Cosmological models in the scalar-tensor theories of gravity, consistent with observational constraints

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Introduction

The presented results are the part of the paper S.V. Chervon, I.V. Fomin, N.A. Koshelev, V.M. Zhuravlev "Scalar-tensor theories of gravity, consistent with observational constraints", which is submitted to "Space, Time and Fundamental Interactions".

Action

$$S = \frac{1}{2\kappa} \int \left\{ F(\phi)R - \omega(\phi) g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - 2V(\phi) \right\} \sqrt{-g} d^4x + S_m. \tag{1}$$

The aim of this work is to study the agreement of the proposed models with constraints on special parameters of cosmological perturbations [1], [2]:

$$n_s = 0.9649 \pm 0.0042, \tag{2}$$

$$r < 0.035, \tag{3}$$

$$\mathcal{P}_s = 2.1 \times 10^{-9}, \tag{4}$$

where n_s is the spectral index of scalar perturbations, $\mathcal{P}_s$ is the power spectrum of scalar perturbations, r is the tensor-to-scalar ratio.

Exponential power-law inflation model

In the paper [3], cosmological models based on scalar-tensor gravity with a function F of the form

$$F(t) = \left(1 + \frac{m}{\lambda t}\right)^2. \tag{5}$$

were considered. It was also assumed

$$H(t) = \lambda + mt^{-1}. \tag{6}$$

This form of Hubble parameter corresponds to the exponential power-law growth of the scale factor

$$a(t) \sim t^m e^{\lambda t}. \tag{7}$$

In [3], where this model was proposed, a special attention is paid to the case $\lambda < 0$. For $\lambda < 0$, the evolution of the model ends with compression. The Universe enters the regime of eternal compression at time

$$t_{cont} = -\frac{m}{\lambda}. \tag{8}$$

At moments

$$t_{1,2} = t_{cont} \mp \sqrt{\frac{t_{cont}}{|\lambda|}}. \tag{9}$$

acceleration is zero

$$\ddot{a}(t_{1,2}) = 0. \tag{10}$$

One can assume that the model is applicable up to time

$$t_1 = t_{cont} - \sqrt{\frac{t_{cont}}{|\lambda|}} = -\frac{m}{\lambda} + \frac{\sqrt{m}}{\lambda}. \tag{11}$$

Let's define the slow-roll parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{m}{(m+\lambda t)^2}, \quad \delta_F \equiv \frac{\dot{F}}{HF} = -2\epsilon, \quad \delta \equiv -\frac{\ddot{H}}{2H\dot{H}} = \frac{1}{m+\lambda t}. \tag{12}$$

The first slow-roll parameter ϵ is growing.

The slow-roll inflation ends at $\epsilon = 1$, which is equivalent to (10). For the moment of time when this occurs, we obtain

$$t_e = -\frac{m}{\lambda} + \frac{\sqrt{m}}{\lambda} = t_1. \tag{13}$$

The moment t_i, which corresponds to ΔN Hubble e-folds, is a solution of the equation

$$\Delta N = m \ln\left(\frac{t_1}{t_i}\right) + \lambda\left(t_1 - t_i\right). \tag{14}$$

This equation can be simplified by introducing the quantities

$$\tau_i = -\lambda t_i, \qquad \tau_1 = -\lambda t_1 = m - \sqrt{m}. \tag{15}$$

In terms of these quantities, the equation takes the form

$$\Delta N = m \ln\left(\frac{\tau_1}{\tau_i}\right) - \tau_1 + \tau_i. \tag{16}$$

For each value $m > 1$ there is a solution. This solution can be expressed in terms of the Lambert W-function:

$$\tau_i = -m \cdot W_0\left(-\left(1 - \frac{1}{\sqrt{m}}\right) e^{-\left(1-\frac{1}{\sqrt{m}}\right)} e^{-\frac{\Delta N}{m}}\right), \tag{17}$$

where $W_0(x)$ is the principal branch of Lambert W-function.

The basic formulae for calculating the parameters of cosmological perturbations are given in [3] (see also [4],[5]).
The spectral index of scalar perturbations is calculated as

$$n_s - 1 = -2\epsilon - \delta_Q, \qquad \delta_Q \equiv \frac{\dot{Q}}{HQ}, \tag{18}$$

where

$$Q = F\frac{2\dot{H}^2 - H\ddot{H}}{\left(H^2 + \dot{H}\right)^2} \tag{19}$$

and the calculation is performed at the moment of Hubble crossing, for $\Delta N \approx 60$ Hubble e-folds before the end of inflation.
In terms of τ_i, one can obtain

$$n_s - 1 = -\frac{1}{m}\frac{1}{1 - \tau_i/m} - \frac{4\tau_i/m}{m\left(1 - \tau_i/m\right)^2 - 1}. \tag{20}$$

This equation allows us to find the spectral index as a function of the parameter m. At $m > 50$, the spectral index of scalar perturbations is consistent with the constraints.

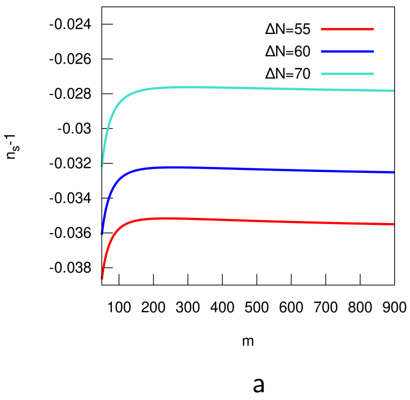
The tensor-to-scalar ratio r is given by

$$r \equiv 16\epsilon_s = 16\frac{Q}{F}. \tag{21}$$

For the model under consideration, in terms of τ_i, this quantity can be written as

$$r = \frac{32\tau_i/m}{\left(m\left(1-\tau_i/m\right)^2 - 1\right)^2}. \tag{22}$$

In particular, for $m = 100$ and $\Delta N = 60$ we obtain $r = 0.00289$. It is evident that these are models with an extremely low tensor-to-scalar ratio, and therefore satisfy the observational constraints.



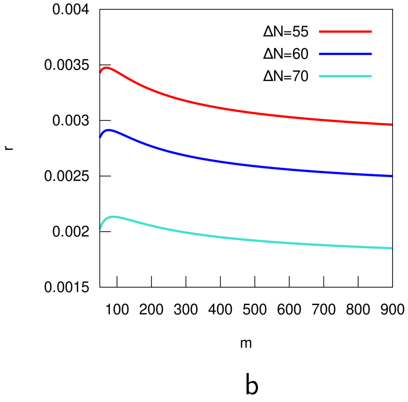


Рис.: Index $n_s - 1$ (a) and tensor-to-scalar ratio r (b) as functions of the parameter m on the interval (50,900) for three values of ΔN.

A model with quasi-exponential inflation of a special type

The paper [6] introduce another class of models of the type (1), which allow vacuum cosmological expansion by the law

$$a(t) \sim e^{\lambda t} e^{-\xi t^2/2}. \tag{23}$$

Such models are obtained at

$$F(t) = \left(\frac{H(t)}{\lambda}\right)^2, \qquad H(t) = \lambda - \xi t, \qquad \lambda > 0. \tag{24}$$

On large times, at

$$t > t_{cont} \equiv \frac{\lambda}{\xi}, \tag{25}$$

there is a compression of the universe. At times

$$t < t_1 \equiv t_{cont} - \frac{1}{\sqrt{\xi}} \tag{26}$$

the expansion has an inflationary character ($\ddot{a} > 0$).
We take t_1 as the end of the slow-roll inflation. At that moment we have $\epsilon(t_1) = 1$.

The calculation of the parameters of cosmological perturbations is based on the previous equations (18), (19), (21).
The obtained results are conveniently written using dimensionless time.

$$\tau_i \equiv \sqrt{\xi} t_i. \tag{27}$$

In particular, we obtain

$$n_s - 1 = -\frac{4}{\left(\frac{\lambda}{\sqrt{\xi}} - \tau_i\right)^2 - 1}, \tag{28}$$

$$r = 2\left(n_s - 1\right)^2. \tag{29}$$

One can see that the tensor-to-scalar ratio turns out to be a quadratic function of $n_s - 1$. Moreover, if $n_s - 1$ satisfies the observational constraints, then r also satisfies corresponding ones.

The obtained result can be rewritten in terms of Hubble e-folds:

$$n_s - 1 = -\frac{2}{\Delta N}. \tag{30}$$

Equations (29) and (30) turn out to be quite sensitive to the number of e-folds between the horizon-crossing and the end of inflation. Assuming $\Delta N = 60$, we obtain

$$n_s - 1 \approx -0.033, \quad r \approx 0.00218, \tag{31}$$

in excellent agreement with observational data.
It remains only to check the possibility of choosing a suitable λ. This value can be chosen by condition

$$\mathcal{P}_s = \frac{H^2}{8\pi^2 Q} = \frac{\lambda^2}{4\pi^2}\Delta N^2 = 2.1 \times 10^{-9}. \tag{32}$$

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