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Superfluid Stars and the Dawn of Quantum Astrophysics

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I. Quantum Astrophysics

1M$ question: **Can astrophysics be quantum?**

There seems to be a contradiction:

Astrophysics mostly deals with large, massive, hot objects

Quantum mechanics mostly deals with tiny, light, cold objects

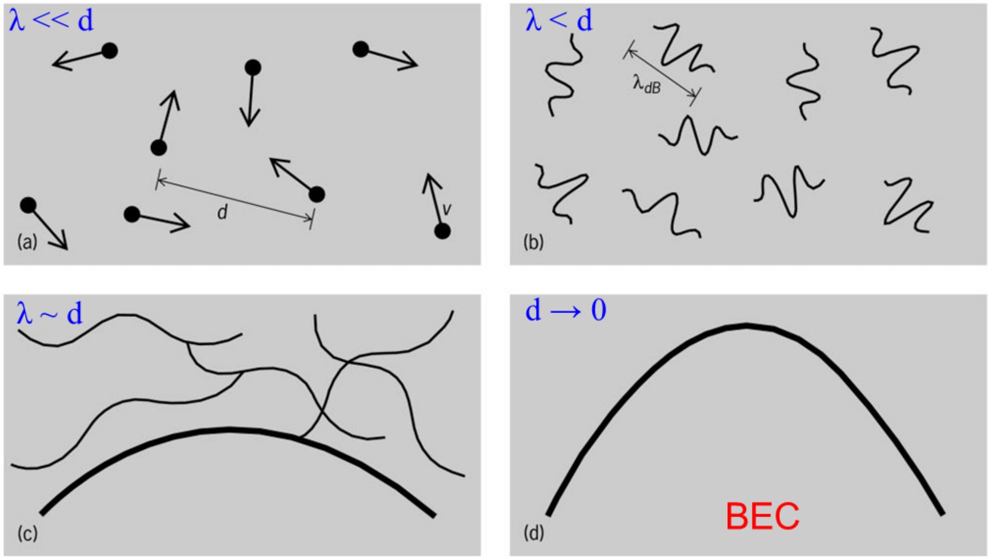
How can these two words appear in the same term?

Criteria for "quantumness" for many-body systems: not size nor mass nor temperature, but the ratio:

de Broglie wavelength / interparticle distance

a.k.a. the "overlap"

<u>Example</u>: Bose-Einstein condensates



The overlap occurs not only at low *T*'s but also at high densities

(this is where quantum astrophysics dwells)

“Semiclassical” vs Quantum Astrophysics

Semiclassical astrophysics isn’t classical anymore but not fully quantum yet. It uses the Pauli exclusion principle as a collapse-preventing agent based on Fermi-Dirac statistics (quantum).

- white dwarf stars - electron degenerate matter, mass limit is 1.4 $M_\odot$ [*Chandrasekhar*]
- neutron stars - neutron degenerate matter, mass limit is 2.2-2.9 $M_\odot$ [*Tolman–Oppenheimer–Volkoff*]

However

- ✓ semiclassical astrophysics does not really use the Heisenberg uncertainty nor quantum evolution equations (SE, von Neumann/q-master, etc)
- ✓ both Chandrasekhar and TOV limits were derived using some classical fluids’ equations of state (expected to mimic quantum systems)

Quantum uncertainty is a big game changer when it comes to astrophysics and here's one **example** why:

In the canonical QM: HUP implies classical trajectories aren't observable (one can't measure the position and tangent vector of a trajectory)

$$\mathrm{Var}(x_i)\ \mathrm{Var}\ (p_i) > 0$$

Similar reasoning can be applied to null hypersurfaces in 4D (a.k.a. *event horizons* a.k.a. *black holes*): $n_\mu n^\mu = 0$, (tangent 4-vectors are null)

If GR is a correct classical limit of some quantum theory then average values of tangent 4-vectors operators

$$\langle \hat{n}_\mu \rangle \langle \hat{n}^\mu \rangle = 0$$

These operators are simultaneously observable if and only if their commutator $\left[\hat{n}_\mu, \hat{n}^\nu\right] \sim [\hat{n}^\mu, \hat{n}^\nu]$ vanishes.

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But in GR $\left[\hat{n}_\mu, \hat{n}^\mu\right] |\Psi\rangle \sim \left[\nabla_\mu, \nabla^\mu\right] |\Psi\rangle = f(\mathrm{R}; \Psi)$ vanishes iff a state vector is a Lorentzian scalar... But it is not – because quantum wavefunctions are Euclidean scalars normalized over the 3D space volume.

Therefore, event horizons aren't QM observable $\mathrm{Var}(n_\mu)\,\mathrm{Var}(n^\nu) \geq \frac{1}{4}\left|\left[\hat{n}_\mu, \hat{n}^\nu\right]\right|^2 > 0.$

What is Quantum Astrophysics

QAP is a discipline which applies the full formalism of QM to astronomy phenomena. Its main features:

- it is based on the quantum uncertainty principle and quantum evolution equations (for state vector and/or density matrices)
- it uses other quantum phenomena which emerge (Pauli exclusion, BEC, superfluidity)

In its strong form, QAP requires a theory of quantum gravity and physical vacuum (such as superfluid vacuum theory).

In its weaker form, QAP requires relativistic QM coupled to classical GR.

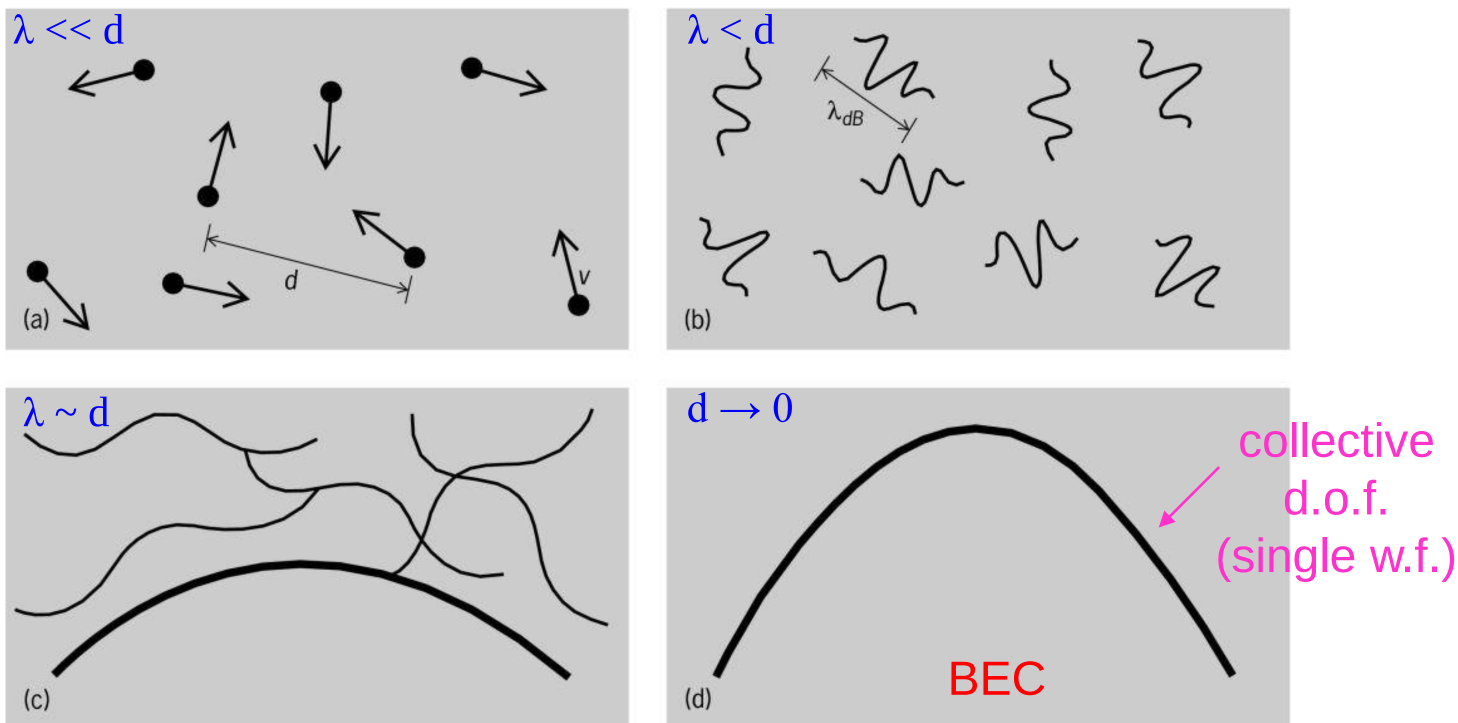
II. Quantum Liquids: Introduction

Q: What is quantum Bose liquid?

A: It is a quantum fluid that consists of the particles that obey the quantum Bose-Einstein statistics (not Fermi–Dirac, not Maxwell–Boltzmann); these particle are called bosons;
spin-statistics theorem: bosons must have integer spin;
examples: BEC, superfluids, etc.

Example 1: **Bose-Einstein condensate**

QM: same-species Bose particles are indistinguishable; particle-wave duality



BEC is an extended continuous quantum object (not anymore a cloud of particles)

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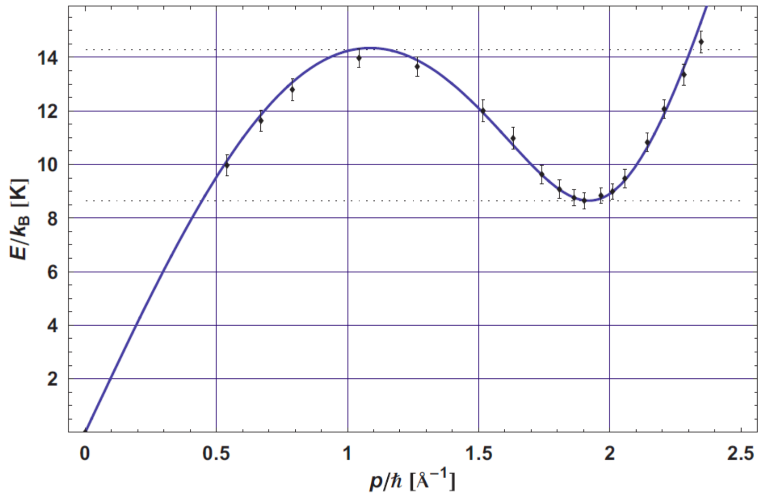
Example 2: **Superfluid**

Superfluid is a quantum Bose liquid in which dissipative fluctuations are suppressed

no dissipations $\Rightarrow$ no friction/drag force $\Rightarrow$ macroscopically behaves like a perfect or ideal fluid



Landau shape for the excitations' spectrum is a criterion

Modern theory of quantum Bose liquids

(BEC and superfluids)

$$\left[-i\hbar\,\partial_t - \frac{\hbar^2}{2m}\vec{\nabla}^2 + V_{\text{ext}}(\vec{x},\, t) + F(|\Psi|^2)\right]\Psi = 0 \qquad \langle\Psi|\Psi\rangle = N$$

where

$$F(\rho) = b\ln(\rho/\bar{\rho}) + \kappa_1\frac{\rho}{\bar{\rho}} + \kappa_2\left(\frac{\rho}{\bar{\rho}}\right)^2 \qquad \rho = n = |\Psi|^2$$

logarithmic term (vacuum?)

Gross-Pitaevskii term (2-body)

Ginzburg-Sobyanin term (3-body)

Advantages of including the logarithmic term:

- ✓ Supported by statistical mechanics (strong interaction: when $K/U << 1$)
- ✓ Non-perturbative
- ✓ Takes into account vacuum effects
- ✓ Fits experimental data (+ resolves some puzzles)

III. Application: Superfluid stars

- **Superfluid stars and Q-balls in curved spacetime**
  K.G. Zloshchastiev, *Low Temp. Phys*. **47** (2021) 89

In the hierarchy of superdense stars, various objects exist, which occupy an intermediate place between neutron stars and black holes.

To a distant observer, such objects would look almost like black holes – however, they have no null surface (horizon)

For which reason they are often aggregated under the name of *compact stars* (CS) and *black hole mimickers* (BHM).

Widely known examples:

- ✓ geons
- ✓ quark stars
- ✓ gravastars
- ✓ boson stars (most popular, but has issues with SM)

In [Zloshchastiev (2021)], it was proposed another type of CS/BHM-type Bose fluid objects – superfluid stars (SFS), which are modelled by scalar field with logarithmic nonlinearity.

Because superfluids are macroscopic quantum (wave-mechanical) objects, the stability of superfluid stars against gravitational collapse is expected to be enhanced by the uncertainty principle, similarly to boson star models.

In [Zloshchastiev (2021)], it was proposed another type of CS/BHM-type Bose fluid objects – superfluid stars (SFS), which are modelled by scalar field with logarithmic nonlinearity.

Because superfluids are macroscopic quantum (wave-mechanical) objects, the stability of superfluid stars against gravitational collapse is expected to be enhanced by the uncertainty principle, similarly to boson star models.

Moreover, superfluidity introduces an additional effect here. The inviscid flow, caused by suppression of dissipative fluctuations, makes the fluid parcels (a.k.a volume elements) more resistant to coming to a full stop and adhering to each other.

Therefore SFS are expected to have a larger degree of resistance to gravitational collapse than the conventional boson stars.

Bonus: unlike conventional boson stars, SFS don't require exotic elementary particles, because superfluids are collective states of matter, which can form from known particles, even fermions.

Following the standard procedure of dealing with Lorentz-symmetric Bose systems: Adopting the units $c = 1$ and metric signature $(-+++)$, we write the Lagrangian of our model as:

$$\boxed{\mathcal{L} = \frac{R}{16\pi G} - \frac{1}{2}\nabla_\mu \phi^* \nabla^\mu \phi - V(\phi, \phi^*)}$$

where the scalar field potential is borrowed from Log-superfluid models:

$$\boxed{V(\phi, \phi^*) = -b\,|\phi|^2 \left[\ln\left(|\phi|^2/a\right) - 1\right]}$$

The corresponding field equations read:

$$\boxed{R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R = 8\pi G T^\mu_\nu}$$

where $$\begin{aligned} T^\mu_\nu &= \frac{1}{2} g^{\mu\sigma}\left(\nabla_\sigma\phi^*\nabla_\nu\phi + \nabla_\sigma\phi\nabla_\nu\phi^*\right) \\ &\quad -\frac{1}{2}\delta^\mu_\nu\left(g^{\alpha\beta}\nabla_\alpha\phi^*\,\nabla_\beta\phi + 2V(\phi,\phi^*)\right) \end{aligned}$$

$$\boxed{\nabla_\mu\nabla^\mu\phi = 2\frac{\partial}{\partial\phi^*}V(\phi,\phi^*) = -2b\ln\left(|\phi|^2/a\right)\phi}$$

We are interested in whether equilibrium field configurations exist. For simplest equilibrium configurations, spacetime metric would be time-independent and likely spherically-symmetric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

while the scalar field must be stationary and spherically-symmetric

$$\phi(r,t) = e^{-i\omega t}\Phi(r)$$

Then field equations simplify to a set of the following ODEs

$$\frac{A'}{A^2 x} + \frac{A-1}{Ax^2} = 2\left[\frac{\Omega^2}{B} + 1 - \ln(\sigma^2/k)\right]\sigma^2 + \frac{(\sigma')^2}{A},$$

$$\frac{B'}{ABx} - \frac{A-1}{Ax^2} = 2\left[\frac{\Omega^2}{B} - 1 + \ln(\sigma^2/k)\right]\sigma^2 + \frac{(\sigma')^2}{A},$$

$$\sigma'' + \left(\frac{2}{x} - \frac{A'}{2A} + \frac{B'}{2B}\right)\sigma' + 2A\left[\frac{\Omega^2}{B} + \ln(\sigma^2/k)\right]\sigma = 0,$$

where $x = r/L,\ L = 1/\sqrt{b},\ \sigma = (4\pi G)^{1/2}\Phi,\ k = 4\pi G a,\ \Omega = \omega/\sqrt{2b},$

These equations are numerically solved, using the shooting method:

- ✓ Values of parameters are chosen in such a way that our scalar field vanishes at spatial infinity, and has no nodes and singularities
- ✓ Then, non-singular finite-mass solutions are expected to describe the lowest-energy bound states
- ✓ Eigenvalue problem for Ω and B_0 with initial conditions:

$$\sigma(0) = \sigma_0, \; \sigma'(0) = 0, \; \mathcal{M}(0) = 0, \text{ and } B(0) = B_0,$$

Once a desired solution is obtained, the total mass can be derived from an asymptotic value of the mass function

$$M = \mathcal{M}(\infty)L/G = \mathcal{M}(\infty)/(G\sqrt{b})$$

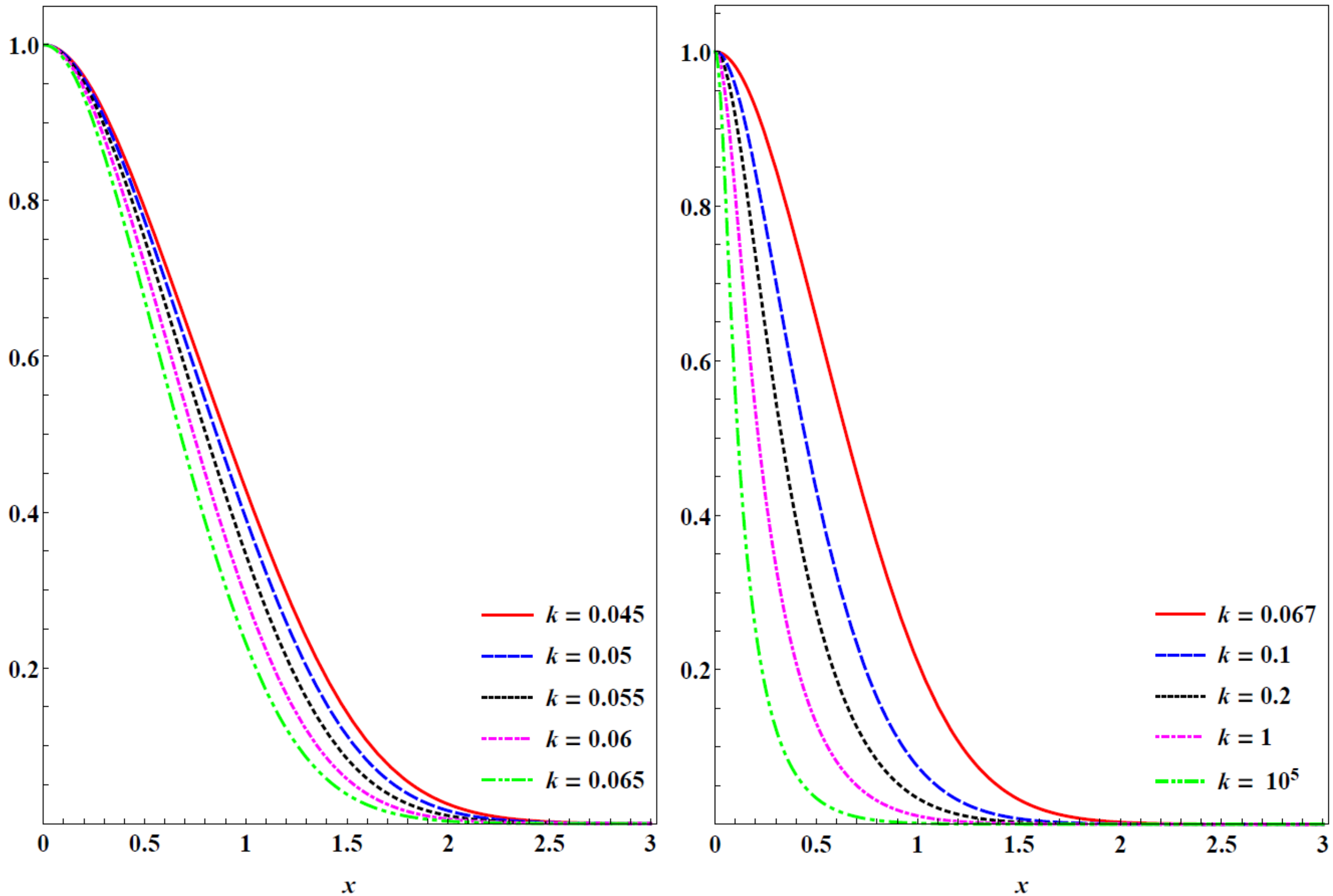
where the mass function is related to the metric via the identity

$$A(x) = [1 - 2\mathcal{M}(x)/x]^{-1}$$

$k = 4\pi G a$

Results: static configurations do exist

Scalar field is localized:

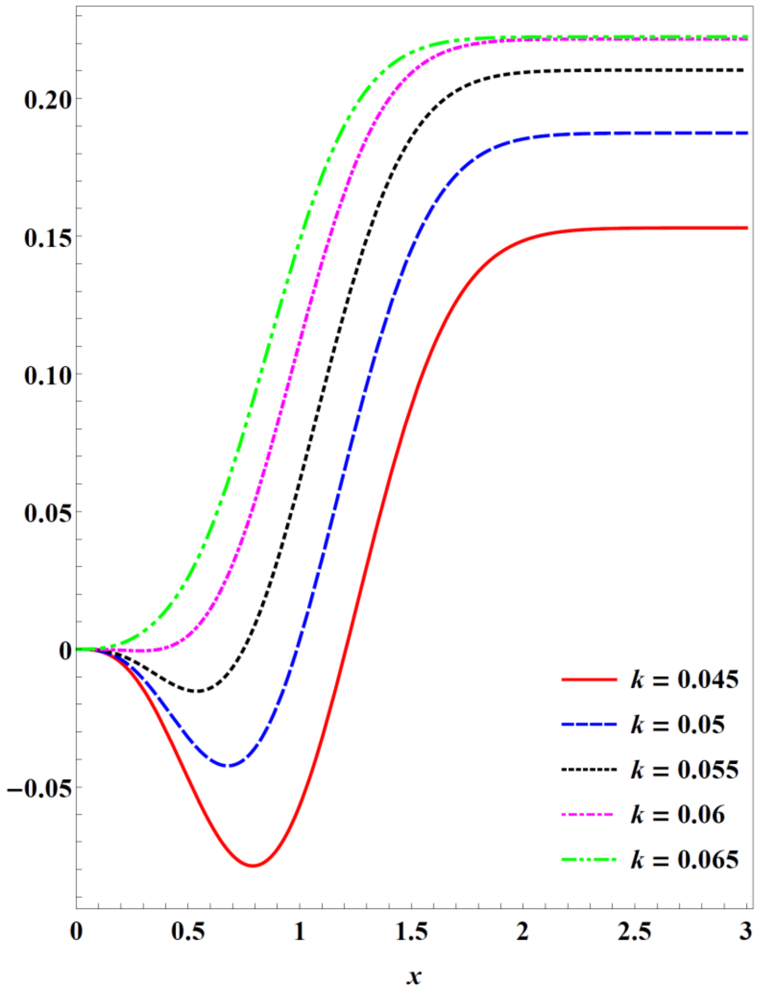


(a) $k < \tilde{k}$ (b) $k > \tilde{k}$

Function $\sigma(x)$ for different values of k, at $\sigma_0 = 1$

$k = 4\pi Ga$

Mass function is asymptotically flat at spatial infinity:



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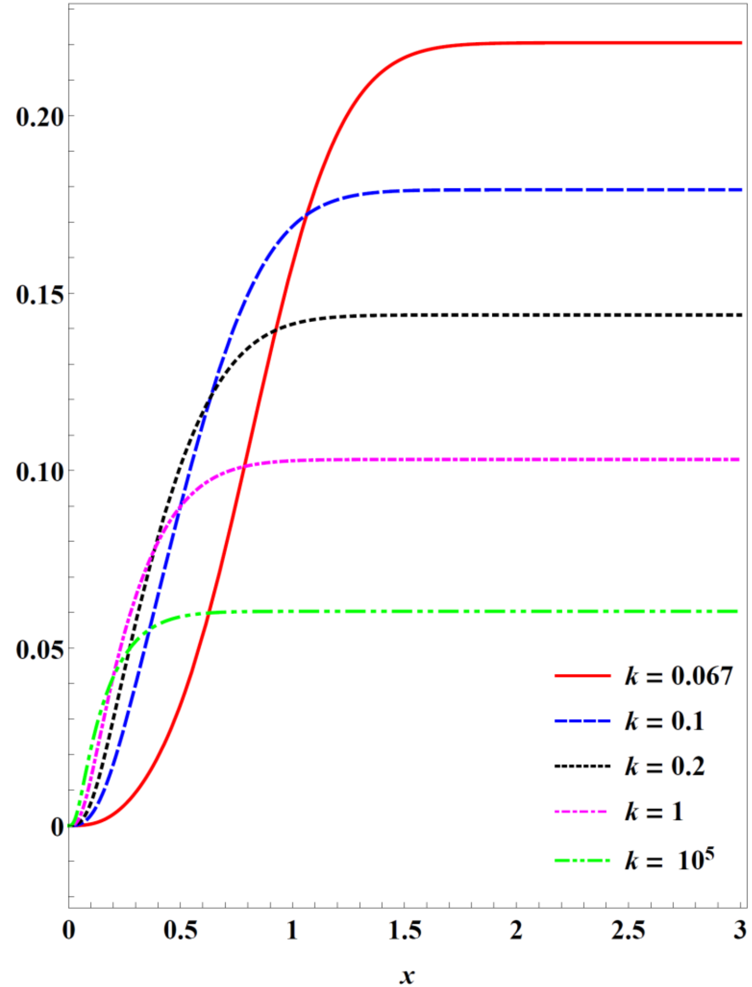
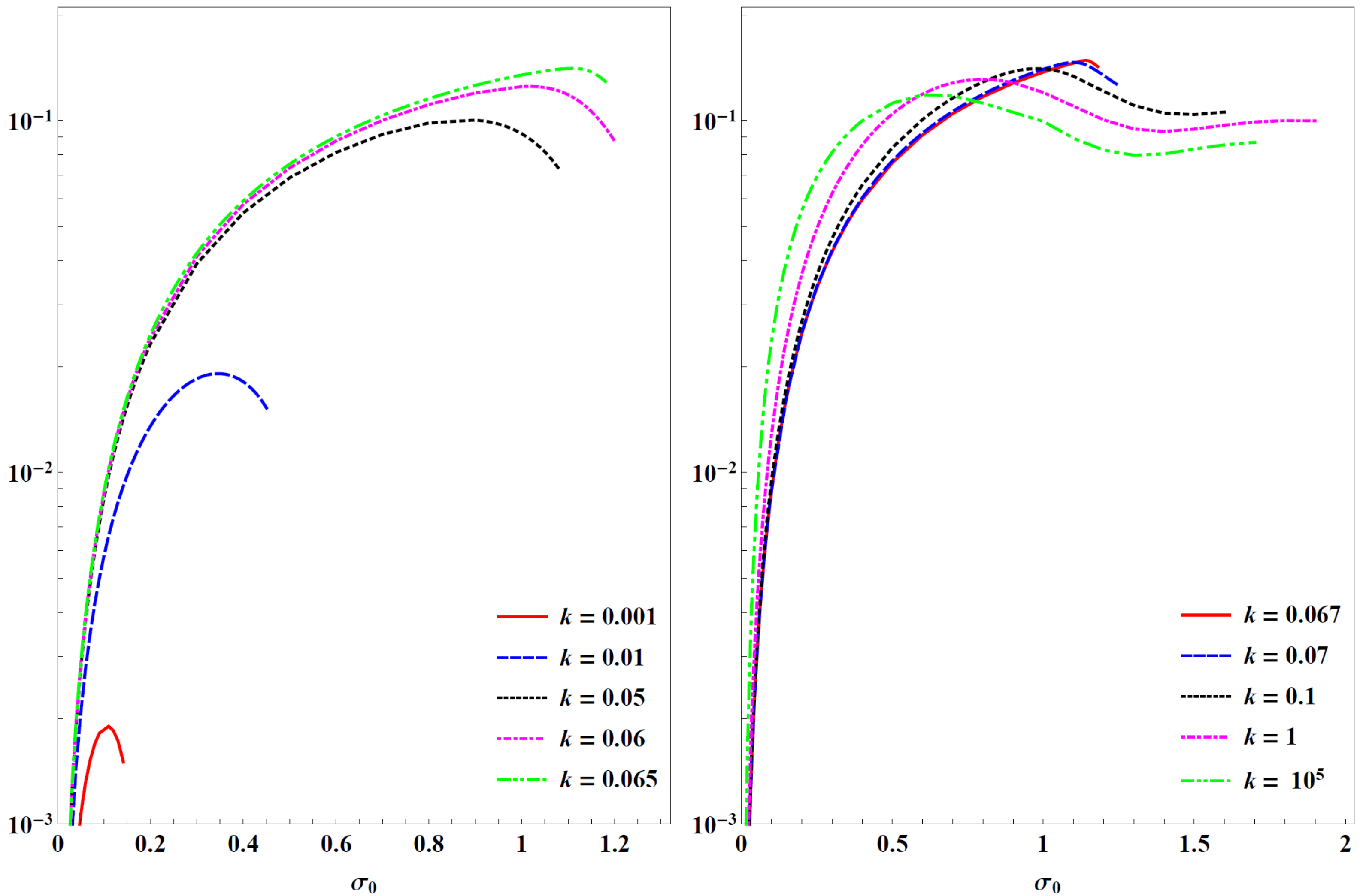


(b) $k > \tilde{k}$

Function $\mathcal{M}(x)$ for different values of k, at $\sigma_0 = 1$

$k = 4\pi Ga$

Compactness

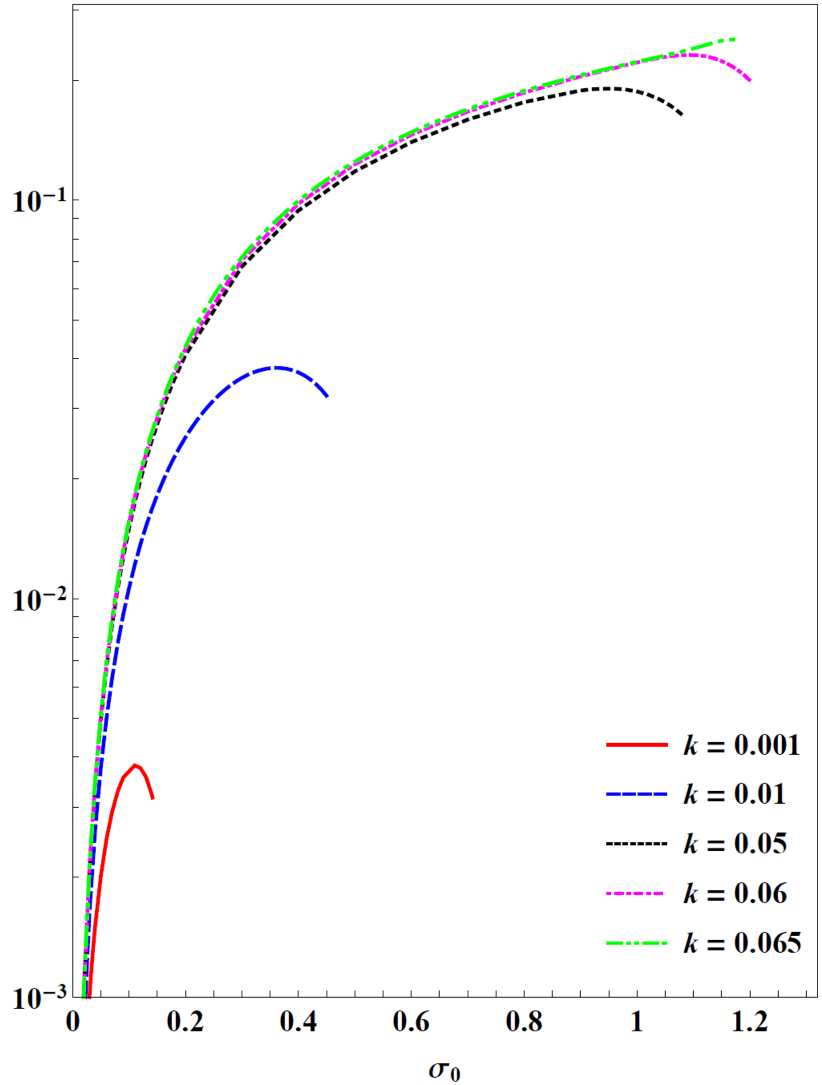


(a) $k < \tilde{k}$ (b) $k > \tilde{k}$

Effective compactness versus central field σ_0, for different values of k.

$k = 4\pi Ga$

Asymptotic value $\mathcal{M}(\infty)$ versus σ_0, for different values of k



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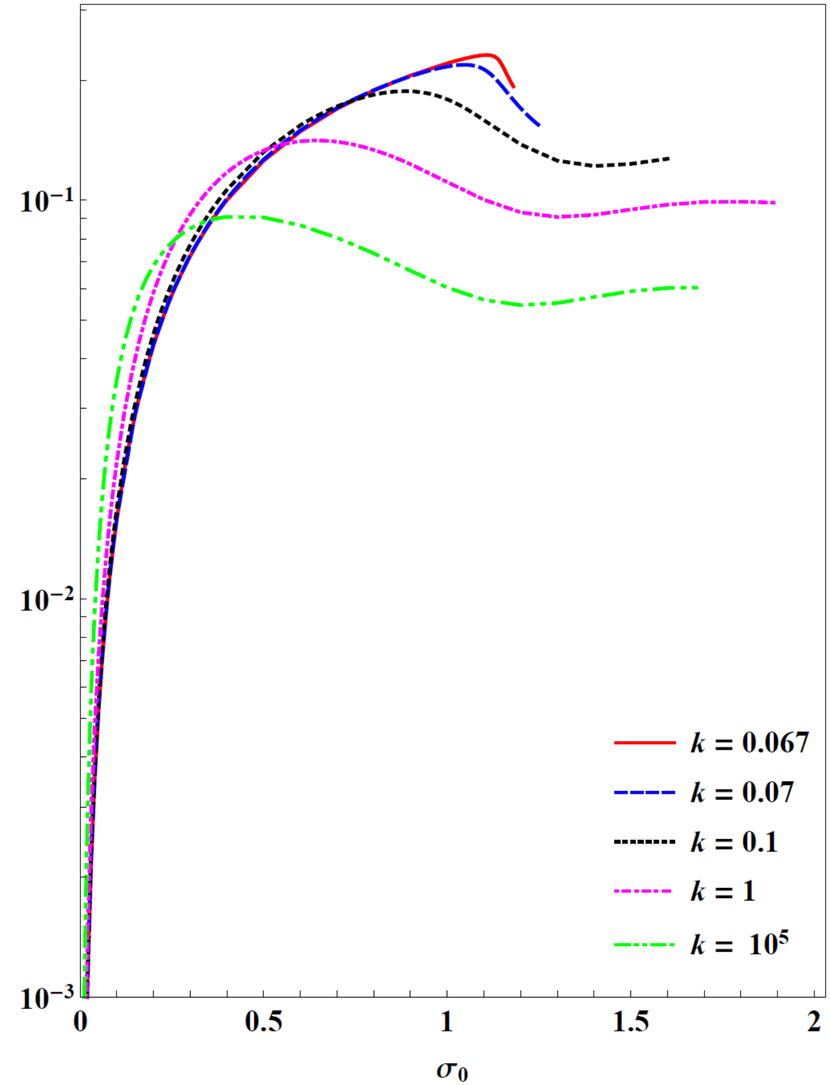
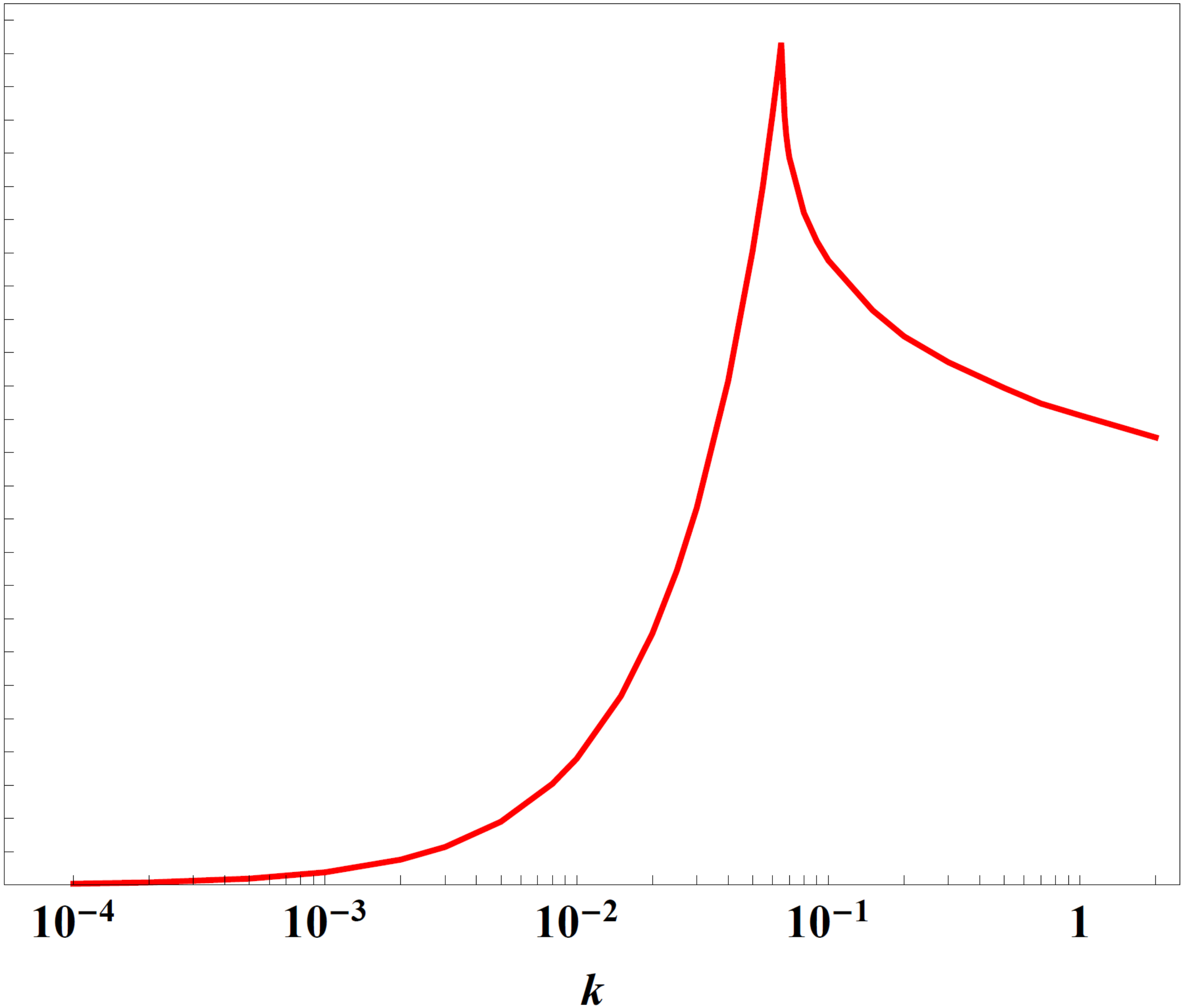


(b) $k > \tilde{k}$

$k = 4\pi Ga$

$\mathcal{M}_{\max} = \max\left[\mathcal{M}(\infty)\right]$ versus k



$$\nabla_\mu \nabla^\mu \phi = 2\frac{\partial}{\partial \phi^*} V(\phi, \phi^*) = -2b \ln\left(|\phi|^2/a\right) \phi$$

Analytical estimates

If b is small, then our model refers to astronomical-scale CS/BHM objects, such as superfluid stars or superfluid cores of neutron stars

For example, let us assume that our superfluid star has the maximum mass which is equal to the mass of Sun $M_\odot \approx 2 \times 10^{33}\,\mathrm{g}$

$$b_\odot \equiv b(M_{\max} = M_\odot) = (4GM_\odot)^{-2} \approx 2.8 \times 10^{-12}\,\mathrm{cm}^{-2}$$

Hence the radius of such a star would be:

$$R_{M_\odot} = \alpha/\sqrt{b_\odot} \approx$$
$$\alpha (2.8 \times 10^{-12})^{-1/2}\,\mathrm{cm} \sim \alpha (6 \times 10^5)\,\mathrm{cm} \lesssim 10\,\mathrm{km}$$

In summary, logarithmic superfluid stars:

- ✓ have BH-similar characteristics – except they don’t have event horizons
- ✓ don’t have an upper mass limit, such as TOV

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