Generation of strong magnetic fields in quark stars driven by the electroweak interaction of quarks

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Outline of the talk

- Basic properties of compact stars: hybrid star (HS) & quark star (QS).
- Problem of the generation of strong large-scale magnetic fields in compact stars: magnetars.
- Main features of our model for the generation of strong magnetic fields in magnetars.
- Parity violating electroweak interaction between quarks.
- Chiral magnetic effect (CME) in the presence of the parity violating interaction.
- Kinetic equations for the spectra of the densities of the magnetic helicity and the magnetic energy, as well as for the chiral imbalance.
- Generation of strong large-scale magnetic fields in dense quark matter of HS/QS driven by the electroweak interaction.
- Summary.

References

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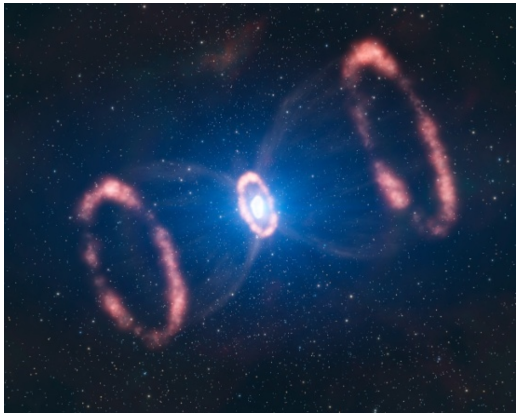
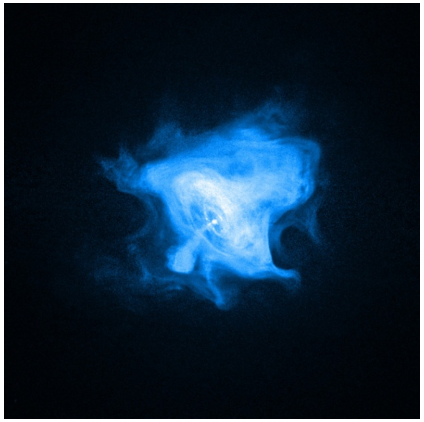
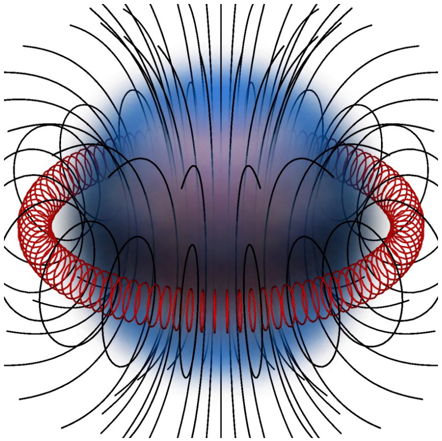
Compact stars and their magnetic fields

- There is a possibility for the formation of quark matter in compact stars.
- Quark matter can exist in the core of a neutron star. In this case the object is called a hybrid star (HS).
- If the strange matter hypothesis by Witten (1984) is valid, the existence of a star entirely composed of quark matter is possible. It is called a quark star (QS) or a strange star.
- HS/QS has the following characteristics: $R_{\rm STAR} = 10$ km, $M_{\rm STAR} = (1.4 - 2)M_{\odot}$, $n = 10^{38}$ cm^{-3}. It consists mainly of u and d quarks with some admixture of s quarks.
- Up to now, no confirmed astronomical observation of HS/QS have been being known.
- Compact star can possess rather strong magnetic fields, ranging from 10^8 G, for old pulsars, to 10^{12} G for young pulsars.
- The origin of such a strong magnetic field $B_0 = 10^{12}$ G can be explained by the magnetic flux conservation: $B_0 = B_{\rm PROTO}(R_{\rm PROTO}/R_{\rm STAR})^2 = 10^{12}$ G, where $B_{\rm PROTO} = (1\text{-}10^2)$ G, $R_{\rm PROTO} \sim R_{\odot} = 7 \times 10^{10}$ cm, $R_{\rm STAR} = 10$ km.

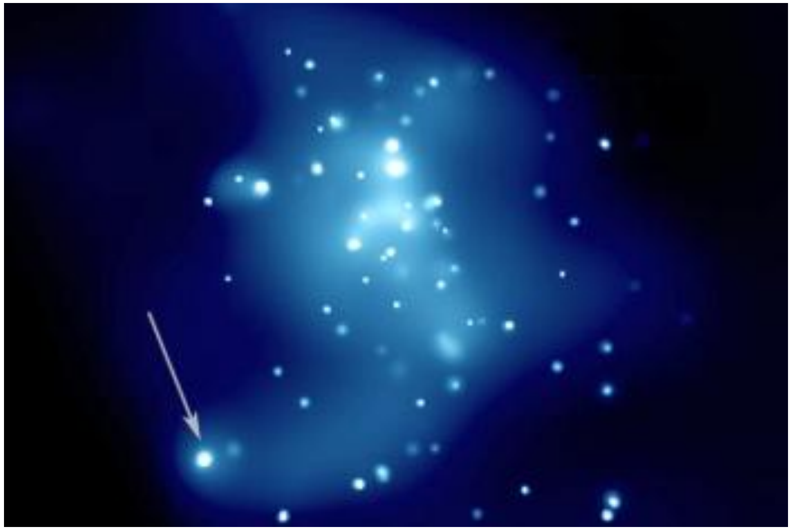


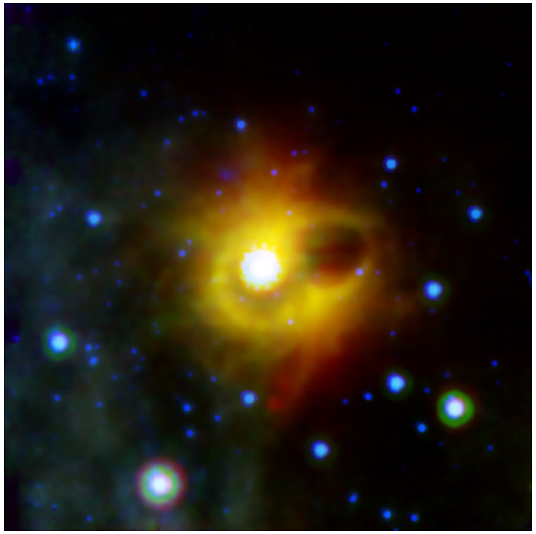




Magnetars

- Anomalous X-ray pulsars and soft gamma-ray repeaters, discovered by Mazets et al. (1979) and Fahlman & Gregory (1981), are supposed to be highly magnetized compact stars with $B > 10^{15}$ G.
- There are several tens of confirmed known magnetars (Mereghetti et al., 2015).
- Woods (2008) showed that number of magnetars should be comparable with that of pulsars.
- Boldin & Popov (2012) claim that there are hidden magnetars, i.e. strong magnetic field exists inside compact stars.
- There are numerous models, such as the turbulent α - Ω dynamo (Duncan & Thompson, 1992), the strong fossil field (Vink & Kuiper, 2006) etc., how to amplify the magnetic field from $B_0 = 10^{12}$ G to $B = 10^{15}$ G.
- Nevertheless the origin of strong magnetic fields in magnetars is still unknown.

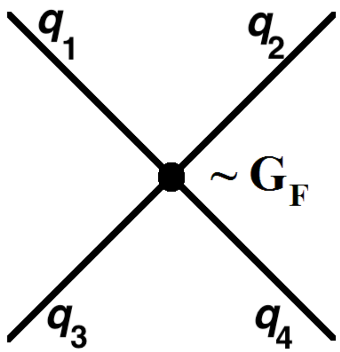




Features of our model for the generation of magnetic fields in magnetars

- Quark matter in HS/QS is highly degenerate: $\mu_u = 239$ MeV and $\mu_d = 301$ MeV
- Dexheimer & Schramm (2010) predict that the chiral phase transition can happen in HS/QS. It means that light u and d quarks become effectively massless.
- It is known that CME can take place when the chiral symmetry is restored.
- Quarks can interact between themselves by the electroweak forces which violate the parity.
- We find the correction to the CME owing to the electroweak interaction of quarks.
- This correction leads to the magnetic field instability, which, in its turn, results in the field growth.
- One can expect the growth of a seed field $B_0 = 10^{12}$ G, typical for young pulsars, to the strengths predicted in magnetars.

Electroweak interaction of electrons with nucleons



We shall describe the electroweak interaction between quarks in the HS/QS matter using the the Fermi approximation.

The amplitude of the *qq* forward scattering involves the Fermi constant $G_F = 1.17 \times 10^{-5}$ GeV^{-2}.

We suppose that HS/QS consists of *u* and *d* quarks. The contribution of *s* quarks, which are massive, can be neglected.

The effective Lagrangian of the interaction between macroscopically nonmoving and unpolarized *u* and *d* quarks reads

$$\mathcal{L} = -\sum_{q=u,d} \bar{q}\left(\gamma_0^L V_{qL} + \gamma_0^R V_{qR}\right) q, \quad \gamma_0^{L,R} = \frac{\gamma_0}{2}\left(1 \mp \gamma^5\right), \quad \xi = \sin^2\theta_W \approx 0.23$$

$$V_{uL} = -\frac{G_F}{\sqrt{2}} n_d \left(1 - \frac{8}{3}\xi + \frac{16}{9}\xi^2 - 2\left|V_{ud}\right|^2\right), \quad V_{uR} = \frac{G_F}{\sqrt{2}} n_d \left(\frac{4}{3}\xi - \frac{16}{9}\xi^2\right),$$

$$V_{dL} = -\frac{G_F}{\sqrt{2}} n_u \left(1 - \frac{10}{3}\xi + \frac{16}{9}\xi^2 - 2\left|V_{ud}\right|^2\right), \quad V_{dR} = \frac{G_F}{\sqrt{2}} n_u \left(\frac{2}{3}\xi - \frac{16}{9}\xi^2\right),$$

Here $V_{ud} = 0.97$ is the element of the Cabbibo- Kobayashi-Maskawa matrix.

Exact solution of the Dirac equation for an electron in magnetic field interacting with electroweak background matter

We shall assume that NS consists mainly of neutrons, i.e. $n_n >> n_p$.

The Dirac equation for an ultrarelativistic electron in electroweak matter under the influence of an external magnetic field $\mathbf{B} = B\,\mathbf{e}_z$, corresponding to $A^\mu = (0,0,Bx,0)$

$$\left[\gamma^\mu\left(i\partial_\mu + eA_\mu\right) - \gamma^0\left(V_L P_L + V_R P_R\right)\right]\psi_e = 0, \quad V_L = \frac{G_F}{\sqrt{2}} n_n\left(1 - 2\sin^2\theta_W\right), \quad V_R = -\frac{G_F}{\sqrt{2}} 2n_n \sin^2\theta_W$$

We solution of this equation can be found by decomposing the electron wave function into left and right chiral projections

$$\psi_e = \psi_L + \psi_R, \; P_{L,R}\psi_{L,R} = \psi_{L,R}, \; \psi_{L,R} = \exp\left(-iE_{L,R}t + ip_y y + ip_z z\right)\psi_{L,R}(x)$$

The basis spinors and the energy levels are

$$\psi_{L,R}^{(n>0)}(x) = \frac{1}{4\pi\sqrt{E_{L,R} - V_{L,R}}}\begin{pmatrix} \sqrt{E_{L,R} - V_{L,R} \mp p_z}\, u_{n-1} \\ \mp i\sqrt{E_{L,R} - V_{L,R} \pm p_z}\, u_n \\ \mp\sqrt{E_{L,R} - V_{L,R} \mp p_z}\, u_{n-1} \\ i\sqrt{E_{L,R} - V_{L,R} \pm p_z}\, u_n \end{pmatrix}, \quad \psi_{L,R}^{(0)}(x) = \frac{1}{2\pi\sqrt{2}}\begin{pmatrix} 0 \\ u_0 \\ 0 \\ \mp u_0 \end{pmatrix}$$

Here $u_n(\eta)$ is the Hermite function and $\eta = \sqrt{eB}x + \frac{p_y}{\sqrt{eB}}$

$$\left(E_{L,R} - V_{L,R}\right)^2 = p_z^2 + 2eBn$$

Chiral magnetic effect in the presence of the electroweakly interacting matter

One can exactly solve the Dirac equation for a massless quark electroweakly interacting with quark matter under the influence of the external magnetic field $\mathbf{B} = B\,\mathbf{e}_z$. Using the solution of this Dirac equation, one can compute the electric current along the magnetic field

$$J_z^{(L,R)} = e_q \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} dp_y \int dp_z \frac{\bar{\psi}_{L,R}\gamma^3\psi_{L,R}}{\exp\left[(E_{qL,R} - \mu_{qL,R})/T\right]+1} + \bar{q}\,\text{contribution}, \quad \psi_q = \psi_L + \psi_R$$

Only quarks at zero Landau level contribute to this current. At zero Landau level, n = 0, $p_z > 0$ for left quarks and antiquarks, $p_z < 0$ for right quarks and antiquarks. At n > 0, $-\infty < p_z < +\infty$. In vector notations the current has the form,

$$\mathbf{J} = \Pi\mathbf{B}, \quad \Pi = \frac{1}{2\pi^2}\sum_{q=u,d} e_q^2\left(\mu_{q5} + V_{q5}\right), \quad \mu_{q5} = \frac{1}{2}\left(\mu_{qR} - \mu_{qL}\right), \quad V_{q5} = \frac{1}{2}\left(V_{qL} - V_{qR}\right) \sim G_F n_q$$

The term in the current, $\sim \mu_{q5}$ reproduces CME. The term $\sim V_{q5}$ is the correction to CME due to electroweak electron-neutron interaction.

Magnetic field instability in presence of the nonzero parameter Π

The Faraday equation results from the Maxwell equations in the MHD approximation ($\sigma_{\text{cond}} >> \omega$). This equation has an unstable solution

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\Pi}{\sigma_{cond}}\left(\nabla \times \mathbf{B}\right) + \frac{1}{\sigma_{cond}}\nabla^2 \mathbf{B}, \quad B(k,t) = B_0 \exp\left[\int_{t_0}^{t}\left(\left|\Pi\right| k - k^2\right)\frac{dt'}{\sigma_{cond}}\right]$$

We define the densities of the spectra of the magnetic helicity and the magnetic energy

$$h(t) = \frac{1}{V}\int d^3x(\mathbf{A}\cdot\mathbf{B}) = \int dk h(k,t), \quad \rho_B(t) = \frac{1}{2}B^2(t) = \int dk \rho_B(k,t)$$

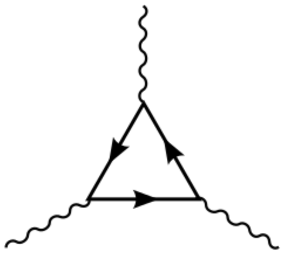
Evolution equations for the spectra are equivalent to the Faraday equation

$$\frac{\partial h(k,t)}{\partial t} = -\frac{2k^2}{\sigma_{cond}} h(k,t) + \frac{4\Pi}{\sigma_{cond}}\rho_B(k,t)$$

$$\frac{\partial \rho_B(k,t)}{\partial t} = -\frac{2k^2}{\sigma_{cond}}\rho_B(k,t) + \frac{\Pi}{\sigma_{cond}} h(k,t)$$

Evolution of the chiral imbalance μ_{5q}

We use the Adler anomaly in QED for the chiral currents



$$\partial_\mu\left(j^\mu_{qR} - j^\mu_{qL}\right) = \frac{e_q^2}{2\pi^2}(\mathbf{E}\cdot\mathbf{B})$$

Integrating the Adler anomaly over the volume of HS/QS and using the Maxwell equations, one gets the conservation law

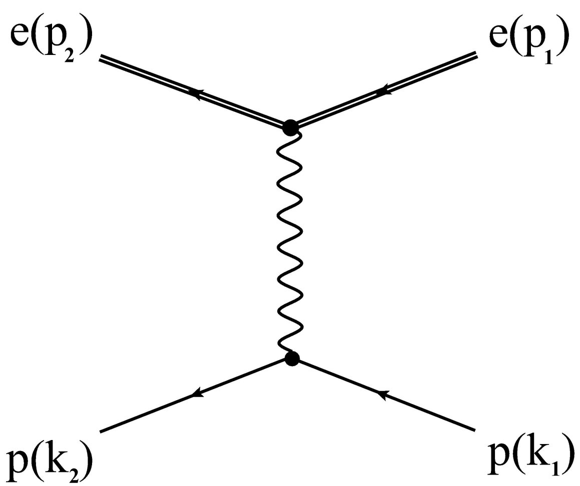
$$\frac{d}{dt}\left[n_{qR} - n_{qL} + \frac{e_q^2}{4\pi^2}h(t)\right] = 0, \quad \frac{dh(t)}{dt} = -\frac{2}{V}\int d^3x\left(\mathbf{E}\cdot\mathbf{B}\right)$$

Thus we derive the kinetic equation for μ_{5q}

$$\frac{d\mu_{5q}}{dt} = -\frac{e_q^2}{2\mu_q^2\sigma_{cond}}\int dk\frac{\partial}{\partial t}h(k,t) - \Gamma_q\mu_{5q}$$

In the last term, we account for the quark helicity flip $q_L \leftrightarrow q_R$ in qq-collisions. This process is allowed due to the nonzero effective quark mass $(m_q)_{eff} \sim \mu_q$.

Helicity flip in dense electroweak matter



- In NS electrons are ultrarelativistic but not massless
- Electron spin will be flipped in scattering off other background fermions
- Collisions of electrons with protons are more frequent than those with electrons and neutrons (Kelly, 1973)

$$e_R(p_1) + p(k_1) \to e_L(p_2) + p(k_2)$$

Exact solutions of Dirac equation for electrons accounting for electroweak interaction with background matter

$$W = \frac{V}{2(2\pi)^8} \int \frac{d^3p_1 d^3p_2 d^3k_1 d^3k_2}{\mathcal{E}_1 \mathcal{E}_2} \delta^4\left(p_1 + k_1 - p_2 - k_2\right) |\mathcal{M}|^2 f_e\left(E_1 - \mu_R\right)\left[1 - f_e\left(E_2 - \mu_L\right)\right] f_p\left(\mathcal{E}_1 - \mu_p\right)\left[1 - f_p\left(\mathcal{E}_2 - \mu_p\right)\right]$$

We assume that electrons are ultrarelativistic and scattering is elastic

$$W = \frac{Ve^4}{32\pi^5} \frac{m_e^2 M_p}{\bar{\mu}_e} T \left[\ln\left(\frac{48\pi}{\alpha_{em}}\right) - 4 \right] (\mu_R - \mu_L), \; \frac{d\mu_5}{dt} = -\Gamma_f \mu_5, \; \Gamma_f = \frac{\alpha_{em}^2}{\pi} \left[\ln\left(\frac{48\pi}{\alpha_{em}}\right) - 4 \right] \left(\frac{m_e}{\bar{\mu}_e}\right)^2 \left(\frac{M_p}{\bar{\mu}_e}\right) T$$

The coefficients in kinetic equations and initial conditions

We consider the initial temperature of HS/QS to be $T_0 \sim (10^8 - 10^9)$ K

The electroweak parameters $V_{u5} = 4.5$ eV and $V_{d5} = 2.9$ eV, which corresponds to the number density of the stellar matter $n = 1.8 \times 10^{38}$ cm^{-3}

The initial Kolmogorov spectrum of the magnetic energy density with $\nu_B = -5/3$

$$\rho_B(k,t_0) = Ck^{\nu_B}, \quad k_{\min} \le k \le k_{\max}, \quad C = \frac{(1+\nu_B)}{2k_{\max}^{1+\nu_B}} B_0^2, \quad B_0 = 10^{12}\,\mathrm{G}$$

Here $k_{\min} = R_{\mathrm{STAR}}^{-1} = 2 \times 10^{-11}$ eV and $k_{\max} = \Lambda_B^{-1}$ is the parameter corresponding to the minimal length scale of the magnetic field.

The initial spectrum of the magnetic helicity density satisfies the relation, $k\,h(k,t_0) = 2r\,\rho_B(k,t_0)$, which accounts for the different initial helicity: $r = 0$ corresponds to nonhelical fields and $r = 1$ to the fields with maximal helicity

The initial chiral imbalance is takes as $\mu_{5q}(t_0) \sim 1$ MeV

The electric conductivity of the quark matter was computed by Heiselberg & Pethick (1993)

$$\sigma_{cond} = 4.64 \times 10^{20} \left(\alpha_s \frac{T}{T_0} \right)^{-5/3} \left(\frac{\mu_0}{300\,\mathrm{MeV}} \right)^{8/3} \mathrm{s}^{-1}$$

Here $\alpha_s \sim 0.1$ is the coupling constant of strong interaction

Energy source providing the magnetic field growth

Despite, in QS/HS, background u and d quarks are degenerate, they have a small but nonzero temperature $T_0 \sim (10^8 - 10^9)$ K at $t_0 = 100$ yr. Thus magnetic field can grow taking energy from the thermal motion of these fermions

Using the energy conservation equations in MHD one can show that the following conservation law is valid: $[\delta\varepsilon_T + \rho_B] = \text{const}$.

$$\varepsilon_T = \varepsilon_0 + \delta\varepsilon_T \qquad \delta\varepsilon_T = \frac{B_{eq}^2}{2} = \left[\mu_u^2 + \mu_d^2\right]\frac{T_0^2}{2}$$

$\delta\varepsilon_T$ is the thermal correction to the internal energy of the degenerate fermion gas

Thermal energy can be transformed to the energy of magnetic field providing the magnetic field growth. Note that the second law of thermodynamics is not violated since the entropy enhances: $dS/dt > 0$

Accounting for the expression for the conductivity of quark matter and the conservation of energy, one should multiply rhs of kinetic equations by the factor.

$$\times\left(1 - \frac{B^2}{B_{eq}^2}\right)^{5/6}$$

This modification allows one to exclude the back reaction of the magnetic field on the background matter and eliminate the excessive growth of the field.

Results of the numerical solution of the kinetic equations: generation of the magnetic field in magnetars

(a) $\Lambda_B = 1$ km, $T_0 = 10^8$ K

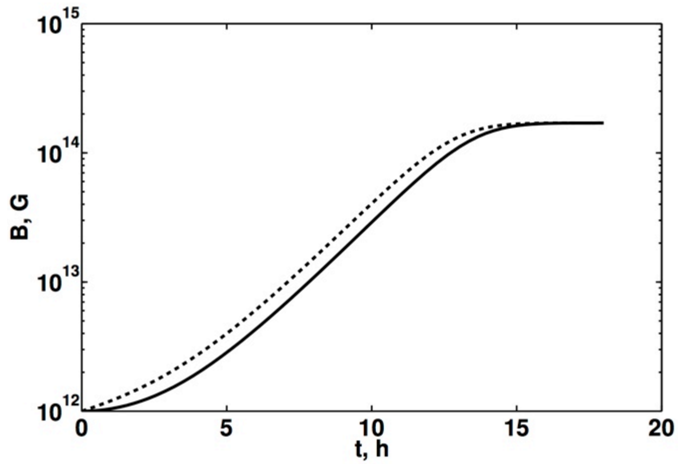
(b) $\Lambda_B = 10^2$ m, $T_0 = 10^8$ K

(c) $\Lambda_B = 1$ km, $T_0 = 10^9$ K

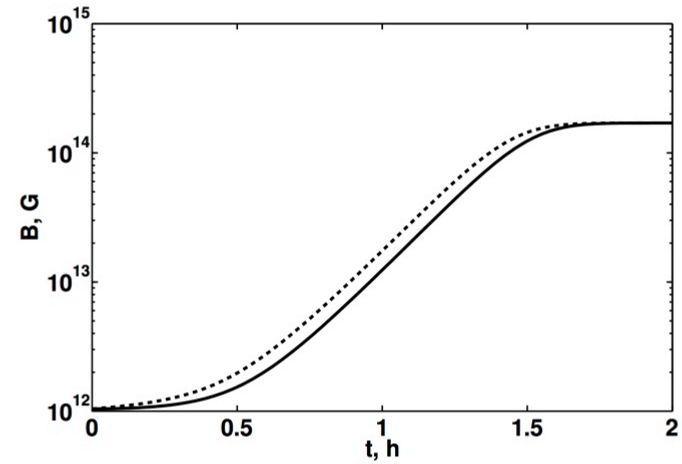
(d) $\Lambda_B = 10^2$ m, $T_0 = 10^9$ K

Solid lines: $r = 0$.

Dashed lines: $r = 1$



(a)



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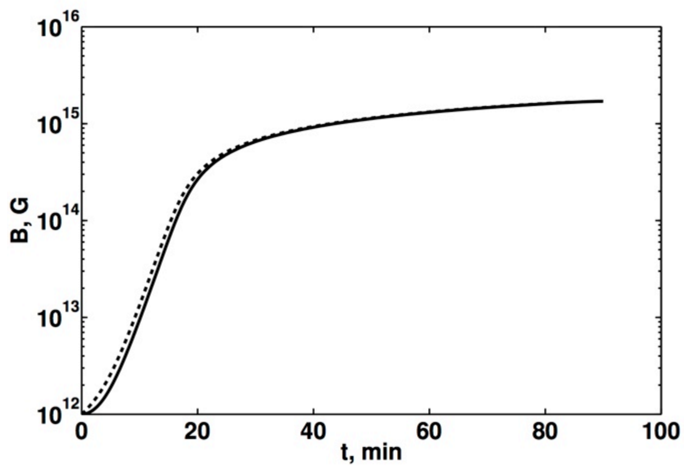

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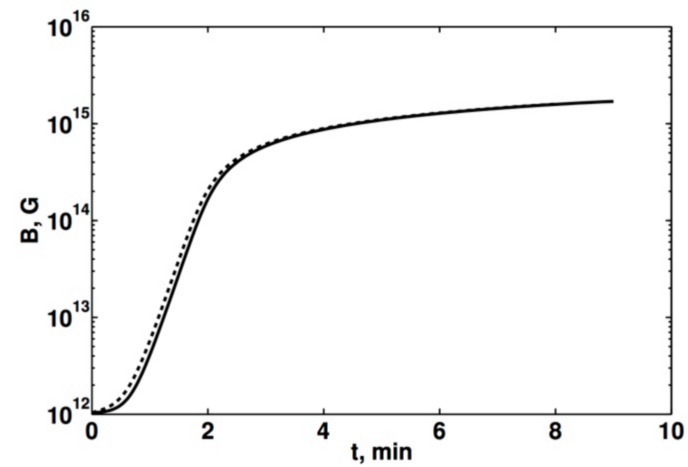

(d)

Discussion

- The maximal magnetic field strength $B_{max} = (10^{14} - 10^{15})$ G depends on the initial thermal energy of HS/QS.
- This field is generated during $t \sim$ (10 min − 10 h) depending on the length scale and the initial temperature.
- Small scale fields in matter with higher T_0 grow faster, since the time of the magnetic field instability $t \sim \sigma_{cond} \Lambda_B/\Pi \sim \Lambda_B/\Pi \, T_0^2$
- We do not take into account the neutrino emission in the HS/QS matter

Summary

- We developed the new model for the generation of magnetic fields in magnetars, where the field is amplified from $B_0 = 10^{12}$ G (typical for young pulsars) to $B_{\text{max}} = (10^{14} - 10^{15})$ G, which is close the field strength predicted in magnetars. The generation of the magnetic field is driven by the electroweak interaction between quarks, violating the parity.
- The magnetic fields generated are large-scale since the length scale of the field is comparable with the star radius $R_{\text{STAR}} \sim 10$ km.
- Besides the generation of strong magnetic fields we predict the generation of the magnetic helicity.
- Our model does not require any special initial conditions, like fast rotation (turbulent dynamo model) or extremely strong seed field (fossil field model). Thus any HS/QS can potentially become a magnetar. This result is in agreement with the analysis of Woods (2008) and Boldin & Popov (2012) that the number of magnetars (and hidden magnetars) should be great

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