# *Minimal analytical model of neutrino distribution function in supernova*


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# Core-collapse supernova

- Supernova (SN) matter is opaque for neutrinos $\Rightarrow$ neutrino interaction with SN matter is an important ingredient of core-collapse supernova models

  [Colgate S. A., White R. H., Astrophys. J., 143, 626 (1966) (idea);
  Boccioli L., Roberti L., Universe, 10, 3 (2024) (modern (last) review)]
- Description of neutrino propagation in SNs is required a self-consistent solution of hydrodynamic and neutrino transport equations
- Boltzmann equation for non-equilibrium neutrino distribution function in SN matter is solved only numerically $\Rightarrow$ it makes difficulties for use of results obtained to other problems
- It is useful to find an analytical approximation of results of numerical simulation

# Neutrino distribution function in SN

- Neutrino propagation in SN is close to spherically symmetric $\Rightarrow$ local non-equilibrium distribution function of neutrinos depends on 4 parameters

$$f_\nu \approx f_\nu(t, r, \varepsilon, \theta) \equiv f_\nu(\varepsilon, \theta)$$

where $t$ is time after a bounce, $r$ is distance from PNS center, $\theta$ is angle between neutrino momentum and radial direction of SN, $\varepsilon$ is neutrino energy

- As $f_\nu(\varepsilon, \theta)$ is dimensionless function, we introduce dimensionless variables

$$x = \varepsilon/\varepsilon_* \,, y = 1 - \cos\theta$$

- $\varepsilon_*$ is the energy parameter and it is connected with average neutrino energy
- Approximation of $f_\nu(\varepsilon, \theta)$ in SN conditions could be factorized as

$$f_\nu(x, y, a, b) = N\,\Psi(x)\,\Phi(y)$$

- $\Psi(x)$ is the energy distribution
- $\Phi(y)$ is the angular distribution

# Neutrino distribution function in SN

- Angular distribution is monotonically decreasing function and has a maximum in radial direction ($y = 0$)

$$\Phi_{max} = \Phi(0)$$

- Parameter $b$ fixes a width of dispersion of $\Phi(y)$
- Maximum of $\Psi(x)$ is determinated by $\varepsilon_*$, $x = \varepsilon/\varepsilon_*$
- Parameter $a$ fixes a width of dispersion of $\Psi(x)$
- $N$ is normalization factor
- Minimal approximation of $f_\nu(\varepsilon, \theta)$ in SN conditions could be presented as 4-parametric approximation

$$f_\nu(\varepsilon, \theta) = N\,\Psi(x, a)\,\Phi(y, b)$$

- Here $N$, $\varepsilon_*$, $a$, $b$, as well as, $f(\varepsilon, \theta)$ depend on $t, r$, but for the simplicity of expressions this dependence is dropped
- Dimensionless spectral distribution: $F(x, a) = x^3\,\Psi(x, a)$
- Connection of $\varepsilon_*$ with average neutrino energy $\overline{\varepsilon}_\nu$

$$\varepsilon_* = \overline{\varepsilon}_\nu \frac{\int_0^\infty x^{-1}\,F(x, a)\,dx}{\int_0^\infty F(x, a)\,dx}$$

# Approximation of neutrino spectral distribution

- Alpha-fit [Keil M. T. and et al, Astrophys. J. 590 (2003)]

$$F_\alpha(x, a_\alpha) = x^{a_\alpha} \exp(-x)\,, \quad a_\alpha \in [1, \infty)$$

- Fermi-like approximation [Janka H.-T., Hillebrandt W., Astron. Astrophys. 224 (1989)]

$$F_F(x, a_F) = \frac{x^3}{\exp(x - a_F) + 1}\,, \quad a_F \in (-\infty, \infty)$$

- Modified Fermi-like approximation [Nadezhin D. K., Otroshchenko I. V., Sov. Astron. 24 (1980)]

$$F_N(x, a_N) = \frac{x^3\ \exp(-a_N\, x^2)}{\exp(x) + 1}\,, \quad a_N \in [0, \infty)$$

- When parameter $a$ increase, the width of spectral distribution $F(x, a)$ decreases

# Approximation of neutrino angular distribution

- Gaussian-like approximation [Dobrynina A. and et al, J. Phys. Conf. Ser. 1690 (2020)]

$$\Phi_G(y, b_G) = \exp\big(-y^2/b_G\big)\,, \quad b_G \in (0, \infty)$$

- Linear approximation

$$\Phi_L(y, b_L) = \big[1 - y/b_L\big]\Theta(b_L - y)\,, \quad b_L \in (0, \infty)$$

- When parameter $b$ increase, the width of angular distribution $\Phi(y, b)$ increases
- Neutrinosphere approximation

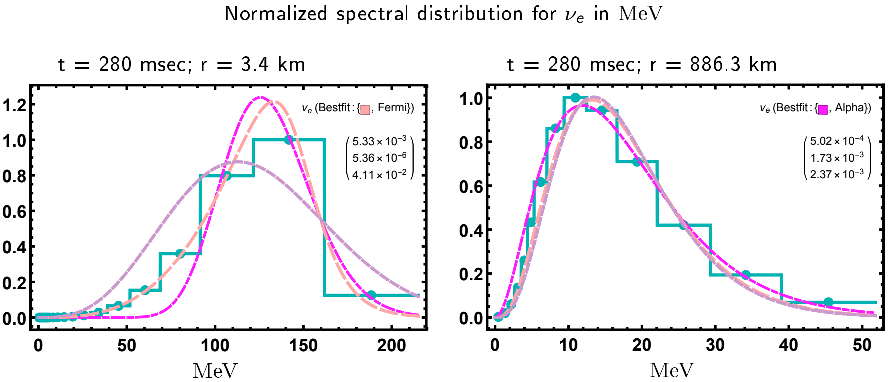
$$f_S(r)|_{r \gg R_\nu} \approx F_{eq}(x)\ \Theta\big(R_\nu^2/(2r^2) - y\big)$$

where $R_\nu$ is the radius of the neutrinosphere.
With growth of $r$ a width of angular distribution decreases

# Data of supernova simulation

- In our analysis we use of the PROMETHEUS-VERTEX code [Hüdepohl L., Ph.D. thesis (2014)]
  - Result of self-consistent solution of 1D hydrodynamic and neutrino transport equations
  - Tangent-ray discretization of the Boltzmann transport equation [Rampp M. and Janka H. T., Astron. Astrophys. 396 (2002)]
  - Results for electron neutrinos $\nu_e$, electron antineutrinos $\overline{\nu}_e$ and all other neutrino types $\nu_x = \nu_{\mu,\tau}, \overline{\nu}_{\mu,\tau}$
  - Results of simulations of explosion of SN progenitors with masses of 11.2, 13.8, 15, 17.8, 20.6 and 25 $M_\odot$ (models s11.2, s13.8, s15.0, s17.8, s20.6, s25.0) [Woosley S. E. and et al, Rev. Mod. Phys. 74 (2002)]
  - Model s15s7b2 with mass of SN progenitors of 15 $M_\odot$ [Woosley S. E. and Weaver T. A., Astrophys. J. Suppl. 101 (1995)]
- Comparison of 1D codes of SN explosion shows good agreement between them [O'Connor E. and et al, J. Phys. G. 45 (2018)] $\Rightarrow$ our conclusions based on PROMETHEUS-VERTEX code will be applicable for other models of SN explosions

# Examples of spectrum approximation

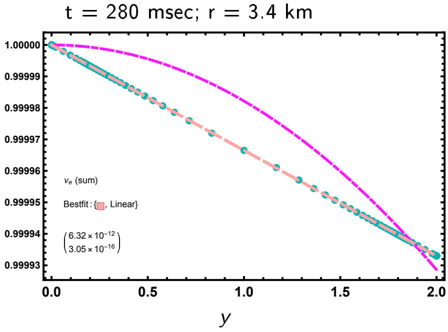


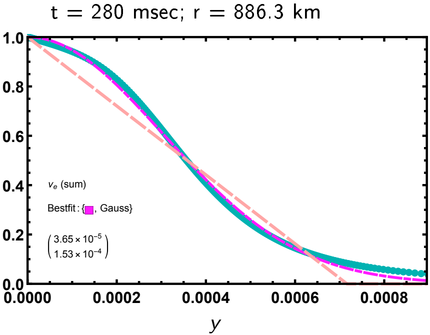

Model s15s7b2 with mass of SN progenitors of 15 $M_\odot$

Cyan line: numerical data;
Magenta line: Alpha-fit;
Pink line: Fermi-like approximation;
Purple line: Modified Fermi-like approximation

# Examples of angle distribution approximation

Neutrino angle distribution as function of $y = 1 - \cos\theta$ for $\nu_e$




Model s15s7b2 with mass of SN progenitors of 15 $M_\odot$

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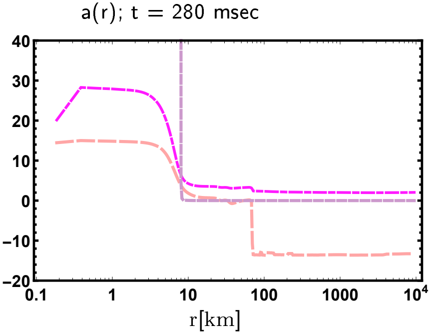
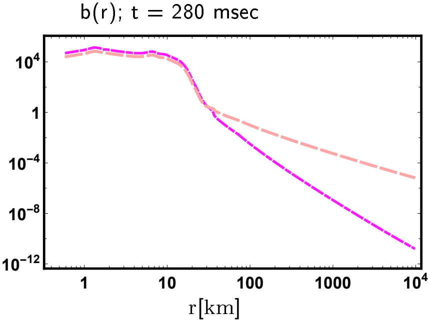
# Examples of spectral and angular parameters

Spectral *a* and angular *b* parameters as function of distance *r*

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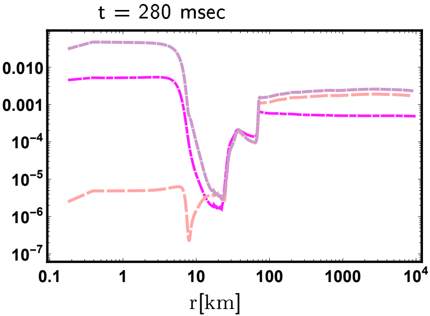
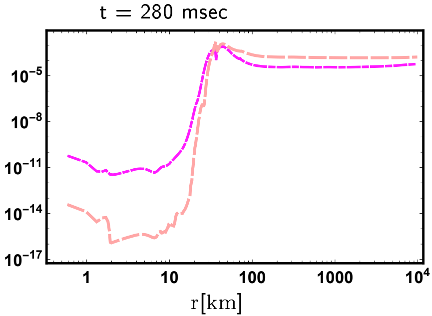
Model s15s7b2 with mass of SN progenitors of 15 $M_\odot$

# Approximation error

Error of spectral and angular approximations as function of distance $r$

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Model s15s7b2 with mass of SN progenitors of 15 $M_\odot$

# Conclusions

- Minimal 4-parametric distribution function of neutrino in core-collapse supernova is considered
- Three spectral and two angular neutrino approximations in SN are investigated
- Parameters of considered approximations are obtained as functions of time after a bounce and distance from PNS center for 7 model of SN with masses from 11.2 $M_{\odot}$ to 25 $M_{\odot}$
- In inner part of supernova the neutrino distribution function can be approximated by Fermi-like approximation for spectral part and liner approximation for angular part
- In outer part of supernova the neutrino distribution function can be approximated by alpha-fit for spectral part and Gaussian-like approximation for angular part
- The behavior of approximation parameters significantly depends on part of the supernova, that is, it correlates with its characteristic spatial scale
- Approximation parameters of the neutrino distribution function change in a supernova in a wide range $\Rightarrow$ it is important to find analytical approximation of them in terms of characteristic spatial scale of SN

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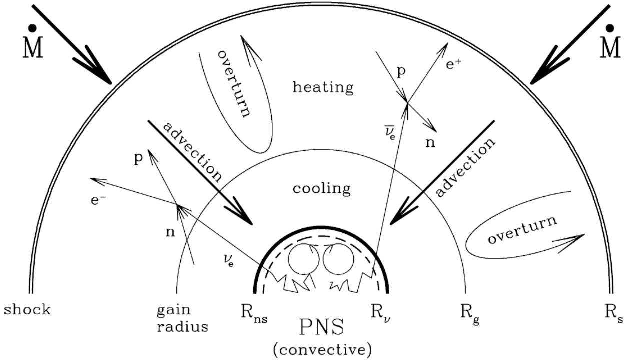
# Supernova spatial scales

Sketch from [Janka H. T., Astron. Astrophys. 368 (2001)]




Hierarchy of SN spatial scales: $R_\nu < R_{ns} < R_{eos} < R_g < R_s$ ($R_\nu \approx R_{ns}$, $R_{eos} \sim R_g$)