Topological defects with power-law tails

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Introduction

- Conditions for existing kinks with power-law asymptotics
- The model $\varphi^8$
- Collective coordinate approximation
- Interaction via Manton’s method
Asymptotics of kinks

Lagrangian density:

\[ \mathcal{L} = \frac{1}{2} \left( \frac{\partial \varphi}{\partial t} \right)^2 - \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 - V(\varphi) \]  \hspace{1cm} (1)

We can rewrite the potential

\[ V(\varphi) = (\varphi - \varphi_i)^{k_i}(\varphi - \varphi_{i+1})^{k_{i+1}} V_1(\varphi) \]  \hspace{1cm} (2)

\( \varphi_i \) and \( \varphi_{i+1} \) are zeros of \( V(\varphi) \) of orders \( k_i \) and \( k_{i+1} \) respectively. \( k_i \) and \( k_{i+1} \) are even; \( V_1(\varphi_i) > 0 \), \( V_1(\varphi_{i+1}) > 0 \).
Asymptotics of kinks

If $k_i > 2$ and $k_{i+1} > 2$

$$\varphi(\bar{\varphi}_i, \bar{\varphi}_{i+1}) \approx \bar{\varphi}_i + C_1 \cdot \frac{1}{|x|^{2/(k_i-2)}}, \quad x \to -\infty; \quad (3)$$

$$\varphi(\bar{\varphi}_i, \bar{\varphi}_{i+1}) \approx \bar{\varphi}_{i+1} - C_2 \cdot \frac{1}{|x|^{2/(k_{i+1}-2)}}, \quad x \to +\infty. \quad (4)$$
The model $\varphi^8$

We consider $\varphi^8$ model with the potential $V(\varphi) = \varphi^4(1 - \varphi^2)^2$.

After inserting this potential into the equation of motion and integrating we obtain two kinks

$$2\sqrt{2}x = -\frac{2}{\varphi} + \ln \frac{1 + \varphi}{1 - \varphi} \quad (5)$$
The model $\varphi$

Consider the kink $\varphi_{(-1,0)}(x)$:

- Asymptotics at $x \to +\infty$: $\varphi_{(-1,0)} \approx -\frac{1}{\sqrt{2x}}$;
- Asymptotics at $x \to -\infty$: $\varphi_{(-1,0)} \approx -1 + \frac{2}{e^2} e^{2\sqrt{2x}}$.

For the kink $\varphi_{(0,1)}(x)$:

- Asymptotics at $x \to -\infty$: $\varphi_{(0,1)} \approx -\frac{1}{\sqrt{2x}}$;
- Asymptotics at $x \to +\infty$: $\varphi_{(0,1)} \approx 1 - \frac{2}{e^2} e^{2\sqrt{2x}}$. 
Collective coordinate approximation

Consider the configurations:

- For power-law tails:
  \[ \varphi(x, \xi) = \varphi(-1,0)(x + \xi) + \varphi(0,-1)(x - \xi); \]

- For exponential tails:
  \[ \varphi(x, \xi) = \varphi(0,1)(x + \xi) + \varphi(1,0)(x - \xi) - 1; \]

\[ U_{\text{eff}}(\xi) = \int_{-\infty}^{+\infty} \left[ \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 + V(\varphi) \right] \, dx, \quad (6) \]

\[ F(\xi) = -\frac{dU_{\text{eff}}}{d\xi}. \quad (7) \]
Manton’s method

\[ F[\varphi] \simeq F_M[\varphi] = \left[ V(\varphi) - \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 \right] \bigg|_{x=0} \tag{8} \]

Asymptotic expressions (\( \varphi = \varphi_{(-1,0)} + \varphi_{(0,-1)} \)):

\begin{itemize}
  \item \( F \sim F_M = \frac{4}{\xi^4} \)
  \item \( U_M = \frac{4}{3\xi^3} \)
\end{itemize}

Asymptotic expressions (\( \varphi = \varphi_{(0,1)} + \varphi_{(1,0)} - 1 \)):

\begin{itemize}
  \item \( F \sim F_M = 64e^{-4(\sqrt{2}\xi+1)} \)
  \item \( U_M = 8\sqrt{2}e^{-4(\sqrt{2}\xi+1)} \)
\end{itemize}
Results (power-law tails)

Power-law tails:

Exponential tails:

Collective coordinate

Manton
Thanks for your attention!
\( \bar{\varphi}_i \) and \( \bar{\varphi}_{i+1} \) are zeros of \( V(\varphi) \) of orders \( k_i \) and \( k_{i+1} \) respectively. \( k_i \) and \( k_{i+1} \) are even; assume \( \bar{\varphi}_i < \bar{\varphi}_{i+1} \).

\[
V(\varphi) = (\varphi - \bar{\varphi}_i)^{k_i} (\varphi - \bar{\varphi}_{i+1})^{k_{i+1}} V_1(\varphi), \tag{9}
\]

\( V_1(\bar{\varphi}_i) > 0, V_1(\bar{\varphi}_{i+1}) > 0; \)

\[
\frac{d\varphi}{dx} = \sqrt{2 V(\varphi)}, \tag{10}
\]

\[
\int d\varphi = \int \frac{d\varphi}{(\varphi - \bar{\varphi}_i)^{k_i/2} (\bar{\varphi}_{i+1} - \varphi)^{k_{i+1}/2} \sqrt{2 V_1(\varphi)}} \approx \frac{1}{(\bar{\varphi}_{i+1} - \bar{\varphi}_i)^{k_{i+1}/2} \sqrt{2 V_1(\bar{\varphi}_i)}} \int \frac{d\varphi}{(\varphi - \bar{\varphi}_i)^{k_i/2}} \tag{11}
\]
The effective force of the interaction can be found by considering the wave momentum $P[\varphi] = -\int_{-\infty}^{b} \frac{\partial \varphi}{\partial t} \frac{\partial \varphi}{\partial x} \, dx$,

$$F[\varphi] = \frac{dP}{dt} = - \left[ \frac{1}{2} \left( \frac{\partial \varphi}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 - V(\varphi) \right] \bigg|_{-\infty}^{b}$$

(12)

$$F[\varphi] \approx F_M[\varphi] = \left[ V(\varphi) - \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 \right] \bigg|_{x=b}$$

(13)