ITEP Lattice

Stern-Gerlach experiment on the lattice

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Why do we study a strong magnetic field?

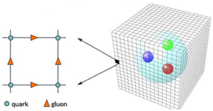
D.E. Kharzeev

Magnetic field of hadronic scale could be formed in :

1. heavy-ion collisions in terrestrial laboratories (RHIC, LHC).
non-central collisions $eB \sim m_\pi^2 - 15m_\pi^2 = (0.02 - 0.3)GeV^2$
ultraperipheral collisions $eB \sim 60m_\pi^2 - 100m_\pi^2 = (1.2 - 2)GeV^2$
2. magnetars $(eB \sim 0.01GeV^2)$,
3. early Universe.

Lattice QCD

$x \Rightarrow an,\ n_i = 0, N_s - 1, \quad i = 1,2,3, \quad n_0 = 1, N_t, n = (n_i, n_0),$
where a is the lattice spacing, $n \in Z$, $N_s^3 \times N_t$ is the lattice volume.



1. Transition to the Euclidean space $x_0 \to it$.
2. Using of discretized version of the Lagrangian instead of it's continuous variant.
$\psi(x) \Rightarrow \psi(n), \quad \partial_\mu \psi(x) \Rightarrow \frac{\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})}{2a} + O(a^2)$
$S_{QCD} \to iS^E_{QCD} \Rightarrow \exp\{iS_{QCD}\} \to \exp\{-S^E_{QCD}\}.$
3. Sampling gluonic ensembles with weight $e^{-S^E_{QCD}}$ using Monte-Carlo methods.

Details of calculations

2-pt correlation function : $C = \langle O_1 O_2 \rangle_A$,
Interpolating operators: $O_{1,2}(\rho) = \psi^\dagger(x)\gamma_i\psi(x)$,
$(i = 1, 2, 3 \longrightarrow x, y, z\ )$.
Calculate the Euclidean correlator:

$$\langle \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma_\nu\psi\rangle_A = -[\gamma_\mu D^{-1}(x,y)\gamma_\nu D^{-1}(y,x)]$$

$$C^{VV}(s_z = \pm 1) = C^{VV}_{xx} + C^{VV}_{yy} \pm i(C^{VV}_{xy} - C^{VV}_{yx})$$

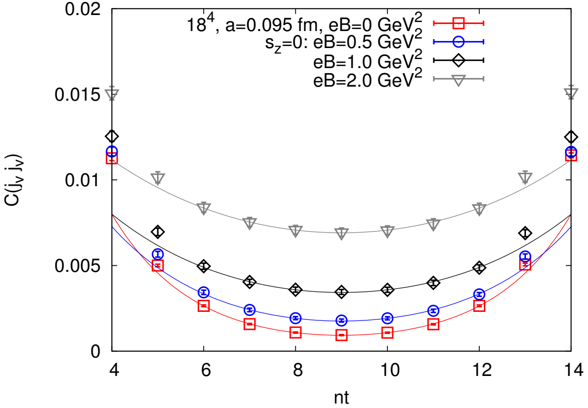
consider zero momentum $\langle p \rangle = 0$ for the ground energy state

$$C^{VV}_{ij}(n_t) = \langle \bar{\psi}(\mathbf{0}, n_t)\gamma_i\psi(\mathbf{0}, n_t)\bar{\psi}(\mathbf{0}, 0)\gamma_j\psi(\mathbf{0}, 0)\rangle_A =$$

$$\sum_k \langle 0|\gamma_i|k\rangle\langle k|\gamma_j|0\rangle e^{-n_t a E_k}$$

We obtain the energies from asymptotic behaviour of the correlator

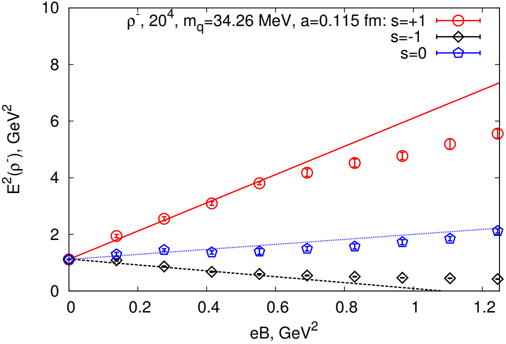
Fitting the correlators



$$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} =$$

$$= 2A_0 e^{-N_T a E_0/2} \cosh((\frac{N_T}{2} - n_t) a E_0$$

Lattice Stern–Gerlach experiment for the ρ mesons



Pointlike particle (lines): $E^2 = (1 + g s_z) qB + m^2$

Non-pointlike particle (dots):

$$E^2 = (1 + g s_z) qB + m^2 - 4\pi m \beta (qB)^2$$

The g-factor of ρ meson

V_{latt}	$m_q(Mev)$	$a(fm)$	g-factor	χ^2/d.o.f.	N_{conf}
18^4	11.99	0.115	2.02 ± 0.11	0.524	250
18^4	17.13	0.115	2.20 ± 0.12	0.766	250
18^4	34.26	0.115	2.10 ± 0.01	0.610	285
18^4	51.39	0.115	2.06 ± 0.13	2.363	250
18^4	$m_q \to 0$	0.115	2.11 ± 0.10	0.805	
18^4	34.26	0.095	2.25 ± 0.08	1.102	200
20^4	34.26	0.115	2.04 ± 0.14	3.101	275

Experiment: $g_{exp} = 2.1 \pm 0.5$ D. G. Gudino and G. T.Sanchez (2015),Int.J. of Mod.Phys.A,30:1550114 arXiv:1305.6345

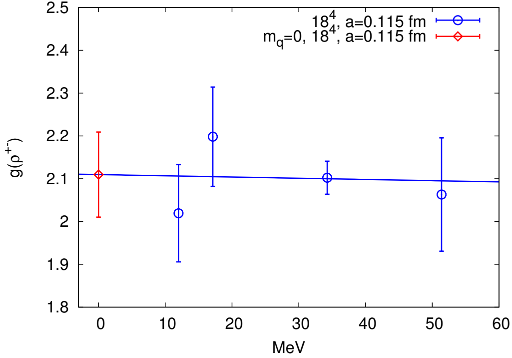
Previous results:

Relativistic quark model : $g \approx 2.37$ A. M. Badalian, Yu. A. Simonov(ITEP), Phys. Rev. D 87, 074012 (2013)

QCD sum rules: $g = 2.4 \pm 0.4$ T. M. Aliev at al., Phys.Lett.B678

Lattice: $g \approx 3.25$ F.X.Lee at al., Phys.Rev. D,78,094502(2008)

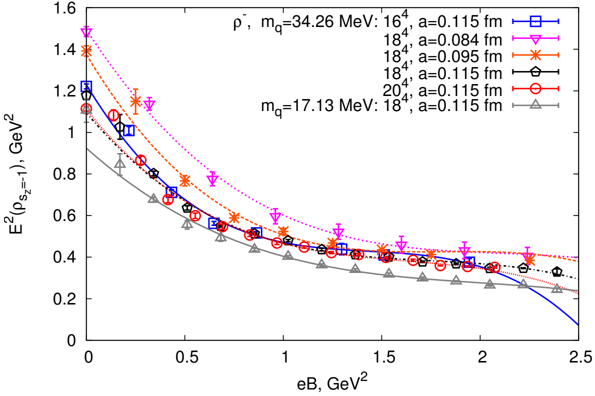
Quark mass dependence of the g-factor of ρ meson



$$m_\rho = c_0 + c_1 \cdot m_u$$

18^4, $a = 0.115\ fm$: $\;m_q \to 0\;\; g = 2.11 \pm 0.10$

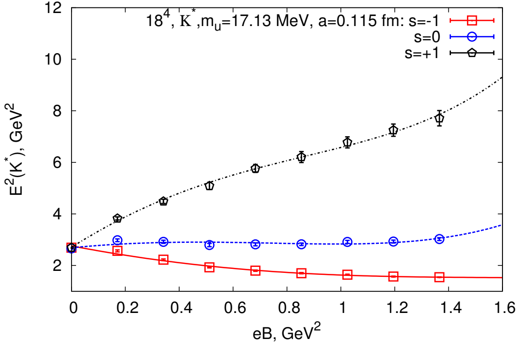
Lack of the charged vector meson tachyonic mode $(E^2 < 0)$



pointlike particle : eB$_c \approx 1GeV^2 \;\; E = 0$

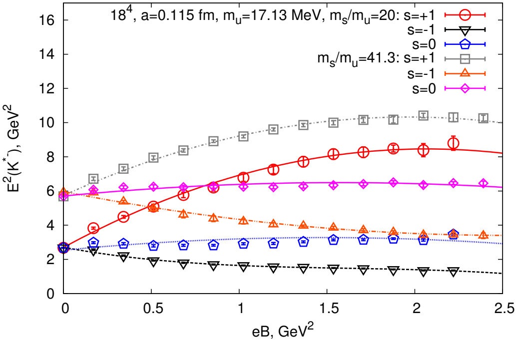
$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m\beta(qB)^2 - 4\pi m\beta^h(qB)^4$$

Lattice Shtern-Gerlach experiment for the charged K* meson



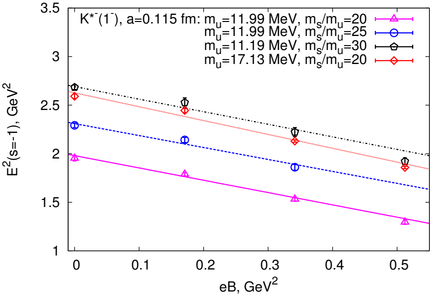
$$E^2 = (1 + g s_z) qB + m^2 - 4\pi m \beta (qB)^2 - 4\pi m (qB)^4$$

Lattice Shtern-Gerlach experiment for the charged K* meson



$$E^2 = (1 + gs_z)qB + m^2 - 4\pi m\beta(qB)^2 - 4\pi m(qB)^4$$

g-factor of charged K* meson



$m_u(Mev)$	m_s/m_u	g-factor	χ^2/d.o.f.	N_{conf}
11.99	20	2.27 ± 0.18	1.845	250
11.99	25	2.23 ± 0.23	1.986	250
11.99	30	2.29 ± 0.19	1.366	250
17.13	20	2.43 ± 0.24	3.282	250

Conclusions:

- Splitting of ground state energy of the ρ and K^* mesons depending on its spin projection on the axis of the external magnetic field
- Lack of the tachyonic mode($E^2 < 0$) of charged ρ mesons
- g-factor of ρ meson has been estimated in the chiral limit
- g-factor of K^* meson has been established (extrapolations in future)

The authors are grateful to FAIR-ITEP supercomputer center where these numerical calculations were performed

Thank you for your attention!
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Details of calculations

Solve Dirac equation numerically

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu)$$

We use the Neuberger overlap operator $D_{overlap}$

$$A_{\mu\,ij} \to A_{\mu\,ij} + A^B_\mu \delta_{ij}$$

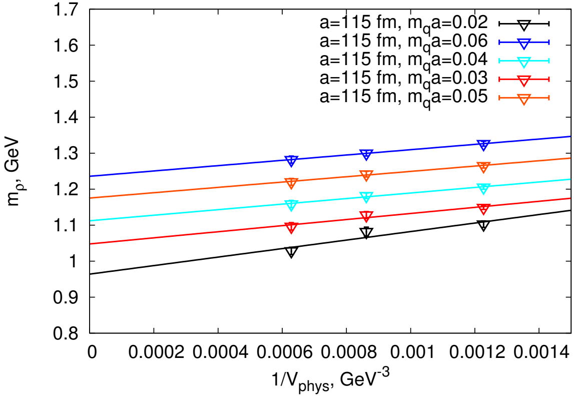
$A^B_\mu(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1})$ - Abelian magnetic field
Calculate the propagators

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}$$

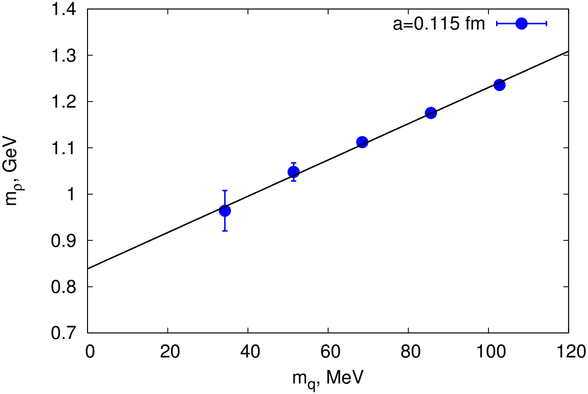
Experiment

Data from BaBar Collaboration for $e^+e^- \longrightarrow \pi^+\pi^-2\pi^0$
Obtained value: $g_{exp} = 2.1 \pm 0.5$

Extrapolation $V_{phys} \to \infty$ for ρ meson

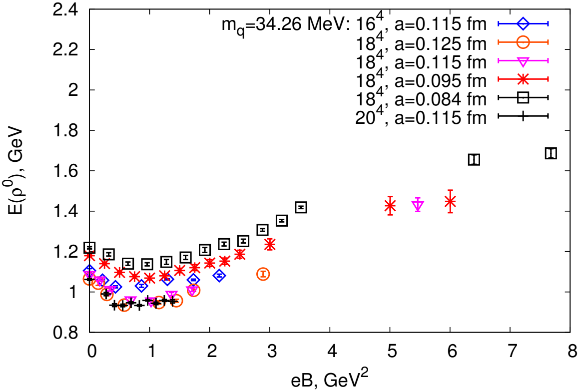


Extrapolation $m_q \to 0$ for ρ meson

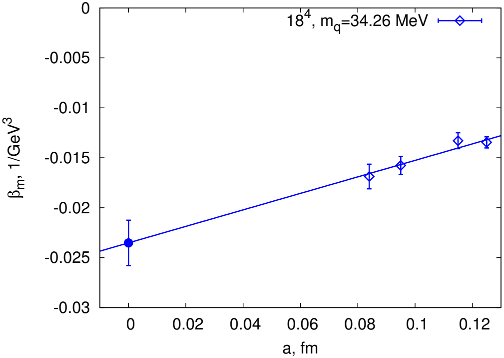


$m_q \to 0$: $\rho(770)$-meson $m_\rho = 839 \pm 11 Mev$

Unpolarized neutral ρ meson



Lattice spacing dependence $\beta_m^{|s_z|=1}(\rho^0)$



$$\beta_m^{|s_z|=1}(\rho^0) = (-0.0235 \pm 0.0023)\ Gev^{-3}$$

$$\beta_m^{|s_z|=1}(\rho^0) = (-1.86 \pm 0.18)\ 10^{-4} fm^3$$

Details of calculations

For the generation of $SU(3)$ gauge configurations the tadpole improved Lüscher-Symanzik action was used

$$S = \beta_{imp} \sum_{plq} S_{plq} - \frac{\beta_{imp}}{20u_0^2} \sum_{rt} S_{rt},$$

where $S_{plq,rt} = (1/3)(1 - U_{plq,rt})$
$u_0 = (W_{1\times 1})^{1/4} = \langle (1/3) U_{plq} \rangle^{1/4}$ is the tadpole factor, calculated at zero temperature (V.G.Bornyakov, E.-M.Ilgenfritz, and M.Müller-Preussker,Phys. Rev. D **72**, 054511 (2005)) .
Anar Rustamov

Details of calculations

Parameters of the ensembles

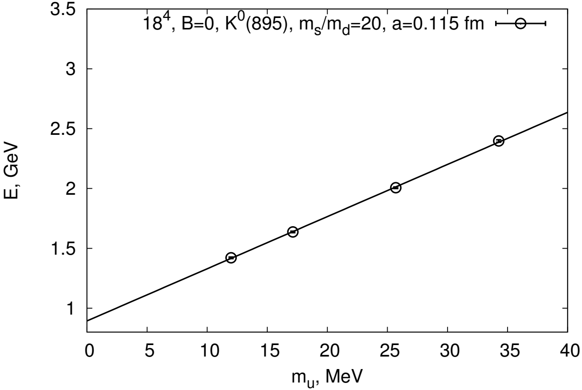
Ensemble	$N_t \times N_s^3$	β_{imp}	$a,$	N_{conf}	$m_u,\ MeV$
A_{16}	16^4	8.20	0.115	245	34.26
A_{18}	18^4	8.10	0.125	250-285	34.26, 17.13 , 11.99
B_{18}	18^4	8.20	0.115	200	34.26
C_{18}	18^4	8.30	0.105	235	34.26
D_{18}	18^4	8.45	0.095	195	34.26
E_{18}	18^4	8.60	0.084	180	34.26
A_{20}	20^4	8.20	0.115	275	34.26
B_{20}	20^4	8.45	0.095	195	34.26

Magnetic polarizability of the ρ^0 meson with nonzero spin

V_{latt}	$a(fm)$	$\beta_m^{m_q=34MeV}$ (Gev^{-3})	Error (Gev^{-3})	χ^2/d.o.f.
18^4	0.084	-0.0169	0.0012	1.235
18^4	0.095	-0.0158	0.0009	0.730
18^4	0.115	-0.0133	0.0008	0.754
18^4	0.125	-0.0135	0.0006	0.832
18^4	$a = 0$ extr.	-0.0235	0.0023	0.561
		$\beta_m^{ch.\ extr}$ (Gev^{-3})		
18^4	0.115	-0.0138	0.0005	2.648
18^4	0.125	-0.0161	0.0025	23.862

Table : The values of magnetic polarizability of the vector ρ^0 meson with nonzero spin for the bare quark mass $m_q = 34.26Mev$, lattice volume 18^4 and various lattice spacings.

Extrapolation $m_q \to 0$ for K^* meson



$m_q \to 0$: $K^*(895)$-meson $m_{K^*} = 894 \pm 12 Mev$