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Stabilization of the extra dimension size in RS model by bulk Higgs field

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Contents

The Randall-Sundrum model

We consider two branes with tension interacting with gravity in a five-dimensional space-time $E = M_4 \times S^1 / Z_2$

In this report the brane separation is assumed to be stabilized by a two-component complex scalar field. On “our” brane it will implement the Higgs mechanism of spontaneous symmetry breaking.

The question about the stabilization of the extra dimension size with the help of the bulk Higgs field was raised earlier in the papers:

- L. Vecchi, “A Natural Hierarchy and a low New Physics scale from a Bulk Higgs,”(2011);
- M. Geller, S. Bar-Shalom and A. Soni, “Higgs-radion unification: Radius stabilization by an SU(2) bulk doublet and the 126 GeV scalar,” (2014).

In the last paper a perturbative stabilizing solution without gauge fields was considered. We attempt to take the gauge fields into account and to find an exact solution.

The background equations

Let us consider the scalar field $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$. The action S of the model:

$$S = S_g + S_\phi + S_{gauge} + S_{brane+SM},$$

gravitational ← S_g; scalar field ← S_ϕ; gauge fields ← S_{gauge}; branes and the SM ← $S_{brane+SM}$

$$S_g = 2M^3 \int d^4x \int_{-L}^{L} dy R\sqrt{g};$$

$$S_\phi = M \int d^4x \int_{-L}^{L} dy \left[(D_M\phi)^+ D^M\phi - V(\phi^+\phi) \right]\sqrt{g};$$

$$S_{gauge} = -\int d^4x \int_{-L}^{L} dy \left[\frac{1}{4p^2} A^a_{MN} A^{a\,MN} + \frac{1}{4q^2} B_{MN} B^{MN} \right]\sqrt{g};$$

$$S_{brane+SM} = -\int_{y=0} d^4x \lambda_1 (\phi^+\phi)\sqrt{-\tilde{g}} + \int_{y=L} d^4x \left[-\lambda_2(\phi^+\phi) + L_{SM-HP}(\phi, \phi^+) \right]\sqrt{-\tilde{g}}.$$

The gauge field:

$SU(2) \to A_M$

$U(1) \to B_M$

A solution is sought in the form, which preserves the Poincare invariance in any four-dimensional subspace $y=const$.

The metric:

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2.$$

The scalar field: $\phi(x, y) = \phi(y).$

The gauge fields:

$$A_\mu(x, y) = 0, \qquad A_4(x, y) = A_4(y),$$
$$B_\mu(x, y) = 0, \qquad B_4(x, y) = B_4(y).$$

By the variation of the action we get the following equations of motion:

$$\frac{1}{2}\left(\phi'^{+}\phi + V + \frac{\lambda_1}{M}\delta(y) + \frac{\lambda_2}{M}\delta(y-L)\right) = 2M^2\left(3A'' - 6(A')^2\right),$$

$$12M^2(A')^2 + \frac{1}{2}\left(V - \phi'^{+}\phi'\right) = 0,$$

$$\frac{dV}{d\phi} + \frac{1}{M}\frac{d\lambda_1}{d\phi}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi}\delta(y-L) = \phi''^{+} - 4A'\phi'^{+},$$

$$\frac{dV}{d\phi^{+}} + \frac{1}{M}\frac{d\lambda_1}{d\phi^{+}}\delta(y) + \frac{1}{M}\frac{d\lambda_2}{d\phi^{+}}\delta(y-L) = \phi'' - 4A'\phi',$$

$$A_M = B_M = 0.$$

The equations for the field fluctuations

In order to build the linearized theory we represent the metric and the fields as follows:

$$g_{MN}(x,y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x,y),$$

$$\phi(x,y) = \phi_0(y) + f(x,y),$$

$$A_M^a(x,y) = 0 + a_M^a(x,y),$$

$$B_M(x,y) = 0 + b_M(x,y).$$

The equations of motion for the scalar fluctuations:

$$\frac{1}{4}\left(\partial_\mu\partial_\nu\tilde{h}-2\partial_\mu\partial_\nu h_{44}\right)-\frac{1}{4}\gamma_{\mu\nu}\left(\partial_\sigma\partial^\sigma\tilde{h}-2\partial_\sigma\partial^\sigma h_{44}-\frac{3}{2}\partial_4\partial_4\tilde{h}\right)-\frac{1}{2}\gamma_{\mu\nu}A'\left(4\partial_4\tilde{h}+3\partial_4 h_{44}\right)+$$

$$+\frac{1}{2}\gamma_{\mu\nu}(A')^2\left(12h_{44}+\tilde{h}\right)-\frac{1}{4}\gamma_{\mu\nu}A''\left(\tilde{h}+6h_{44}\right)+\frac{1}{\sqrt{2M}}\gamma_{\mu\nu}\left[\phi_0'^{+}f'+\left(\phi''^{+}-4A'\phi'^{+}\right)f\right]=0,$$

$$\frac{3}{4}\partial_4\partial_\mu\tilde{h}-3A'\partial_\mu h_{44}+\sqrt{\frac{2}{M}}\phi_0'^{+}\partial_\mu f=0,$$

$$\frac{3}{4}\partial_\mu\partial^\mu\tilde{h}+3A'\partial_4\tilde{h}+\frac{1}{2M^2}Vh_{44}+\sqrt{\frac{2}{M}}\left[\phi_0'^{+}f'-\frac{1}{2}\left(\frac{dV}{d\phi}f+f^{+}\frac{dV}{d\phi^{+}}\right)\right]=0,$$

$$M\left(\partial_M\partial^M f+4A'f'+\frac{d^2V}{d\phi^{+}d\phi}f+f^{+}\frac{d^2V}{\left(d\phi^{+}\right)^2}\right)-\frac{1}{\sqrt{2M}}\left[\frac{1}{2}\phi_0'\left(\partial_4\tilde{h}+\partial_4 h_{44}\right)+\left(\phi_0''-4A'\phi_0'\right)h_{44}\right]+$$

$$+\left(\frac{d^2\lambda_1}{d\phi^{+}d\phi}f+f^{+}\frac{d^2\lambda_1}{\left(d\phi^{+}\right)^2}\right)\delta(y)+\left(\frac{d^2\lambda_2}{d\phi^{+}d\phi}f+f^{+}\frac{d^2\lambda_2}{\left(d\phi^{+}\right)^2}\right)\delta(y-L)+$$

$$+\frac{1}{2\sqrt{2M^3}}\left[\frac{d\lambda_1}{d\phi^{+}}\delta(y)+\frac{d\lambda_2}{d\phi^{+}}\delta(y-L)\right]h_{44}=0.$$

And the h.c. equation

The radion-like interactions of the Higgs boson

Here the Higgs boson = the radion. Consequently, it has the interaction with the energy-momentum tensor untypical for the usual Higgs boson:

$$S \supset -\frac{1}{\sqrt{8M^3}}\int dx\int_{-L}^{L} dy T^{MN} h_{MN}\sqrt{\gamma} = -\frac{1}{\sqrt{8M^3}}\int dx\int_{-L}^{L} dy\left(\frac{1}{2}T_{\mu}^{\mu} + T_{44}\right)e^{-2A}g$$

where $g = e^{-2A}h_{44}.$

Let us consider the vector field Z_M:

$$L_Z = -\frac{1}{4} Z_{MN} Z^{MN} + \frac{1}{2}\phi_0^2 Z_M Z^M .$$

The mode decomposition (if Z_4=0):

$$Z_\mu(x,y) = \sum_n z_\mu^n(x) Z_n(y).$$

Consequently, the interaction term:

$$S \supset \frac{1}{\sqrt{8M^3}} \sum_n \int dx \int_{-L}^{L} dy \left(\frac{1}{4}\eta^{\rho\sigma} z_{\mu\rho}^n(x) z_{\nu\sigma}^n(x) - z_\mu^n(x) \sum_m z_\nu^m(x) B_{nm} + m_n^2 z_\mu^n(x) z_\nu^n(x) \right) \eta^{\mu\nu} e^{-2A} g Z_n^2(y)$$

$$B_{nm} = \int_{-L}^{L} dy e^{-2A} \phi_0^2(y) Z_n(y) Z_m(y)$$

The KK number non-conservation!

Specific example of an exact background solution

Let us choose an ansatz:

$$V = \frac{1}{4}\frac{dW}{d\phi}\frac{dW}{d\phi^{+}} - \frac{1}{24M^{2}}\left(W(\phi^{+}\phi)\right)^{2},$$

$$\phi'(y) = sign(y)\frac{1}{2}\frac{dW}{d\phi^{+}}, \qquad \phi'^{+}(y) = sign(y)\frac{1}{2}\frac{dW}{d\phi},$$

$$A'(y) = sign(y)\frac{1}{24M^{2}}W(\phi^{+}\phi), \qquad W = 24M^{2}k - 2u\phi^{+}\phi.$$

The brane potentials:

$$\lambda_{1}(\phi^{+}\phi) = MW(\phi^{+}\phi) + \beta_{1}\left(\phi^{+}\phi - \frac{\upsilon_{1}^{2}}{2}\right)^{2},$$

$$\lambda_{2}(\phi^{+}\phi) = -MW(\phi^{+}\phi) + \beta_{2}\left(\phi^{+}\phi - \frac{\upsilon_{2}^{2}}{2}\right)^{2}.$$

A Higgs-like potential

We finally get:

$$\phi(y) = \begin{pmatrix} 0 \\ \frac{\upsilon}{\sqrt{2}} e^{-u(|y|-L)} \end{pmatrix}, \quad \upsilon = 246\,GeV,$$

$$A(y) = k(|y|-L) + \frac{\upsilon^2}{96M^2}\left(e^{-2u(|y|-L)} - 1\right).$$

The interbrane distance is defined by the boundary conditions for the scalar field and is expressed in terms of the parameters of the model by the relation:

$$L = \frac{1}{u}\ln\left(\frac{\upsilon_1}{\upsilon}\right),$$

so we have the size of the extra dimension stabilized.

An estimate of the model parameters

After the mode decomposition of g we get the equation in the Sturm-Liouville form:

$$\frac{d}{dy}\left(\frac{e^{2A}}{(\phi_2')^2}g_n'\right)-\frac{e^{2A}}{6M^2}g_n=-\mu_n^2 g_n\frac{e^{4A}}{(\phi_2')^2},\qquad y\in(0,L)$$

and the boundary conditions on the branes:

$$\left.\left(\frac{1}{4M}\frac{d^2\lambda_1}{d\phi_2^2}-\frac{\phi_2''}{\phi_2'}\right)g_n'+\mu_n^2 e^{2A}g_n\right|_{y=+0}=0,$$

$$\left.\left(\frac{1}{4M}\frac{d^2\lambda_2}{d\phi_2^2}+\frac{\phi_2''}{\phi_2'}\right)g_n'-\mu_n^2 e^{2A}g_n\right|_{y=L-0}=0.$$

In case $uL<<1$ for the lowest excitation of the scalar field identified with the Higgs boson we have

$$m_H^2 = \frac{\upsilon^2 u^2}{3M^2} \frac{\beta_2 \upsilon^2 - uM}{\beta_2 \upsilon^2 + uk}.$$

If we choose $M = 2TeV$ and $\beta_2 \to \infty$ we get the model parameters as follows:

$$u \approx 1.76 TeV, \qquad \phi_1 = 345 TeV,$$

$$k \approx 186 TeV, \qquad L = 0.2 TeV^{-1} \approx 2 \cdot 10^{-18} cm.$$

The coupling of the Higgs boson to the trace of the SM energy-momentum tensor:

$$L \supset \varepsilon_H g_0 T_\mu^{\ \mu}, \qquad \varepsilon_H = -\sqrt{\frac{k}{24M^3}} \sim 1 TeV^{-1}.$$

Conclusion

- The stabilization of the interbrane distance in the RS model and the spontaneous symmetry breaking on "our" brane are explained simultaneously with help of the bulk Higgs field.
- The background equations of motion are found and it is shown that the bulk gauge fields vanish.
- The second variation Lagrangian is derived and the equations of motion for the field fluctuations are obtained. The scalar fluctuations of the gauge fields vanish.

- The Higgs boson is the radion at the same time, so it now has an interaction with the energy-momentum tensor. This interactions with the bulk fields are studied and it is shown that the KK number conservation is violated.
- The specific example of the exact stabilizing vacuum solution is found.
- Based on it, the possible values of the model parameters are estimated, which give the correct value of the Higgs boson mass.

Thank you for your attention!