Anisotropic flow analyses with multiparticle azimuthal correlations

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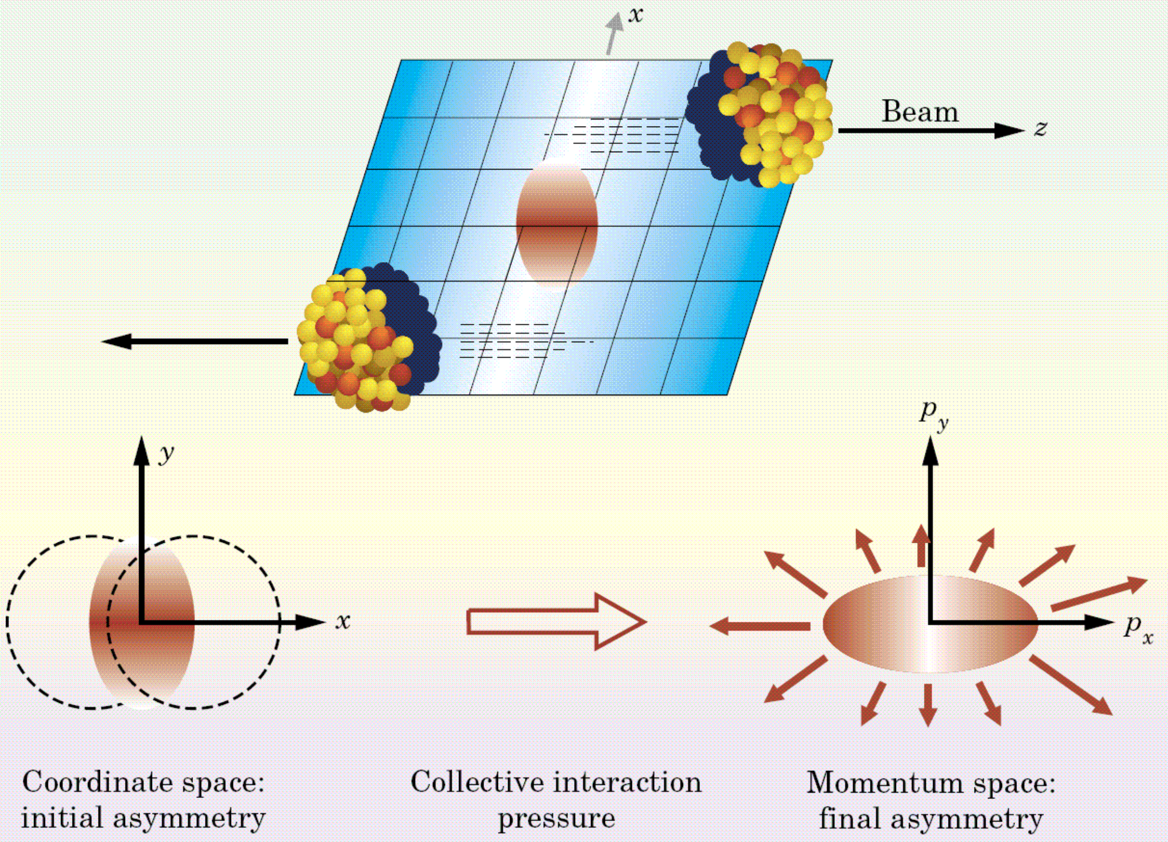
Moscow, 13/10/2016
'The 2nd International Conference on Particle Physics and Astrophysics'

Outline

- Introduction
  - Anisotropic flow
  - Multiparticle azimuthal correlations
- New flow observables: Symmetric Cumulants
- Unresolved problems
- Next

Anisotropic flow

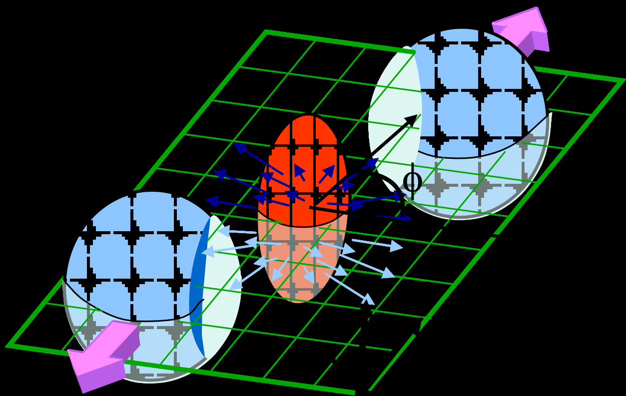


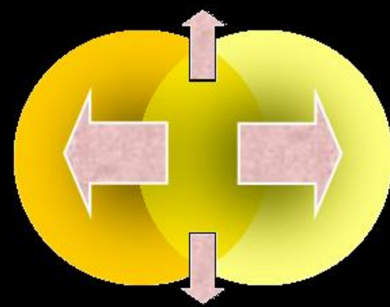
- The transfer of initial anisotropy in coordinate space into the final anisotropy in momentum space via interactions between the constituents is the anisotropic flow phenomenon

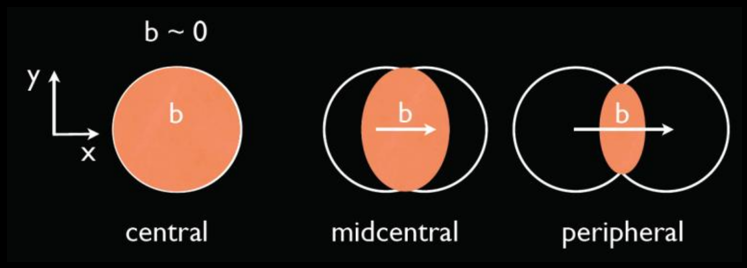
Transfer of anisotropy

- **Thermalisation** ⇔ large number of mutual interactions among constituents
- Large number of mutual interactions ⇔ large number of interacting particles confined to a small volume
- Large number of interacting particles confined to a small volume ⇔ **heavy-ion collisions**
  - It is much less probable that thermalisation will be reached in collisions of lighter objects (e.g. in p+p collisions)
  - Once we have a thermalized medium we can start naturally to speak about thermodynamic concepts like temperature, pressure, equation of state, etc.

How to quantify flow?







- S. Voloshin and Y. Zhang, Z.Phys.C70:665-672,1996: **Fourier series**

$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\mathrm{RP}}\right)\right)\right)$$

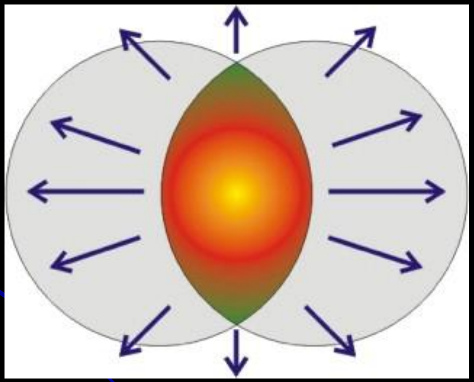
$$v_n = \langle\cos(n(\phi-\Psi_{\mathrm{RP}}))\rangle$$

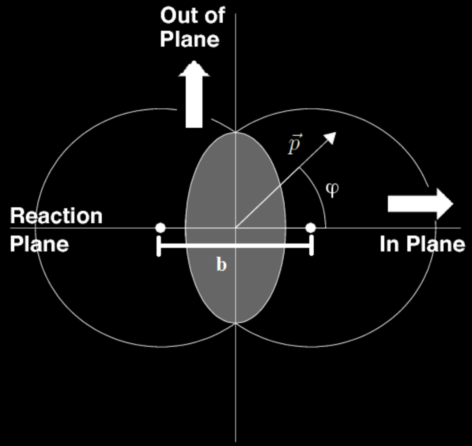
- Harmonics v_n quantify anisotropic flow
  - v_1 is directed flow, v_2 is elliptic flow, v_3 is triangular flow, etc.

The 'flow principle'

- Can we estimate the amplitudes v_n without the explicit knowledge of symmetry planes?

$$v_n = \langle \cos(n(\phi - \Psi_{\mathrm{RP}})) \rangle$$

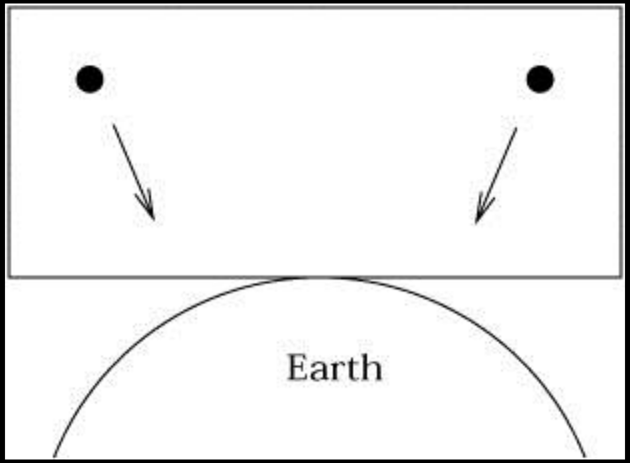




- **The 'flow principle':** Correlations among produced particles are induced solely by correlation of each particle to the reaction plane

Analogy with gravity

- Falling bodies appear to be **correlated** in gravitational field due to correlation of each body with the common center of gravity



- Geometry of massive body **=>** gravitational field
- Geometry of heavy-ion collision **=>** the pressure gradients
  - Particle trajectories are the same whether they would be emitted simultaneously or one-by-one: **statistical independence**

Correlation techniques

- As an outcome of 'flow principle' we have **factorization**

event average

particle average

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\rm RP}-(\phi_2-\Psi_{\rm RP}))} \right\rangle\!\right\rangle$$
$$= \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\rm RP})} \right\rangle \left\langle e^{-in(\phi_2-\Psi_{\rm RP})} \right\rangle\!\right\rangle = \langle v_n^2 \rangle$$

- Estimating **higher order moments** v_n^k
- Behind the scene: Factorization of joint multivariate p.d.f.

$$f(\varphi_1,\ldots,\varphi_n) = f_{\varphi_1}(\varphi_1)\cdots f_{\varphi_n}(\varphi_n)$$

- If the measured azimuthal correlators have contribution only from flow correlations, factorization works exactly to all orders

Correlation techniques

- We have to correlate different particles, self-correlations are useless (yet dominant!) contribution in averages

$$\langle 2\rangle \equiv \langle \cos n(\phi_1-\phi_2)\rangle\ , \qquad \phi_1 \neq \phi_2$$

$$\langle 4\rangle \equiv \langle \cos n(\phi_1+\phi_2-\phi_3-\phi_4)\rangle\ , \qquad \phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_4$$

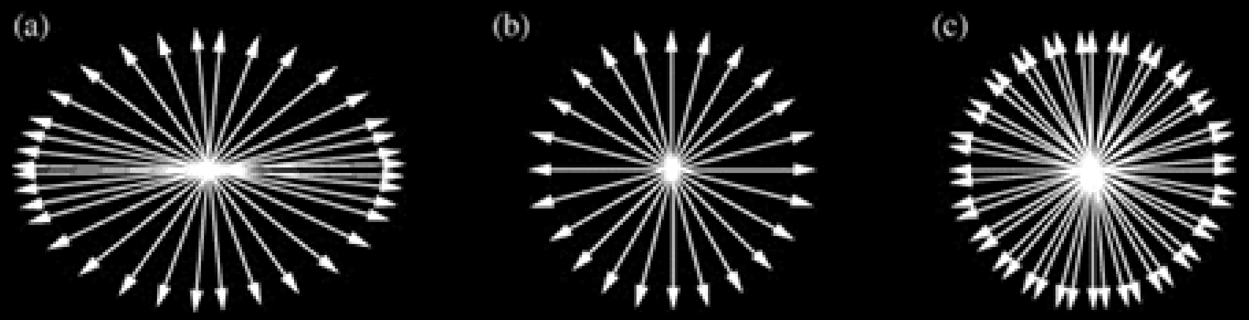
- Only isotropic correlators are non-trivial
- Analytic result:

$$\langle \cos(n_1\varphi_1+\cdots+n_k\varphi_k)\rangle = v_{n_1}\cdots v_{n_k}\cos(n_1\Psi_{n_1}+\cdots+n_k\Psi_{n_k})$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, PRC **84** 034910 (2011)

Nonflow

- ‘Direct correlations’, a.k.a. **nonflow**



Flow | No flow | Nonflow

- Nonflow: Typically all sources of correlations in momentum space among produced particles which ‘have nothing to do’ with the reaction plane orientation
  - Generally involve only a small subset of the produced particles
  - Factorization of underlying multivariate p.d.f. is broken

$$f(\varphi_1, \ldots, \varphi_n) \neq f_{\varphi_1}(\varphi_1) \cdots f_{\varphi_n}(\varphi_n)$$

Cumulants

- Concrete example: What are **$v_n\{2\}$** and **$v_n\{4\}$**?

$$\sqrt{c_n\{2\}} = \sqrt{\langle\langle 2\rangle\rangle} = v_n\,,$$
$$\sqrt[4]{-c_n\{4\}} = \sqrt[4]{-\langle\langle 4\rangle\rangle + 2\cdot\langle\langle 2\rangle\rangle^2} = \sqrt[4]{-v_n^4 + 2v_n^4} = v_n$$

- In an actual experiment due to nonflow and event-by-event flow fluctuations the above lines are not exact, therefore estimates of v_n from 2- and 4-particle cumulants will be **systematically different**
  - This systematic difference is indicated with separate notations:

$$v_n\{2\} \equiv \sqrt{c_n\{2\}}\,,$$
$$v_n\{4\} \equiv \sqrt[4]{-c_n\{4\}}$$

Generic case

- Generic definition of m-particle azimuthal correlation:

$$\langle m \rangle_{n_1,n_2,\ldots,n_m} = \frac{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1 \\ k_1\neq k_2\neq\ldots\neq k_m}}^{M} w_{k_1} w_{k_2} \cdots w_{k_m} \, e^{i(n_1\varphi_{k_1}+n_2\varphi_{k_2}+\cdots+n_m\varphi_{k_m})}}{\sum\limits_{\substack{k_1,k_2,\ldots,k_m=1 \\ k_1\neq k_2\neq\ldots\neq k_m}}^{M} w_{k_1} w_{k_2} \cdots w_{k_m}}$$

- We want all possible multiparticle correlations:
  - Measured exactly and efficiently free from autocorrelations
  - Corrected for various detector inefficiencies

Analytic solutions

- Analytic standalone formulas in terms of weighted *Q*-vectors

$$Q_{n,p} \equiv \sum_{k=1}^{M} w_k^p \, e^{in\varphi_k}$$

- Exact and efficient solutions for the problem of autocorrelations for the most general case
- Example: Generic solution for 4-particle azimuthal correlation

$$\begin{aligned}\mathrm{Num}\,\langle 4\rangle_{n_1,n_2,n_3,n_4} \;=\;& Q_{n_1,1}Q_{n_2,1}Q_{n_3,1}Q_{n_4,1} - Q_{n_1+n_2,2}Q_{n_3,1}Q_{n_4,1} - Q_{n_2,1}Q_{n_1+n_3,2}Q_{n_4,1}\\ &- Q_{n_1,1}Q_{n_2+n_3,2}Q_{n_4,1} + 2Q_{n_1+n_2+n_3,3}Q_{n_4,1} - Q_{n_2,1}Q_{n_3,1}Q_{n_1+n_4,2}\\ &+ Q_{n_2+n_3,2}Q_{n_1+n_4,2} - Q_{n_1,1}Q_{n_3,1}Q_{n_2+n_4,2} + Q_{n_1+n_3,2}Q_{n_2+n_4,2}\\ &+ 2Q_{n_3,1}Q_{n_1+n_2+n_4,3} - Q_{n_1,1}Q_{n_2,1}Q_{n_3+n_4,2} + Q_{n_1+n_2,2}Q_{n_3+n_4,2}\\ &+ 2Q_{n_2,1}Q_{n_1+n_3+n_4,3} + 2Q_{n_1,1}Q_{n_2+n_3+n_4,3} - 6Q_{n_1+n_2+n_3+n_4,4}\end{aligned}$$

AB, C. H. Christensen, K. Gulbrandsen, A. Hansen, Y. Zhou, Phys. Rev. C **89**, 064904 (2014)

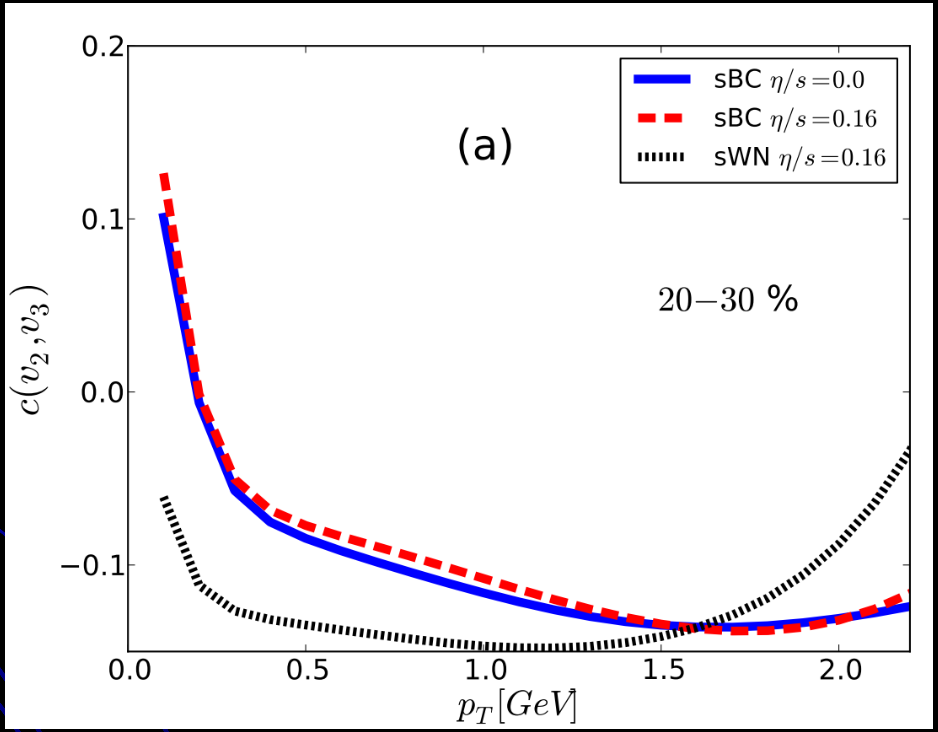
New flow observables: Symmetric Cumulants

New flow observables

- Fluctuations of individual flow harmonics and symmetry plane correlations have been measured
- **Next step:** What is the relationship between fluctuations of individual flow harmonics?
  - Not correlated?
  - Correlated?
  - Anti-correlated?
- **Two key questions:**
  - Are correlations of different harmonics more sensitive to system properties than the individual flow harmonics?
  - Can correlations of different harmonics disentangle the contribution of initial stages to flow from contributions of other stages?

New flow observables

- Niemi *et al* addressed the question of whether event-by-event fluctuations of v_n and v_m ($n \neq m$) are correlated or not



H. Niemi *et al*, Phys. Rev. C **87** (2013) no.5, 054901

Symmetric Cumulants (SC)

- How to quantify if event-by-event amplitude fluctuations of harmonics v_n and v_m ($n \neq m$) are correlated or not?
- **Step 1:** Mathematical prescription available to isolate cumulant from multi-particle correlator:

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle\end{aligned}$$

R. Kubo, Journal of the Physical Society of Japan, Vol. 17, No. 7 (1962)

- **Step 2:** Analytic result which relates first moment of multi-particle correlator to flow degrees of freedom:

$$\langle \cos(n_1 \varphi_1 + \cdots + n_k \varphi_k) \rangle = v_{n_1} \cdots v_{n_k} \cos(n_1 \Psi_{n_1} + \cdots + n_k \Psi_{n_k})$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **84** 034910 (2011)

Symmetric Cumulants (SC)

- **Step 3:** Generic framework to calculate analytically any multi-particle correlator with Q-vectors:
  - For any number of particles
  - For any choice of harmonics
  - Corrected for detector inefficiencies with particle weights

AB, C. H. Christensen, K. Gulbrandsen, A. Hansen, Y. Zhou, Phys. Rev. C **89**, 064904 (2014)

- **Step 4:** Out of the plethora of independent multi-particle correlators which are now available experimentally, select the ones to which a physical message can be attached

Symmetric Cumulants (SC)

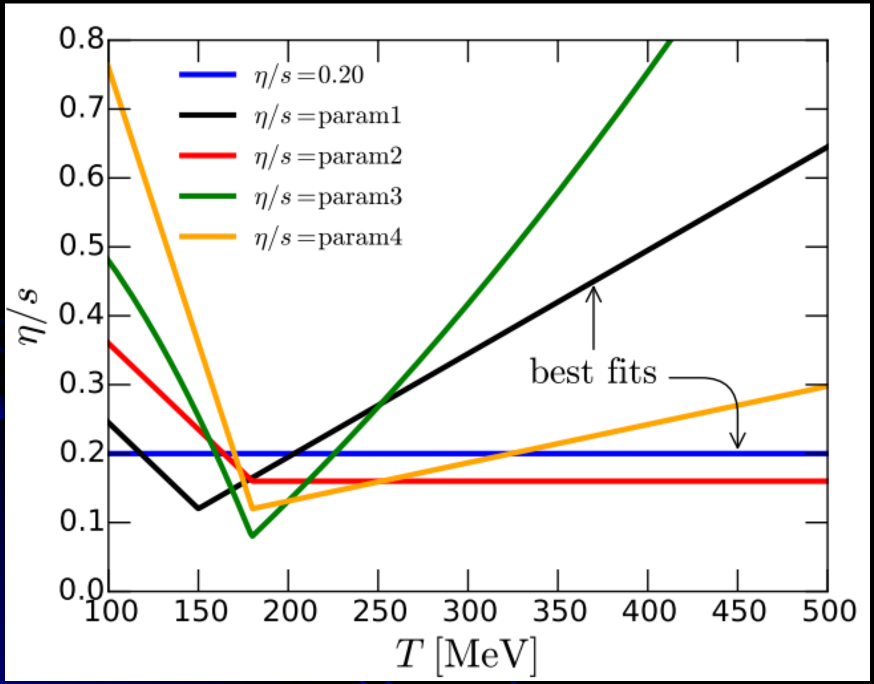
- Consider the following new 4-particle observable a.k.a. 'Symmetric Cumulant' **SC(*m*,*n*):**

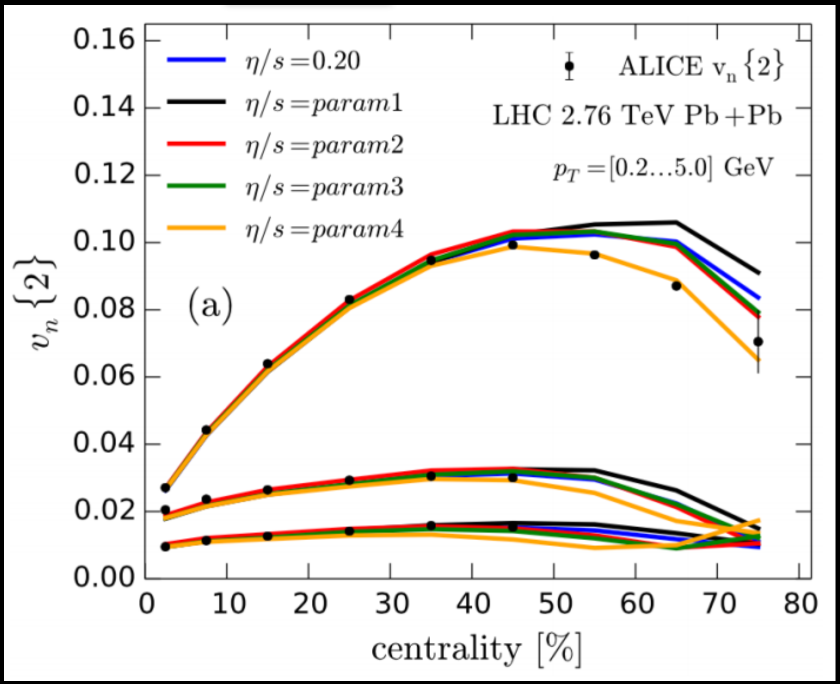
$$\begin{aligned}\langle\langle\cos(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)\rangle\rangle_c &= \langle\langle\cos(m\varphi_1+n\varphi_2-m\varphi_3-n\varphi_4)\rangle\rangle \\ &\quad -\langle\langle\cos[m(\varphi_1-\varphi_2)]\rangle\rangle\langle\langle\cos[n(\varphi_1-\varphi_2)]\rangle\rangle \\ &= \left\langle v_m^2 v_n^2\right\rangle - \left\langle v_m^2\right\rangle\left\langle v_n^2\right\rangle\end{aligned}$$

- If not zero, $\langle v_m^2 v_n^2\rangle$ cannot be factorized, i.e. e-b-e amplitude fluctuations of v_n and v_m are correlated
  Section IV C in AB, C. H. Christensen, K. Gulbrandsen, A. Hansen, Y. Zhou, Phys. Rev. C **89**, 064904 (2014)
- By construction not sensitive to:
  - Magnitudes of constant flow harmonics
  - Inter-correlations of various symmetry planes

η/s(T)

- Study of temperature dependence of transport coefficients has just begun
- This state-of-the-art model quantitatively describes the Run 1 LHC data
  - Example: Centrality dependence of individual flow harmonics v_n

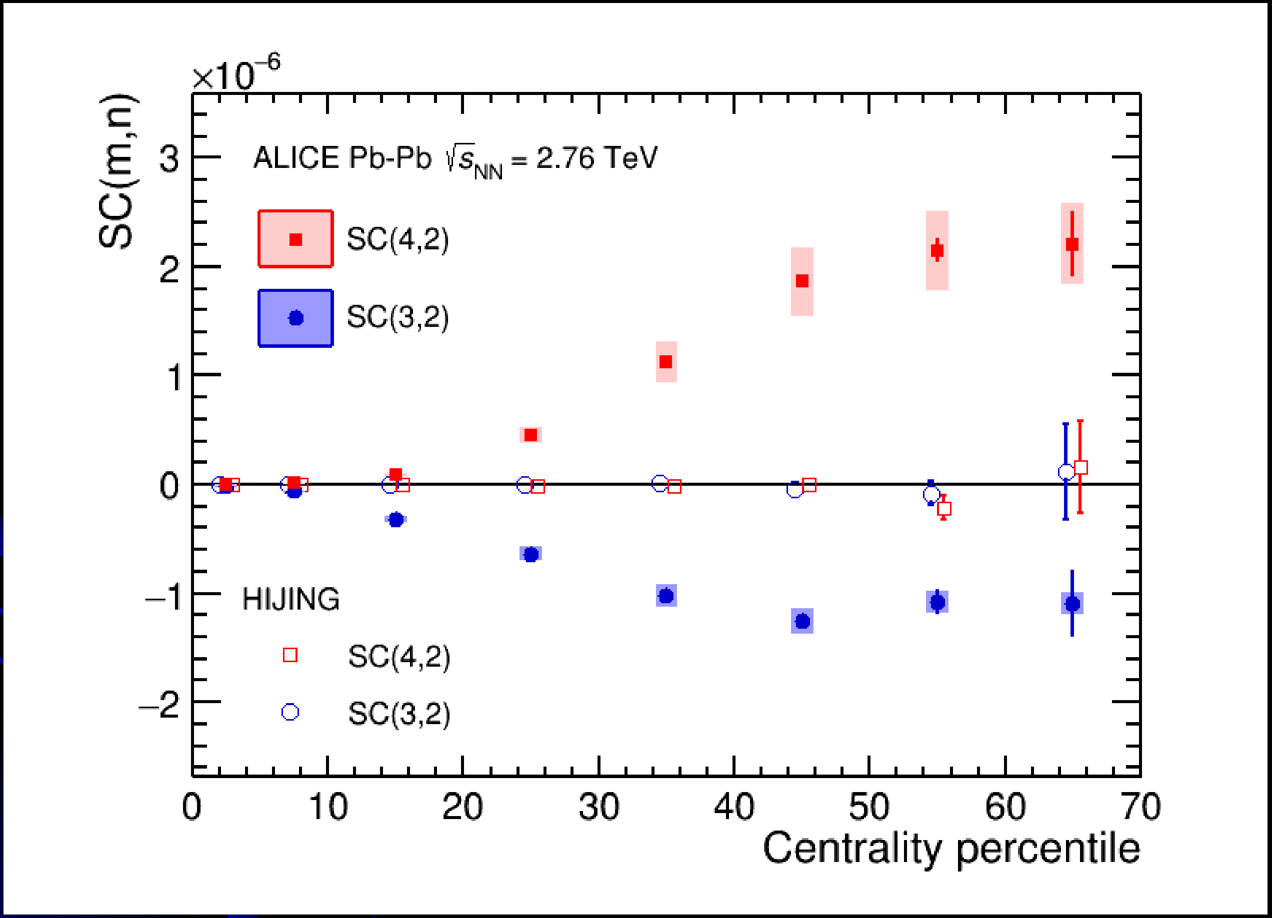




H. Niemi, K. J. Eskola, R. Paatelainen, Phys. Rev. C 93, 024907 (2016)

SC(3,2) and SC(4,2)

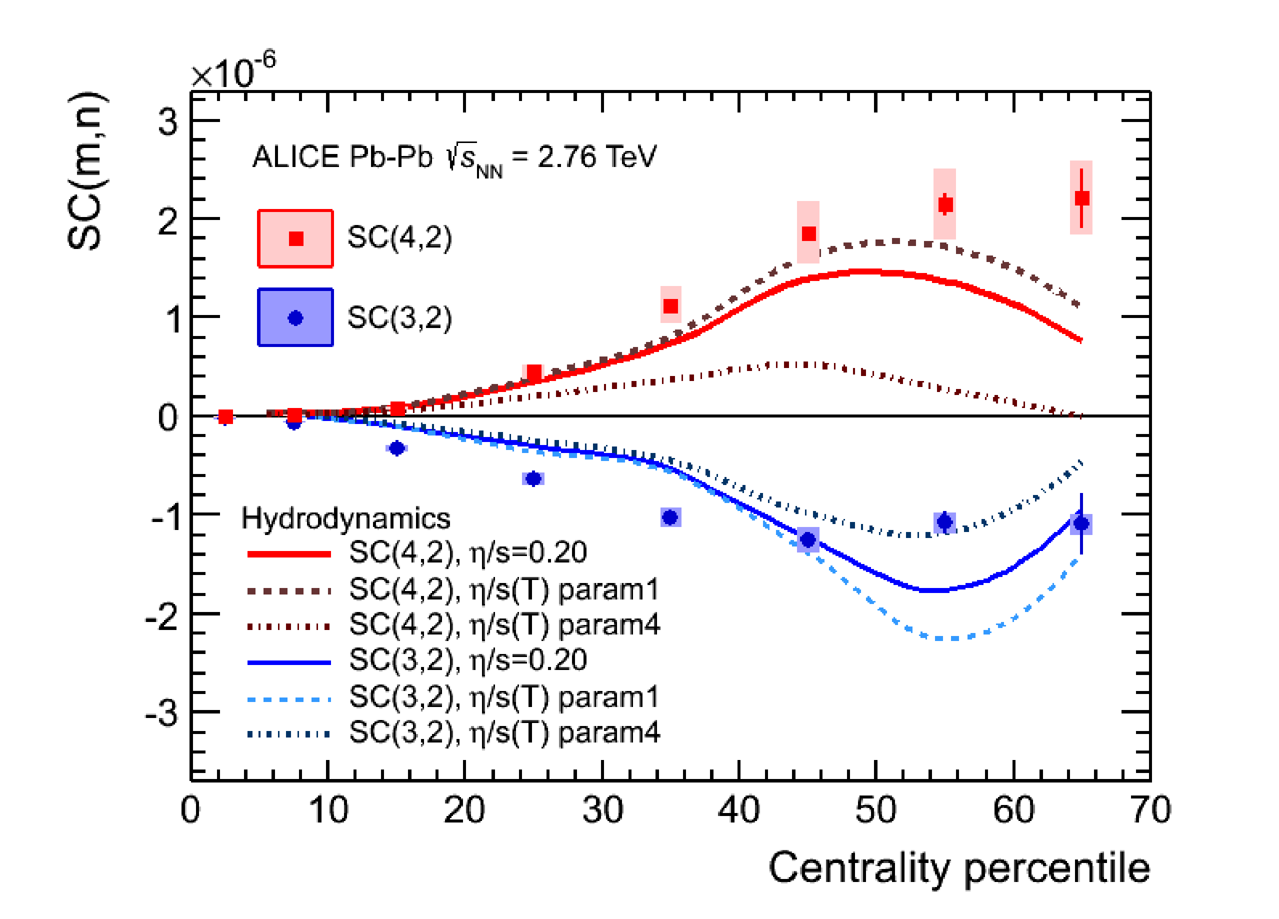
- Demonstrating robustness against nonflow



ALICE, arXiv:1604.07663, accepted by PRL

SC(3,2) and SC(4,2)

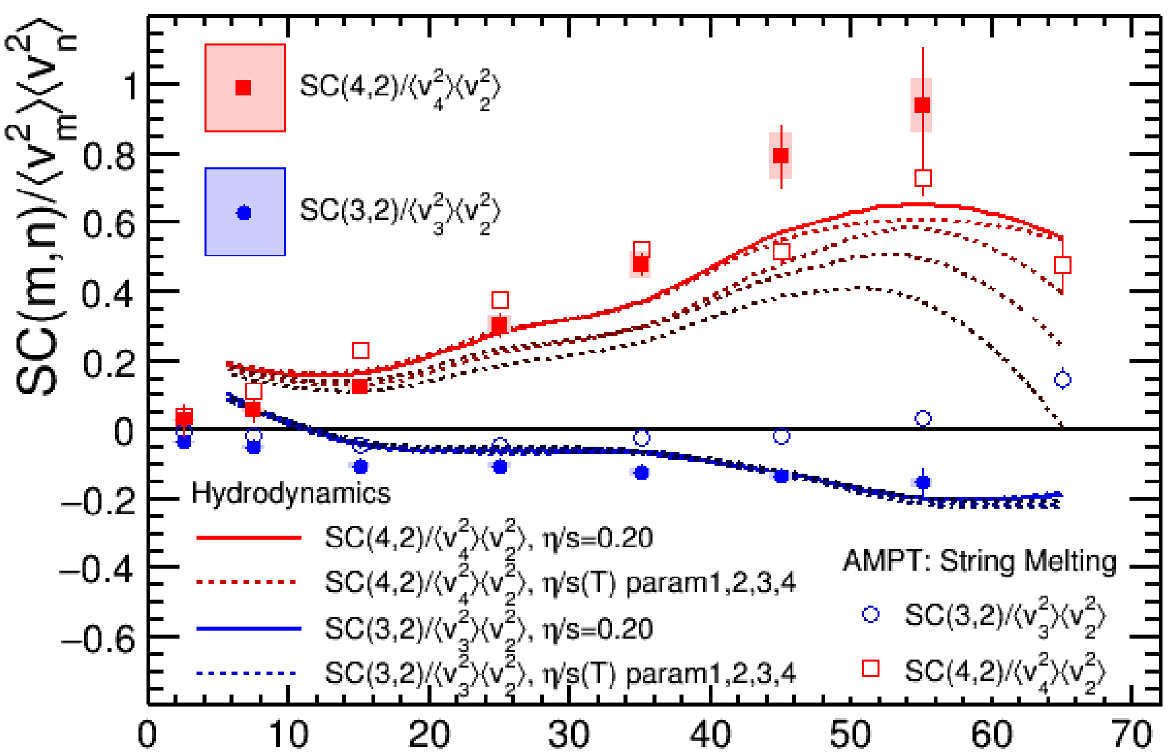
- Demonstrating sensitivity to different η/s(T) parametrizations



ALICE, arXiv:1604.07663, accepted by PRL

Normalized SC observables

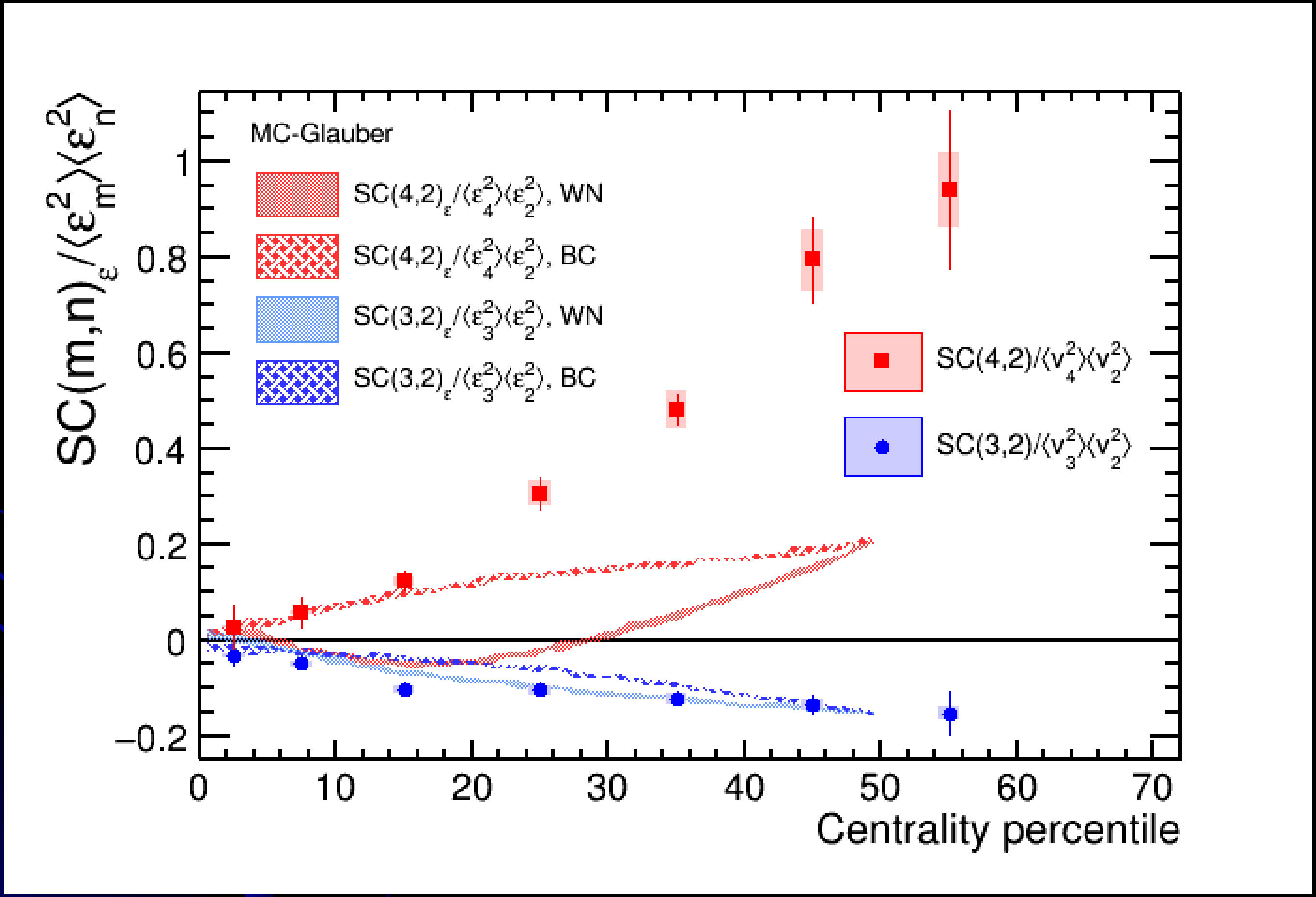
- Normalized SC(3,2) is sensitive mainly (only?) to initial conditions!!



ALICE, arXiv:1604.07663, accepted by PRL

Normalized SC observables

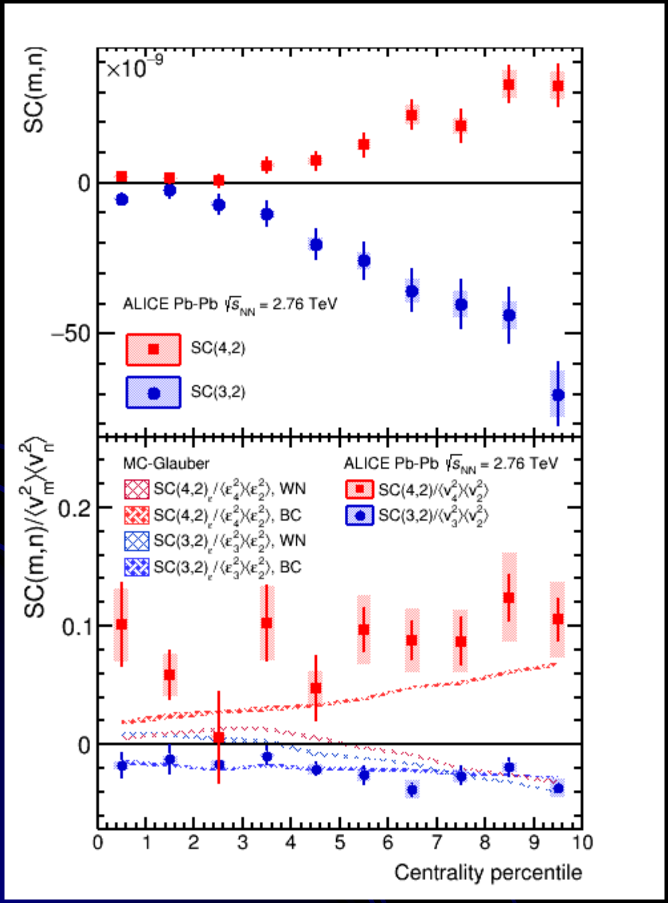
- Constraining the initial conditions with SC's in coordinate space:



ALICE, arXiv:1604.07663, accepted by PRL

SC in different regime

- Geometry-dominated regime (non-central collisions)
- Fluctuation-dominated regime (ultra-central collisions)



- The strength of the (anti)-correlations in ultra-central collisions exhibits a **different centrality dependence** than for the wider centrality range in non-central collisions

ALICE, arXiv:1604.07663, accepted by PRL

Unresolved problems

Sensitivity

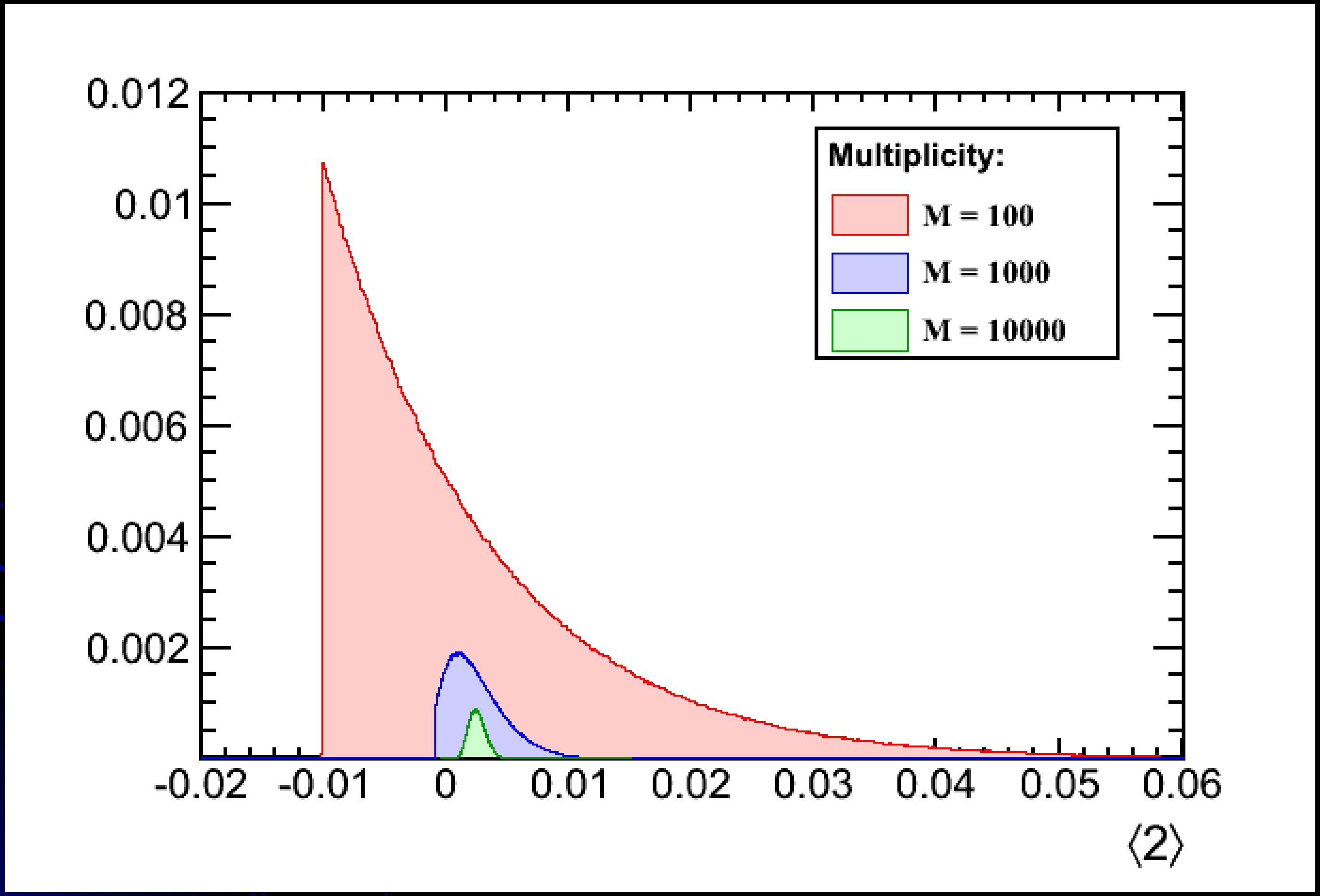
- When only flow correlations are present, and if flow harmonic v was estimated with k-particle correlator, for the data set having N events, each of which has M particles, to leading order:

$$\sigma_v \sim \frac{1}{\sqrt{N}}\frac{1}{M^{k/2}}\frac{1}{v^{k-1}}$$

- In the heavy-ion collisions with a large elliptic flow and large multiplicity, this scaling is a 'great news'
- **In collisions of small systems, use correlation techniques in flow analysis at your own peril**

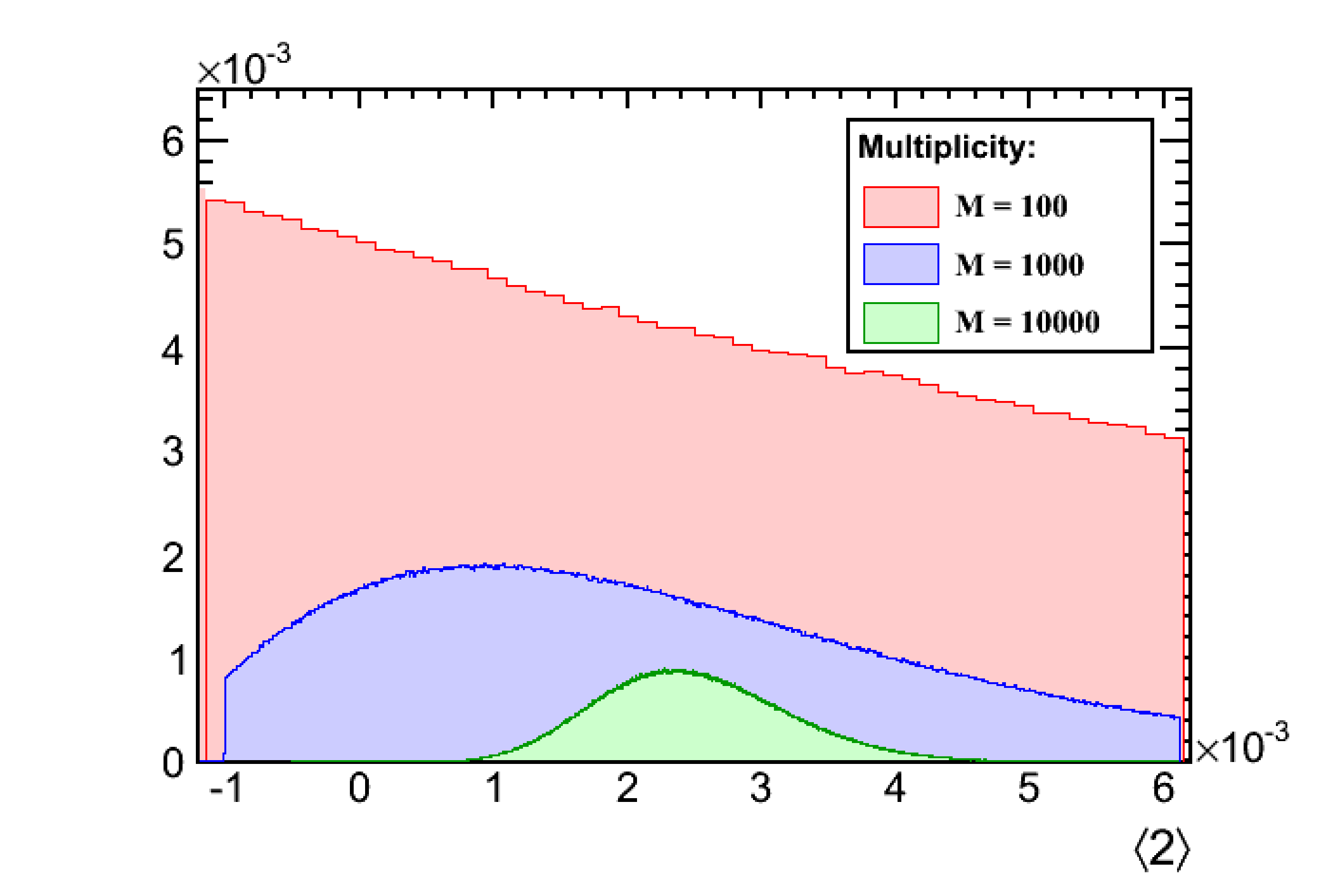
Resolution

- Fixed input $v_2 = 0.05$



Resolution

- All three distributions have mean value of $v_2{}^2 = 0.05^2 = 0.0025$
  - But for small M the resolution is just scary

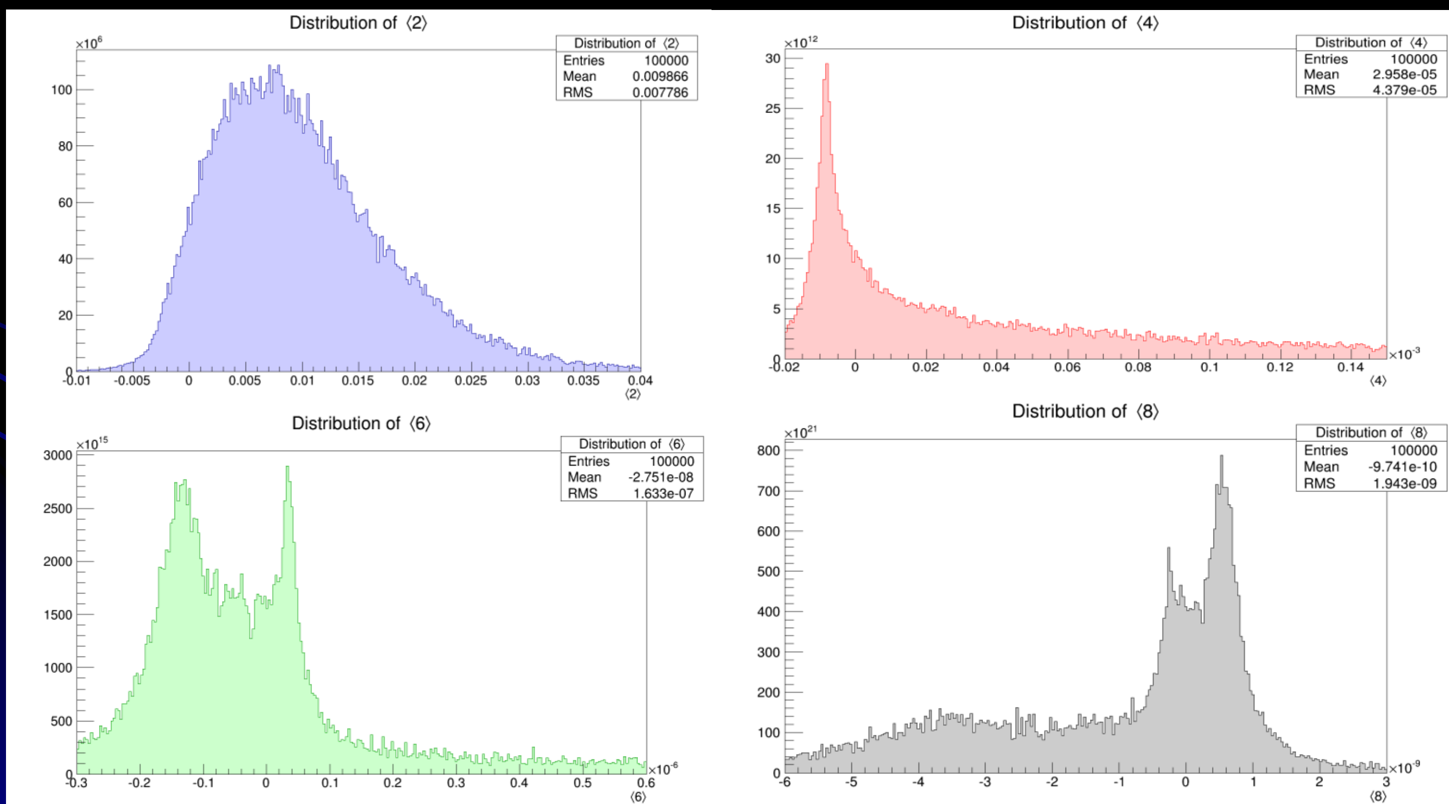


Higher order moments

- Can only first moments of correlations really tell us everything?

$$\langle\cos(n_1\varphi_1+\cdots+n_k\varphi_k)\rangle = v_{n_1}\cdots v_{n_k}\cos(n_1\Psi_{n_1}+\cdots+n_k\Psi_{n_k})$$

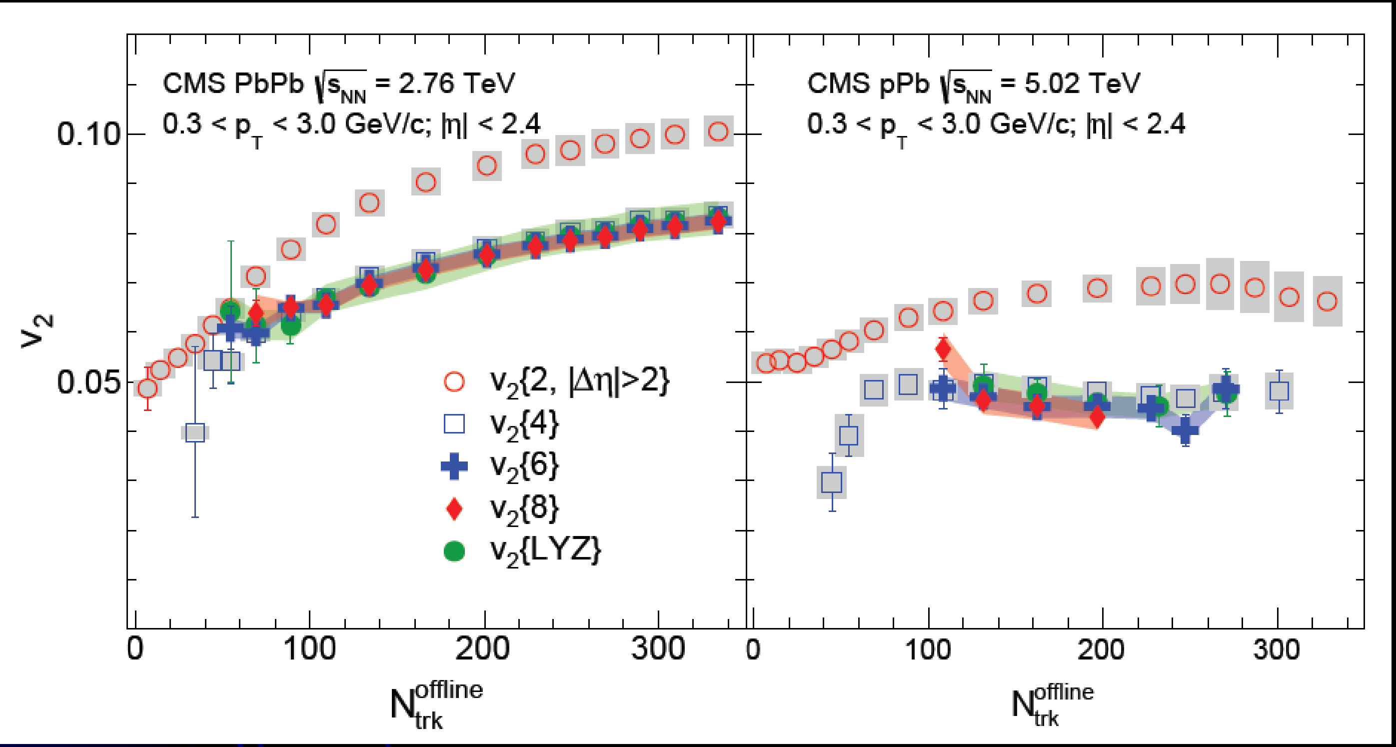
- Fixed input $v_2 = 0.10$



Attractor?

- If correlation techniques exhibit an attractor, then the two results below in Pb-Pb and p-Pb couldn't be more different when it comes to physics

CMS, Phys. Rev. Lett. 115 (1) (2015) 012301

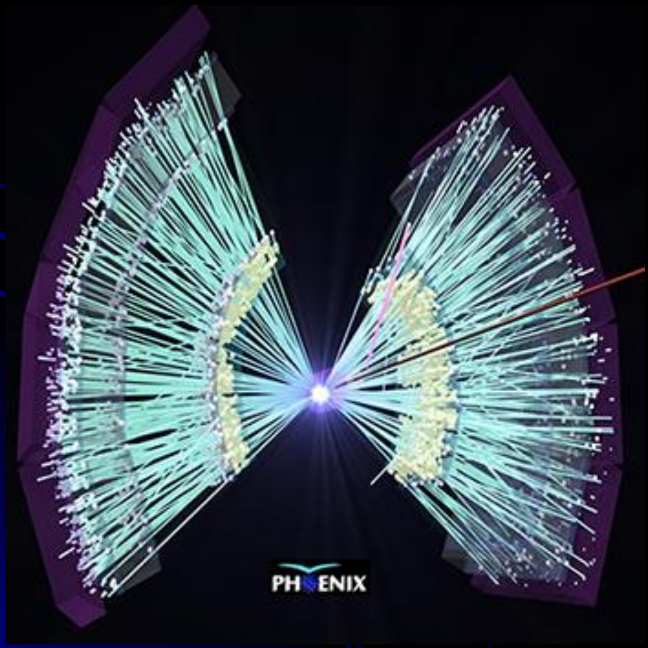


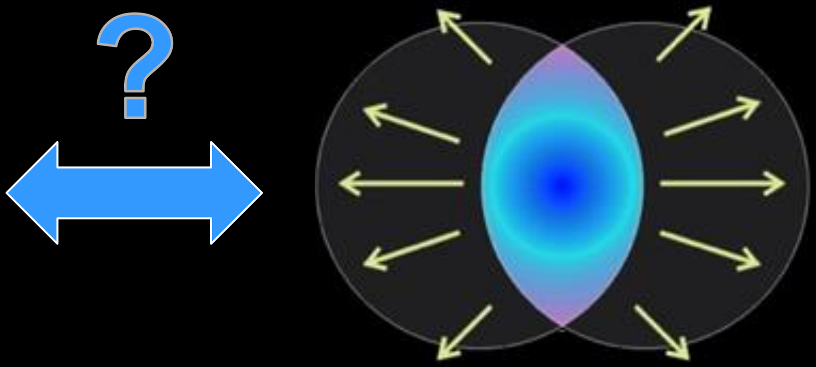
Most likely, the devil is in the detail (as always...)

AB, arXiv:1605.06160

Non-uniform acceptance

- If a detector has non-uniform acceptance in azimuthal angle, then in each event we have trivial anisotropies in momentum distributions of detected particles
  - Clearly this has nothing to do with anisotropic flow!
  - Can we disentangle ‘detector holes’ from flow anisotropy?





Non-uniform acceptance

- **Acceptance corrections** for cumulants, leading order:

$$
\begin{aligned}
c_n\{2\} &= \langle\langle 2\rangle\rangle - \left[\langle\langle \cos n\phi_1\rangle\rangle^2 + \langle\langle \sin n\phi_1\rangle\rangle^2\right] \\
c_n\{4\} &= \langle\langle 4\rangle\rangle - 2\cdot\langle\langle 2\rangle\rangle^2 - \\
&- 4\cdot\langle\langle \cos n\phi_1\rangle\rangle\,\langle\langle \cos n(\phi_1-\phi_2-\phi_3)\rangle\rangle \\
&+ 4\cdot\langle\langle \sin n\phi_1\rangle\rangle\,\langle\langle \sin n(\phi_1-\phi_2-\phi_3)\rangle\rangle \\
&- \langle\langle \cos n(\phi_1+\phi_2)\rangle\rangle^2 - \langle\langle \sin n(\phi_1+\phi_2)\rangle\rangle^2 \\
&+ 4\cdot\langle\langle \cos n(\phi_1+\phi_2)\rangle\rangle \\
&\times \left[\langle\langle \cos n\phi_1\rangle\rangle^2 - \langle\langle \sin n\phi_1\rangle\rangle^2\right] \\
&+ 8\cdot\langle\langle \sin n(\phi_1+\phi_2)\rangle\rangle\,\langle\langle \sin n\phi_1\rangle\rangle\,\langle\langle \cos n\phi_1\rangle\rangle \\
&+ 8\cdot\langle\langle \cos n(\phi_1-\phi_2)\rangle\rangle \\
&\times \left[\langle\langle \cos n\phi_1\rangle\rangle^2 + \langle\langle \sin n\phi_1\rangle\rangle^2\right] \\
&- 6\cdot\left[\langle\langle \cos n\phi_1\rangle\rangle^2 + \langle\langle \sin n\phi_1\rangle\rangle^2\right]^2
\end{aligned}
$$

Next

Coming next

- ‘Flow trigger’
  - Can we generalize Event Shape Engineering?

    J. Schukraft, A. Timmins and S. A. Voloshin, Phys. Lett. B 719 (2013) 394
- ‘Life beyond SC’
  - Can we generalize Symmetric Cumulants?
- ‘Genuine Symmetry Plane Correlations’
  - Results are around for some time, but...

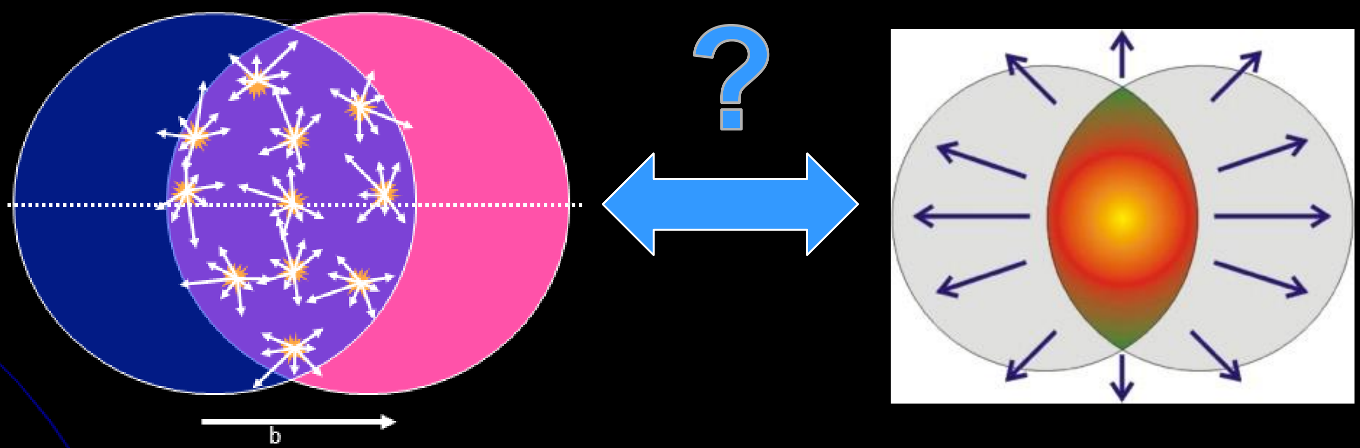
    G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. C **90** (2014) no.2, 024905
- Femtoscopy with multiparticle correlations

Thanks!

Backup slides

Transfer of anisotropy

- Two conceptually different notions of anisotropy:
  - **Coordinate space anisotropy:** Is the volume containing the interacting particles produced in a heavy-ion collision anisotropic or not?
  - **Momentum space anisotropy:** Is the final-state azimuthal distribution of resulting particles recorded in the detector anisotropic or not?
- **A priori** these two anisotropies are **unrelated**



made by Mike Lisa

Necessary and sufficient conditions

- What are the necessary and sufficient conditions for anisotropic flow development?
- Example **necessary** conditions:
  - Initial anisotropy in the interaction region of two colliding ions
  - Small shear viscosity of produced medium
  - ...
- The **sufficient** condition:
  - Anisotropic pressure gradients developed in a strongly interacting medium
- Anisotropic flow is sensitive both to the details of **initial stages** and to the details of **system properties**

New flow observables

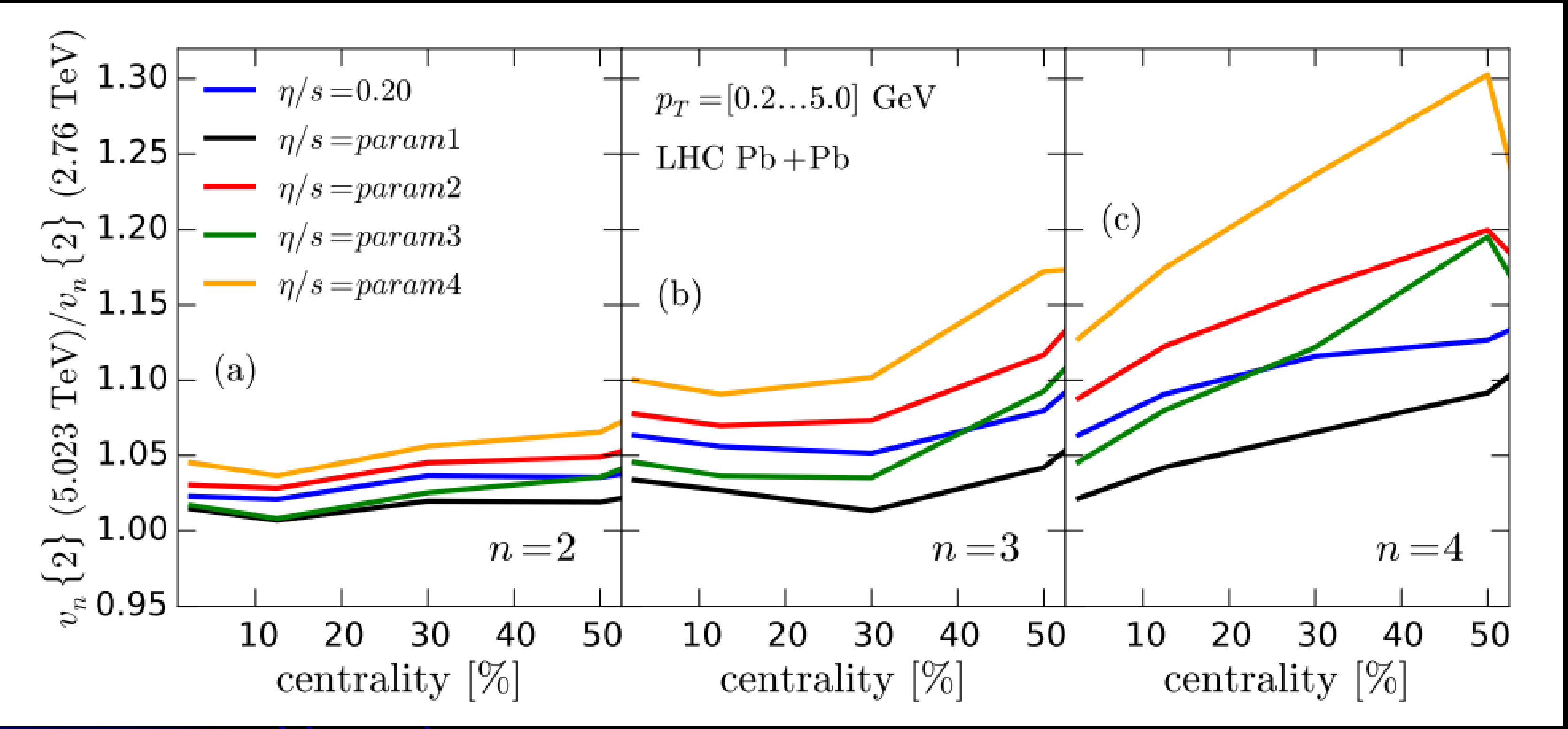
- Niemi *et al* have used **linear correlation coefficient** as an observable in their studies:

$$c(a,b) = \left\langle \frac{(a - \langle a \rangle_{\rm ev})(b - \langle b \rangle_{\rm ev})}{\sigma_a \sigma_b} \right\rangle_{\rm ev}$$

- Not accessible experimentally
- Accessible: Higher order flow moments obtained with correlation techniques

η/s(T): Run 1 *vs.* Run 2

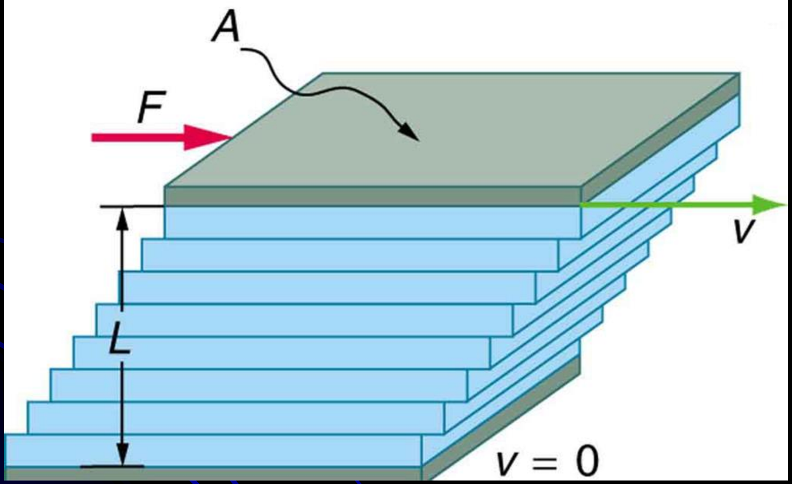
- Compared to the Run 1 LHC measurements, higher harmonics will show bigger and non-trivial increase as a function of centrality

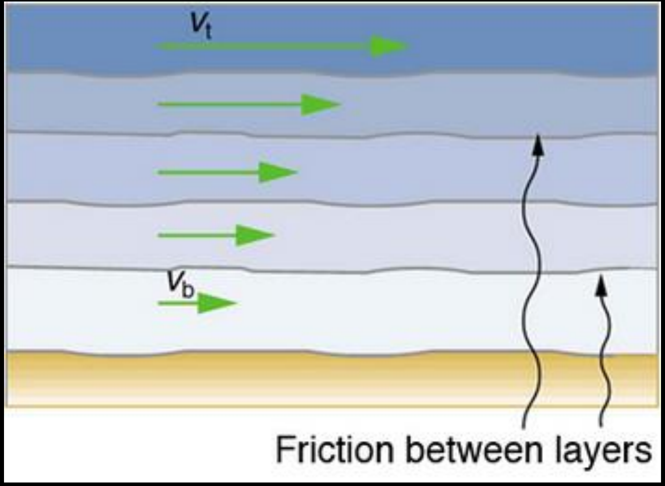


H. Niemi, K. J. Eskola, R. Paatelainen, and K. Tuominen, Phys. Rev. C **93**, 014912 (2016)

System properties

- By measuring event-by-event anisotropies in the resulting momentum distribution of detected particles, we can probe the properties of produced matter
- Example: **Shear viscosity**

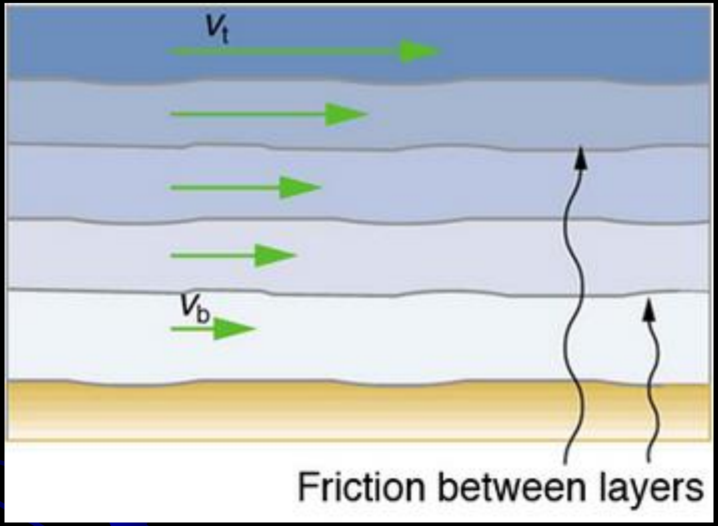


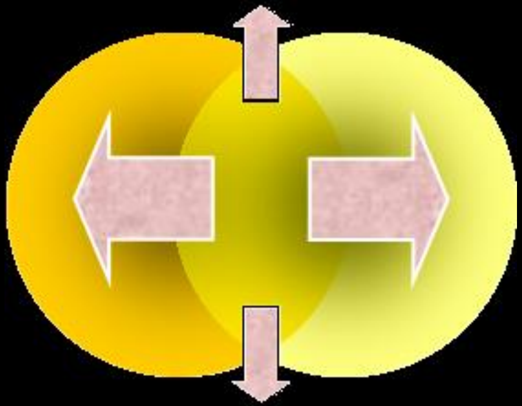


- Shear viscosity characterizes quantitatively the resistance of the liquid or gas to the parallel displacement of its neighbouring layers

Shear viscosity

- Shear viscosity 'fights' against anisotropic flow





- Perfect liquid ⇔ kinematic shear viscosity negligible ⇔ anisotropic flow develops easily
- The ratio of shear viscosity to entropy density (η/s) has a lower bound: **1/4π** (obtained in strong-coupling calculations based on the AdS/CFT conjecture)

P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 94 (2005) 111601, arXiv:hep-th/0405231

Correlations *vs.* Q-vectors

- Calculated 'by hand' all the way up to 8-p correlations:

$$
\begin{aligned}
\langle 8\rangle &\equiv \langle\cos(n(\phi_1+\phi_2+\phi_3+\phi_4-\phi_5-\phi_6-\phi_7-\phi_8))\rangle\\
&= \frac{1}{\binom{M}{8}8!}\sum_{\substack{i,j,k,l,m,n,o,p=1\\(i\neq j\neq k\neq l\neq m\neq n\neq o\neq p)}}^{M} e^{in(\phi_i+\phi_j+\phi_k+\phi_l-\phi_m-\phi_n-\phi_o-\phi_p)}\\
&= \frac{1}{\binom{M}{8}8!}\times\big[\,|Q_n|^8-12\cdot Q_{2n}Q_nQ_nQ_n^*Q_n^*Q_n^*Q_n^*\\
&\quad+6\cdot Q_{2n}Q_{2n}Q_n^*Q_n^*Q_n^*Q_n^*+16\cdot Q_{3n}Q_nQ_n^*Q_n^*Q_n^*Q_n^*\\
&\quad-96\cdot Q_{3n}Q_nQ_{2n}^*Q_n^*Q_n^*-12\cdot Q_{4n}Q_n^*Q_n^*Q_n^*Q_n^*\\
&\quad-36\cdot Q_{2n}Q_{2n}Q_{2n}^*Q_n^*Q_n^*+96(M-6)\cdot Q_{2n}Q_nQ_n^*Q_n^*Q_n^*\\
&\quad+72\cdot Q_{4n}Q_{2n}^*Q_n^*Q_n^*+48\cdot Q_{3n}Q_nQ_{2n}^*Q_{2n}^*\\
&\quad-64(M-6)\cdot Q_{3n}Q_n^*Q_n^*Q_n^*+192(M-6)\cdot Q_{3n}Q_{2n}^*Q_n^*\\
&\quad-96\cdot Q_{4n}Q_{3n}^*Q_n^*-36\cdot Q_{4n}Q_{2n}^*Q_{2n}^*\\
&\quad-144(M-7)(M-4)Q_{2n}Q_n^*Q_n^*+36\,|Q_{4n}|^2+64\,|Q_{3n}|^2\,|Q_n|^2\\
&\quad-64(M-6)\,|Q_{3n}|^2+9\,|Q_{2n}|^4+36\,|Q_n|^4\,|Q_{2n}|^2-144(M-6)\,|Q_{2n}|^2\,|Q_n|^2\\
&\quad+72(M-7)(M-4)(|Q_{2n}|^2+|Q_n|^4)-16(M-6)\,|Q_n|^6\\
&\quad-96(M-7)(M-6)(M-2)\,|Q_n|^2\\
&\quad+24M(M-7)(M-6)(M-5)\big]
\end{aligned}
$$