The effect of inclusion of Δ resonances in relativistic mean-field model with scaled hadron masses and coupling constants

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Introduction

- The equation of state (EoS) of strongly interacting baryonic matter for various baryon number densities n, temperatures T and isospin asymmetries $\beta = (n_n - n_p)/n$ is required for a description of:
  - finite nuclei ($T = 0$, $n \simeq n_0, \beta \ll 1$)
  - heavy ion collisions (HIC) ($0 < T < 100 - 200\,\mathrm{MeV}$, $0 < n < 5 - 10\,n_0$, $\beta \ll 1$)
  - neutron stars (NS) ($T = 0$, $0 < n \lesssim 10\,n_0$, $0 < \beta < 1$)

  A vast number of constraints follow from the experimental results in these areas. The EoS at $T = 0$ should:
  - Reproduce bulk properties of nuclei
  - Pass the constraints for the pressure at $T = 0$ from analyses of flows and kaon production in HIC
  - Support the existence of NSs with masses $> M$[PSR J0348+0432] $=$ $2.01 \pm 0.04\,M_\odot$ – the maximum precisely measured NS mass
  - Not contradict the NS cooling data
  - Describe the known ratio of baryon and gravitational masses for PSR J0737-3039(B).

EoS models

There are many EoSs of two types:

Microscopic approaches

- Many-body theories starting from potentials which reproduce scattering phases in vacuum
- Robust for low n, large uncertainties already at $n \simeq n_0$
- Non-relativistic approaches $\Rightarrow$ breaking of causality at higher densities

Phenomenological approaches

current work

- Models with parameters (coupling constants, meson masses, etc.) adjusted to reproduce the observable properties
- Relativistic framework $\Rightarrow$ causality preserved

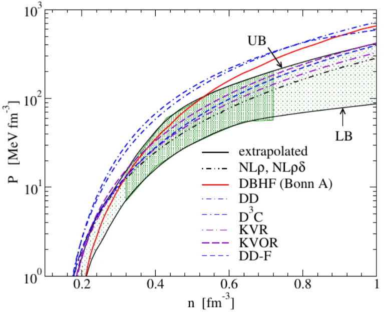
Phenomenological relativistic mean-field models: successfully described finite nuclei, HIC and neutron stars

Flow & maximum mass constraint

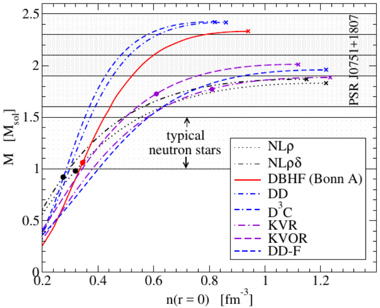
Constraint for the pressure at $T = 0$ in ISM, obtained from analyses of transverse and elliptic flows in HICs

[P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)]

Flow constraint – soft EoSs



Maximum NS mass constraint – stiff EoS

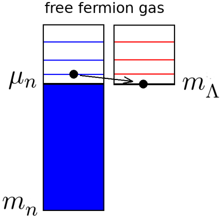


figures from [T. Klahn et al. PRC74 (2006)]

Additional flexibility needed

- New interaction terms in the Lagrangian
- Density-dependent coupling constants – needs an additional procedure for restoring self-consistency
- Field-dependent coupling constants ← current work

Hyperon puzzle



If one uses hyperon potentials consistent with hypernuclear data, then with increasing density already at $n \gtrsim 2 \div 3\, n_0$ conversions

$$p + e^- \longleftrightarrow \Lambda + \nu_e,$$
$$n + e^- \longleftrightarrow \Sigma^- + \nu_e$$
$$\ldots$$

becomes energetically favorable.
[H. Diapo, B.-J. Schaefer and J. Wambach PRC81 (2010), J. Schaffner-Bielich NPA804 (2008)]

Chemical equilibrium condition (Q_B – electric charge of a baryon B):

$$\mu_B = \mu_N - Q_B \mu_e$$

The appearance of new species results in a softening of the EoS and maximum NS mass decrease

Hyperon puzzle

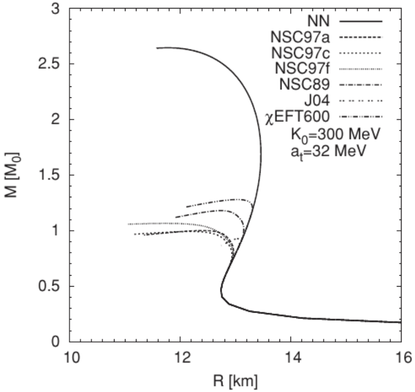


figure from H. Diapo, B.-J. Schaefer and J. Wambach PRC81 (2010)

In a majority of realistic models the maximum NS mass decreases below the observed values

Problem can be resolved in relativistic mean-field (RMF) models by taking into account a hadron mass and couplings in-medium modifications

[K. A. M., E. E. Kolomeitsev and D. N. Voskresensky NPA 950 (2016)]

Δ-resonance puzzle?

A recent work* showed that an appearance of $\Delta(1232)$-resonances is possible in the NS medium

Same problem as for hyperons: EoS softens $\Rightarrow$ maximum NS mass can decrease below the observed limit

How does the inclusion of Δ-resonances change the EoS in the RMF model with scaled hadron masses and couplings?

* A. Drago et al. Phys. Rev. C **90** (2014)

Generalized relativistic mean-field model

E. E. Kolomeitsev, D.N. Voskresensky, NPA 759 (2005)
K. A. M., E. E. Kolomeitsev, D.N. Voskresensky, Phys. Lett. B 748 (2015),
present work

$$
\begin{aligned}
&\mathcal{L} = \mathcal{L}_{\rm bar} + \mathcal{L}_{\rm mes} + \mathcal{L}_l,\\
&\mathcal{L}_{\rm bar} = \sum_{i=b\cup r} (\bar{\Psi}_i \left(iD_\mu^{(i)}\gamma^\mu - m_i\Phi_i(\sigma)\right)\Psi_i,\\
&D_\mu^{(i)} = \partial_\mu + ig_{\omega i}\chi_{\omega i}(\sigma)\omega_\mu + ig_{\rho i}\chi_{\rho i}(\sigma)\vec{t}\vec{\rho}_\mu + ig_{\phi i}\chi_{\phi i}(\sigma)\phi_\mu,\\
&\{b\} = (N, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}, \Delta^-, \Delta^0, \Delta^+, \Delta^{++})\\
&\mathcal{L}_{\rm mes} = \frac{\partial_\mu\sigma\partial^\mu\sigma}{2} - \frac{m_\sigma^2\Phi_\sigma^2(\sigma)\sigma^2}{2} - U(\sigma) +\\
&+\frac{m_\omega^2\Phi_\omega^2(\sigma)\omega_\mu\omega^\mu}{2} - \frac{\omega_{\mu\nu}\omega^{\mu\nu}}{4} + \frac{m_\rho^2\Phi_\rho^2(\sigma)\vec{\rho}_\mu\vec{\rho}^\mu}{2} - \frac{\rho_{\mu\nu}\rho^{\mu\nu}}{4} +\\
&+\frac{m_\phi^2\Phi_\phi^2(\sigma)\phi_\mu\phi^\mu}{2} - \frac{\phi_{\mu\nu}\phi^{\mu\nu}}{4},\\
&\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \quad \vec{\rho}_{\mu\nu} = \partial_\nu\vec{\rho}_\mu - \partial_\mu\vec{\rho}_\nu, \quad \phi_{\mu\nu} = \partial_\nu\phi_\mu - \partial_\mu\phi_\nu,\\
&\mathcal{L}_l = \sum_l \bar{\psi}_l(i\partial_\mu\gamma^\mu - m_l)\psi_l, \quad \{l\} = (e,\, \mu).
\end{aligned}
$$

Mean-field approximation

Meson fields are threated as classical fields:

$$\sigma \to \langle\sigma\rangle \equiv \bar{\sigma}, \quad \omega^\mu \to \langle\omega^\mu\rangle \equiv (\omega_0, \vec{0}), \quad \rho_i^\mu \to \langle\rho_i^\mu\rangle = \delta_{i3}(\rho_0, \vec{0})$$

Meson mean-field values can be obtained by averaging the equations of motion by the ground state (analogically for ϕ meson):

$$\omega_0 = \frac{1}{m_\omega^{*2}} \sum_B g_{\omega B}\chi_{\omega B}(\sigma) n_B, \quad \rho_0 = \frac{1}{m_\rho^{*2}} \sum_B g_{\rho B}\chi_{\rho B}(\sigma) t_{3B} n_B,$$

$$\phi_0 = \frac{1}{m_\phi^{*2}} \sum_B g_{\phi B}\chi_{\phi B}(\sigma) n_B$$

Energy density functional

$$E = -\mathcal{L} = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b (2S_b+1)\int_0^{p_{\mathrm{F},b}} \frac{p^2\,dp}{2\pi^2}\sqrt{p^2 + m_b^2\Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu}\int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2+m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho.$$

Scaling functions

Redefine the scalar field $f = g_{\sigma N}\chi_{\sigma N}(\sigma)\sigma/m_N$,

Without finite size effects only $\eta_m = \Phi_m^2(f)/\chi_{mB}^2(f)\,, \quad m = \sigma, \omega, \rho, \phi$

$\Phi_N(f) = \Phi_m(f) = 1 - f$, all hadron masses scale the same way

$\Phi_H(f) = \Phi_N(x_{\sigma H}(\chi_{\sigma H}(f)/\chi_{\sigma N}(f))f m_N/m_H)\,,$

$x_{mB} = g_{mB}/g_{mN}$

Energy density functional

$$E = -\mathcal{L} = \frac{m_N^4 f^2}{2C_\sigma^2}\eta_\sigma(f) + \frac{C_\omega^2}{2m_N^2\eta_\omega(f)}\Big(\sum_b x_{\omega b} n_b\Big)^2 +$$

$$+\frac{C_\rho^2}{2m_N^2\eta_\rho(f)}\Big(\sum_b x_{\rho b} t_{3b} n_b\Big)^2 + \frac{C_\omega^2}{2m_N^2\eta_\phi(f)}\frac{m_\omega^2}{m_\phi^2}\Big(\sum_H x_{\phi H} n_H\Big)^2 +$$

$$+\sum_b (2S_b+1)\int_0^{p_{\mathrm{F},b}} \frac{p^2\,dp}{2\pi^2}\sqrt{p^2 + m_b^2\Phi_b^2(f)} + E_l,$$

$$E_l = \sum_{l=e,\mu}\int_0^{p_{\mathrm{F},l}} \frac{p^2 dp}{\pi^2}\sqrt{p^2+m_l^2}\,, \quad C_i = \frac{g_{iN} m_N}{m_i}, \quad i = \sigma, \omega, \rho.$$

⊕ Equation of motion: $\dfrac{\partial E}{\partial f} = 0 \Rightarrow f(n_B)$.

⊕ Beta-equilibrium and electrical neutrality conditions (Q_i-electric charge of particle species i):

$$\left.\begin{array}{c} \mu_n = \mu_b - Q_b\mu_e \\ \sum\limits_{i=b,l} Q_i n_i = 0 \end{array}\right\} \Rightarrow n_B(n), n_l(n)$$

Saturation properties

For comparing the EoS at the saturation:

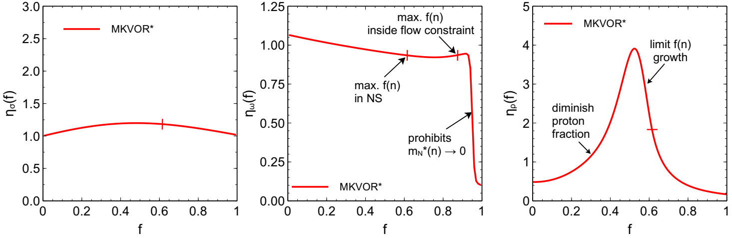
$$\mathcal{E} = \mathcal{E}_{\rm bind} + \frac{K}{18}\epsilon^2 - \frac{K^{'}}{162}\epsilon^3 + ... + \beta^2\left(J + \frac{L}{3}\epsilon + \frac{K_{\rm sym}}{18}\epsilon^2 ...\right),$$
$$\epsilon = (n - n_0)/n_0, \quad \beta = [(n_n - n_p)/n_0]_{n_0}$$

From the data the following values are deduced:

$$n_0 = 0.16 \pm 0.015\,{\rm fm}^{-3}, \quad \mathcal{E}_{\rm bind} = -15.6 \pm 0.6 \text{ MeV}, \quad K = 240 \pm 20 \text{ MeV},$$
$$J = 28 - 33\,{\rm MeV}, \quad m_N^* = 0.7 - 0.9\, m_N$$

This allows to determine $C_\sigma, C_\omega,\ C_\rho$ and 2 parameters of $\eta_\sigma(f)$

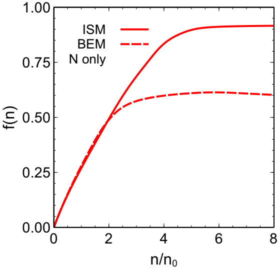
Scaling functions in our model (MKVOR*)



Rapid decrease of scaling functions results in limiting the $f(n)$ growth

("cut"method, cf. [K.A.M., D. N. Voskresensky, E. E. Kolomeitsev PRC 92 (2015)])

Different $f(n)$ behaviors in iso-symmetric (ISM, $n_n = n_p$) and beta-equilibrium (BEM) matter.



Hyperon coupling constants

Vector meson couplings – from SU(6) symmetry:

$$g_{\omega\Lambda} = g_{\omega\Sigma} = 2g_{\omega\Xi} = \frac{2}{3} g_{\omega N}\,, \quad g_{\rho\Lambda} = 0\,, \quad g_{\rho\Sigma} = 2g_{\rho\Xi} = 2g_{\rho N}\,,$$

$$2g_{\phi\Lambda} = 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3} g_{\omega N}\,, \quad g_{\phi N} = 0.$$

Scalar meson couplings – from the hyperon binding energies in nuclear matter at $n = n_0$:

$$U^H(n_0) = \frac{C_\omega^2}{m_N^2} x_{\omega H} n_0 - x_{\sigma H} \left[m_N - m_N^*(n_0)\right],$$

$$U^\Lambda(n_0) = -28\,\mathrm{MeV}, \quad U^\Sigma(n_0) = 30\,\mathrm{MeV}, \quad U^\Xi(n_0) = -15\,\mathrm{MeV}\,.$$

Hyperon scalings

(label: $H\phi$) Vacuum couplings with ϕ, but m_ϕ^* changes in the same way as other hadrons

$$\chi_{\phi B}(f) = 1, \quad \Phi_\phi(f) = 1 - f, \quad \eta_\phi = (1-f)^2$$

Inclusion of Δ-isobars

Coupling constants

Coupling constants with vector mesons equal to nucleons' in the SU(6) symmetry assumption (quark counting):

$$g_{\omega\Delta} = g_{\omega N}, \quad g_{\rho\Delta} = g_{\rho N}, \quad g_{\phi\Delta} = 0$$

Δ coupling with the scalar meson is deduced from the Δ potential at the saturation density:

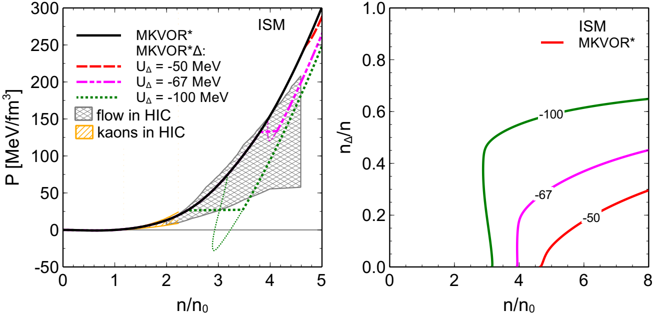
$$U_\Delta(n_0) = -x_{\sigma\Delta} m_N \, f_0 + x_{\omega\Delta} C_\omega^2 (n_0/m_N^2).$$

U_Δ is poorly constrained by data.
From the quark counting follows* $U_\Delta \simeq U_N$.
We allow for a variation of parameters and consider values
$-50\,\mathrm{MeV} > U_\Delta > -100\,\mathrm{MeV}$ in our analysis.
We label models with Δ included by Δ

* A.B. Migdal, E.E. Saperstein, M.A. Troitsky and D.N. Voskresensky, Phys. Rept. **192** (1990)
F. Riek, M. F. M. Lutz and C. L. Korpa, Phys. Rev. C **80** (2009)

ISM: Flow constraint

Without Δ the constraint is passed for $n < 4\,n_0$ in our model
$-56\,\mathrm{MeV} < U_\Delta$ – second order phase transition (PT)
$U_\Delta < -56\,\mathrm{MeV}$ – first-order PT
$-83\,\mathrm{MeV} < U_\Delta < -65\,\mathrm{MeV}$ – pressure curve lies fully within the constraint.
$U_\Delta \lesssim -95\,\mathrm{MeV}$ – metastable state appears($P = 0$)
Δ in ISM help to pass the flow constraint!



Can manifest itself in HICs provided $-U_\Delta$ is sufficiently large

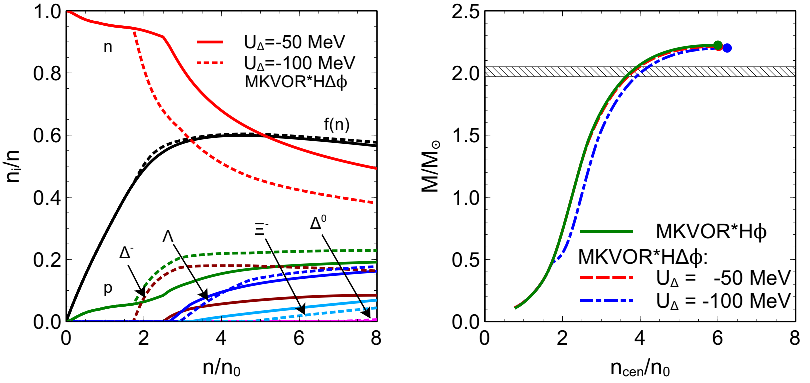
Kaon production constraint – [W. G. Lynch et al. Prog. Part. Nucl. Phys. 62 (2009)]

Maximum mass: hyperons and Δ

MKVOR*Hϕ – successful model with hyperons

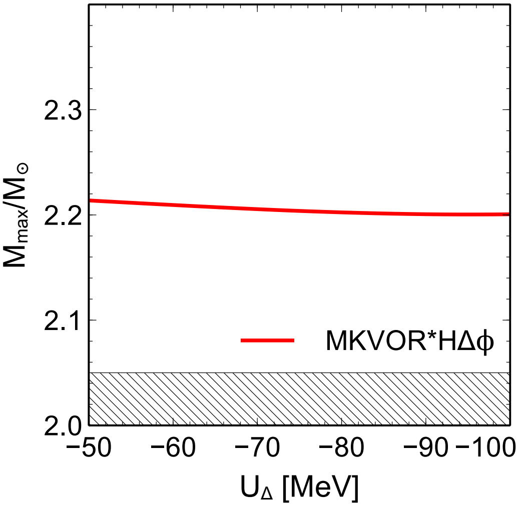
[K. A. M., E. E. Kolomeitsev and D. N. Voskresensky NPA 950 (2016)]

MKVOR*H$\Delta\phi$ – the same model with Δ included



Δ modify particle fractions, which should significantly affect transport properties of NS medium

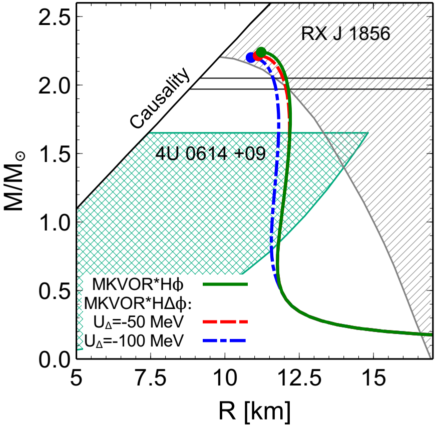
Maximum mass: U_Δ dependence



Mass-radius relation

Constraints from:

- $M[\mathrm{PSR\,J0348+0432}] = 2.01 \pm 0.04 M_\odot$
- Quasi-periodical oscillations of 4U 0614+091
- Isolated NS thermal radiation of RX J1856



For $U_\Delta = -100\,\mathrm{MeV}$ the radius of NSs with $M = 1.5\,M_\odot$ decreases by $\simeq 1\,\mathrm{km}$, while NS mass decrease is $\simeq 0.04\,M_\odot$

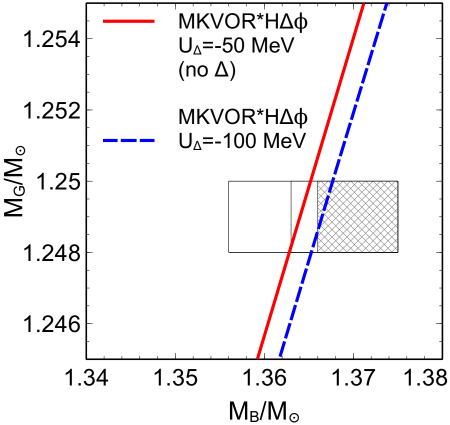
Gravitational vs baryon mass constraint

Pulsar J0737-3039B: Electron capture SN [P. Podsiadlowski et al. Mon. Not. R. Astron. Soc. 361 (2005) 1243.]

$M_G = 1.249 \pm 0.001 M_{\odot}, \quad M_B = 1.366 - 1.375 M_{\odot}$

dashed box – assuming no mass loss

two rectangles – assuming 0.3 & 1% mass loss



If Δ appear at lower densities, the constraint is passed better!

Summary

Conclusion

- In our model an appearance of Δ-resonances is energetically favorable for densities $n \simeq 2 - 3\, n_0$ depending on the value of U_Δ
- In the iso-symmetric matter for $U_\Delta > -56\,\mathrm{MeV}$ Δ appear by a third-order phase transition and for $U_\Delta < -56\,\mathrm{MeV}$ by a first-order phase transition. Flow constraint would be passed better if $-83\,\mathrm{MeV} < U_\Delta < -65\,\mathrm{MeV}$
- In neutron star mater the presence of Δ noticeably changes particle fractions and NS radii for strong attractive U_Δ, but doesn't lead to a significant decrease of a maximum NS mass. Maximum mass constraint is still satisfied.

$\Rightarrow$ Within our model Δ and hyperon puzzles are solved.

Further development of the model

- Incorporating a possibility of meson (ρ^-, π, K) condensation.
- Generalization for finite temperatures for a description of supernova explosions and heavy-ion collisions.