The effect of tidal deformations on the gravitational wave signal during neutron star mergers

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Motivation

- 2015 year - direct registration of the gravitational signal;
- 2017 year - Nobel prize for LIGO and gravitatinal waves detecton;
- Three large gravitational wave interferometers are planned to be launched in the 2030s;
- it will help to get closer to understanding the equation of state for neutron stars;
- development of high energy physics.

Figure: Logos of key projects in gravitational wave research

The aim of this research

- To analyze the current results obtained for modeling the gravitational signal from merging neutron stars;
- divide the process of obtaining the form of gravitational waves into stages;
- implement calculation of $x(t)$, $\omega(t)$ in maxima.

Stages of model development for neutron star

- BBH: PN for inspiral + BoB model
- BBH: Combining methods to create a basic waveform
- BNS: adding tidal effects via model NRTidal
- BNS: verification with the results of numerical simulation
- BNS: lifting the restriction on the independence of the type of functions ϕ and A from parameters Λ (tidal deformability)

Basic concepts and formulas

- PN-model: Correction of Newton's equation by powers of $x = (v/c)^2$
- BoB-model: obtaining system parameters from end object parameters;
- NRTidal-model: adds corrections for tides to analytical formulas, is implemented in the library of algorithms from LIGO and VIRGO, uses assumptions about the equation of state of objects.
- Parameter h (tidal correction strain)
- k_2 – Love number, describes how much a star is deformed by tidal forces;
- Parameter $\Lambda = \frac{2}{3}k_2(\frac{c^2 R}{GM})^5$ (dimensionless tidal deformability) – scales this deformation based on the radius and mass of the star

Basic formulas

- $$M\frac{dx}{dt}|^{6PN} = \frac{64}{5}\eta x^5(1+\sum_{k=2}^{12} a_{k/2}x^{k/2}) \tag{1}$$
- $$M\frac{d\Phi_{orb}}{dt} = M\omega_{orb} = x^{3/2} \tag{2}$$
- $$\Omega_{hyb}(t) = (1-\sigma(t))_{PN}(t-t_t) + \sigma(t)\Omega_{BoB}(t-t_i) \tag{3}$$
- adding tidal effects to tension:
$$h_T(t) = A_T(t)e^{-2i\phi_T(t)} \tag{4}$$
- Parameter $\tilde{\Lambda}_A = \frac{16}{13}\frac{(M_A+12M_B)M_A^4\Lambda_A}{M^5}$

Practical part

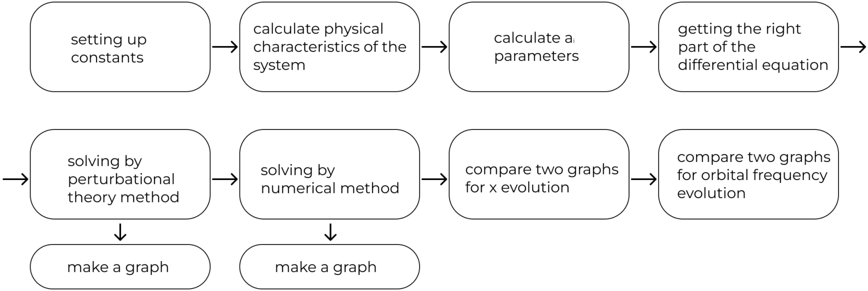

Figure: the way code on maxima works

Graphs for x(t)

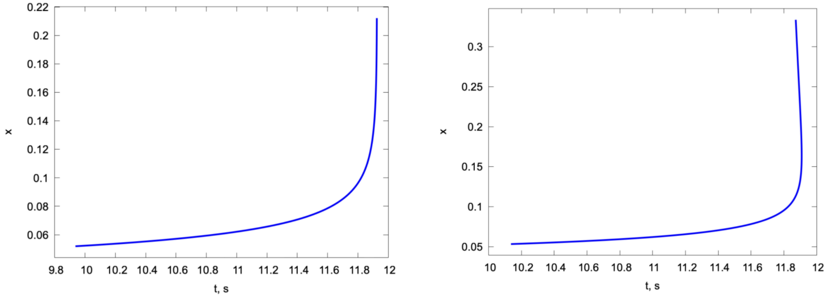

Figure: Graphs for x(t) and (a) Runge-Kutta method and (b) perturbation theory method

Comparison of two methods

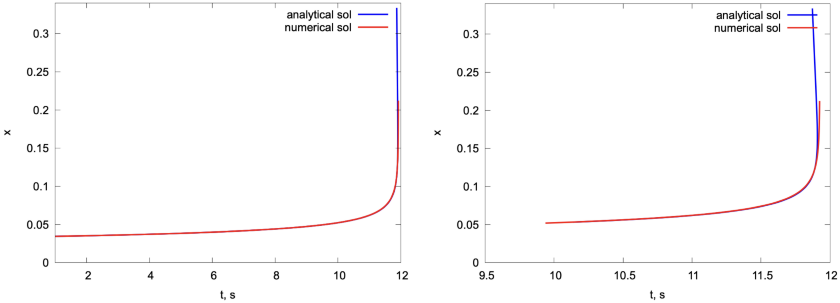

Figure: graphs for x(t): comparison

Graphs for $\omega(t)$

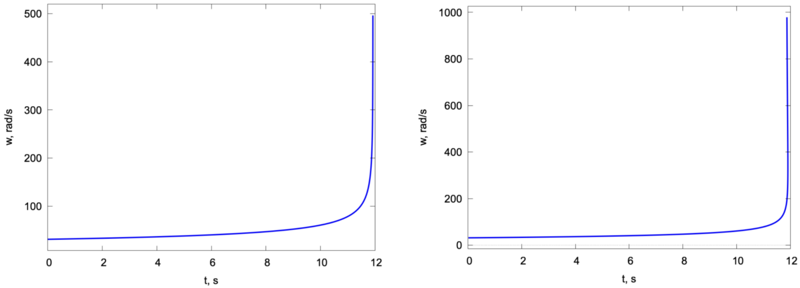

Figure: Graphs for $\omega(t)$ for (a) Runge-Kutta method and (б) perturbation theory method

Graphs for $r_{sep}(t)$

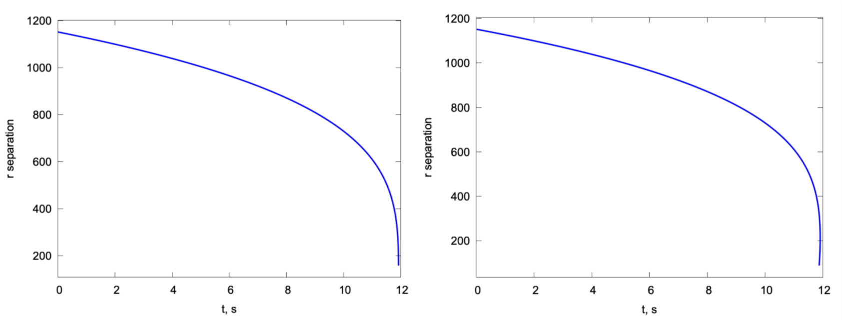

Figure: Graphs for $r_{sep}(t)$ for (a) Runge-Kutta method and (б) perturbation theory method

Results

- An analysis of modern results on modeling the shape of the gravitational signal from the merger of neutron stars is carried out, taking into account tidal effects.
- The work of reproducing and verifying the results of the original article is divided into stages.
- As part of the implementation of the first stage, the coefficients for the differential equation for the post-Newtonian parameter were calculated.
- These coefficients are compared with the coefficients from the original article.
- The equation was solved using the Runge-Kutta method (numerically) and the perturbation theory method, that is, analytically.
- The results of the solutions were compared with the solution presented in the original article.
- The evolution of the cyclic frequency was calculated using both numerical and analytical methods.

Future plans

- It is necessary to continue this work by calculating the shape of the gravitational wave signal from merging black holes.
- The next step will be to apply the code to a neutron star system, but before that it will be necessary to add tidal effects to the code, as described in the theoretical section of this paper.
- It is also necessary to evaluate the errors of all the methods used in order to better understand which of them gives a more accurate result.

Key concepts

Neutron stars

collapsed core of a massive supergiant star (8-20 solar masses).

Black Holes

Objects predicted by Schwarzschild, who found the first modern solution of Einstein's equation. Their specific feature is the existence of singularity and event horizon, which is the boundary of no escape (the velocity needed to escape exceeds the velocity of light)

Gravitational waves

- are caused by the orbits of massive binary objects such as black holes and neutron stars and spread out as ripples in the fabric of space-time;
- predicted by Einstein as a consequence of general relativity theory in 1916;
- 14.09.2015 detected by LIGO for the first time.