm{a}^2}\right) + \Lambda + \rho = 0,$$

$$-M_{\mathrm{Pl}}^2\left(2\dot{H} + 3H^2 + \frac{K}{\mathrm{a}^2}\right) - \frac{1}{2}\varepsilon\,\psi^2 - \alpha\,\psi^2\left(\dot{H} + \frac{3}{2}H^2 - \frac{K}{\mathrm{a}^2} + 2H\frac{\dot{\psi}}{\psi}\right) + \Lambda - p = 0.$$

The scalar field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}\left(\mathrm{a}^3\left(3\alpha\left(H^2 + \frac{K}{\mathrm{a}^2}\right) - \varepsilon\right)\psi\right) = 0,$$

where $\psi = \dot{\Phi}$, and $\Phi = \Phi(t)$ is a homogeneous scalar field

Cosmological models

The first integral of the scalar field equation:

$$\mathrm{a}^3\left(3\alpha\left(H^2+\frac{K}{\mathrm{a}^2}\right)-\varepsilon\right)\psi = C,$$

where C is the Noether charge associated with the shift symmetry $\Phi \to \Phi + \Phi_0$.

Let $C = 0$. One finds in this case two different solutions:

GR branch: $\psi = 0 \quad \Longrightarrow \quad H^2+\frac{K}{\mathrm{a}^2}=\frac{\Lambda+\rho}{3M_{\mathrm{Pl}}^2}$

Screening branch: $H^2+\frac{K}{\mathrm{a}^2}=\frac{\varepsilon}{3\alpha} \quad \Longrightarrow \quad \psi^2=\frac{\alpha\,(\Lambda+\rho)-\varepsilon\,M_{\mathrm{Pl}}^2}{\alpha\,(\varepsilon-3\alpha\,K/\mathrm{a}^2)}$

NOTICE: The role of the cosmological constant in the screening solution is played by $\varepsilon/3\alpha$ while the Λ-term is screened and makes no contribution to the universe acceleration.

Note also that the matter density ρ is screened in the same sense.

Let $C \neq 0$, then

$$\psi = \frac{C}{\mathrm{a}^3\,\left[3\alpha\left(H^2+\frac{K}{\mathrm{a}^2}\right)-\varepsilon\right]},$$

and the modified Friedmann equation reads

$$3M_{\mathrm{Pl}}^2\left(H^2+\frac{K}{\mathrm{a}^2}\right) = \frac{C^2\left[\varepsilon-3\alpha\left(3H^2+\frac{K}{\mathrm{a}^2}\right)\right]}{2a^6\,\left[\varepsilon-3\alpha\left(H^2+\frac{K}{\mathrm{a}^2}\right)\right]^2}+\Lambda+\rho.$$

Introducing dimensionless values and density parameters

$$H^2 = H_0^2\,y,\;\; \mathrm{a}=\mathrm{a}_0\,a\,,\;\; \rho_{\mathrm{cr}} = 3M_{\mathrm{Pl}}^2 H_0^2\,,\;\; \zeta = \frac{\varepsilon}{3\alpha\,H_0^2}\,,$$

$$\Omega_0 = \frac{\Lambda}{\rho_{\mathrm{cr}}},\;\; \Omega_2 = -\frac{K}{H_0^2\mathrm{a}_0^2},\;\; \Omega_6 = \frac{C^2}{6\alpha\,\mathrm{a}_0^6\,H_0^2\,\rho_{\mathrm{cr}}},\;\; \rho = \rho_{\mathrm{cr}}\left(\frac{\Omega_4}{a^4}+\frac{\Omega_3}{a^3}\right)$$

gives

the master equation:

$$y = \Omega_0 + \frac{\Omega_2}{a^2}+\frac{\Omega_3}{a^3}+\frac{\Omega_4}{a^4}+\frac{\Omega_6\left[\zeta-3y+\frac{\Omega_2}{a^2}\right]}{a^6\left[\zeta-y+\frac{\Omega_2}{a^2}\right]^2}$$

Asymptotical behavior: Late time limit $a \to \infty$

GR branch:

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{(\zeta - 3\,\Omega_0)\,\Omega_6}{(\,\Omega_0 - \zeta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right) \Longrightarrow \quad H^2 \to \Lambda/3$$

Notice: The GR solution is stable (no ghost) if and only if $\zeta > \Omega_0$.

Screening branches:

$$y_\pm = \zeta + \frac{\Omega_2}{a^2} \pm \frac{\chi}{(\,\Omega_0 - \zeta)\,a^3} \pm \frac{\Omega_2\Omega_6}{\chi\,a^5} - \frac{\Omega_6(\zeta - 3\Omega_0) \pm \Omega_3\chi}{2(\Omega_0 - \zeta)^2\,a^6} + \mathcal{O}\left(\frac{1}{a^7}\right)$$

$$\Longrightarrow \quad H^2 \to \varepsilon/3\alpha$$

Notice: The screening solutions are stable (no ghost) if and only if $0 < \zeta < \Omega_0$.

Asymptotical behavior: The limit $a \to 0$

GR branch:

$$y = \frac{\Omega_4}{a^4} + \frac{\Omega_3}{a^3} + \frac{\Omega_2\Omega_4 - 3\Omega_6}{\Omega_4 a^2} + \frac{3\Omega_3\Omega_6}{\Omega_4 a} + \mathcal{O}(1)$$

Notice: The GR solution is unstable

Screening branch:

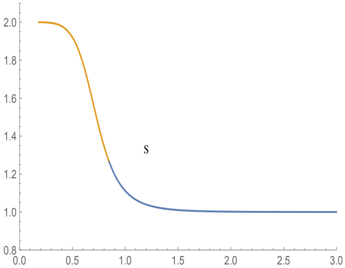
$$y_+ = \frac{3\Omega_6}{\Omega_4\, a^2} - \frac{3\Omega_3\Omega_6}{\Omega_4^2\, a} + \frac{5}{3}\zeta + \frac{3\Omega_6\Omega_3^2 + 9\Omega_6^2}{\Omega_4^3} + \mathcal{O}(a),$$

$$y_- = \frac{\zeta}{3} + \frac{4\,\zeta^2}{27\ \Omega_6}\left(\Omega_4\, a^2 + \Omega_3\, a^3\right) + \mathcal{O}(a^4)$$

Notice: Both screening solutions are stable

Global behavior

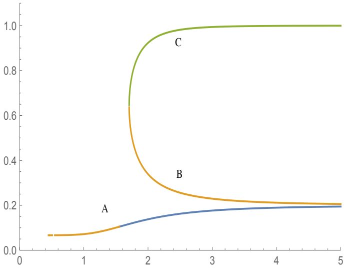
$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_2 = 0$, $\Omega_3 = \Omega_4 = 0$ and for $\zeta = 6$

Global behavior

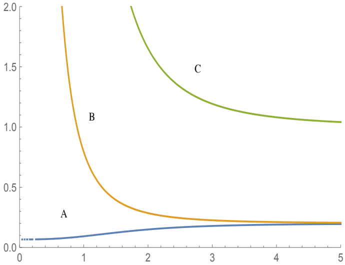
$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6\left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6\left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1,\ \Omega_2 = 0,\ \Omega_3 = \Omega_4 = 0,\ \zeta = 0.2$

Global behavior

$$y = \Omega_0 + \frac{\Omega_2}{a^2} + \frac{\Omega_3}{a^3} + \frac{\Omega_4}{a^4} + \frac{\Omega_6 \left[\zeta - 3y + \frac{\Omega_2}{a^2}\right]}{a^6 \left[\zeta - y + \frac{\Omega_2}{a^2}\right]^2}$$



Solutions $y(a)$ for $\Omega_0 = \Omega_6 = 1$, $\Omega_3 = 5$, $\Omega_4 = 0$, $\zeta = 0.2$. One has $\Omega_2 = 0$.

Conclusions

- The theory with nonminimal kinetic coupling admits various cosmological solutions.
- Ghost-free solutions exist if $\alpha \geq 0$ and $\varepsilon \geq 0$.
- The no-ghost conditions eliminate many solutions, as for example the bounces or the "emerging time" solutions.
- For $\zeta > \Omega_0$ there exists a ghost-free solution. It describes a universe with the standard late time dynamic dominated by the Λ-term, radiation and dust. At early times the matter effects are totally screened and the universe expands with a constant Hubble rate determined by ε/α. Since it contains two independent parameters ζ and $\Omega_0 \sim \Lambda$ in the asymptotics, this solution can have an hierarchy between the Hubble scales at the early and late times. However, at late times it is not screening and dominated by Λ, thus invoking again the cosmological constant problem.

Conclusions

- For $0 < \zeta < \Omega_0$ there exist two ghost-free solutions, A and B. The solution A is sourced by the scalar field, with or without the matter, while the solution B exists only when the matter is present. They both show the screening because their late time behaviour is controlled by $\zeta \sim \varepsilon/\alpha$ and not by Λ. Therefore, they could in principle describe the late time acceleration while circumventing the cosmological constant problem, and one might probably find arguments justifying that ε/α should be small. At the same time, these solutions cannot describe the early inflationary phase. Indeed, the near singularity behaviour of the solution B does not correspond to inflation, while the solution A does show an inflationary phase, but with essentially the same Hubble rate as at late times, hence there is no hierarchy between the two Hubble scales.

THANKS FOR YOUR ATTENTION!