Topological defects with power-law tails

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Introduction

- Conditions for existing kinks with power-law asymptotics
- The model φ^8
- Collective coordinate approximation
- Interaction via Manton's method

Asymptotics of kinks

Lagrangian density:

$$\mathscr{L} = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - V(\varphi) \qquad (1)$$

We can rewrite the potential

$$V(\varphi) = (\varphi - \bar{\varphi}_i)^{k_i}(\varphi - \bar{\varphi}_{i+1})^{k_{i+1}}\, V_1(\varphi) \qquad (2)$$

$\bar{\varphi}_i$ and $\bar{\varphi}_{i+1}$ are zeros of $V(\varphi)$ of orders k_i and k_{i+1} respectively.
k_i and k_{i+1} are even; $V_1(\bar{\varphi}_i) > 0$, $V_1(\bar{\varphi}_{i+1}) > 0$.

Asymptotics of kinks

If $k_i > 2$ and $k_{i+1} > 2$

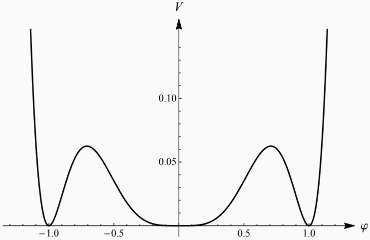
$$\varphi_{(\bar{\varphi}_i,\bar{\varphi}_{i+1})} \approx \bar{\varphi}_i + C_1 \cdot \frac{1}{|x|^{2/(k_i-2)}}, \quad x \to -\infty; \tag{3}$$

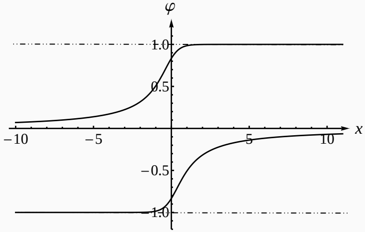
$$\varphi_{(\bar{\varphi}_i,\bar{\varphi}_{i+1})} \approx \bar{\varphi}_{i+1} - C_2 \cdot \frac{1}{|x|^{2/(k_{i+1}-2)}}, \quad x \to +\infty. \tag{4}$$

We consider φ^8 model with the potential $V(\varphi) = \varphi^4(1-\varphi^2)^2$.

After inserting this potential into the equation of motion and integrating we obtain two kinks

$$2\sqrt{2}x = -\frac{2}{\varphi} + ln\frac{1+\varphi}{1-\varphi} \qquad (5)$$





The model φ^8

Consider the kink $\varphi_{(-1,0)}(x)$:

- Asymptotics at $x \to +\infty$: $\varphi_{(-1,0)} \approx -\frac{1}{\sqrt{2}x}$;
- Asymptotics at $x \to -\infty$: $\varphi_{(-1,0)} \approx -1 + \frac{2}{e^2}\, e^{2\sqrt{2}x}$.

For the kink $\varphi_{(0,1)}(x)$:

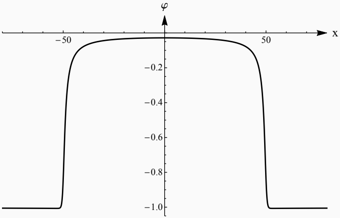
- Asymptotics at $x \to -\infty$: $\varphi_{(0,1)} \approx -\frac{1}{\sqrt{2}x}$;
- Asymptotics at $x \to +\infty$: $\varphi_{(0,1)} \approx 1 - \frac{2}{e^2}\, e^{2\sqrt{2}x}$.

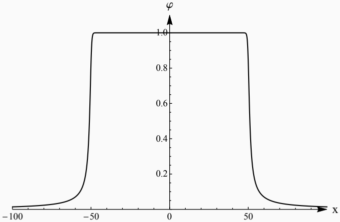
Consider the configurations:

- For power-law tails: $\varphi(x,\xi) = \varphi_{(-1,0)}(x+\xi) + \varphi_{(0,-1)}(x-\xi)$;
- For exponential tails: $\varphi(x,\xi) = \varphi_{(0,1)}(x+\xi) + \varphi_{(1,0)}(x-\xi) - 1$;

$$U_{eff}(\xi) = \int_{-\infty}^{+\infty} \left[\frac{1}{2} \left(\frac{\partial\varphi}{\partial x} \right)^2 + V(\varphi) \right] dx, \quad (6)$$

$$F(\xi) = -dU_{eff}/d\xi. \quad (7)$$





Manton's method

$$F[\varphi] \simeq F_M[\varphi] = \left[V(\varphi) - \frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 \right]\Bigg|_{x=0} \quad (8)$$

Asymptotic expressions ($\varphi = \varphi_{(-1,0)} + \varphi_{(0,-1)}$):

- $F \sim F_M = \frac{4}{\xi^4}$
- $U_M = \frac{4}{3\xi^3}$

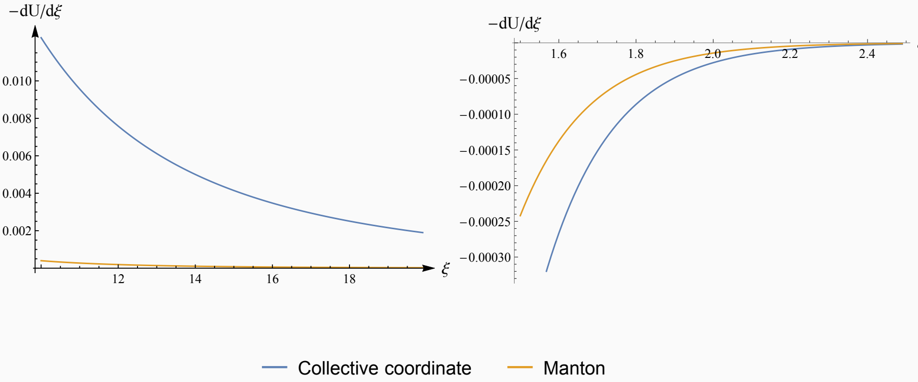
Asymptotic expressions ($\varphi = \varphi_{(0,1)} + \varphi_{(1,0)} - 1$):

- $F \sim F_M = 64e^{-4(\sqrt{2}\xi+1)}$
- $U_M = 8\sqrt{2}e^{-4(\sqrt{2}\xi+1)}$

Results (power-law tails)

Power-law tails:

Exponential tails:



Thanks for your attention!

Some formulas

$\bar{\varphi}_i$ and $\bar{\varphi}_{i+1}$ are zeros of $V(\varphi)$ of orders k_i and k_{i+1} respectively. k_i and k_{i+1} are even; assume $\bar{\varphi}_i < \bar{\varphi}_{i+1}$.

$$V(\varphi) = (\varphi - \bar{\varphi}_i)^{k_i} (\varphi - \bar{\varphi}_{i+1})^{k_{i+1}} \; V_1(\varphi), \tag{9}$$

$V_1(\bar{\varphi}_i) > 0$, $V_1(\bar{\varphi}_{i+1}) > 0$;

$$\frac{d\varphi}{dx} = \sqrt{2\,V(\varphi)}, \tag{10}$$

$$\begin{aligned} \int dx = \int \frac{d\varphi}{(\varphi - \bar{\varphi}_i)^{k_i/2} (\bar{\varphi}_{i+1} - \varphi)^{k_{i+1}/2} \sqrt{2\,V_1(\varphi)}} \approx \\ \approx \frac{1}{(\bar{\varphi}_{i+1} - \bar{\varphi}_i)^{k_{i+1}/2} \sqrt{2\,V_1(\bar{\varphi}_i)}} \int \frac{d\varphi}{(\varphi - \bar{\varphi}_i)^{k_i/2}} \end{aligned} \tag{11}$$

Some formulas

The effective force of the interaction can be found by considering the wave momentum $P[\varphi] = -\int_{-\infty}^{b} \frac{\partial \varphi}{\partial t}\frac{\partial \varphi}{\partial x}\,dx$;

$$F[\varphi] = \frac{dP}{dt} = -\left[\frac{1}{2}\left(\frac{\partial \varphi}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2 - V(\varphi)\right]\Bigg|_{-\infty}^{b} \tag{12}$$

$$F[\varphi] \simeq F_M[\varphi] = \left[V(\varphi) - \frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2\right]\Bigg|_{x=b} \tag{13}$$