On stability of exponential cosmological solutions with non-static volume factor in the Einstein-Gauss-Bonnet model

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1 Introduction

This talk is devoted to D-dimensional gravitational model with the so-called Gauss-Bonnet term. It is governed by the action

$$S = \int_M d^D z \sqrt{|g|} \{ \alpha_1 (R[g] - 2\Lambda) + \alpha_2 \mathcal{L}_2[g] \}, \tag{1.1}$$

where $g = g_{MN} dz^M \otimes dz^N$ is the metric defined on the manifold M, $\dim M = D$, $|g| = |\det(g_{MN})|$ and

$$\mathcal{L}_2 = R_{MNPQ} R^{MNPQ} - 4 R_{MN} R^{MN} + R^2 \tag{1.2}$$

is the quadratic "Gauss-Bonnet term" and Λ is cosmological term. Here α_1 and α_2 are non-zero constants. The appearance of the Gauss-Bonnet term was motivated by string theory (Zwiebach,Gross, Witten, Fradkin, Tseytlin, ...).

At present, the so-called Einstein-Gauss-Bonnet (EGB) gravitational model which is governed by the action (1.1) and its modifications are intensively used in cosmology, e.g. for explanation of accelerating expansion of the Universe following from supernovae (type Ia) observational data. Here we consider the cosmological solutions with diagonal metrics governed by n scale factors depending upon one variable, where $n > 3$; $D = n + 1$. We study the stability of solutions with exponential dependence of scale factors with respect to the synchronous time variable t

$$a_i(t) \sim \exp{(v^i t)}, \tag{1.3}$$

$i = 1, \dots, n$. In our analysis we restrict ourselves by a class of perturbations which depend on t and do not disturb the diagonal form of the metric.

For possible physical applications solutions describing an exponential isotropic expansion of 3-dimensional flat factor-space, i.e. with

$$v^1 = v^2 = v^3 = H > 0, \tag{1.4}$$

and small enough variation of the effective gravitational constant G are of interest. We remind that G (for $4d$ metric in Jordan frame is proportional to the inverse volume scale factor of the internal space. Due to experimental data, the variation of G is allowed at the level of 10^{-13} per year and less. The most stringent limitation on G-dot (coming from the set of ephemerides) was obtained in ref. [1] (Pitjeva, 2013)

$$\dot{G}/G = (0.16 \pm 0.6) \cdot 10^{-13} \text{ year}^{-1} \tag{1.5}$$

allowed at 95% confidence (2-σ).

In multidimensional cosmology

$$G = G^J_{eff}(t) \sim (\prod_{i=4}^{n} a_i(t))^{-1}$$

is four-dimensional effective gravitational constant which appear in (multidimensional analogue of) the so-called Brans-Dicke-Jordan (or simply Jordan) frame. In this case the physical 4-dimensional metric $g^{(4)}$ is defined as 4-dimensional section of the multidimensional metric g, i.e. $g^{(4)} = g^{(4,J)}$, where

$$g = g^{(4,J)} + \sum_{i=4}^{n} a_i^2(t) dy^i \otimes dy^i$$

.

When the Einstein-Pauli (or simply Einstein) frame is used, we put $g^{(4)} = g^{(4,E)} = (\prod_{i=4}^{n} a_i(t)) g^{(4,J)}$ and hence we get the effective gravitational constant to be an exact constant: $G^E_{eff} = G^J_{eff}(t) \prod_{i=4}^{n} a_i(t) =$ const (Rainer, Zhuk, 1999).

2 The model

2.1 The set-up

Here we consider the manifold

$$M = (t_-, t_+) \times M_1 \times \ldots \times M_n, \tag{2.1}$$

with the metric

$$g = -e^{2\gamma(t)} dt \otimes dt + \sum_{i=1}^{n} e^{2\beta^i(t)} dy^i \otimes dy^i, \tag{2.2}$$

where $i = 1, \ldots, n$; $M_1, ..., M_n$ are one-dimensional manifolds (either $\mathbb{R}$ or S^1) and $n > 3$. The functions $\gamma(t)$ and $\beta^i(t)$, $i = 1, \ldots, n$, are smooth on (t_-, t_+).

For physical applications we put $M_1 = M_2 = M_3 = \mathbb{R}$, while $M_4, ..., M_n$ may be considered to be compact ones (i.e. coinciding with S^1).

The integrand in (1.1), when the metric (2.2) is substituted, reads as follows

$$\sqrt{|g|}\{\alpha_1 R[g] + \alpha_2 \mathcal{L}_2[g]\} = L + \frac{df}{dt}, \tag{2.3}$$

where

$$L = \alpha_1 (e^{-\gamma+\gamma_0} G_{ij} \dot{\beta}^i \dot{\beta}^j - 2\Lambda e^{\gamma+\gamma_0}) - \frac{1}{3}\alpha_2 e^{-3\gamma+\gamma_0} G_{ijkl} \dot{\beta}^i \dot{\beta}^j \dot{\beta}^k \dot{\beta}^l, \tag{2.4}$$

$\gamma_0 = \sum_{i=1}^{n} \beta^i$ and

$$G_{ij} = \delta_{ij} - 1, \tag{2.5}$$

$$G_{ijkl} = G_{ij} G_{ik} G_{il} G_{jk} G_{jl} G_{kl} \tag{2.6}$$

are respectively the components of two metrics on $\mathbb{R}^n$ [4, 5]. The first one is "minisupermetric" - 2-metric of pseudo-Euclidean signature and the second one is the Finslerian 4-metric [4, 5]. Here we denote $\dot{A} = dA/dt$ etc. The function $f(t)$ in (2.3) is irrelevant for our consideration (see [4, 5]).

The equations of motion corresponding to the action (1.1) have the following form

$$\mathcal{E}_{MN} = \alpha_1 \mathcal{E}^{(1)}_{MN} + \alpha_2 \mathcal{E}^{(2)}_{MN} = 0, \qquad (2.7)$$

where

$$\mathcal{E}^{(1)}_{MN} = R_{MN} - \frac{1}{2} R g_{MN} + \Lambda g_{MN}, \qquad (2.8)$$

$$\begin{aligned} \mathcal{E}^{(2)}_{MN} &= 2(R_{MPQS} R_N{}^{PQS} - 2 R_{MP} R_N{}^P \\ &-2 R_{MPNQ} R^{PQ} + R R_{MN}) - \frac{1}{2} \mathcal{L}_2 g_{MN}. \end{aligned} \qquad (2.9)$$

It may be shown that the field eqs. (2.7) for the metric (2.2) are equivalent to the Lagrange equations corresponding to the Lagrangian L from (2.4).

Thus, eqs. (2.7) read as follows

$$\alpha_1 (G_{ij} \dot{\beta}^i \dot{\beta}^j + 2\Lambda e^{2\gamma}) - \alpha_2 e^{-2\gamma} G_{ijkl} \dot{\beta}^i \dot{\beta}^j \dot{\beta}^k \dot{\beta}^l = 0, \qquad (2.10)$$

$$\frac{d}{dt}[2\alpha_1 G_{ij} e^{-\gamma+\gamma_0} \dot{\beta}^j - \frac{4}{3} \alpha_2 e^{-3\gamma+\gamma_0} G_{ijkl} \dot{\beta}^j \dot{\beta}^k \dot{\beta}^l] - L = 0, \qquad (2.11)$$

$i = 1, \ldots, n$; and L is defined in (2.4).

Now we put $\gamma = 0$. By introducing "Hubble-like" variables $h^i = \dot{\beta}^i$, eqs. (2.10) and (2.11) may be rewritten as follows

$$E = E(h) \equiv G_{ij} h^i h^j + 2\Lambda - \alpha G_{ijkl} h^i h^j h^k h^l = 0, \tag{2.12}$$

$$U_i = U_i(\dot{h}, h) \equiv \frac{dL_i}{dt} + (\sum_{j=1}^{n} h^j) L_i - L_0 = 0, \tag{2.13}$$

where $\alpha = \alpha_1/\alpha_2$,

$$L_0 = G_{ij} h^i h^j - 2\Lambda - \frac{1}{3} \alpha G_{ijkl} h^i h^j h^k h^l, \tag{2.14}$$

and

$$L_i = L_i(h) = 2 G_{ij} h^j - \frac{4}{3} \alpha G_{ijkl} h^j h^k h^l, \tag{2.15}$$

$i = 1, \dots, n$. Thus, we are led to the autonomous system of the first-order differential equations on $h^1(t), \dots, h^n(t)$ (see [4, 5] for $\Lambda = 0$). Due to (2.12) we have $L_0 = \frac{2}{3}(G_{ij} h^i h^j - 4\Lambda)$.

In what follows we will use instead of (2.12), (2.13) an equivalent set of equations: (2.12) and

$$Y_i = Y_i(\dot{h}, h) \equiv \frac{dL_i}{dt} + (\sum_{j=1}^{n} h^j) L_i - \frac{2}{3}(G_{ij} h^i h^j - 4\Lambda) = 0. \tag{2.16}$$

We note that the following identity is valid $U_i(\dot{h}, h) = Y_i(\dot{h}, h) - \frac{1}{3} E(h)$.

2.2 Polynomial equations for solutions with constant h^i

Let us consider the following solutions to eqs. (2.12) and (2.16)

$$h^i(t) = v^i, \tag{2.17}$$

with constant v^i, which correspond to the solutions

$$\beta^i = v^i t + \beta^i_0, \tag{2.18}$$

where β^i_0 are constants, $i = 1, \ldots, n$.

In this case we obtain for the metric

$$g = -dt \otimes dt + \sum_{i=1}^{n} B_i^2 e^{2v^i t} dy^i \otimes dy^i, \tag{2.19}$$

where $B_i > 0$ are arbitrary constants.

For the fixed point $v = (v^i)$ we have the set of polynomial equations

$$E = E(v) = G_{ij} v^i v^j + 2\Lambda - \alpha G_{ijkl} v^i v^j v^k v^l = 0, \tag{2.20}$$

$$Y_i = Y_i(\mathbf{0}, v) = (\sum_{j=1}^{n} v^j) L_i(v) - \frac{2}{3} G_{kj} v^k v^j + \frac{8}{3}\Lambda = 0, \tag{2.21}$$

where L_i is defined in (2.15), $i = 1, \ldots, n$. For $n > 3$ this is the set of forth-order polynomial equations.

Proposition 1. [4] *For any solution $v = (v^1, \ldots, v^n)$ to polynomial eqs. (2.20), (2.21) with $n > 3$ there are no more than three different numbers among $v^1, ..., v^n$, if $\sum_{i=1}^{n} v^i \neq 0$.*

Here we deal with the ansatz which contain two Hubble parameters

$$v = (v^i) = (H, \dots, H, h, \dots, h) \tag{2.22}$$

where H appears m-times and h appears l-times, $n = m + l$.

with two restrictions imposed

$$mH + lh \neq 0, \qquad H \neq h. \tag{2.23}$$

In what follows we adopt the following agreement for indices: $\mu, \nu, \dots = 1, \dots, m$; $\alpha, \beta, \dots = m + 1, \dots, n$. Thus, $v^\mu = H$ and $v^\alpha = h$.

In this case the set of $n + 1$ eqs. (2.12), (2.13) is equivalent to the set of two equations

$$\begin{aligned} E = mH^2 + lh^2 - (mH + lh)^2 + 2\Lambda - \alpha[m(m-1)(m-2)(m-3)H^4 \\ +4m(m-1)(m-2)lH^3h + 6m(m-1)l(l-1)H^2h^2 \\ +4ml(l-1)(l-2)Hh^3 + l(l-1)(l-2)(l-3)h^4] = 0, \qquad (2.24) \\ 1 + 2\alpha Q(H, h) = 0, \qquad (2.25) \end{aligned}$$

where

$$Q(H, h) = (m-1)(m-2)H^2 + 2(m-1)(l-1)Hh + (l-1)(l-2)h^2. \tag{2.26}$$

For general scheme of reduction see [7] (Chirkov et al).

3 Stability of fixed point solutions $h^i(t) = v^i$

Here we study the stability of static solutions $h^i(t) = v^i$ to eqs. (2.12) and (2.13) in linear approximation in pertubations. We put

$$h^i(t) = v^i + \delta h^i(t), \tag{3.1}$$

$i = 1, \dots, n$. By substitution (3.1) into eqs. (2.12) and (2.13) we obtain in linear approximation the following relations for perturbations δh^i

$$C_i(v)\delta h^i = 0, \tag{3.2}$$

$$L_{ij}(v)\delta \dot{h}^j = B_{ij}(v)\delta h^j, \tag{3.3}$$

where

$$C_i = C_i(v) = 2v_i - 4\alpha G_{ijks} v^j v^k v^s, \tag{3.4}$$

$$L_{ij} = L_{ij}(v) = 2G_{ij} - 4\alpha G_{ijks} v^k v^s, \tag{3.5}$$

$$B_{ij} = B_{ij}(v) = -(\sum_{k=1}^{n} v^k) L_{ij}(v) - L_i(v) + \frac{4}{3} v_j. \tag{3.6}$$

We remind that $v_i = G_{ij} v^j$, $L_i(v) = 2v_i - \frac{4}{3}\alpha G_{ijks} v^j v^k v^s$ and $i, j, k, s = 1, \dots, n$.

We put the following restriction on the matrix $L = (L_{ij}(v))$

$$\text{(R)} \quad \det(L_{ij}(v)) \neq 0, \tag{3.7}$$

i.e. the matrix L should be invertible.

Here we restrict ourselves by exponential solutions (2.19) with non-static volume factor, which is proportional to $\exp(\sum_{i=1}^{n} v^i t)$, i.e. we put

$$K = K(v) = \sum_{i=1}^{n} v^i \neq 0. \tag{3.8}$$

Then it may proved that eq. (3.3) reads

$$L_{ij}(v)\delta \dot{h}^j = -(\sum_{k=1}^{n} v^k) L_{ij} \delta h^j, \tag{3.9}$$

or, equivalently,

$$\delta \dot{h}^i = -(\sum_{k=1}^{n} v^k) \delta h^i, \tag{3.10}$$

$i = 1, \dots, n$. Here we used the restriction (3.7).

Thus, the set of linear equations on perturbations (3.2), (3.3) is equivalent to the set of linear eqs. (3.2), (3.10), which has the following solution

$$\delta h^i = A^i \exp(-K(v)t), \tag{3.11}$$

$$\sum_{i=1}^{n} C_i(v) A^i = 0. \tag{3.12}$$

$i = 1, \dots, n$. We remind that $K(v) = \sum_{k=1}^{n} v^k$.

Due to (3.11) that the following proposition is valid.

Proposition 2. *The fixed point solution* $(h^i(t)) = (v^i)$ *(*$i = 1, \dots, n$; $n > 3$*) to eqs. (2.12), (2.13) obeying restrictions (3.7), (3.8) is stable under perturbations (3.1) (as* $t \to +\infty$*) if* $K(v) = \sum_{k=1}^{n} v^k > 0$ *and it is unstable (as* $t \to +\infty$*) if* $K(v) = \sum_{k=1}^{n} v^k < 0$.

Now let us consider the matrix (3.5) for the anisotropic case (2.22) with two Hubble parameters obeying (2.23).

For the the ansatz (2.22) we obtain

$$L_{\mu\nu} = G_{\mu\nu}(2 + 4\alpha S_{HH}), \tag{3.13}$$

$$L_{\mu\alpha} = L_{\alpha\mu} = -2 - 4\alpha S_{Hh}, \tag{3.14}$$

$$L_{\alpha\beta} = G_{\alpha\beta}(2 + 4\alpha S_{hh}). \tag{3.15}$$

Here S_{HH}, S_{Hh} and S_{hh} are defined ($S_{ij} = G_{ijks}v^k v^s$) as follows

$$S_{HH} = (m-2)(m-3)H^2 + 2(m-2)lHh + l(l-1)h^2, \tag{3.16}$$

$$S_{Hh} = (m-1)(m-2)H^2 + 2(m-1)(l-1)Hh + (l-1)(l-2)h^2, \tag{3.17}$$

$$S_{hh} = m(m-1)H^2 + 2m(l-2)Hh + (l-2)(l-3)h^2. \tag{3.18}$$

Here we denote: $S_{\mu\nu} = S_{HH}$ for $\mu \neq \nu$; $S_{\mu\alpha} = S_{\alpha\mu} = S_{Hh}$; $S_{\alpha\beta} = S_{hh}$ for $\alpha \neq \beta$.

But here we have a remarkable coincidence (see (2.26))

$$Q(H,h) = S_{Hh}, \tag{3.19}$$

which implies $L_{\mu\alpha} = L_{\alpha\mu} = 0$ due to eq. (2.25). Thus under restrictions (2.23) assumed the matrix (L_{ij}) has a block-diagonal form

$$(L_{ij}) = \mathrm{diag}(L_{\mu\nu}, L_{\alpha\beta}). \tag{3.20}$$

This matrix is invertible if and only if $m > 1$, $l > 1$ and

$$S_{HH} \neq -\frac{1}{2\alpha}, \qquad S_{hh} \neq -\frac{1}{2\alpha}. \tag{3.21}$$

We remind that $m \times m$ matrix $(G_{\mu\nu})$ and $l \times l$ matrix $(G_{\alpha\beta})$ are invertible only for $m > 1$ and $l > 1$, respectively.

4 Examples

Here we consider several examples of exponential solutions and analyse their stability.

4.1 Solution for $m = 3$, $l = 2$ and $\Lambda = 0$.

Let us consider the case $m = 3$, $l = 2$, $\Lambda = 0$. We have the following solution to the set of polynomial eqs. (2.24), (2.25) with $H > 0$:

$$H = \frac{1}{6}(7 + 4 \cdot 10^{1/3} + 10^{2/3})^{1/2} \alpha^{-1/2} \approx 0.750173 \alpha^{-1/2}, \tag{4.1}$$

$$h = -\frac{1}{6}(7 - 0.5 \cdot 10^{1/3} + 10^{2/3})^{1/2} \alpha^{-1/2} \approx -0.541715 \alpha^{-1/2}. \tag{4.2}$$

It the approximate form this solution was found earlier by D. Ratanov (RUDN), in analytic form (different from (4.1), (4.2)) it was obtained in [6].

Using (3.16) and (3.18) we get

$$S_{HH} = 2h(2H + h) \approx -1.038610 \alpha^{-1}, \qquad S_{hh} = 6H^2 \approx 3.376557 \alpha^{-1}. \tag{4.3}$$

Relations (3.21) are valid and hence the first restriction (3.7) is satisfied. The second restriction (3.8) is also satisfied since $K(v) = 3H + 2h > 0$. Thus, due to Proposition 2, the solution is stable in agreement with [9].

4.2 Solution for $m = l = 3$ and $\Lambda = 0$

Now we consider solutions with $m = 3$, $l = 3$ and $\Lambda = 0$. There are two solutions to eqs. (2.24), (2.25) with $H > 0$:

$$H_1 = \frac{1}{4}(\sqrt{5} - 1)\alpha^{-1/2}, \qquad h_1 = \frac{1}{4}(-\sqrt{5} - 1)\alpha^{-1/2}, \tag{4.4}$$

and

$$H_2 = \frac{1}{4}(\sqrt{5} + 1)\alpha^{-1/2}, \qquad h_2 = \frac{1}{4}(-\sqrt{5} + 1)\alpha^{-1/2}. \tag{4.5}$$

For the first solution we get

$$S_{HH} = \frac{3}{4}(\sqrt{5} + 1)\alpha^{-1}, \qquad S_{hh} = \frac{3}{4}(-\sqrt{5} + 1)\alpha^{-1}, \tag{4.6}$$

while for the second one we obtain

$$S_{HH} = \frac{3}{4}(-\sqrt{5} + 1)\alpha^{-1}, \qquad S_{hh} = \frac{3}{4}(\sqrt{5} + 1)\alpha^{-1}. \tag{4.7}$$

In both cases relations (3.21) are satisfied and hence the first restriction (3.7) is valid. The second restriction (3.8) is also valid for any of these solutions since $K(v_1) = 3H_1 + 3h_1 = -\frac{3}{2}\alpha^{-1/2} < 0$ and $K(v_2) = 3H_2 + 3h_2 = \frac{3}{2}\alpha^{-1/2} > 0$. According to Proposition 2 the first solution (4.4) is unstable, while the second one (4.5) is stable.

4.3 Solution for $m = 11$, $l = 16$ and $\Lambda = 0$

For $\Lambda = 0$ the solution (2.19) with $v = (v^i)$ from (2.22), $m = 11$, $l = 16$ and

$$H = \frac{1}{\sqrt{15\alpha}}, \qquad h = -\frac{1}{2\sqrt{15\alpha}} \tag{4.8}$$

was found in [8]. This solution describes the zero variation of the effective cosmological constant G.

The calculations give us

$$S_{HH} = -\frac{4}{5}\alpha^{-1}, \quad S_{hh} = \frac{1}{10}\alpha^{-1}. \tag{4.9}$$

Due to (3.21) the symmetric matrix (L_{ij}), which has a block-diagonal form, is invertible, i.e. the condition (3.7) is satisfied.

We find $(C_i) = (C_\mu = 12H, C_\alpha = 18H)$. From (3.11) we get the following solution for perturbations

$$\delta h^i = A^i \exp(-3Ht), \tag{4.10}$$

$$2\sum_{\mu=1}^{11} A^\mu + 3\sum_{\alpha=12}^{27} A^\alpha = 0, \tag{4.11}$$

where $H = \frac{1}{\sqrt{15\alpha}}$, $i = 1, \ldots, 27$. Thus, the solution (4.8) is stable, as $t \to +\infty$.

4.4 Solution for $m = 15$, $l = 6$ and $\Lambda = 0$

Now we consider another exponential solution (2.19) from [8] with $v = (v^i)$ from (2.22), $m = 15$, $l = 6$, $\Lambda = 0$ and

$$H = \frac{1}{6}\alpha^{-1/2}, \qquad h = -\frac{1}{3}\alpha^{-1/2}. \tag{4.12}$$

We get

$$S_{HH} = -\alpha^{-1}, \qquad S_{hh} = \frac{1}{2}\alpha^{-1}. \tag{4.13}$$

According to (3.21) the symmetric block-diagonal matrix (L_{ij}) is non-degenerate one.

We get $(C_i) = (C_\mu = \frac{14}{3}, C_\alpha = \frac{20}{3})$. Due to (3.11) the solution for perturbations reads

$$\delta h^i = A^i \exp(-3Ht) = A^i \exp(-\frac{1}{2}\alpha^{-1/2}t), \tag{4.14}$$

$$7\sum_{\mu=1}^{15} A^\mu + 10\sum_{\alpha=16}^{21} A^\alpha = 0, \tag{4.15}$$

$i = 1, \dots, 21$. Hence, the solution (4.12) is stable as $t \to +\infty$.

4.5 Solutions with $m \geq 3$, $l > 1$ and certain $\Lambda > 0$

Here we consider the following solution to eqs. (2.24), (2.25) for $m > 2$, $l > 1$ and $\alpha < 0$:

$$H^2 = -\frac{1}{2\alpha(m-1)(m-2)}, \qquad h = 0, \tag{4.16}$$

which is valid for

$$\Lambda = -\frac{m(m+1)}{8\alpha(m-1)(m-2)} > 0. \tag{4.17}$$

We get from (3.16) and (3.18)

$$S_{HH} = (m-2)(m-3)H^2 = -\frac{m-3}{2\alpha(m-1)} \neq -\frac{1}{2\alpha} \tag{4.18}$$

and

$$S_{hh} = m(m-1)H^2 = -\frac{m}{2\alpha(m-2)} \neq -\frac{1}{2\alpha}, \tag{4.19}$$

which implies the fulfilment of the restriction (3.7) (here $m > 2$ and $l > 1$). Since $K(v) = mH$ we get from Proposition 2 that the cosmological solution (2.19) with H, h from (4.16) is stable for $H > 0$ and unstable for $H < 0$.

6 - examples of solutions with $G = const$ and $\Lambda \neq 0$ in V. D. Ivashchuk, On Stable Exponential Solutions in Einstein-Gauss-Bonnet Cosmology with Zero Variation of G, Gravitation and Cosmology, Vol. 22, No. 4, pp. 329-332 (2016) (in press).

4.6 A subclass of solutions with zero variation of G

The 4d effective gravitational constant is proportional to inverse volume scale factor of the internal space, i.e.

$$G \sim \prod_{i=4}^{n} [a_i(t)]^{-1}, \tag{4.20}$$

where $a_i(t) = \exp(\beta^i(t))$.

For the solutions (2.19) we obtain the following relations

$$G(t) = G(0) \exp\left(-K_{int} t\right), \quad K_{int}(v) = \sum_{i=4}^{n} v^i, \tag{4.21}$$

which imply

$$\frac{\dot{G}}{G} = -K_{int}(v). \tag{4.22}$$

Now, let us consider a subclass of cosmological solutions (2.19) which obey restriction (3.7) and describe an exponential isotropic expansion of 3-dimensional flat factor-space with $v^1 = v^2 = v^3 = H > 0$ with zero variation of G. Then we get from (4.22) $K_{int}(v) = 0$ and hence $K(v) = \sum_{i=1}^{n} v^i = 3H + K_{int}(v) = 3H > 0$. According to Proposition 2 any solution from this subclass is stable. Three solutions from the previous subsection: (4.8), (4.12) and (4.16) with $m = 3$ (and $l > 1$) belong to this subclass.

5 Conclusions

We have considered the $(n+1)$-dimensional Einstein-Gauss-Bonnet (EGB) model with the Λ-term. By using the ansatz with diagonal cosmological metrics, we have studied the stability of solutions with exponential dependence of scale factors $a_i \sim \exp(v^i t)$, $i = 1, \dots, n$, with respect to synchronous time variable t in dimension $D > 4$.

The problem was reduced to the analysis of stability of the fixed point solutions $h^i(t) = v^i$ to eqs. (2.12) and (2.16), where $h^i(t)$ are Hubble-like parameters.

In this paper a set of equations for perturbations δh^i was considered (in linear approximation) and general solution to these equations was found. We have proved (in Proposition 2) that the solutions with non-static volume factor, i.e. with $K(v) = \sum_{k=1}^{n} v^k \neq 0$, which obey restriction (3.7), are stable if $K(v) > 0$ while they are unstable if $K(v) < 0$.

We have also proved (in Proposition 1) that for any exponential solution with $v = (v^1, ..., v^n)$ there are no more than three different numbers among $v^1, ..., v^n$, if $\sum_{i=1}^{n} v^i \neq 0$.

Here we have presented several examples of stable cosmological solutions with exponential behavior of scale factors. We have also shown that general solutions with $v^1 = v^2 = v^3 = H > 0$ and zero variation of the effective gravitational constant are stable if the restriction (3.7) is obeyed.

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Thank you for your attention!