New slow-roll approximations for inflation in Einstein-Gauss-Bonnet gravity

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Introduction

- There are the observational bounds on the inflationary parameters: the amplitude of scalar perturbations $A_s = (2.10 \pm 0.03) \times 10^{-9}$, the spectral index $n_s = 0.9654 \pm 0.0040$ and the tensor-to-scalar ratio $r < 0.028$.
- The models with the minimally coupled scalar field and the quadratic and quartic potential give rise too large value of ***r*** compared to the observed. An unusually large coupling constant is required to agree with observations of the models with the nonminimal coupling with the Higgs field.
- It is intresting that the addition to the system the non-minimal coupling to the Gauss-Bonnet term allows to reduce ***r*** to the observation range, while it does not change n_s.

Introduction

Many inflationary models with the Gauss-Bonnet term (**Z.-K. Guo** and **D. J. Schwarz**, 2010) include the function $\xi(\varphi)=C/V(\varphi)$, where ***C*** is a constant. In the case of monomial potential the function $\xi(\varphi)$ rapidly raises at the end of inflation, so the slow-roll approximation is violated and there are problems with the exit from the inflation (**C. van de Bruck** and **C. Longden**, 2016). Such situation is typical for the inflationary models with the Gauss-Bonnet term (**E. O. Pozdeeva**, 2021).

This problem can be solved by the addition of a small positive constant ***Λ*** to $\xi(\varphi)$**:** $\xi(\varphi)=C/(V(\varphi)+\Lambda)$, where the constants ***C*** and ***Λ*** are positive.

Goal of the work

The goal of the present work was to compare different slow-roll approximations with the numerical calculations and find more suitable from them for the description of the inflation in the models of Einstein-Gauss-Bonnet gravity with following Lagrangian

$$L=\sqrt{-g}\left[U_0 R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V(\varphi)-\frac{1}{2}\xi(\varphi)\tilde{G}\right],$$

where $\tilde{G}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-R_{\mu\nu}R^{\mu\nu}+R^2$ – Gauss-Bonnet term,

$$V(\varphi)=V_0\varphi^n,\ \ \xi(\varphi)=\frac{\xi_n}{V(\varphi)+\Lambda},\ V_0>0,\ n>0,\ \xi_n>0,\ \Lambda>0\ .$$

The FLRW metric $ds^2=-dt^2+a^2(t)\,dl^2$
and Planck units are used $c=\hbar=1$.

Methods of the investigation

Methods of the numerical integration,

algebraic methods

are applyed in this work.

Main equations

Equations of the gravitation and the scalar fields are derived by varying the action with the Lagrangian

$$12\,H^2\left(U_0-2\,\xi_\varphi\dot{\varphi}\,H\right)=\varphi^2+2\,V(\varphi), \qquad (1)$$

$$4\,\dot{H}\left(U_0-2\,\xi_\varphi\,\dot{\varphi}\,H\right)=-\varphi^2+4\,H^2\left(\xi_{\varphi\varphi}\dot{\varphi}^2+\xi_\varphi\,\dot{\varphi}-\xi_\varphi\,\dot{\varphi}\,H\right), \qquad (2)$$

$$\ddot{\varphi}+3\,H\,\dot{\varphi}=-V_\varphi-12\,H^2\,\xi_\varphi\left(\dot{H}+H^2\right), \qquad (3)$$

where $\dfrac{dV}{d\varphi}=V_\varphi,\quad \dfrac{d\xi}{d\varphi}=\xi_\varphi,\quad \dfrac{d^2\xi}{d\varphi^2}=\xi_{\varphi\varphi},$

$$\boldsymbol{U_0=\frac{1}{16\pi G}=\frac{1}{{M_{Pl}}^2}}.$$

7 The slow-roll and the inflation parameters, the scalar perturbation propagation speed c_R^2 and the gravitation wave propagation speed c_T^2.

1. The slow-roll parameters:

$$\varepsilon_1=-\frac{\dot{H}}{H^2},\qquad \varepsilon_2=\frac{d\ln|\varepsilon_1|}{dN},\qquad \delta_1=\frac{2}{U_0}\xi_\varphi\dot{\varphi}H,\qquad \delta_2=\frac{d\ln|\delta_1|}{dN}.$$

Here $N = ln\ a$.

2. The inflation parameters:

$$r=8|2\varepsilon_1-\delta_1|,\qquad A_s=\frac{H^2}{\pi^2 U_0 r},\qquad n_s=1-2\varepsilon_1-\frac{2\varepsilon_1\varepsilon_2-\delta_1\delta_2}{2\varepsilon_1-\delta_1}.$$

3. $$c_R^2=1-\frac{\delta_1^2\left[2\varepsilon_1+\frac{1}{2}\delta_1(1-5\varepsilon_1-\delta_2)\right]}{(1-\delta_1)^2\left[2\varepsilon_1-\delta_1(1+\varepsilon_1-\delta_2)+\frac{3\delta_1^2}{2(1-\delta_1)}\right]},\qquad c_T^2=1+\frac{\delta_1(1-\varepsilon_1-\delta_2)}{(1-\delta_1)}.$$

The standard approximation

1. In order to find **the slow-roll parameters** we solve the system of the leading order equations

$$H^2(\varphi)\approx\frac{V}{6U_0},\qquad \dot H\approx-\frac{\dot\varphi^2}{4U_0}-\frac{\xi_\varphi H^3\dot\varphi}{U_0},\qquad \dot\varphi\approx-\frac{V_\varphi+12\xi_\varphi H^4}{3H}.$$

Using $V_{eff}(\varphi)=-\frac{U_0{}^2}{V(\varphi)}+\frac{1}{2}\xi(\varphi)$, we express (see **S.Yu. Vernov** et al, 2019, 2020).

$$\varepsilon_1(\varphi)=\frac{V_\varphi}{U_0}V_{eff\,\varphi},\qquad \varepsilon_2(\varphi)=-\frac{2V}{U_0}V_{eff\,\varphi}\left[\frac{V_{\varphi\varphi}}{V_\varphi}+\frac{V_{eff\,\varphi\varphi}}{V_{eff\,\varphi}}\right],$$

$$\delta_1(\varphi)=-\frac{2V^2}{3U_0{}^3}\xi_\varphi V_{eff\,\varphi},\qquad \delta_2(\varphi)=-\frac{2V}{U_0}V_{eff\,\varphi}\left[2\frac{V_\varphi}{V}+\frac{V_{eff\,\varphi\varphi}}{V_{eff\,\varphi}}+\frac{\xi_{\varphi\varphi}}{\xi_\varphi}\right],$$

2. The inflation parameters:

$$r(\varphi)=16\frac{V^2}{U_0{}^3}\left(V_{eff\,\varphi}\right)^2,\qquad A_s(\varphi)=\frac{U_0}{96\pi^2V\left(V_{eff}\right)^2},\qquad n_s(\varphi)=1+\frac{2}{U_0}\left(2VV_{eff\,\varphi\varphi}+V_\varphi V_{eff\,\varphi}\right).$$

The approximation I

In order to find **the slow-roll parameters** we solve the reduced system of the field equations

$$\begin{cases} H^2(\varphi,\delta_1)\approx\dfrac{V}{6(U_0-2\xi_\varphi\dot{\varphi}H)}=\dfrac{V}{6U_0(1-\delta_1)}\approx\dfrac{V}{6U_0}(1+\delta_1), \\ \dot{H}\approx-\dfrac{\dot{\varphi}^2+4H^3\xi_\varphi\dot{\varphi}}{4(U_0-2\xi_\varphi\dot{\varphi}H)}=\dfrac{\delta_1}{4\mathrm{U}_0(1-\delta_1)}\left(\dfrac{U_0\delta_1}{4\xi_\varphi{}^2H^2}+2H^2\right)\approx-\dfrac{H^2\delta_1}{2}-\dfrac{U_0\delta_1{}^2}{16\xi_\varphi{}^2H^2}-\dfrac{H^2\delta_1{}^2}{2}, \\ 3\dot{\varphi}H\approx-V_\varphi-12\xi_\varphi H^2(\dot{H}+H^2) \end{cases}$$

$$\boxed{\delta_1(\varphi)=-\frac{2\xi_\varphi\left(3U_0{}^2V_\varphi+\xi_\varphi V^2\right)}{3\left(3U_0{}^3+\xi_\varphi{}^2V^2\right)},}\qquad H^2(\varphi)\approx\frac{V\left(9U_0{}^3-6U_0{}^2\xi_\varphi V_\varphi+\xi_\varphi{}^2V^2\right)}{18U_0\left(3U_0{}^3+\xi_\varphi{}^2V^2\right)},$$

$$\dot{H}=\frac{\dot{\varphi}H}{2H^2}\frac{d(H^2)}{d\varphi}\quad\Rightarrow\quad\varepsilon_1=-\frac{\dot{H}}{H^2}=-\frac{\dot{\varphi}H}{2H^4}\frac{d(H^2)}{d\varphi}$$

$$\boxed{\begin{aligned}\varepsilon_1(\varphi)=&\frac{6U_0{}^2(3U_0{}^2V_\varphi+\xi_\varphi V^2)}{V\left(9U_0{}^3-6U_0{}^2\xi_\varphi V_\varphi+\xi_\varphi{}^2V^2\right)}\times\\ &\times\left(\frac{V_\varphi}{2V}+\frac{\xi_\varphi\xi_{\varphi\varphi}V^2+\xi_\varphi{}^2VV_\varphi-3U_0{}^2\xi_{\varphi\varphi}V_\varphi-3U_0{}^2\xi_\varphi V_{\varphi\varphi}}{9U_0{}^3-6U_0{}^2\xi_\varphi V_\varphi+\xi_\varphi{}^2V^2}-\frac{\xi_\varphi\xi_{\varphi\varphi}V^2+\xi_\varphi{}^2VV_\varphi}{3U_0{}^3+\xi_\varphi{}^2V^2}\right).\end{aligned}}$$

The approximation II

In order to find **the slow-roll parameters** we solve the reduced system of the field equations

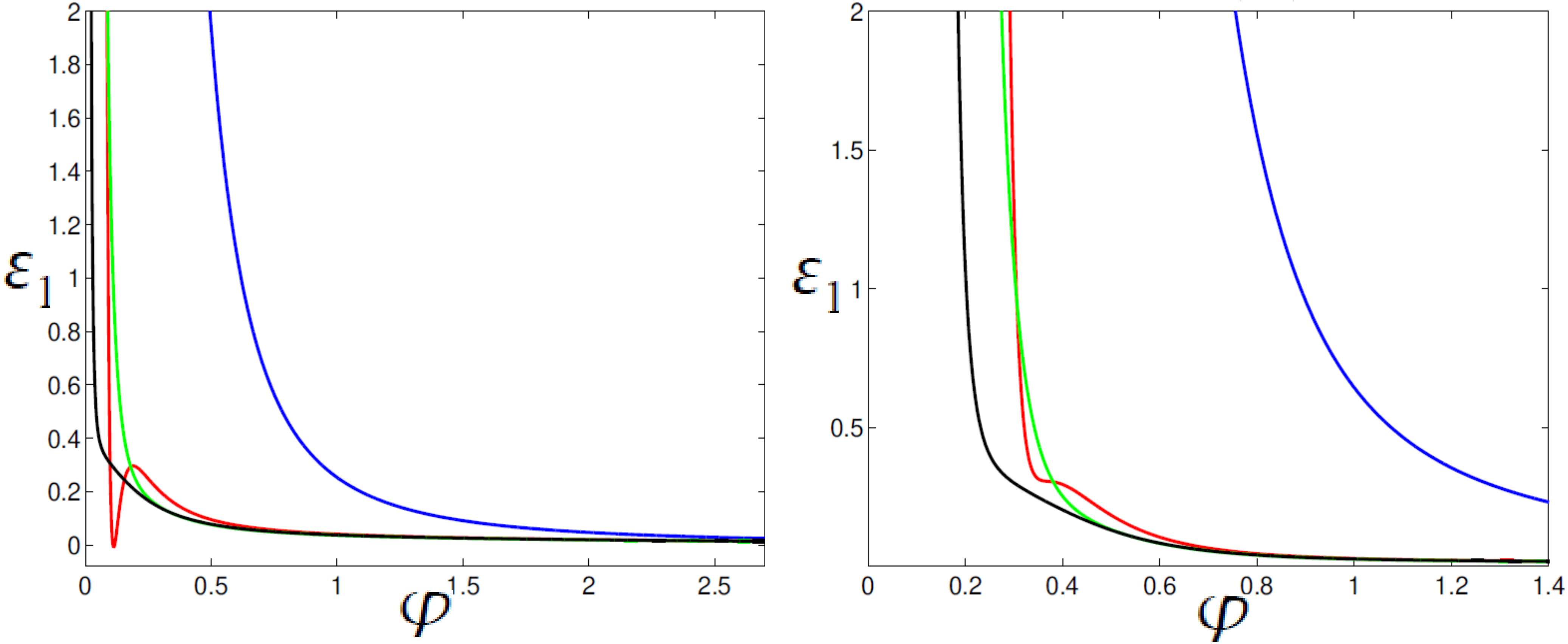
$$\begin{cases} H^2(\varphi,\delta_1)\approx\dfrac{V}{6(U_0-2\xi_\varphi\dot{\varphi}H)}=\dfrac{V}{6U_0(1-\delta_1)}, \\ \dot{H}=\dfrac{\dot{\varphi}H}{2H^2}\dfrac{d(H^2)}{d\varphi}, \\ 3\dot{\varphi}H\approx -V_\varphi-12\xi_\varphi H^2(\dot{H}+H^2) \end{cases}$$

$$\boxed{\delta_1(\varphi)=-\frac{2\xi_\varphi\left(3U_0^2V_\varphi+\xi_\varphi V^2\right)}{9U_0^2\left(U_0-\xi_\varphi V\right)},}\qquad H^2(\varphi)\approx\frac{3U_0V\left(U_0-\xi_\varphi V\right)}{2\left(9U_0^3-3U_0^2\xi_\varphi V_\varphi+2\xi_\varphi^2V^2\right)},$$

$$\dot{H}=\frac{\dot{\varphi}H}{2H^2}\frac{d(H^2)}{d\varphi}\quad\Rightarrow\quad \varepsilon_1=-\frac{\dot{H}}{H^2}=-\frac{\dot{\varphi}H}{2H^4}\frac{d(H^2)}{d\varphi}$$

$$\boxed{\begin{aligned}\varepsilon_1(\varphi)=&\frac{(3U_0^2V_\varphi+\xi_\varphi V^2)\left(9U_0^3-6U_0^2\xi_\varphi V_\varphi+\xi_\varphi^2V^2\right)}{27U_0^2V\left(U_0-\xi_\varphi V_\varphi\right)^2}\times\\ &\times\left(\frac{V_\varphi}{V}+\frac{\xi_{\varphi\varphi}V_\varphi+\xi_\varphi V_{\varphi\varphi}}{\xi_\varphi V_\varphi-U_0}+\frac{3U_0^2\xi_{\varphi\varphi}V_\varphi+3U_0^2\xi_\varphi V_{\varphi\varphi}-4\xi_\varphi\xi_{\varphi\varphi}V^2-4\xi_\varphi^2VV_\varphi}{9U_0^3-6U_0^2\xi_\varphi V_\varphi+\xi_\varphi^2V^2}\right).\end{aligned}}$$

Fig. 1. The dependence ε_1 on the scalar field φ for the model $V(\varphi)=V_0\varphi^n$, $\xi(\varphi)=\dfrac{\xi_n}{V(\varphi)+\Lambda}$.



$$n=2,\qquad \xi_2=0.6885\,M_{Pl}^{\ 4},$$

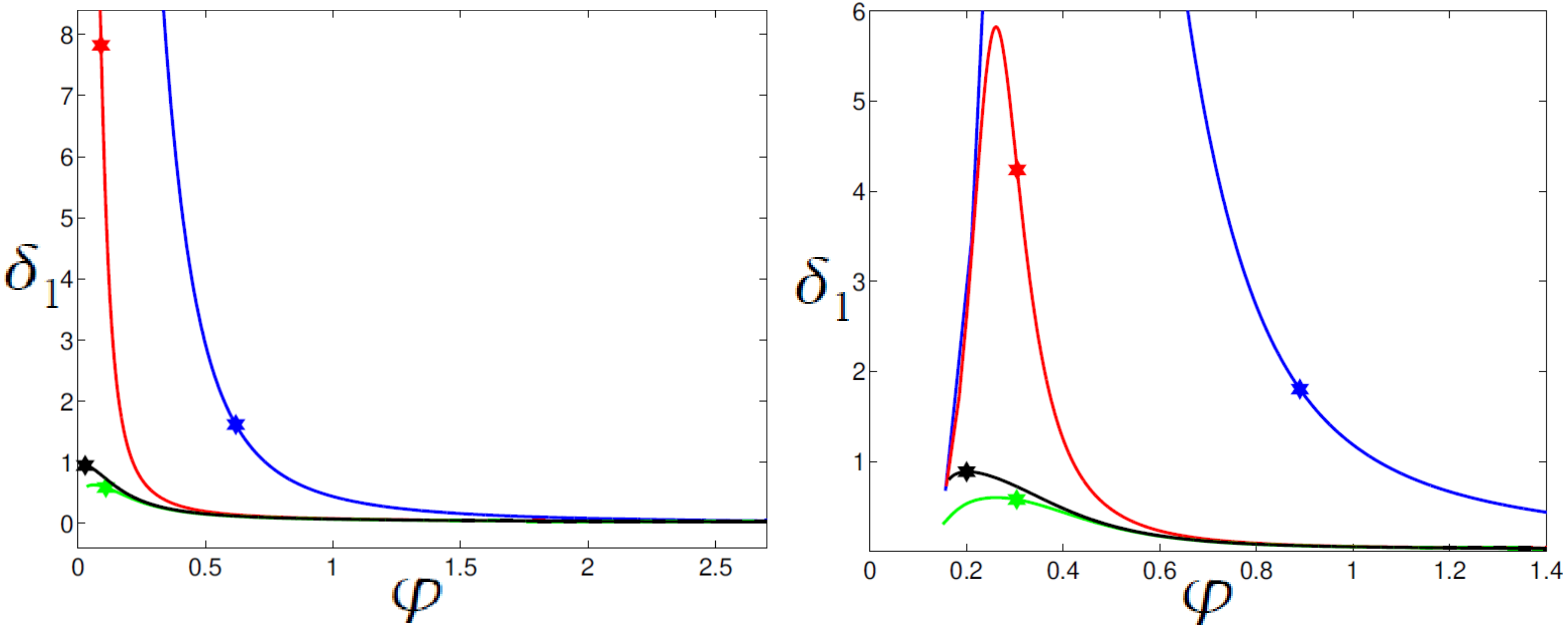
$$\Lambda=1.0125\cdot10^{-12}M_{Pl}^{\ 4},\qquad V_0=4.05\cdot10^{-11}M_{Pl}^{\ 4},$$

$$n=4,\qquad \xi_4=0.714\,M_{Pl}^{\ 4},$$

$$\Lambda=5.95\cdot10^{-13}M_{Pl}^{\ 4},\qquad V_0=3.4\cdot10^{-11},$$

$$U_0=\frac{M_{Pl}^{\ 2}}{2}=\frac{1}{2}$$

Fig. 2. The dependence δ_1 on the scalar field φ for the model $V(\varphi)=V_0\varphi^n$, $\xi(\varphi)=\dfrac{\xi_n}{V(\varphi)+\Lambda}$.



$n=2$, $\xi_2=0.6885\,M_{Pl}{}^4$,

$\Lambda=1.0125\cdot10^{-12}M_{Pl}{}^4$, $V_0=4.05\cdot10^{-11}M_{Pl}{}^4$,

$n=4$, $\xi_4=0.714\,M_{Pl}{}^4$,

$\Lambda=5.95\cdot10^{-13}M_{Pl}{}^4$, $V_0=3.4\cdot10^{-11}$,

$$U_0=\frac{M_{Pl}{}^2}{2}=\frac{1}{2}$$

Table 1, 2. Numerical and approximate values of the parameters for the model $V(\varphi)=V_0\varphi^n$, n=2.

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Parameter	Numerical result	Standard approximation	Approximation I	Approximation II
$\phi_0/M_{\rm Pl}$	2.7565	4.8472	2.9757	2.7082
$10^9 A_s(\phi_0)$	2.097	6.696	2.491	1.985
$n_s(\phi_0)$	0.965	0.971	0.967	0.965
$r(\phi_0)$	0.0102	0.0096	0.0099	0.0104
$\phi_{\rm end}/M_{\rm Pl}$	0.0294	0.6184	0.0906	0.1097
$\delta_1(\phi_{\rm end})$	0.950	1.62	7.82	0.590
$N(\phi_{\rm end})$	65.0	65.0	65.0	65.0

2

Parameter	Standard approximation	Approximation I	Approximation II
$\phi_{\rm in}/M_{\rm Pl}$	3.6589	2.7912	2.7676
$10^9 A_s(\phi_{\rm in})$	2.10	2.10	2.10
$n_s(\phi_{\rm in})$	0.947	0.965	0.966
$r(\phi_{\rm in})$	0.0174	0.0104	0.0102
$N(\phi_{\rm end}) - N(\phi_{\rm in})$	35.1	60.0	66.6

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Table 3, 4. Numerical and approximate values of the parameters for the model $V(\varphi)=V_0\varphi^n$, *n=4*.

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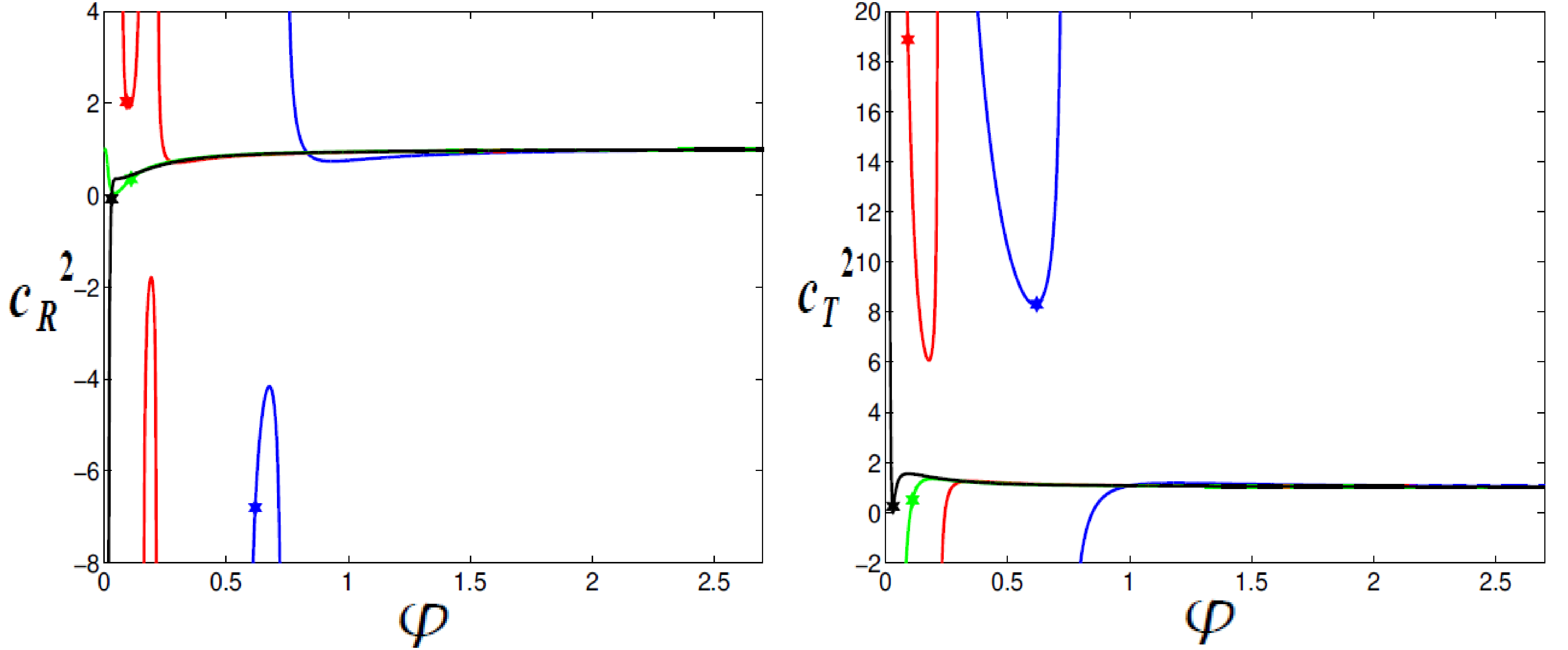
Parameter	Numerical result	Standard approximation	Approximation I	Approximation II
$\phi_0/M_{\rm Pl}$	1.4019	4.9705	1.4898	1.3974
$10^9 A_s(\phi_0)$	2.096	117.2	2.599	2.017
$n_s(\phi_0)$	0.965	0.953	0.965	0.965
$r(\phi_0)$	0.0044	0.0120	0.0045	0.0045
$\phi_{\rm end}/M_{\rm Pl}$	0.2017	0.8899	0.3048	0.3037
$\delta_1(\phi_{\rm end})$	0.885	1.80	4.23	0.577
$N(\phi_{\rm end})$	60.6	60.6	60.6	60.6

4

Parameter	Standard approximation	Approximation I	Approximation II
$\phi_{\rm in}/M_{\rm Pl}$	2.5555	1.4104	1.4116
$10^9 A_s(\phi_{\rm in})$	2.10	2.10	2.10
$n_s(\phi_{\rm in})$	0.817	0.964	0.965
$r(\phi_{\rm in})$	0.0466	0.0045	0.0045
$N(\phi_{\rm end}) - N(\phi_{\rm in})$	13.5	54.6	61.8

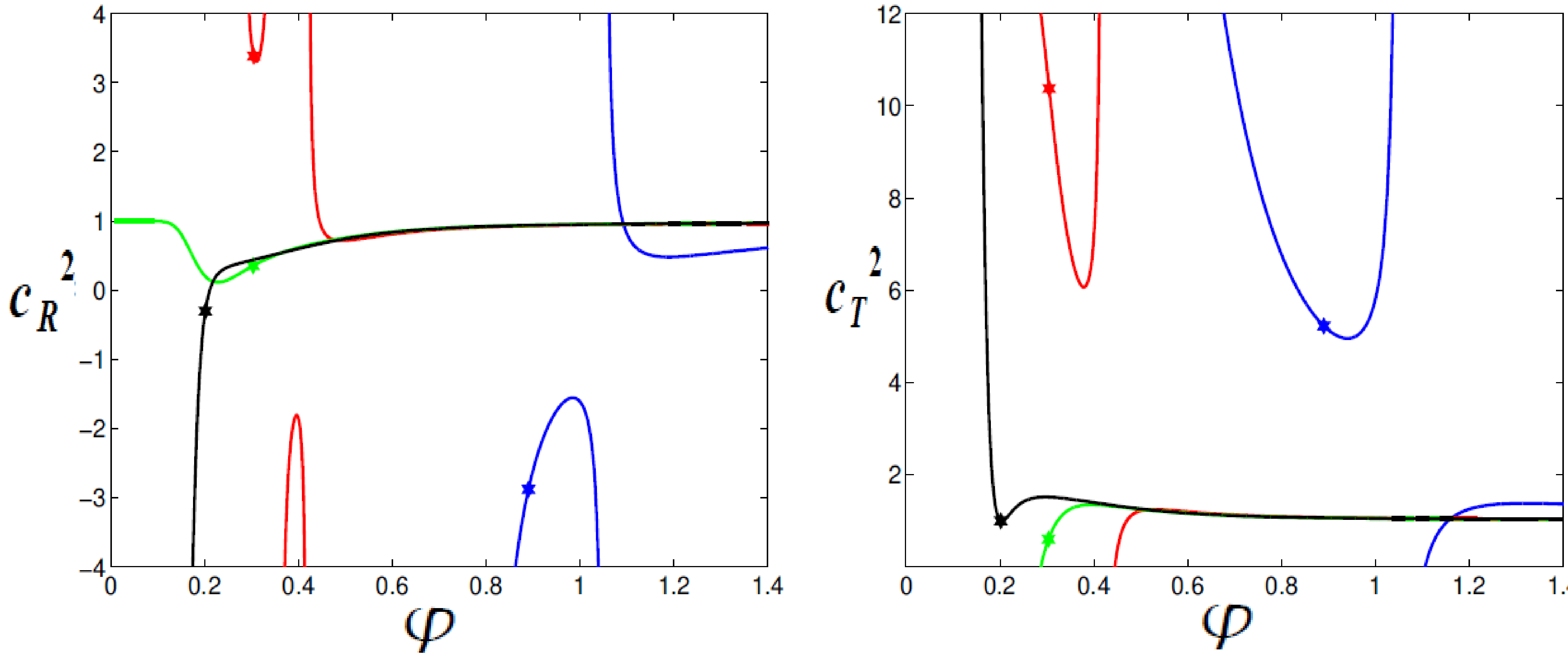
15 **Fig. 3. The dependence c_R^2 and c_T^2 on the scalar field φ for the model $V(\varphi) = V_0\varphi^n$, $\xi(\varphi) = \dfrac{\xi_n}{V(\varphi)+\Lambda}$.**



$$n = 2, \quad \xi_2 = 0.6885\, M_{Pl}^4, \quad \Lambda = 1.0125\cdot 10^{-12} M_{Pl}^4, \quad V_0 = 4.05\cdot 10^{-11} M_{Pl}^4,$$

$$U_0 = \frac{M_{Pl}^2}{2} = \frac{1}{2}$$

Fig. 4. The dependence $c_R^{\,2}$ and $c_T^{\,2}$ on the scalar field φ for the model $V(\varphi)=V_0\varphi^n$, $\xi(\varphi)=\dfrac{\xi_n}{V(\varphi)+\Lambda}$.



$$n=4,\quad \xi_4=0.714\,M_{Pl}^{\;4},\quad \Lambda=5.95\cdot10^{-13}\,M_{Pl}^{\;4},\quad V_0=3.4\cdot10^{-11},$$

$$U_0=\frac{M_{Pl}^{\;2}}{2}=\frac{1}{2}$$

Conclusion

1. We have investigated the inationary models in Einstein-Gauss-Bonnet gravity with the potential $V(\varphi)=V_0\varphi^n$ and the nonminimal coupling to the Gauss-Bonnet term $\xi(\varphi)=\dfrac{\xi_n}{V(\varphi)+\Lambda}$.

2. The several numerical scenarios have been found for ***n=2*** and ***n=4***, which agree good with the observations.

3. We have shown that the **standard slow-roll approximation** used earlier **are not suitable** for the calculations of the inflation parameters in these models and we have constructed **two new higher accuracy slow-roll approximations** giving us the results, which close enough to the numerical solutions and the observational data.

Thanks for attention!

Proposals of the work send to

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