Multidimensional gravity, black holes and mirror stars

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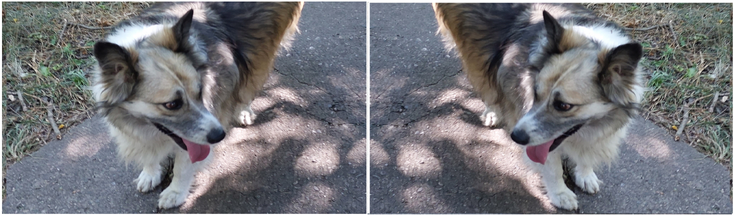

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Plan of the talk

- Compact strong-gravity objects in GR and beyond
- Reflection properties of many objects, echoes
- Multidimensional gravity. Spherically symmetric space-times, unusual horizons
- Example: a 5D Schwarzschild BH and a mirror horizon
- More general scalar-electrovacuum space-times
- Mirror stars in 5D GR with nonlinear electrodynamics
- Concluding remarks

Compact strong-gravity objects

- **Black holes:**
  Extremal and non-extremal, singular and regular, D-dimensional etc.
- **Boson stars:** scalar or vector fields, real or complex...
  "Quasi-BHs": field configurations close to BHs but without horizons
  Naked singularities, superspinars
- Compact stellar models with **anisotropic fluids** of any origin
  (models with extreme compactness close to BHs)
- **Wormholes:** static or dynamic ones, the exotic matter problem
- "Gravitational vacuum stars" (**gravastars**), related to QFT effects:
  dS/AdS cores, Schwarzschild exteriors, thin shells
- One more opportunity: **"mirror stars"**

Reflection phenomena, echoes

"To model different compact objects, one usually puts a partially reflecting surface slightly away from the location of the would-be horizon, such that the reflective surface is within the photon sphere, mimicking the ringdown waveform of a black hole"

(S. Biswas et al., arXiv: 2307.04836)

"Upon each interaction with the light ring, a fraction exits to outside observers, giving rise to a series of echoes of ever-decreasing amplitude."

(V. Cardoso, P. Pani, "Testing the nature of dark compact objects: a status report," arXiv: 1904.05363)

"Some studies have shown evidence of the echo signals from several binary black hole merger events.... On the other hand, the other studies have shown the low significance of such signals from various events in the first, second and third observing runs (O1, O2 and O3)."

(Nami Uchikata et al.,"Searching for gravitational wave echoes from black hole binary events in the third observing run of LIGO, Virgo, and KAGRA collaborations," arXiv: 2309.01894.)

Here: some more theoretical examples of reflecting compact objects.

Consider static, spherically symmetric metrics in a space-time $\mathbb{M}_D = \mathbb{M}_0^{(4)} \times \prod \mathbb{M}_i$ with Kaluza-Klein-like extra dimensions,

$$ds_D^2 = A(x)dt^2 - \frac{dx^2}{A(x)} - r^2(x)d\Omega^2 + \sum_i \mathrm{e}^{2\beta_i(u)} ds_i^2, \qquad (1)$$

where the factor spaces $\mathbb{M}_i$ $(\dim M_i = d_i)$ are compact and very small. There are quite a lot of BH solutions of multidimensional gravity, above all, GR.[1] Their **horizons** in (1): $A(x) = 0$ (regular zeros of $A(x)$).

Now: Suppose that among $\mathbb{M}_i$ there is some extra **1D** factor space parametrized by a variable v. Let us look what can happen if we consider the same BH metric (1) but replace

$$\boxed{dt \longleftrightarrow dv.}$$

The metric (1) will be again a solution to the same field equations, but the whole picture of space-time will drastically change! How? Let us try to make it clear using a simple example: 5D Schwarzschild space-time.

[1] For reviews see, e.g., G.T. Horowitz, T. Wiseman, arXiv: 1107.5563; S. Tomizawa, H. Ishihara, arXiv: 0903.3555; V.D. Ivashchuk, V.N. Melnikov, arXiv: gr-qc/0002085.

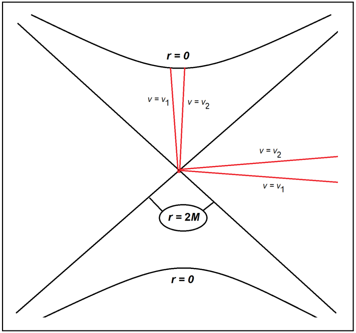
A 5D Schwarzschild BH and its "exotic" analog

5D Schw.: $ds_5^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 + \eta_v dv^2,$

where $\eta_v = \pm 1$ (v either timelike or spacelike). After the replacement:

$$ds_5^2 = \eta_v \left(1 - \frac{2m}{r}\right) dv^2 + dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2. \qquad (2)$$

If $\boxed{\eta_v = 1}$, compactness of v
$\Rightarrow$ a singularity at $r = 2m$.



If $\boxed{\eta_v = -1}$ (spacelike extra dimension), then at $r = 2m$, signature $(-+---) \mapsto (+++--)$. So we need to consider the neighborhood of $r = 2m$ in more detail, taking into account the compactness of the variable v, i.e., identification of its two (rather close) values.

Transform (2) to coordinates (x, y) making the metric manifestly nonsingular:

$$r - 2m = \frac{x^2 + y^2}{8m}, \qquad v = 4m \arctan\big(y/x\big),$$

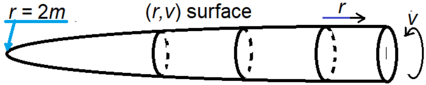
Then near $r = 2m$ the (r, v) surface metric is

$$ds_2^2(r, v) \approx \frac{r - 2m}{2m} dv^2 + \frac{2m}{r - 2m} dr^2 = dx^2 + dy^2. \qquad (3)$$

The horizon $r = 2m$ becomes the origin $x = y = 0$, where (3) is locally flat. The v coordinate behaves as a polar angle.

[Related: conformal mapping with the analytic function $\log z$, $z = x + \mathrm{i}y$, then $v \propto \arg z$; thus the origin = branch-point singularity.]

Compact extra dimension $v \quad \Rightarrow \quad 0 \le v < 2\pi\ell$, with ℓ = small length.



A classical geodesic passing $r = 2m$ returns back at another value of v.
A quantum particle with v-independent $\Psi(x^\mu)$ must simply return
$\Rightarrow$ the surface $r = 2m$ acts like a mirror!

5D Einstein-Maxwell, with F_{ab} of three types, with nonzero components: (i) electric ($F_{rt} = \partial_r A_t$), (ii) magnetic ($F_{\theta\phi} = \partial_\theta A_\phi$), (iii) quasiscalar ($F_{rv} = \partial_r A_v$).

Examples of mirror-star solutions (special cases of general solutions)

Electric: only a BH solution, $F_{tx} = \dfrac{q}{x^2(1+p/x)}$;

$$ds_5^2 = \frac{1-2k/x}{(1+p/x)^2}dt^2 - \left(1+\frac{p}{x}\right)\left[\frac{dx^2}{1-2k/x} + x^2 d\Omega^2 - \eta_v dv^2\right]. \qquad (4)$$

Magnetic: $F_{\theta\phi} = q\sin\theta$;

$$ds_5^2 = -\frac{1-2k/x}{1+p/x}dv^2 + \frac{dt^2}{1+p/x} - \left(1+\frac{p}{x}\right)^2 \left[\frac{dx^2}{1-2k/x} + x^2 d\Omega^2\right]. \qquad (5)$$

Quasiscalar: $F_{xv} = \dfrac{q^2}{x^2(1+p/x)(k+p-kp/x^2)}$;

$$ds_5^2 = -\frac{q^2(1-2k/x)}{(k+p-kp/x)^2}dv^2 + \left(1+\frac{p}{x}\right)\left[dt^2 - \frac{dx^2}{1-2k/x} - x^2 d\Omega^2\right]. \qquad (6)$$

In all cases q is the corresponding charge, $p = \sqrt{k^2+q^2} - k$, and $k > 0$ is an integration constant related to the mass.

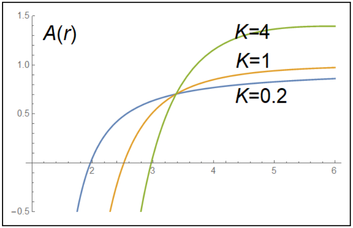
In 4D, static, spherically symmetric solutions (electric or magnetic) for GR+NED are obtained easily, (see, e.g., Pellicer and Torrence, 1969; K.B., arXiv: gr-qc/0006014, etc.), with many examples of BH solutions.

In 5D the problem is much harder, but some examples have been obtained. Here is one in 5D GR + NED ($L_{\rm NED}$ = function of $F = F_{ab}F^{ab}$), with a radial magnetic field (here, $L_{\rm NED}$ is given by a very large expression):

$$ds_5^2 = -A(r)dv^2 + \left(1 - \frac{m}{r}\right)^2 dt^2 - \frac{16r^4 dr^2}{(2r-m)^4 A(r)} - r^2 d\Omega^2, \qquad (7)$$

where $A(r) = \left(1 - \frac{m}{r}\right)\left\{1 - 2K\left[\frac{4m^2 - 10mr + 5r^2}{(r-m)^2(2r-m)} + \frac{6}{m}\log\frac{2(r-m)}{2r-m}\right]\right\}$.

Here m is the mass, and K is an integration constant determining the position of the mirror horizon $r_{\rm hor} > m$.



The plots show $A(r)$ for some values of K, assuming $m = 1$. The mirror horizons are located at zeros of $A(r)$ at $r > 1$, that is, outside a would-be BH horizon.

Conclusion and outlook

- We have constructed some examples of 5D models of objects with nonpenetrable horizons that reflect falling particles ([blue **"mirror stars'**'). The mentioned electrovacuum models are special cases of much more general and complicated D-dimensional solutions obtained previously (e.g., K.B., arXiv: gr-qc/9505020).
- It is clear that such solutions to multidimensional gravity equations are almost as numerous as BH ones.
- It is also clear that if such objects do exist, the falling matter does not disappear from our sight (as happens with BHs) but is stored in their environment.
- Many further studies are necessary with mirror star solutions:
  - a search for new solutions of interest (not only spherical),
  - their stability,
  - quasinormal modes,
  - particle motion in their field,
  - weak and strong lensing, etc.

THANK YOU!