Recent nonperturbative insights into the QCD coupling and confinement: The holographic light-front perspective

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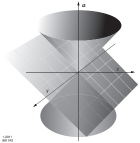

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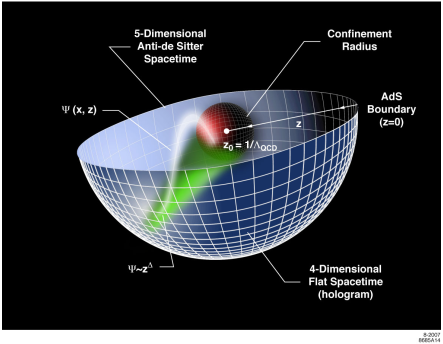
Introduction: The holographic principle

Holographic principle (1993): Duality between a strongly coupled quantum-mechanical system with many degrees of freedom and a gravity theory (the bulk)

Prototype: AdS/CFT duality (1997)

Anti-de Sitter AdS_{d+1} is the maximally symmetric $d+1$ space with negative constant curvature and a d-dimensional flat space boundary: Minkowski spacetime

$$
\begin{aligned}
ds^2 &= g_{MN} dx^M dx^N \\
&= \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)
\end{aligned}
$$



Ideal tool to describe QCD in the IR with an infinite number of degrees of freedom

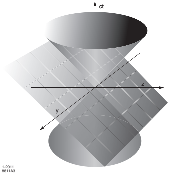
But maximal AdS space implies a Conformal Field Theory in its asymptotic boundary, therefore no discrete spectrum

To introduce an IR scale and confinement in the gravity dual the constant curvature of AdS should be modified the large z region

Note: z is the holographic coordinate and R is the AdS radius

Front-form QCD and the gauge/gravity duality

Front-form QCD: The boundary theory



Light-front (LF) quantization uses the null plane $x^+ = x^0 + x^3 = 0$ tangent to the light cone (Dirac 1949), thus without reference to a specific Lorentz frame

Evolution in LF time x^+ is given by the Hamiltonian equation

$$i \frac{\partial}{\partial x^+} |\psi\rangle = P^- |\psi\rangle, \qquad P^- |\psi\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+} |\psi\rangle,$$

for a hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, where P^- is a dynamical generator (it contains the interactions) and P^+ and $\mathbf{P}_\perp$ are kinematical

Hadron mass spectra from LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^+ P^- - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \qquad |\psi\rangle = \sum_n^\infty \psi_n |n\rangle$$

The LF Fock expansion is the sum of the N-parton states above the valence state

$$|\psi\rangle_p = \psi_{uud/p} |uud\rangle + \psi_{uudg/p} |uudg\rangle + \psi_{uudq\bar{q}/p} |uudq\bar{q}\rangle + \psi_{uudggg} |uudggg\rangle + \cdots$$

Constituent Counting Rules (1973): determine the valence quark configuration at high momentum transfer (Brodsky, Farrar and Matveev, Muradian, Tavkhelidze)

In the IR the valence configuration is no longer decoupled from the Fock expansion and all states above the valence state contribute to the dynamics of confinement: Problem with an infinite number of degrees of freedom and complexity

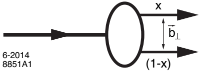
Semiclassical approximation to light-front QCD

GdT and S. J. Brodsky, PRL **102**, 081601 (2009)

Starting from the QCD Lagrangian we write the LF Hamiltonian operator P^- in terms of quark and gluon dynamical fields

For a $q\bar{q}$ state we factor out the longitudinal $X(x)$ and orbital $e^{iL\theta}$ dependence from ψ

$$\psi(x, \zeta, \varphi) = e^{iL\theta} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



where $\zeta^2 = x(1-x)b_\perp^2$ is the invariant transverse separation between two quarks and L their relative LF orbital angular momentum

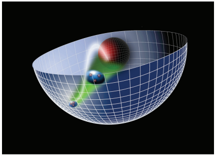
Chiral limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and the LF invariant equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ becomes a wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Critical value $L = 0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE in QCD
- The effective potential U includes all interactions, including those from the higher Fock states

Gravity dual: The bulk theory

GdT and S. J. Brodsky, PRL **102**, 081601 (2009)
GdT, H. G. Dosch and S. J. Brodsky, PRD **87**, 075005 (2013)



We start with the AdS_{d+1} action for a tensor-J field $\Phi_{N_1 \dots N_J}$

$$S = \int d^d x \, dz \sqrt{g} \, e^{\varphi(z)} \left(D_M \Phi_J D^M \Phi_J - \mu^2 \Phi_J^2 \right)$$

in presence of a dilaton φ to modify the IR region of AdS

Variation off the AdS action leads to the WE

$$\left[- \frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \frac{(\mu R)^2}{z^2} \right] \Phi_J(z) = M^2 \Phi_J(z)$$

Upon the substitution $\Phi_J(z) = z^{(d-1)/2-J} e^{-\varphi(z)/2} \, \phi(z)$ we separate kinematics and dynamics to bring the AdS equation into its Schrödinger form

Mapping the bulk WE into physical Minkowski space

$$\left(- \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right) \phi(z) = M^2 \phi(z)$$

we find the QCD LFWE provided that $z \to \zeta$, $(\mu R)^2 \to L^2 - (d/2 - J)^2$, with the LF confinement potential given in terms of the IR modification of the ADS geometry

$$U(z) = z^{\gamma} e^{-\varphi(z)/2} \, \partial_z \left(z^{-\gamma} \partial_z e^{\varphi(z)/2} \right) \qquad \gamma = d - 1 - 2J$$

Superconformal symmetry and emergence of a mass scale

S. J. Brodsky, GdT, H. G. Dosch, PLB **729**, 3 (2015)
GdT, H. G. Dosch, S. J. Brodsky, PRD **91**, 045040 (2015)
H. G. Dosch, GdT, S. J. Brodsky, PRD **91**, 085016 (2015)

No mass scale appears in the QCD Lagrangian and its action is conformal invariant: What sets the scale of the confining dynamics of the boundary theory?

In the dual theory confinement is determined by the deformation of AdS space in the IR, thus a geometric problem

We follow the procedure of de Alfaro, Fubini and Furlan (1976) for breaking conformal symmetry in the Hamiltonian but keeping the action conformal invariant

Extension to superconformal symmetry following Fubini and Rabinovici (1984) leads to

$$\left(-\frac{d^2}{dz^2} - \frac{1 - 4(f + \frac{1}{2})^2}{4z^2} + \lambda^2 z^2 + 2\lambda \left(f - \tfrac{1}{2}\right)\right)\phi_+ = M^2 \phi_+$$

$$\left(-\frac{d^2}{dz^2} - \frac{1 - 4(f - \frac{1}{2})^2}{4z^2} + \lambda^2 z^2 + 2\lambda \left(f + \tfrac{1}{2}\right)\right)\phi_- = M^2 \phi_-$$

where f is dimensionless and λ has the dimension of $[M^2]$

Solving $U(z)$ in terms of $\varphi(z)$ we obtain for $d = 4$

$$\boxed{\varphi(z) = \lambda z^2}$$

neglecting backreaction from the metric

Light-front mapping and baryons

GdT, H. G. Dosch, S. Brodsky, PRD **91**, 045040 (2015)

Upon substitution in the superconformal equations

$$z \mapsto \zeta, \quad \phi_1 \mapsto \psi_-, \quad \phi_2 \mapsto \psi_+$$
$$f \mapsto L + \tfrac{1}{2}, \quad \lambda \to \kappa^2$$

we determine exactly the nucleon potential in
GdT, Dosch and Brodsky, PRD **87**, 075005 (2013)

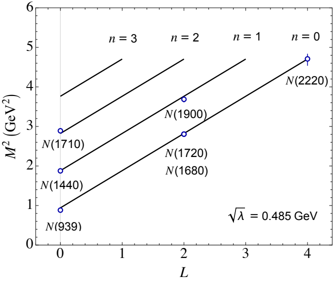
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L+1)\right)\psi_+ = M^2\psi_+$$
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2 L\right)\psi_- = M^2\psi_-$$

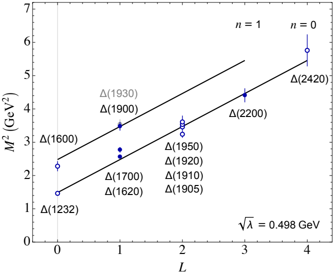
Eigenvalues

$$M^2 = 4\kappa^2(n+L+1)$$

Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\kappa^2\zeta^2/2} L_n^{L+1}(\kappa^2\zeta^2)$$





See also: Abidin and Carlson (2009) and Gutsche, Lyubovitskij, Schmidt and Vega (2012)

Superconformal meson-baryon symmetry

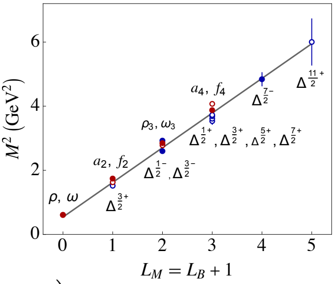
H. G. Dosch, GdT, S. J. Brodsky, PRD **91**, 085016 (2015)

Mapping the bulk WE into physical space

$$z \mapsto \zeta, \quad \phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

$$f \mapsto L_M - \tfrac{1}{2} = L_B + \tfrac{1}{2}, \quad \lambda \mapsto \kappa_B^2 = \kappa_M^2$$

we find the semiclassical LF meson/baryon bound-state equations



$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \kappa_M^4\,\zeta^2 + 2\kappa_M^2(L_M - 1)\right)\phi_M = M^2\,\phi_M$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \kappa_B^4\,\zeta^2 + 2\kappa_B^2(L_B + 1)\right)\phi_B = M^2\,\phi_B$$

with eigenvalues $M_M^2 = 4\kappa_M^2(n + L_M)$ and $M_B^2 = 4\kappa_B^2(n + L_M + 1)$

Superconformal QM imposes the conditions $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\boxed{L_M = L_B + 1}$

L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator diquark cluster in the baryon

Note: Additional term $2\kappa^2 S$ for internal spin, $S = 0, 1$

The strong coupling and the IR-UV transition domain

Geometric coupling in the bulk

We can write the dilaton term in the AdS action in terms of a geometric coupling $g_s^2(z)$, the inverse of the dilaton term,

$$e^{\varphi(z)} = \frac{1}{g_s^2(z)}$$

The coupling $g_s(z)$ is known as the asymptotic string coupling in the classical gravity approximation: It encodes the strong dynamics in the IR

The confinement potential can also be written in terms of the coupling $g_s(z)$

$$U(z) = z^{\gamma} g_s(z)\, \partial_z \left(z^{-\gamma}\, \partial_z g_s^{-1}(z)\right), \qquad \gamma = d - 1 - 2J$$

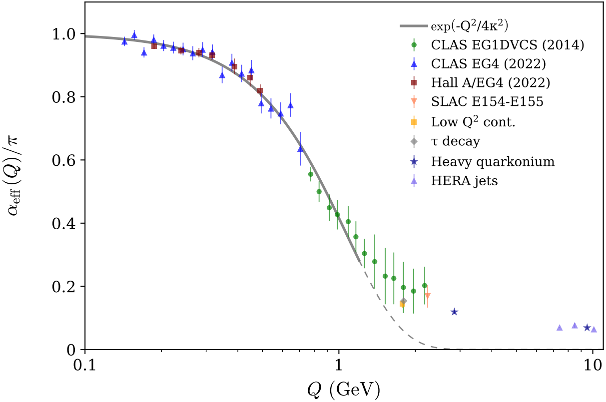
The physical IR coupling $\alpha_{\rm eff}(Q^2)$ measured at $Q^2 > 0$ is the Fourier transform of the bulk coupling $\alpha_s(z) = g_s^2(z)/4\pi$, integrated in the transverse LF plane

$$\alpha_{\rm eff}(Q^2) \equiv \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s(\zeta)$$
$$= \alpha_{\rm eff}(0) e^{-Q^2/4\kappa^2}$$

where $\alpha_s(\zeta) \sim e^{-\kappa^2\zeta^2}$ from the LF mapping $z \to \zeta$, $\lambda \to \kappa^2$, the result found in

S. J. Brodsky, GdT and A. Deur, PRD **81**, 096010 (2010)

Comparison with experiment: IR domain



HLFQCD prediction (2010) for $\alpha_{\mathrm{eff}}(Q^2)$ compared with experimental results (including recent JLab data) in the g_1 scheme where $\alpha_{\mathrm{eff}}(0) = \pi$. The grey curve corresponds to $\kappa = 0.534$ GeV extracted from the hadron spectrum. The IR gaussian regime is valid up to $Q^2 \simeq 4\kappa^2 \simeq 1\ \mathrm{GeV}^2$. For $Q^2 > 4\kappa^2$ the semiclassical approximation is no longer valid and quantum corrections become important

Continuity of physical observables in QFT

Transition between the nonperturbative exponential fall off of the coupling in the IR and the logarithmic decrease in the UV domain from asymptotic freedom

Point matching: Matching α_s and $\beta(\alpha_s)$ at single point $Q_0 \simeq 1$ GeV gives a good description of IR and UV data and avoids the Landau singularities by construction
A. Deur, S. J. Brodsky, and GdT, PLB 750, 528 (2015); 757, 275 (2016)

Present approach: Use analyticity properties of QFT which require the continuity of physical observables to describe the continuous transition between IR and UV

We choose an effective charge (Grunberg) defined by an observable required by the analytic construction of the present approach

We choose the effective charge in the g_1 scheme defined by the Bjorken sum rule

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\left[g_1^p(x,Q^2) - g_1^n(x,Q^2)\right], \quad \alpha_{g_1}(0) = \pi$$

which is well measured in the IR and UV and is a physical observable

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QCD Running Coupling in the Nonperturbative and Near-Perturbative Regimes

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Extension of the effective coupling to the near-IR region

Our starting point is the analytic expression

$$\alpha_{\rm eff}(Q^2) = \alpha_{\rm eff}(0)\, \exp\left(-\int_0^{Q^2} \frac{du}{4\kappa^2 + u\, \ln\left(\frac{u}{\Lambda^2}\right)}\right)$$

which evolves between the IR nonperturbative exponential form

$$\alpha_{\rm eff}(Q^2) \to e^{-Q^2/4\kappa^2}, \quad \text{for} \quad Q^2 \ll 4\kappa^2,$$

and the logarithmic Q^2 dependence characteristic of asymptotic freedom

$$\alpha_{\rm eff}(Q^2) \to \frac{1}{\ln\left(Q^2/\Lambda^2\right)}, \quad \text{for} \quad Q^2 \gg 4\kappa^2$$

Question: How does the IR confinement scale κ relates to the log evolution scale Λ ?

Question: How to remove the IR singularities which have frustrated previous attempts to describe the IR-UV transition region?

Both questions are related and their solution stems from the study of the flow of singularities into the complex Q^2-plane ($Q^2 \geq 0$)

Singularity flows and confinement

Analytic continuation of $\alpha_{\rm eff}(Q^2)$ generates the flow of singularities in the complex Q^2-plane which determines the possible solutions

The actual flow follows from the equation

$$4\kappa^2 + Q^2 \ln\left(\frac{Q^2}{\Lambda^2}\right) = 0$$

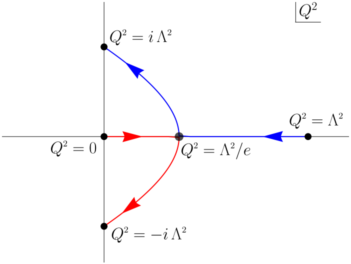
Its solutions are given in terms of the Lambert function $W_k(z)$

$$Q_u^2(\kappa^2) = \Lambda^2 \exp\left[W_0\left(-\frac{4\kappa^2}{\Lambda^2}\right)\right]$$

and

$$Q_l^2(\kappa^2) = \Lambda^2 \exp\left[W_{-1}\left(-\frac{4\kappa^2}{\Lambda^2}\right)\right]$$

for fixed Λ, where u and l are the solutions in the upper and lower Q^2 half-planes



κ^2	Q_u^2	Q_l^2
0	Λ^2	0
$\Lambda^2/4e$	Λ^2/e	Λ^2/e
$\pi\Lambda^2/8$	$i\Lambda^2$	$-i\Lambda^2$

It is also useful to express κ as a function of Q^2 along the solution locus

$$\kappa^2(Q^2) = -\frac{1}{4} Q^2 \ln\left(\frac{Q^2}{\Lambda^2}\right), \quad \text{with} \quad Q^2 \to Q^2_{u,l}$$

For $\kappa^2 \leq \Lambda^2/4e$ α_{eff} is not defined: Singularities located in the real axis where integration is done

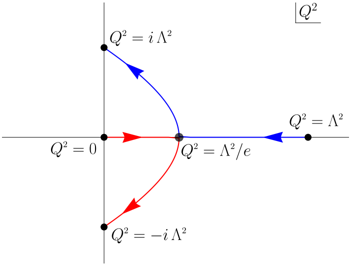
Critical value $\kappa^2 = \Lambda^2/4e$ is the maximal value of κ along the real axis. It corresponds to the bifurcation point $Q^2 = \Lambda^2/e$ where a double pole is split into two complex conjugate singularities

For $\kappa^2 > \Lambda^2/4e$ the effective coupling $\alpha_{\rm eff}$ can be computed for any value of Q^2

The maximal separation in the imaginary axis is an upper bound of κ for a given Λ

$$\kappa^2 = \frac{\pi}{8}\Lambda^2,$$

It leads to a unique relation between the scales relevant at large and short distances



κ^2	Q_u^2	Q_l^2
0	Λ^2	0
$\Lambda^2/4e$	Λ^2/e	Λ^2/e
$\pi\Lambda^2/8$	$i\Lambda^2$	$-i\Lambda^2$

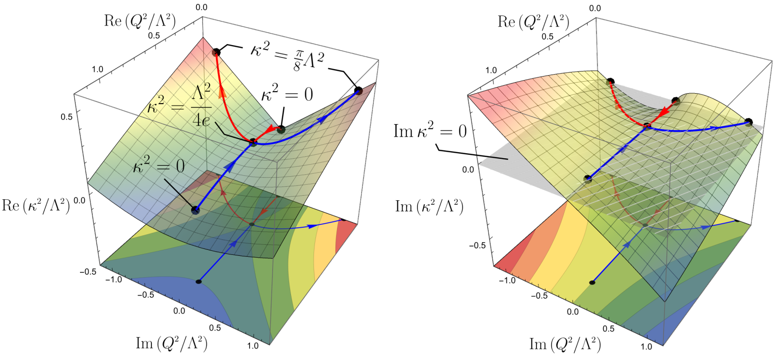
It also implies that α_{eff} is an holomorphic function in the full $Q^2 > 0$ complex plane: It corresponds to maximal analyticity, namely the largest possible domain in the Q^2-Euclidean plane, compatible with general principles of QFT for an observable

The Cauchy-Riemann differential equations

$$\nabla^2 \mathrm{Re}\, \kappa^2 = 0, \quad \nabla^2 \mathrm{Im}\, \kappa^2 = 0$$

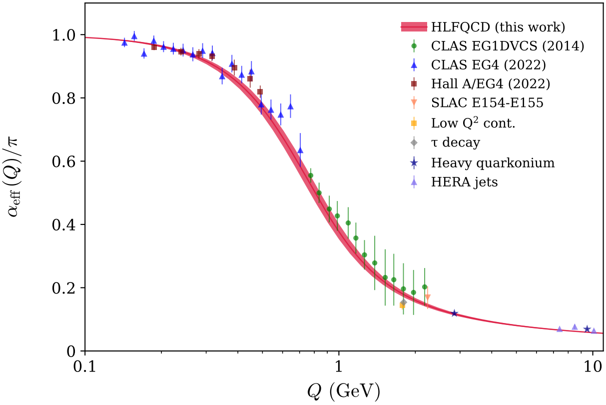
imply that a maximum of $\mathrm{Re}\, \kappa^2$ along the real Q^2-axis corresponds to a minimum of $\mathrm{Re}\, \kappa^2$ along the imaginary direction, thus to a saddle point

The bifurcation point $Q^2 = \Lambda^2/e$ for the critical value $\kappa^2 = \Lambda^2/4e$ is a saddle point !



Singularity flow for the Re and Im components of κ^2. The bifurcation point of the flow in the Q^2-plane is a saddle point for $\mathrm{Re}\, \kappa^2$ (left). The intersection of the surface $\mathrm{Im}\, \kappa^2$ with the plane $\mathrm{Im}\, \kappa^2 = 0$ illustrates that κ^2 is real along the flow (right)

Comparison with experiment: IR-UV transition domain



Comparison of the effective strong coupling α_{eff} with experimental results (including recent JLab data) in the g_1 scheme where $\alpha_{\rm eff}(0) = \pi$. The red band corresponds to $\kappa = 0.534 \pm 0.025$ GeV extracted from the hadron spectrum and to maximal analyticity

Conclusion and outlook

We have presented a brief overview of key theoretical points, unique connections and physical facts that support the HLFQCD approach and ideas

This new approach to hadron physics leads to universal Regge trajectories and supersymmetry relations between mesons, baryons and tetraquarks, an effective supersymmetry

We have extended previous holographic results for the strong coupling into the IR-UV transition domain. The analyticity-based approach leads to a continuous transition as required by observables in QFT. The procedure removes the IR singularities which have frustrated previous attempts to describe the coupling in IR-UV transition region. It allows to describe the available data in this domain

Extension of the present nonperturbative results to the deep UV domain, presently under consideration, combines exact results from the renormalization group for non-Abelian gauge theories with the model presented. Our analysis leads to precise constraints between the deep IR and UV behavior and yields to a description of the effective coupling from 0 to 2 TeV in agreement with experiment