The effect of inclusion of Δ resonances in relativistic mean-field model with scaled hadron masses and coupling constants

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Introduction

The equation of state (EoS) of strongly interacting baryonic matter for various baryon number densities $n$, temperatures $T$ and isospin asymmetries $\beta = (n_n - n_p)/n$ is required for a description of:

- finite nuclei ($T=0$, $n \simeq n_0$, $\beta \ll 1$)
- heavy ion collisions (HIC) ($0 < T < 100 - 200$ MeV, $0 < n < 5 - 10 n_0$, $\beta \ll 1$)
- neutron stars (NS) ($T=0$, $0 < n \lesssim 10 n_0$, $0 < \beta < 1$)

A vast number of constraints follow from the experimental results in these areas. The EoS at $T=0$ should:

- Reproduce bulk properties of nuclei
- Pass the constraints for the pressure at $T=0$ from analyses of flows and kaon production in HIC
- Support the existence of NSs with masses $> M[\text{PSR J0348+0432}] = 2.01 \pm 0.04 M_\odot$ – the maximum precisely measured NS mass
- Not contradict the NS cooling data
- Describe the known ratio of baryon and gravitational masses for PSR J0737-3039(B).
EoS models

There are many EoSs of two types:

**Microscopic approaches**
- Many-body theories starting from potentials which reproduce scattering phases in vacuum
- Robust for low $n$, large uncertainties already at $n \approx n_0$
- Non-relativistic approaches ⇒ breaking of causality at higher densities

**Phenomenological approaches**

**current work**
- Models with parameters (coupling constants, meson masses, etc.) adjusted to reproduce the observable properties
- Relativistic framework ⇒ causality preserved

Phenomenological relativistic mean-field models: successfully described finite nuclei, HIC and neutron stars
Flow & maximum mass constraint

Constraint for the pressure at $T = 0$ in ISM, obtained from analyses of transverse and elliptic flows in HICs


Flow constraint – soft EoSs

Maximum NS mass constraint – stiff EoS

Additional flexibility needed

- New interaction terms in the Lagrangian
- Density-dependent coupling constants – needs an additional procedure for restoring self-consistency
- Field-dependent coupling constants ← current work
If one uses hyperon potentials consistent with hypernuclear data, then with increasing density already at $n \gtrsim 2 \div 3 n_0$ conversions

$\begin{align*}
    p + e^- & \leftrightarrow \Lambda + \nu_e, \\
    n + e^- & \leftrightarrow \Sigma^- + \nu_e \\
    \ldots
\end{align*}$

becomes energetically favorable.

[H. Diapo, B.-J. Schaefer and J. Wambach
PRC81 (2010), J. Schaffner-Bielich NPA804 (2008)]

Chemical equilibrium condition ($Q_B$ – electric charge of a baryon $B$):

$$\mu_B = \mu_N - Q_B \mu_e$$

The appearance of new species results in a softening of the EoS and maximum NS mass decrease
In a majority of realistic models the maximum NS mass decreases below the observed values. Problem can be resolved in relativistic mean-field (RMF) models by taking into account a hadron mass and couplings in-medium modifications [K. A. M., E. E. Kolomeitsev and D. N. Voskresensky NPA 950 (2016)]
Δ-resonance puzzle?

A recent work* showed that an appearance of Δ(1232)-resonances is possible in the NS medium. Same problem as for hyperons: EoS softens ⇒ maximum NS mass can decrease below the observed limit.

How does the inclusion of Δ-resonances change the EoS in the RMF model with scaled hadron masses and couplings?

Generalized relativistic mean-field model

E. E. Kolomeitsev, D.N. Voskresensky, NPA 759 (2005)
present work

\[ \mathcal{L} = \mathcal{L}_{\text{bar}} + \mathcal{L}_{\text{mes}} + \mathcal{L}_{l}, \]

\[ \mathcal{L}_{\text{bar}} = \sum_{i=\text{b, r}} (\bar{\Psi}_i \left( iD^{(i)}_\mu \gamma^\mu - m_i \Phi_i(\sigma) \right) \Psi_i, \]

\[ D^{(i)}_\mu = \partial_\mu + ig_{\omega i} \chi_{\omega i}(\sigma) \omega_\mu + ig_{\rho i} \chi_{\rho i}(\sigma) \vec{\rho}_\mu + ig_{\phi i} \chi_{\phi i}(\sigma) \phi_\mu, \]

\[ \{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^{-,0}, \Delta^{+,0}, \Delta^{++,0}) \]

\[ \mathcal{L}_{\text{mes}} = \frac{\partial_\mu \sigma \partial^\mu \sigma}{2} - \frac{m^2_\sigma \Phi^2_\sigma(\sigma) \sigma^2}{2} - U(\sigma) + \]

\[ + \frac{m^2_\omega \Phi^2_\omega(\sigma) \omega_\mu \omega^\mu}{2} - \frac{\omega_{\mu\nu} \omega^{\mu\nu}}{2} + \frac{m^2_\rho \Phi^2_\rho(\sigma) \vec{\rho}_\mu \vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu} \rho^{\mu\nu}}{4} + \]

\[ + \frac{m^2_\phi \Phi^2_\phi(\sigma) \phi_\mu \phi^\mu}{2} - \frac{\phi_{\mu\nu} \phi^{\mu\nu}}{4}, \]

\[ \omega_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu \vec{\rho}_\mu - \partial_\mu \vec{\rho}_\nu, \quad \phi_{\mu\nu} = \partial_\nu \phi_\mu - \partial_\mu \phi_\nu, \]

\[ \mathcal{L}_{l} = \sum_l \bar{\psi}_l (i\partial_\mu \gamma^\mu - m_l) \psi_l, \quad \{l\} = (e, \mu). \]
Mean-field approximation

Meson fields are threatened as classical fields:

\[ \sigma \rightarrow \langle \sigma \rangle \equiv \bar{\sigma}, \quad \omega^\mu \rightarrow \langle \omega^\mu \rangle \equiv (\omega_0, \vec{0}), \quad \rho^\mu_i \rightarrow \langle \rho^\mu_i \rangle = \delta_{i3}(\rho_0, \vec{0}) \]

Meson mean-field values can be obtained by averaging the equations of motion by the ground state (analogically for \( \phi \) meson):

\[
\begin{align*}
\omega_0 &= \frac{1}{m_\omega^*} \sum_B g_{\omega B} \chi_{\omega B}(\sigma) n_B, \\
\rho_0 &= \frac{1}{m_\rho^*} \sum_B g_{\rho B} \chi_{\rho B}(\sigma) t_{3B} n_B,
\end{align*}
\]

\[
\phi_0 = \frac{1}{m_\phi^*} \sum_B g_{\phi B} \chi_{\phi B}(\sigma) n_B
\]
Energy density functional

\[ E = -\mathcal{L} = \frac{m_N^4 f^2}{2C_{\sigma}^2} \eta_\sigma(f) + \frac{C_\omega^2}{2m_N^2 \eta_\omega(f)} \left( \sum_b x_{\omega b} n_b \right)^2 + \]

\[ + \frac{C_\rho^2}{2m_N^2 \eta_\rho(f)} \left( \sum_b x_{\rho b} t_{3b} n_b \right)^2 + \frac{C_\omega^2}{2m_N^2 \eta_\phi(f)} \frac{m_\omega^2}{m_\phi^2} \left( \sum_H x_{\phi H} n_H \right)^2 + \]

\[ + \sum_b \left( 2S_b + 1 \right) \int_0^{p_{F,b}} \frac{p^2 dp}{2\pi^2} \sqrt{p^2 + m_b^2 \Phi_b^2(f)} + E_l, \]

\[ E_l = \sum_{l=e,\mu} \int_0^{p_{F,l}} \frac{p^2 dp}{\pi^2} \sqrt{p^2 + m_l^2}, \quad C_i = \frac{g_i N m_N}{m_i}, \quad i = \sigma, \omega, \rho. \]

Scaling functions

Redefine the scalar field \( f = g_N \chi_N(\sigma) \sigma / m_N, \)

Without finite size effects only \( \eta_m = \Phi_m^2(f) / \chi_{mB}^2(f), \quad m = \sigma, \omega, \rho, \phi \)

\( \Phi_N(f) = \Phi_m(f) = 1 - f, \) all hadron masses scale the same way

\( \Phi_H(f) = \Phi_N(x_{\sigma H}(\chi_{\sigma H}(f) / \chi_N(f)) f m_N / m_H), \)

\( x_{mB} = g_{mB} / g_{mN} \)
Energy density functional

\[ E = -\mathcal{L} = \frac{m_N^4 f^2}{2C_\sigma^2} \eta_\sigma(f) + \frac{C_\omega^2}{2m_N^2 \eta_\omega(f)} \left( \sum_b x_{\omega b} n_b \right)^2 + \]

\[ + \frac{C_\rho^2}{2m_N^2 \eta_\rho(f)} \left( \sum_b x_{\rho b} t_{3b} n_b \right)^2 + \frac{C_\omega^2}{2m_N^2 \eta_\phi(f)} m_\omega^2 \left( \sum_H x_{\phi H} n_H \right)^2 + \]

\[ + \sum_b \left( 2S_b + 1 \right) \int_0^{p_{F,b}} \frac{p^2 dp}{2\pi^2} \sqrt{p^2 + m_b^2 \Phi_b^2(f)} + E_l, \]

\[ E_l = \sum_{l=e,\mu} \int_0^{p_{F,l}} \frac{p^2 dp}{\pi^2} \sqrt{p^2 + m_l^2}, \quad C_i = \frac{g_i N m_N}{m_i}, \quad i = \sigma, \omega, \rho. \]

\[ \implies \text{Equation of motion:} \quad \frac{\partial E}{\partial f} = 0 \Rightarrow f(n_B). \]

\[ \implies \text{Beta-equilibrium and electrical neutrality conditions (} Q_i \text{-electric charge of particle species } i): \]

\[ \mu_n = \mu_b - Q_b \mu_e \]

\[ \sum_{i=b,l} Q_i n_i = 0 \quad \Rightarrow n_B(n), n_l(n) \]
Saturation properties

For comparing the EoS at the saturation:
\[ \mathcal{E} = \mathcal{E}_{\text{bind}} + \frac{K}{18} \epsilon^2 - \frac{K'}{162} \epsilon^3 + \ldots + \beta^2 \left( J + \frac{L}{3} \epsilon + \frac{K_{\text{sym}}}{18} \epsilon^2 \ldots \right), \]
\[ \epsilon = (n - n_0)/n_0, \quad \beta = [(n_n - n_p)/n_0] n_0 \]

From the data the following values are deduced:
\[ n_0 = 0.16 \pm 0.015 \text{ fm}^{-3}, \quad \mathcal{E}_{\text{bind}} = -15.6 \pm 0.6 \text{ MeV}, \quad K = 240 \pm 20 \text{ MeV}, \]
\[ J = 28 - 33 \text{ MeV}, \quad m_N^* = 0.7 - 0.9 m_N \]

This allows to determine \( C_\sigma, C_\omega, C_\rho \) and 2 parameters of \( \eta_\sigma(f) \)
Scaling functions in our model (MKVOR*)


Different $f(n)$ behaviors in iso-symmetric (ISM, $n_n = n_p$) and beta-equilibrium (BEM) matter.
Hyperon coupling constants

Vector meson couplings – from SU(6) symmetry:

\[ g_{\omega \Lambda} = g_{\omega \Sigma} = 2g_{\omega \Xi} = \frac{2}{3}g_{\omega N}, \quad g_{\rho \Lambda} = 0, \quad g_{\rho \Sigma} = 2g_{\rho \Xi} = 2g_{\rho N}, \]

\[ 2g_{\phi \Lambda} = 2g_{\phi \Sigma} = g_{\phi \Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N}, \quad g_{\phi N} = 0. \]

Scalar meson couplings – from the hyperon binding energies in nuclear matter at \( n = n_0 \):

\[ U^H(n_0) = \frac{C^2}{m_N^2} \frac{\omega}{2} x_{\omega H} n_0 - x_{\sigma H} [m_N - m_N^*(n_0)], \]

\[ U^\Lambda(n_0) = -28 \text{ MeV}, \quad U^\Sigma(n_0) = 30 \text{ MeV}, \quad U^\Xi(n_0) = -15 \text{ MeV}. \]

Hyperon scalings

(label: \( H\phi \)) Vacuum couplings with \( \phi \), but \( m^*_\phi \) changes in the same way as other hadrons

\[ \chi_{\phi B}(f) = 1, \quad \Phi_\phi(f) = 1 - f, \quad \eta_\phi = (1 - f)^2 \]
Inclusion of $\Delta$-isobars

**Coupling constants**

Coupling constants with vector mesons equal to nucleons’ in the SU(6) symmetry assumption (quark counting):

\[ g_\omega \Delta = g_\omega N, \quad g_\rho \Delta = g_\rho N, \quad g_\phi \Delta = 0 \]

$\Delta$ coupling with the scalar meson is deduced from the $\Delta$ potential at the saturation density:

\[ U_\Delta(n_0) = -x_{\sigma \Delta} m_N f_0 + x_{\omega \Delta} C^2_\omega (n_0/m_N^2). \]

$U_\Delta$ is poorly constrained by data.

From the quark counting follows* $U_\Delta \simeq U_N$.

We allow for a variation of parameters and consider values $-50 \text{ MeV} > U_\Delta > -100 \text{ MeV}$ in our analysis.

We label models with $\Delta$ included by $\Delta$

ISM: Flow constraint

Without $\Delta$ the constraint is passed for $n < 4n_0$ in our model
- $-56 \text{ MeV} < U_\Delta$ – second order phase transition (PT)
- $U_\Delta < -56 \text{ MeV}$ – first-order PT
- $-83 \text{ MeV} < U_\Delta < -65 \text{ MeV}$ – pressure curve lies fully within the constraint.
- $U_\Delta \lesssim -95 \text{ MeV}$ – metastable state appears ($P = 0$)

$\Delta$ in ISM help to pass the flow constraint!

Can manifest itself in HICs provided $-U_\Delta$ is sufficiently large

Maximum mass: hyperons and $\Delta$

MKVOR$^*H\phi$ – successful model with hyperons

[K. A. M., E. E. Kolomeitsev and D. N. Voskresensky NPA 950 (2016)]

MKVOR$^*H\Delta\phi$ – the same model with $\Delta$ included

$\Delta$ modify particle fractions, which should significantly affect transport properties of NS medium
Maximum mass: $U_{\Delta}$ dependence

\[
\frac{M_{\text{max}}}{M_{\odot}}
\]

\[
U_{\Delta} \quad [\text{MeV}]
\]

\[
-50 \quad -60 \quad -70 \quad -80 \quad -90 \quad -100
\]

\[
2.0 \quad 2.1 \quad 2.2 \quad 2.3
\]

\[
\text{MKVOR}^*H_{\Delta \phi}
\]
Mass-radius relation

Constraints from:

- \( M[\text{PSR J0348 + 0432}] = 2.01 \pm 0.04 M_{\odot} \)
- Quasi-periodical oscillations of 4U 0614+091
- Isolated NS thermal radiation of RX J1856

For \( U_\Delta = -100 \text{ MeV} \) the radius of NSs with \( M = 1.5 \, M_{\odot} \) decreases by \( \approx 1 \text{ km} \), while NS mass decrease is \( \approx 0.04 \, M_{\odot} \).
Gravitational vs baryon mass constraint


\[ M_G = 1.249 \pm 0.001 M_\odot, \quad M_B = 1.366 - 1.375 M_\odot \]
dashed box – assuming no mass loss
two rectangles – assuming 0.3 & 1% mass loss

If \( \Delta \) appear at lower densities, the constraint is passed better!
Summary

Conclusion

- In our model an appearance of $\Delta$-resonances is energetically favorable for densities $n \simeq 2 - 3 n_0$ depending on the value of $U_\Delta$
- In the iso-symmetric matter for $U_\Delta > -56 \text{ MeV}$ $\Delta$ appear by a third-order phase transition and for $U_\Delta < -56 \text{ MeV}$ by a first-order phase transition. Flow constraint would be passed better if $-83 \text{ MeV} < U_\Delta < -65 \text{ MeV}$
- In neutron star matter the presence of $\Delta$ noticeably changes particle fractions and NS radii for strong attractive $U_\Delta$, but doesn’t lead to a significant decrease of a maximum NS mass. Maximum mass constraint is still satisfied.

⇒ Within our model $\Delta$ and hyperon puzzles are solved.

Further development of the model

- Incorporating a possibility of meson ($\rho^-$, $\pi$, $K$) condensation.
- Generalization for finite temperatures for a description of supernova explosions and heavy-ion collisions.