Oblique projectors in image morphology

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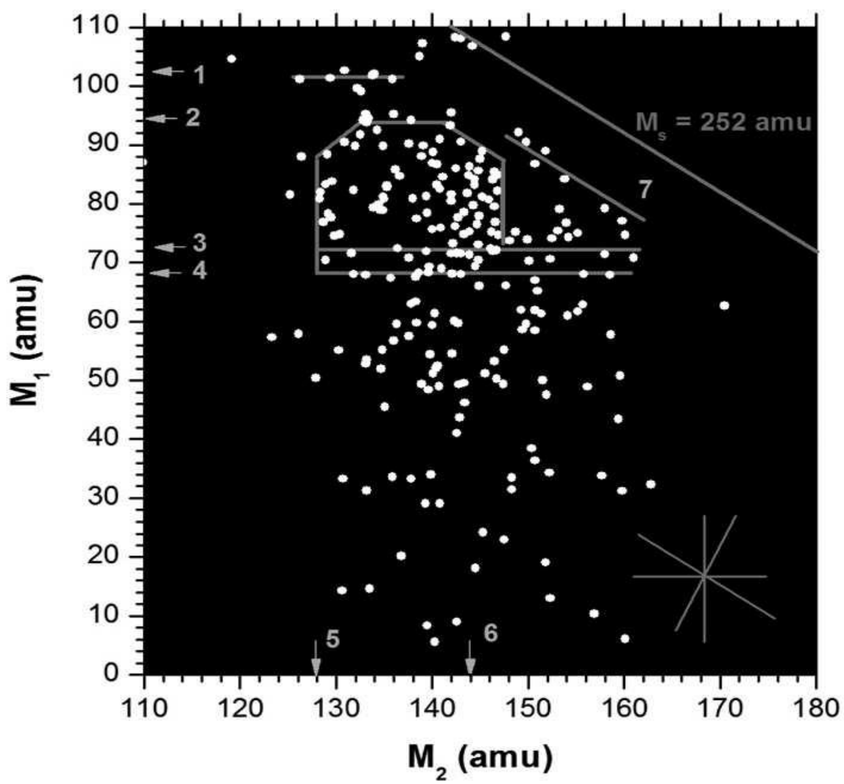
Research objectives

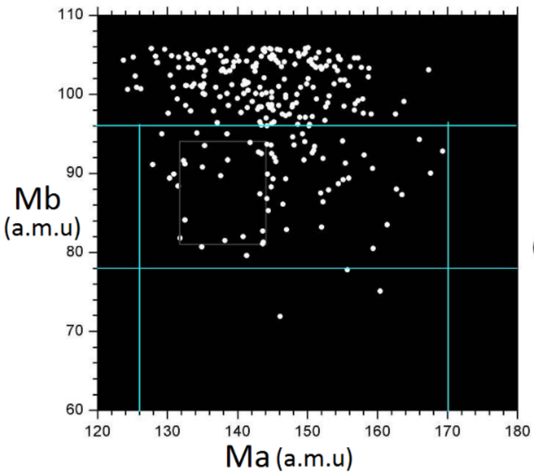
- Spontaneous $_{252}$Cf fission

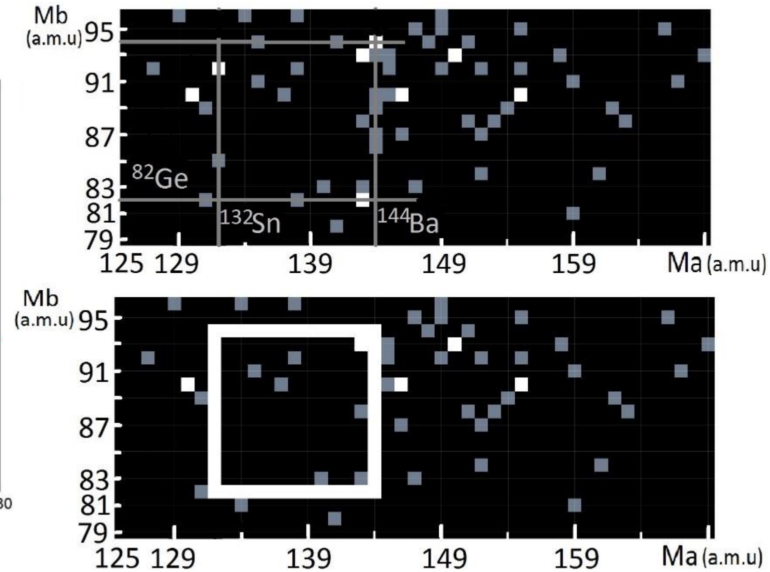


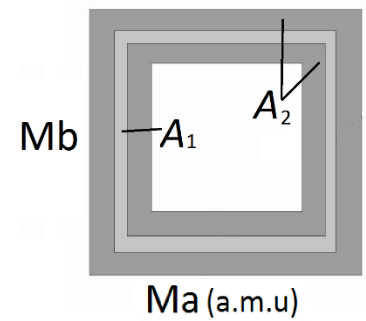
Application of mathematical methods of morphological image analysis for experimental data from nuclear fission for linear-point structure revealing

- RFBR project №14-07-00409 A
- RFBR project №14-07-00441 A

Previous report: searching for linear structures in the mass correlation distribution







- The value of functional t(g) characterizes the "distance" between a given image g and an image of the "square-like" form. Functional t(g) is invariant with respect to brightness and contrast transformations.
- The decision rule has the following form: a hypothesis H="there is a square-like structure" is accepted if, by a shift and scale transformation, a fragment f_ω can be found such that t(f_ω) ≤ A. A is an empirically determined constant, and it is rejected (an alternative K = "there is no square-like structures" is accepted) if such a fragment is absent.

What is Morphological Analysis?

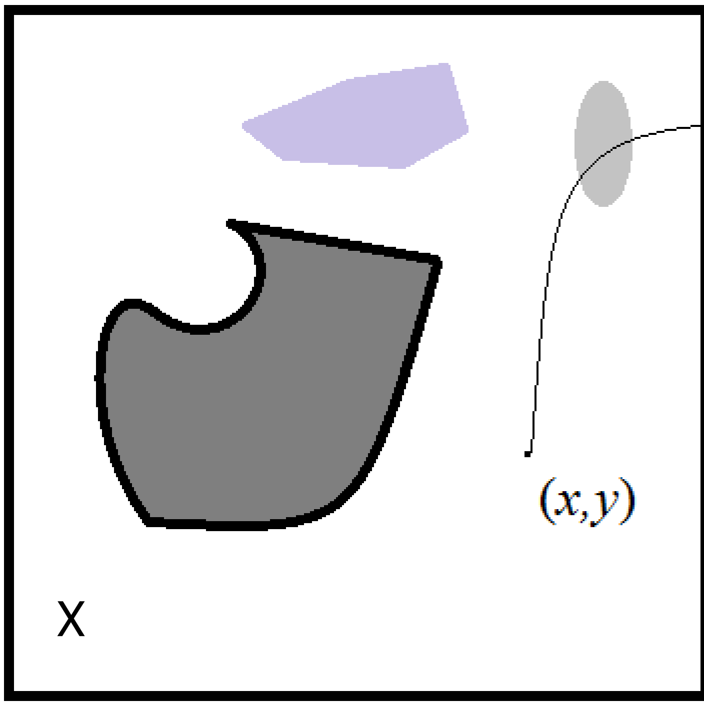
- Methods for solving problems of recognition, classification of objects, building estimates for their parameters, feature extraction from “scenes” using their “images”;
- They are invariant to changes in the conditions of registration of signals;
- They are based on the notion of the “signal shape” or “image shape”.

What is Shape?

- Simply speaking, it is an object that is present in all images of a given scene irrespective of the conditions of its registration



Mathematical model of the object's image



f(x , y)* – image brightness in the point *(x,y)* of the field of view, *X

$$\int_X f^2(x,y)\,dxdy < \infty$$

***X*- field of view**

What is shape?

- Let $\mathcal{S}$ be a scene, $\mathcal{K}$ *is a set of it's image registration conditions,* $\mathcal{V}_s$ (*SHAPE*) *is a set of different scene S images obtained under different registration conditions* $k \in \mathcal{K}$:
- $\mathcal{V}_s = \{\mathbf{f}(\cdot, k) \in E,\ k \in \mathcal{K}\}$ – shape of images of a scene $\mathcal{S}$.

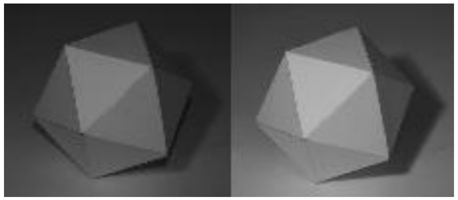
$\mathbf{f}$

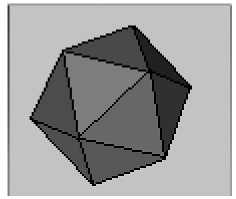
g_1

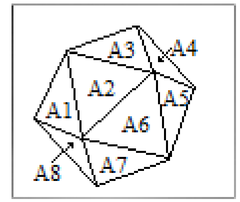
g_2

When conditions of observation change, the brightness of pixels varies according to this rule: $g_i(x, y) = F(\mathbf{f}(x, y))$

Shape of a piece-wise image







It is a class

$$\mathbf{f}(x) = \sum_{i=1}^{N} c\chi_i(x),\ x \in X,\ c_i \in (-\infty, +\infty),\ i = 1, \dots, N,$$

$$\chi_i(x) = \begin{cases} 1, & x \in A_i, \\ 0, & x \notin A_i, \end{cases} \qquad i = 1, \dots, n.$$

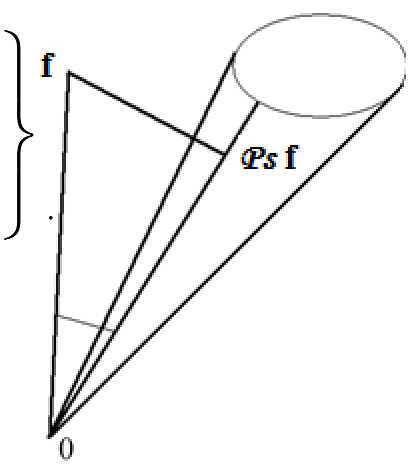
How to compute orthogonal projection

- If $\mathcal{V}_s$ is a closed convex set in $\mathcal{E}$, then orthogonal projector $\mathcal{P}_s$ on $\mathcal{V}_s$ is defined as a solution of the best approximation problem:
- $\|\mathcal{P}_s\mathbf{f} - \mathbf{f}\|^2 = \inf\{\|\mathbf{g} - \mathbf{f}\|^2 \mid \mathbf{g} \in \mathcal{V}_s\}$, $\|\mathcal{P}_s\mathbf{f} - \mathbf{f}\|^2$ - “distance” from $\mathbf{f}$ to images $\mathcal{S}$ by shape,

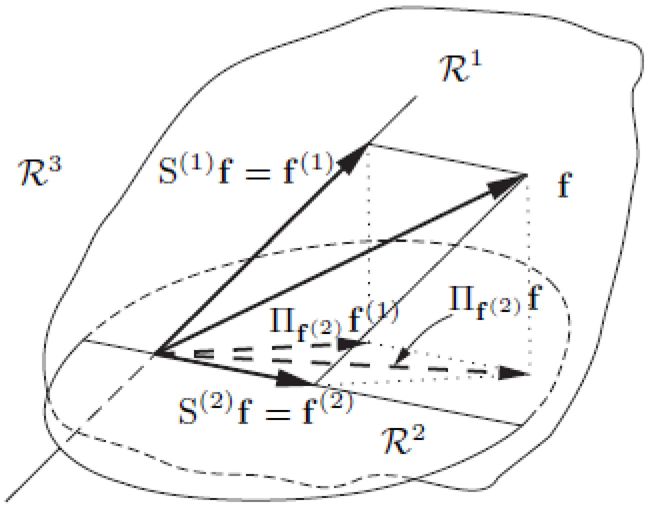
if $\mathcal{V}_s = \left\{ \mathbf{f}(x) = \sum_{i=1}^{N} c_i \chi_i(x), c_i \in (-\infty, \infty), i = 1, \ldots, N \right\}$

then

$$P_s\mathbf{f} = \sum_{i=1}^{N} \frac{(\mathbf{f}, \chi_i)}{\|\chi_i\|^2} \chi_i,$$



Oblique projection



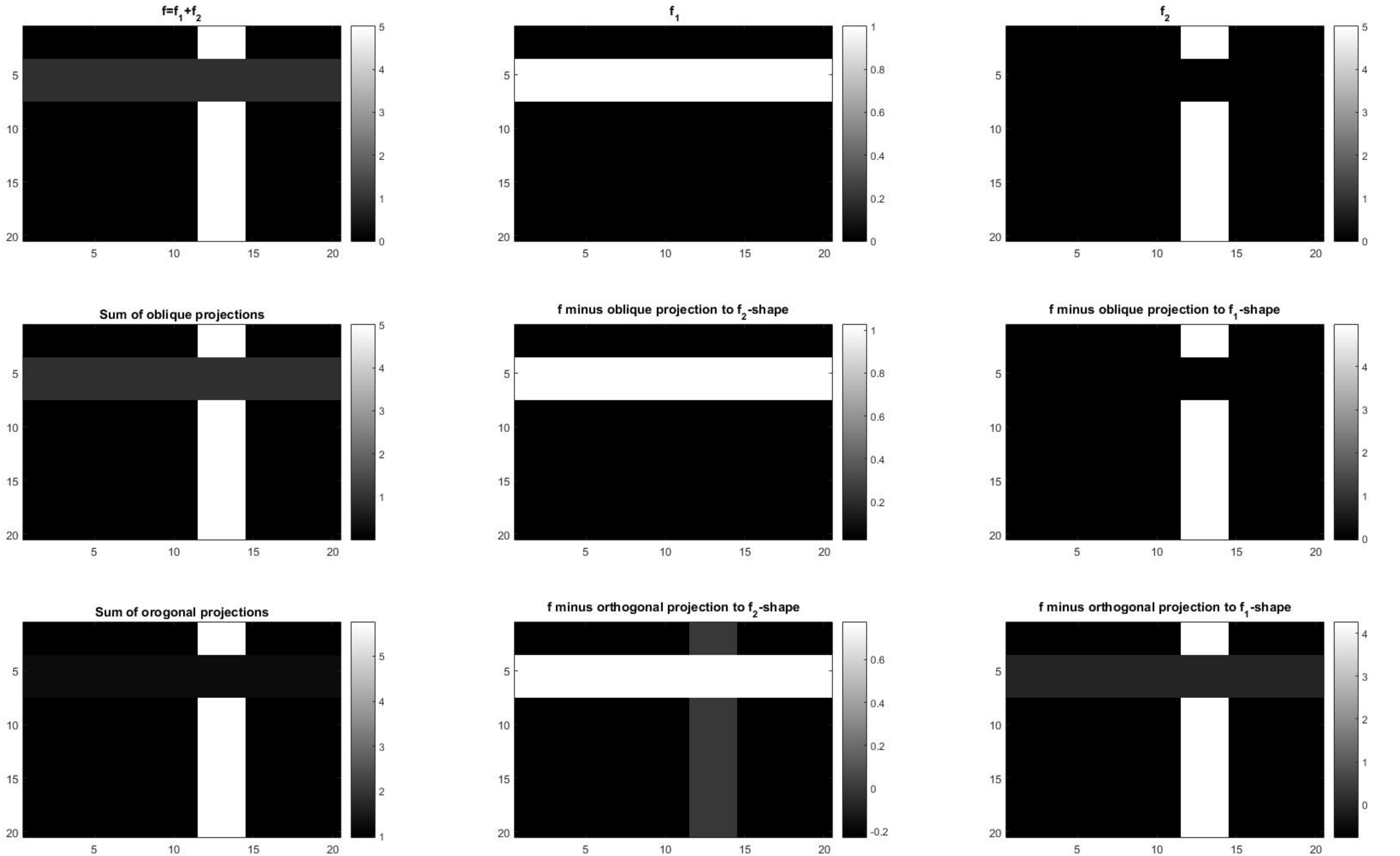
$$\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2, \quad \mathbf{f}_1 \in R^1, \mathbf{f}_2 \in R^2,$$

$$S^{(1)}\mathbf{f} = \mathbf{f}_1 \in R^1, S^{(2)}\mathbf{f} = \mathbf{f}_2 \in R^2 \Rightarrow \mathbf{f} = S^{(1)}\mathbf{f} + S^{(2)}\mathbf{f}$$

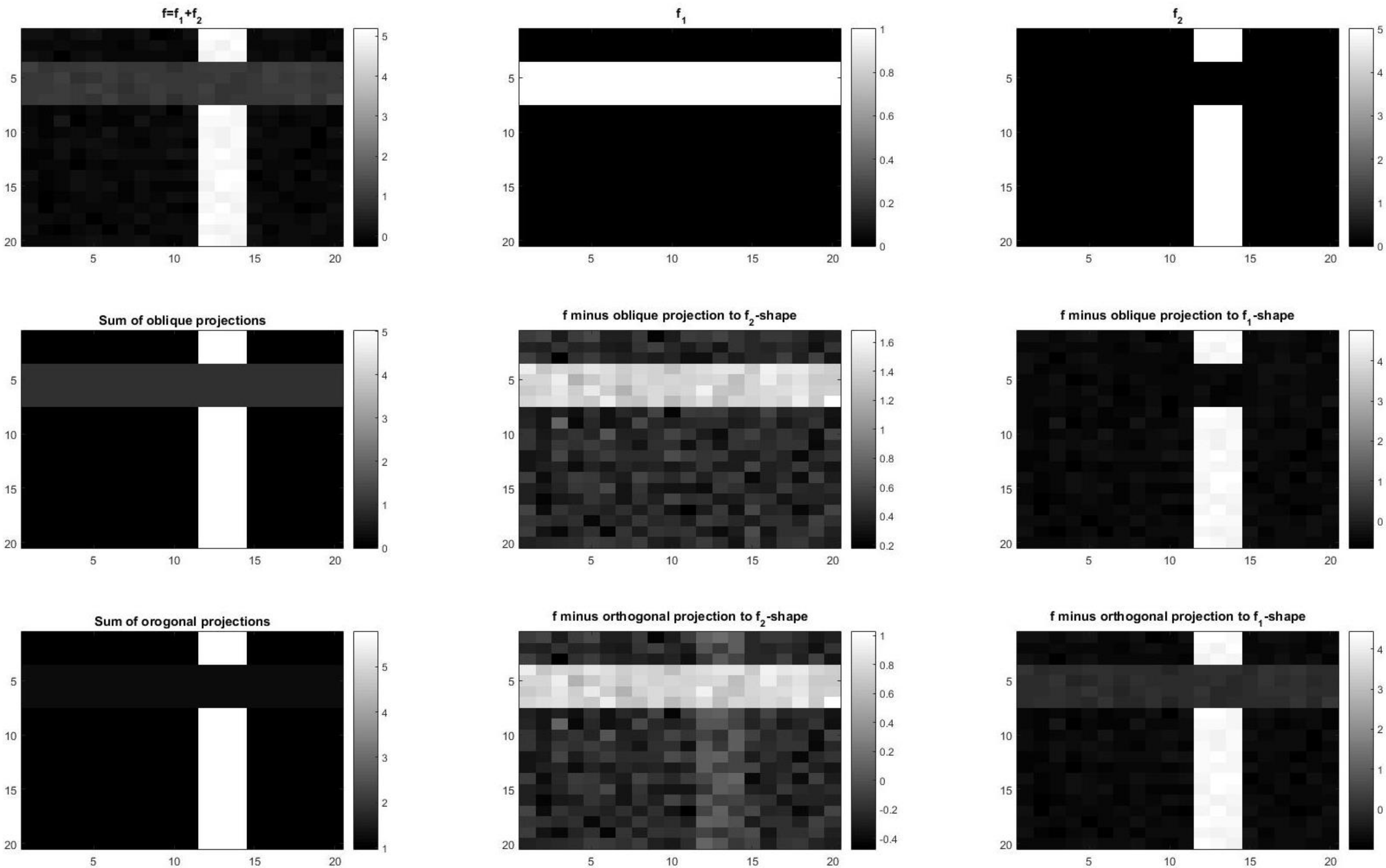
$$\Pi^{(1)}\mathbf{f} = \Pi^{(1)}\mathbf{f}_1 + \Pi^{(1)}\mathbf{f}_2 = \mathbf{f}_1 + \Pi^{(1)}\mathbf{f}_2,$$

$$\Pi^{(2)}\mathbf{f} = \Pi^{(2)}\mathbf{f}_1 + \Pi^{(2)}\mathbf{f}_2 = \mathbf{f}_2 + \Pi^{(2)}\mathbf{f}_1$$

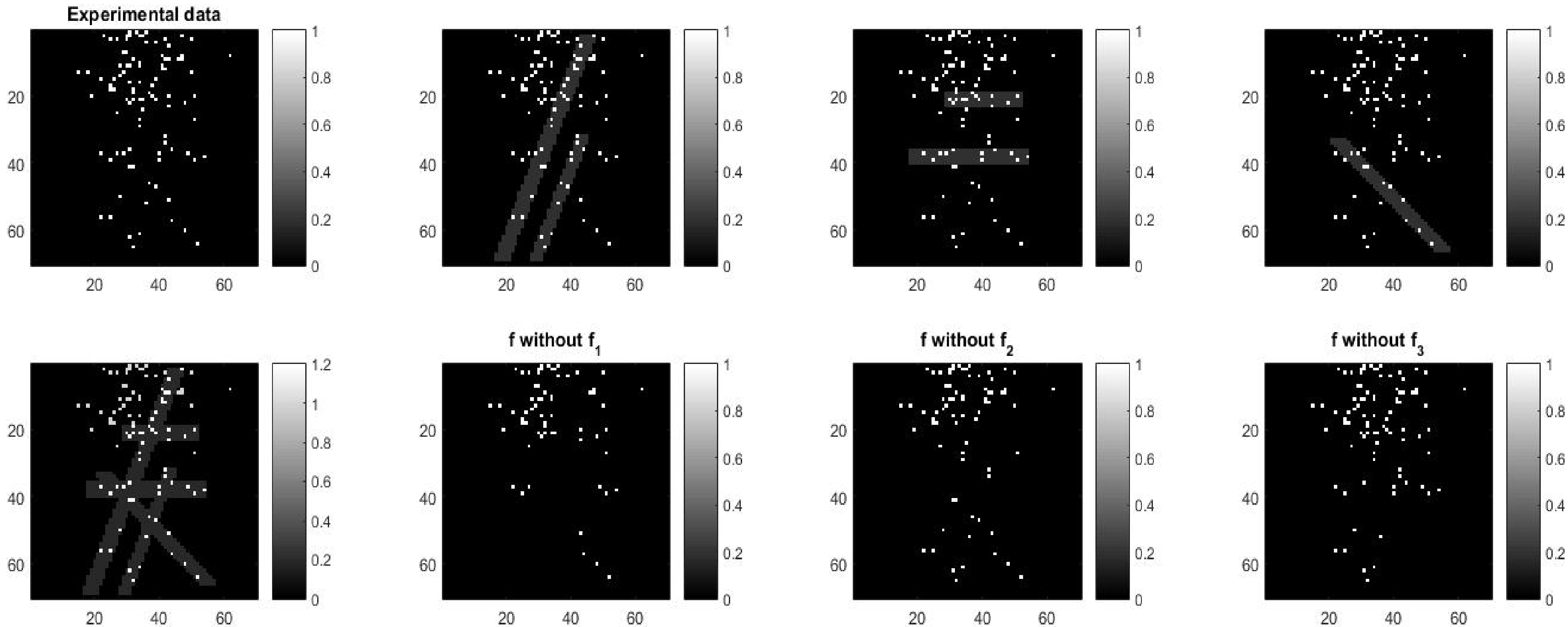
Model data without noise



Model data with noise



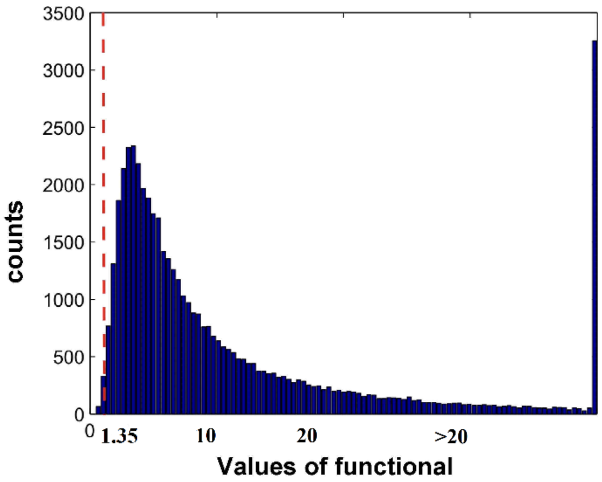
Experimental data



$$\frac{\left\|\mathbf{f} - S_1\mathbf{f} - S_2\mathbf{f} - S_3\mathbf{f}\right\|^2}{\left\|S_1\mathbf{f} + S_2\mathbf{f} + S_3\mathbf{f} - \mathcal{P}_0\mathbf{f}\right\|^2} < A$$

Estimation of reliability

- This probability is the probability of false acceptance of a hypothesis against the nearest "homogeneous field of view" alternative. It estimates an upper bound for the probability of false acceptance of the hypothesis against the alternative that such a fragment is absent.
- This criterion is analogous to the principle of the locally homogeneous strongest criterion.
- The probability that the square-like structure found on real data is generated by the noise is ~0.01.



The spectrum of the values of functional (1) obtained on the basis of model data. The dotted line shows the threshold value of A, for which P(t ≤ 40) = 0.01.

Estimated reliability ~ 99%
Hough transform reliability ~97%

Thanks for your attention!

- 1. Estimation of reliability of linear point structures revealed in two-dimensional distributions of experimental data / O. V. Falomkina, Y. V. Pyatkov, Y. P. Pyt'ev, D. V. Kamanin // *Journal of Physics: Conference Series (JPCS)*. — 2016. — Vol. 675. — P. 042001.
- 2. *Pyt'ev Y. P.* Oblique projectors and relative forms in image morphology // *Journal of Computational Mathematics and Mathematical Physics*. — 2013. — Vol. 53, no. 12. — P. 1916–1937.
- 3. Kayalar, S. & Weinert, H.L. Oblique projections: Formulas, algorithms, and error bounds // Math. Control Signal Systems (1989) 2: 33. doi:10.1007/BF02551360