Primordial black holes mergers

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V. Stasenko and K. Belotsky, MNRAS, 526, 4308 (2023), 2307.12924
V. Stasenko, PRD 109 (2024) 12, 123546, 2403.11325

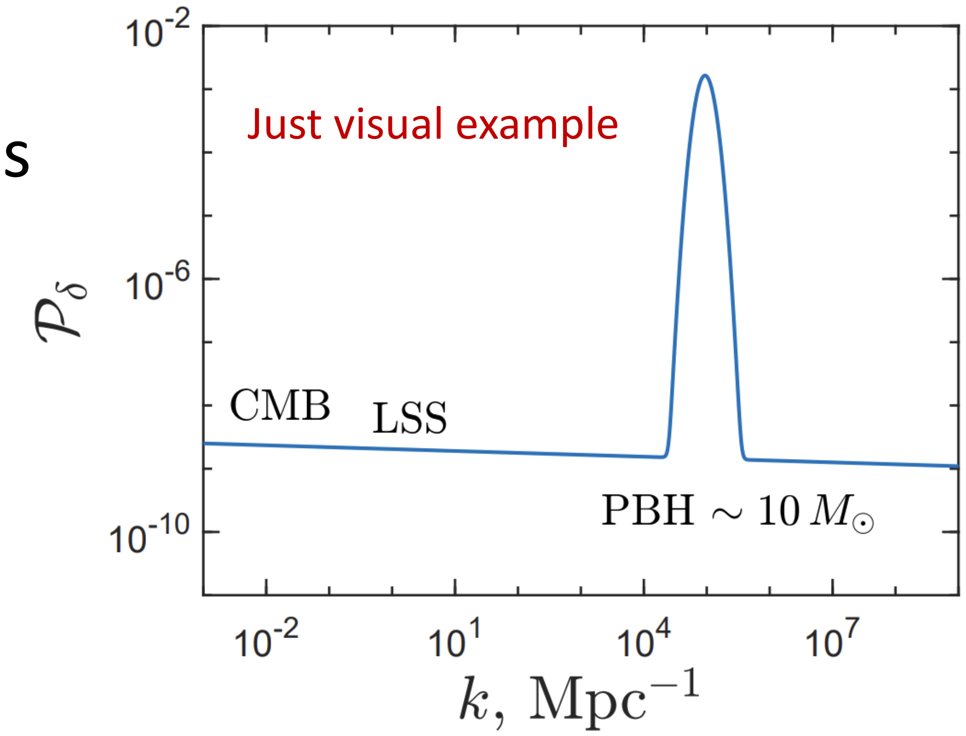
Outline

- Introduction to PBH
- Dark matter candidate
- PBH mergers and LIGO-Virgo-KAGRA observations
- Clustering of PBHs and its impact on the merger rate
- Impact of the clustering on the merger rate
- High redshift PBH mergers

Introduction

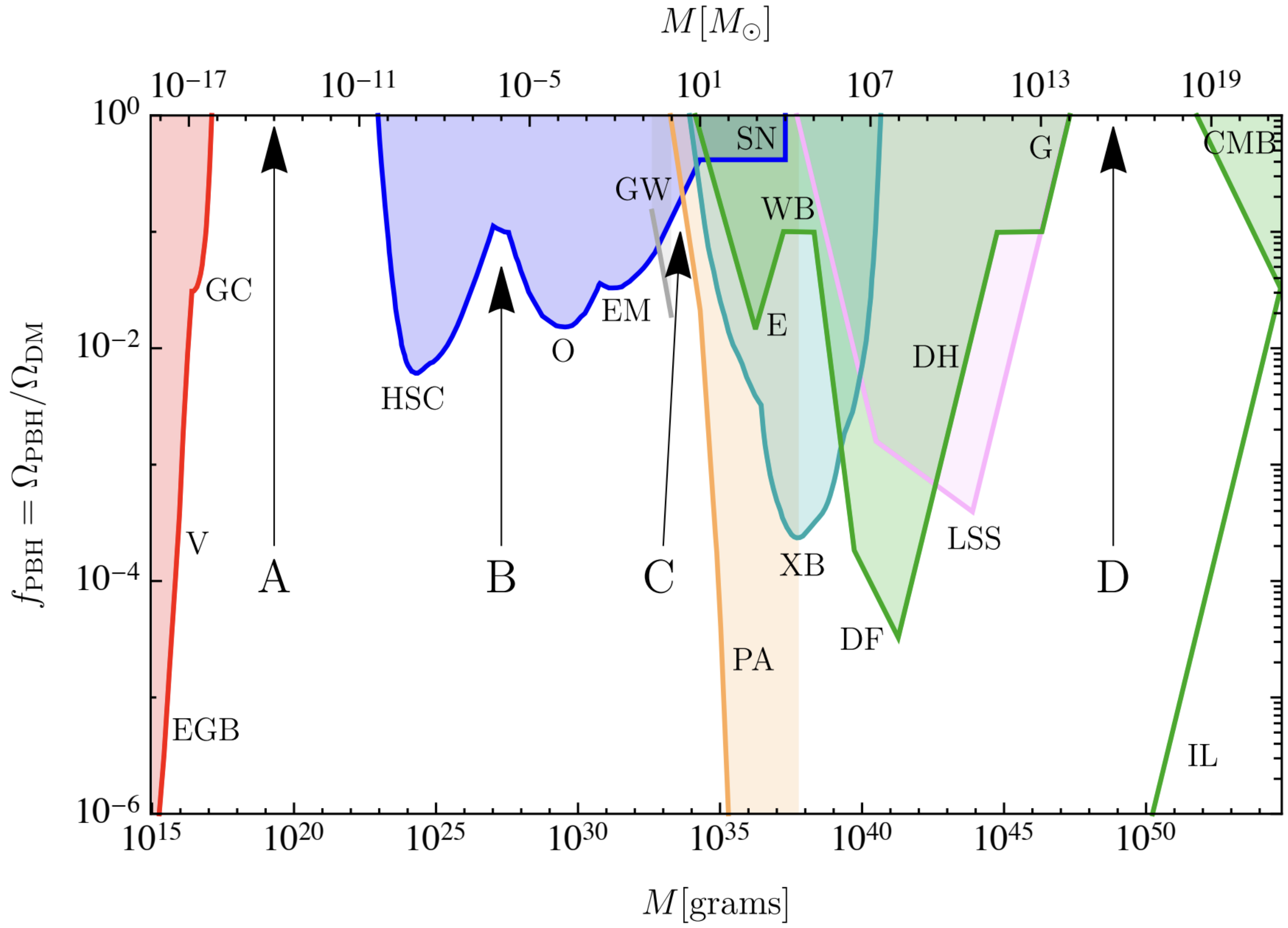
- Zeldovich and Novikov (1967); Hawking … (1971)
- PBHs arise from the collapse of density fluctuations $\delta\rho/\rho \sim 1$ on horizon scale

The simplest scenario: these fluctuations are generated by inflation



Although other possibilities are also considered: the collapse of domain walls, cosmic strings, ...

PBH as dark matter



These constraints are model dependent!

We will assume PBH $m \sim 10\, M_\odot$

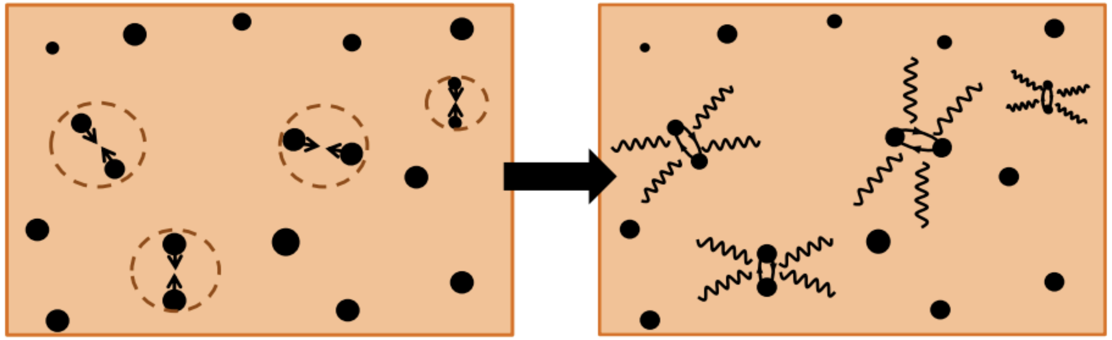
two-component dark matter – PBH + DM particles and $f = \Omega_{PBH}/\Omega_{DM} = 0.1$

B. Carr, F. Kuhnel, Primordial Black Holes as Dark Matter: Recent Developments, 2006.02838

PBH binaries

There are two main mechanisms for the formation of binaries.

1) Detachment from the Hubble flow at RD stage – early binaries. *Sasaki et al., Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914, 1603.08338*

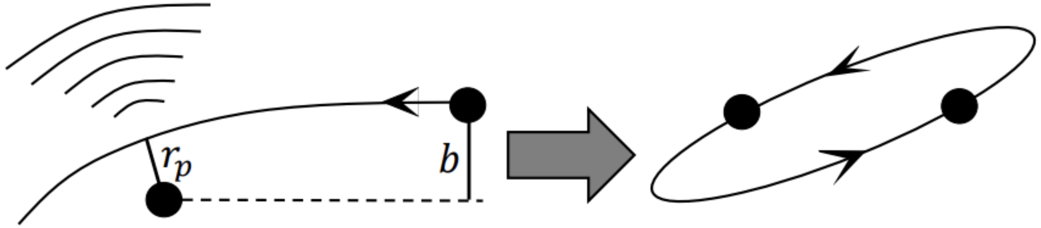


Distribution of early binary parameters

$$dP = \frac{3}{2}\left(\frac{f}{\bar{x}}\right)^{3/2}\frac{\sqrt{a}}{j^2}da\,dj$$

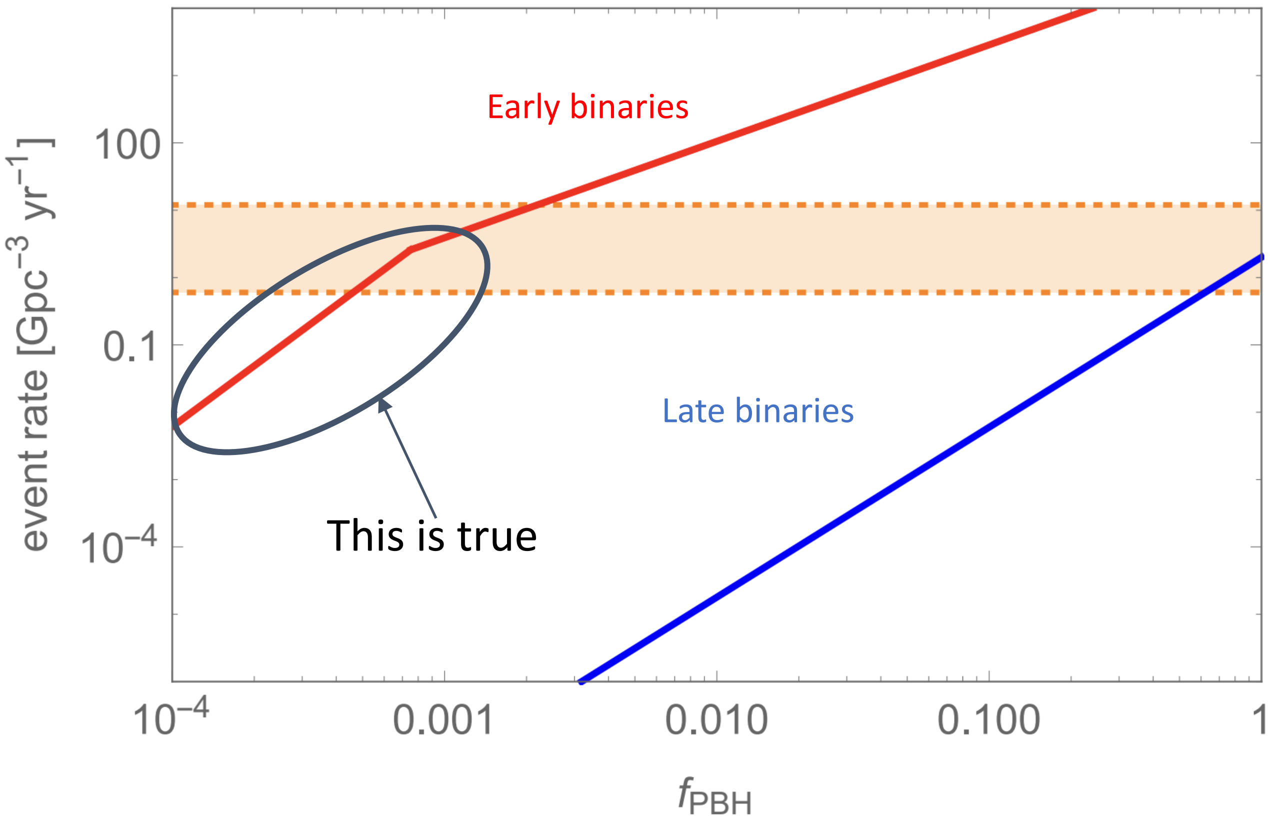
$$j = \sqrt{1-e^2}$$

$$j \ll 1$$



2) Dynamic channel in modern structures – late binaries. *Bird et al., Did LIGO detect dark matter? 1603.00464.*

LVK constraints



Sasaki et al., Primordial Black Holes - Perspectives in Gravitational Wave Astronomy, 1801.05235

From the distribution $dP = \frac{3}{2}\left(\frac{f}{x}\right)^{3/2}\frac{\sqrt{a}}{j^2}dadj$

→ the merger rate can be calculated $\mathcal{R}$

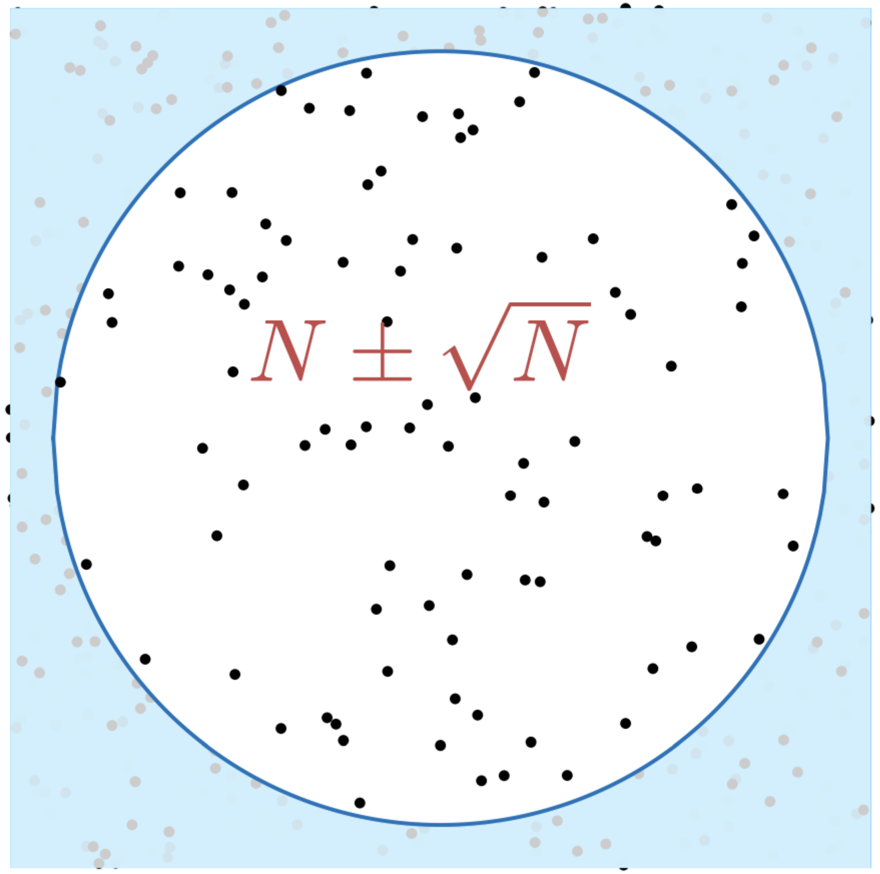
The red line is correct if the binaries are not perturbed from formation to merging.

It is also assumed that the contribution of late binaries is negligible. This will be shown to be wrong as well.

Poisson clustering of PBH

Due to their random distribution, the PBHs create density fluctuations. On small scales, these fluctuations dominate over inflationary ones.

$\delta \sim f_{PBH}/\sqrt{N}$



The power spectrum changes (*Afshordi et al. 2003, astro-ph/0302035*)

$$P_{PBH} = T_{iso}^2 \frac{m f_{PBH}}{\Omega_{DM} \rho_c} = \frac{9}{4}(1+z_{eq})^2 \frac{m f_{PBH}}{\Omega_{DM} \rho_c}$$

variance

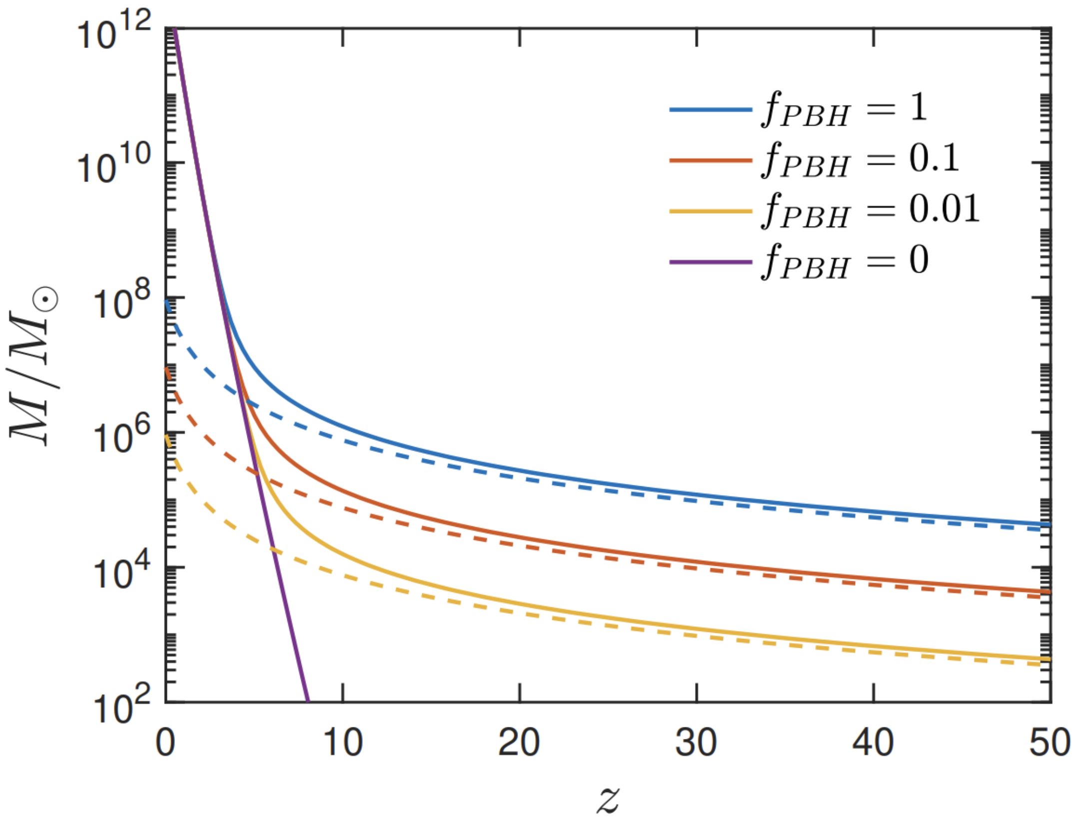
$$\sigma_M^2(R,z) = D^2(z) \int \frac{dk}{2\pi^2} k^2 P(k) W^2(kR)$$

$$\sigma_M^2(M) = \frac{9\, m f_{PBH}}{4\, M} \left(\frac{1+z_{eq}}{1+z} \right)^2$$

If we consider only the PBH (large redshifts)

Inman and Ali-Haïmoud, Early structure formation in primordial black hole cosmologies, 1907.08129

Characteristic halo mass PBH + DM



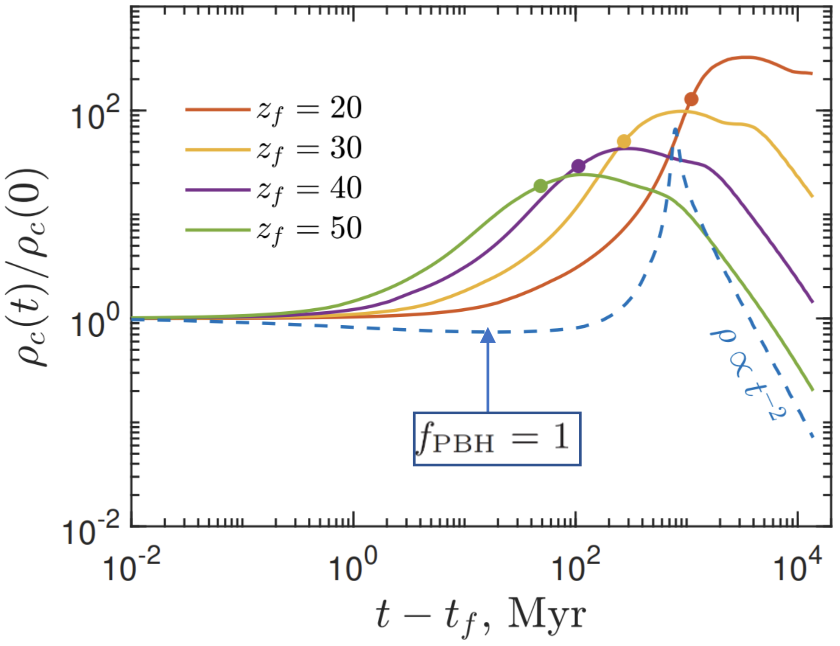
Define as $\sigma(M_{\rm ch}) = \delta_c = 1.69$

dashed lines $M_{\rm ch} = \frac{9\, m f_{\rm PBH}}{4\,\delta_c^2}\left(\frac{1+z_{\rm eq}}{1+z}\right)^2$

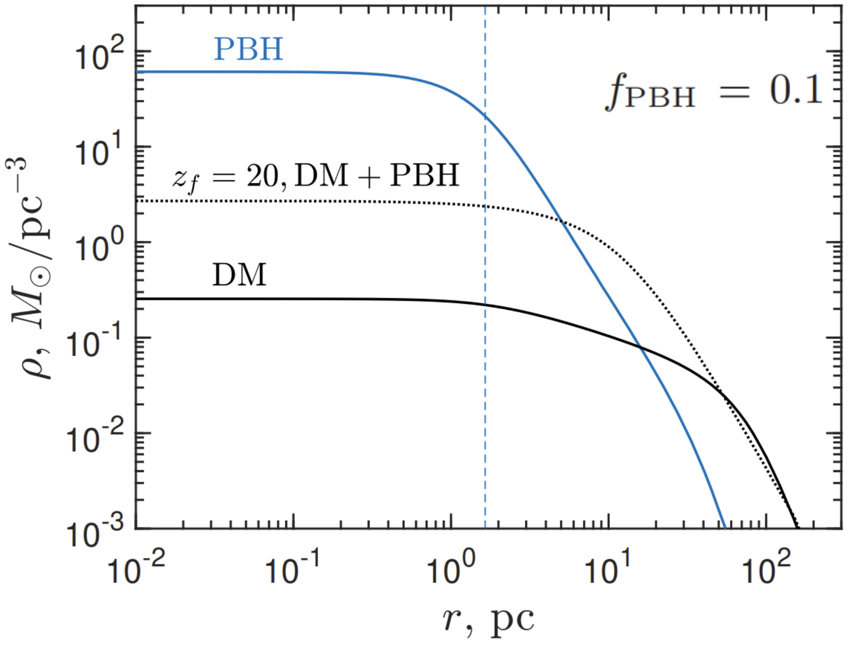
In such structures, early binaries actively scatter with other PBHs.

Evolution of early halos

Early halos evolve like globular clusters – N-body problem -> solving the Fokker-Planck kinetic equation



Evolution of the central density of PBHs in halos formed at different z_f и $f_{\rm PBH} = 0.1$



The halo density profile at the present moment. The dotted line is at the formation.

The evolution of the halo leads to the formation of dense clusters of PBHs. In such clusters, the binaries are actively perturbed! The collapse of the core is stopped by the formation of binaries in three-body interactions

PBH mergers

The perturbation mechanism is that during the core collapse, the binary interacts with the single PBH -> its angular momentum increases *(Jedamzik 2006.11172)*

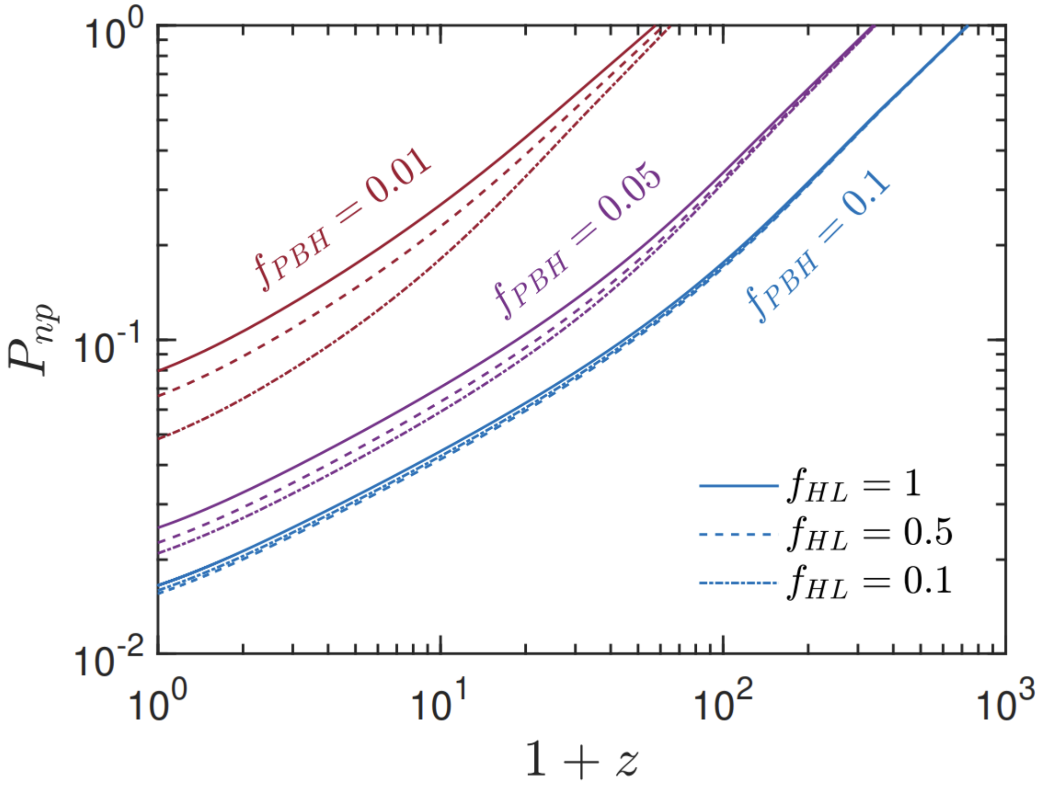
$$t_{\rm mer} = \frac{3\,c^5 a^4 j^7}{170\,G^3 m^3}, \quad j \ll 1$$

Early binaries

The fraction of unperturbed binaries *(Vaskonen and Veermäe 2019, 1908.09752)*

$$P_{\rm np}(z) = 1 - \sum_{N=3}^{N_{\rm crit}(z)} p_N(z_f) - \sum_{N' > N_{\rm crit}(z)} \left(\sum_{N=3}^{N_{\rm crit}(z)} \tilde{p}_N(z_f) \right) p_{N'}(z_f)$$

Roughly speaking, those binaries that have not clustered will merge.



The following chain arises: a binary enters in the halo of DM + PBH -> a cluster of PBHs with a high density is formed -> the probability of a binary perturbation = 1 in the cluster

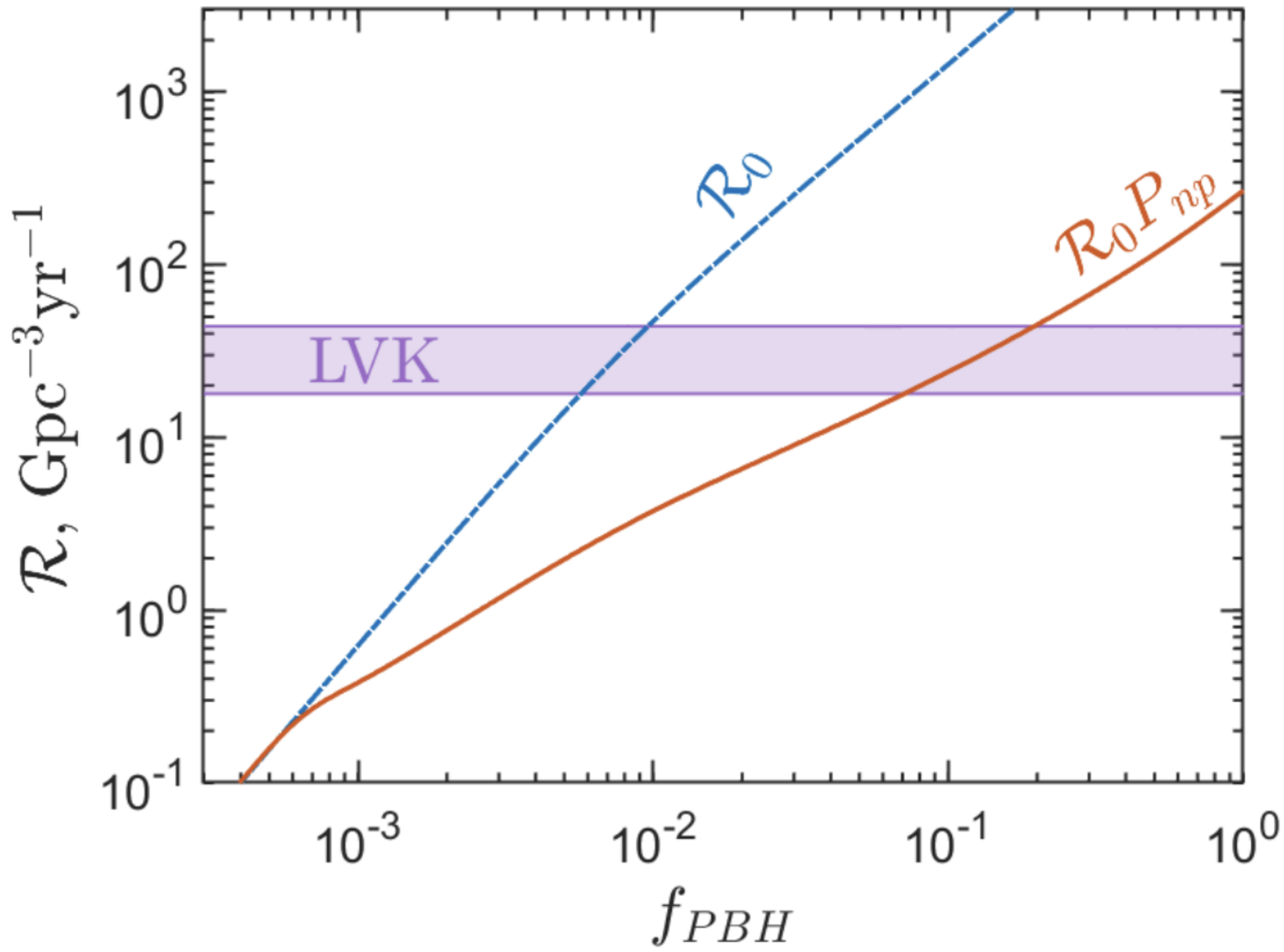
PBH mergers

The merger rate of unperturbed binaries

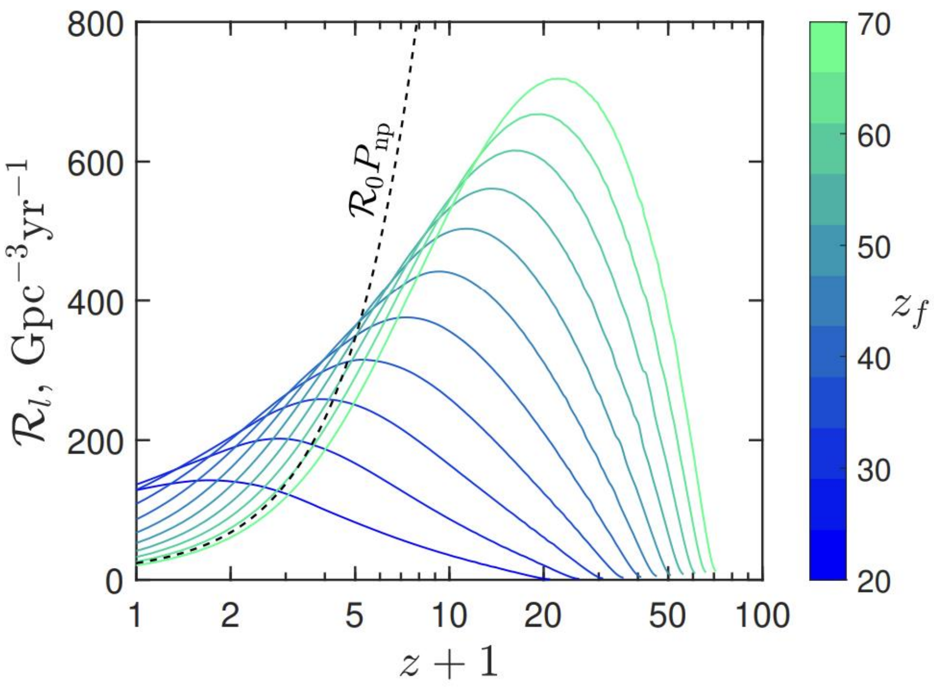
$$\mathcal{R} = \mathcal{R}_0 P_{\rm np}$$

The rate of early binary mergers is significantly suppressed. Constraints are relaxed to the level of microlensing constraints $f_{PBH} \leq 0.1$

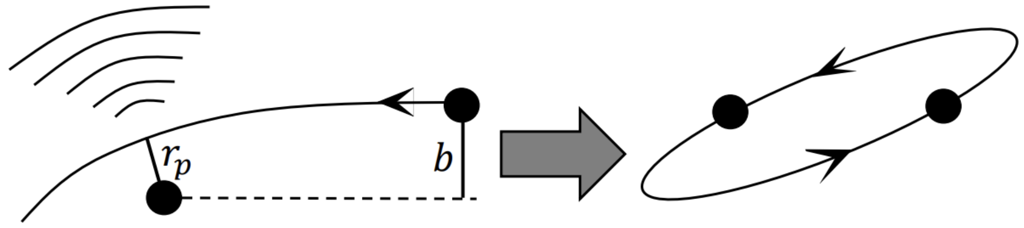
Purple band – data GWTC-3 LVK 2111.03634



In clusters the probability of mergers late binaries increases



The rate of late binary mergers over time with “frozen” halo mass functions z_f, is also assumed to be $w = 1$. There will be a maximum if all halos are formed at $z_f = 20 - 30$



Cross-section of this process $\Sigma = \pi R_s^2 \left(\frac{85\pi}{3}\right)^{2/7} \left(\frac{v_{\rm rel}}{c}\right)^{-18/7}$

Merger rate per halo $\Gamma_{\rm h} = \frac{2\pi}{m^2} \int dr\, r^2 \rho_{\rm PBH}^2 \langle \Sigma v_{rel} \rangle$

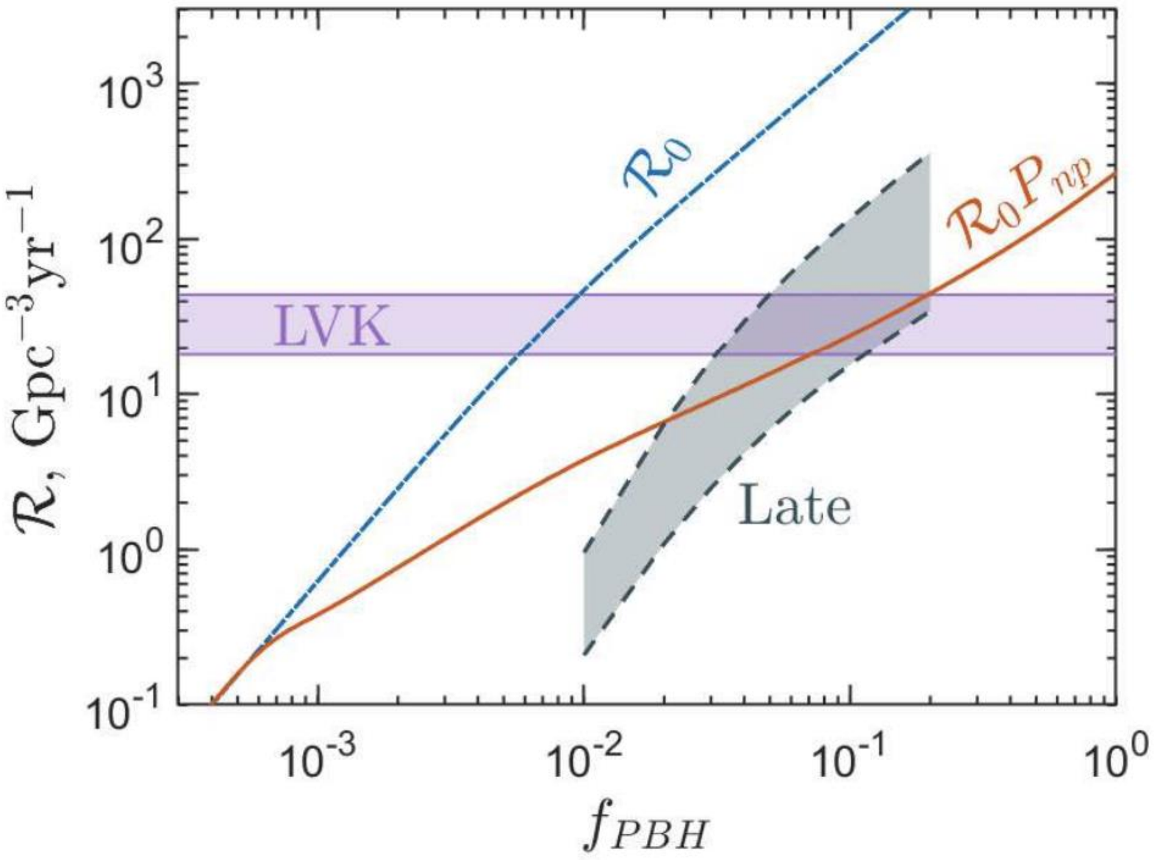
Total merger rate of late binaries

$$\mathcal{R}_1(z) = \int w(M, z_f) \frac{dn}{dM}(z_f) \Gamma_{\rm h}(z)\, dM$$

The fraction of surviving halos by moment z formed at z_f

The mass function of the halo formed at z_f

PBH merger rate



Late binaries

$$\mathcal{R}_1(z) = \int w(M, z_f) \frac{dn}{dM}(z_f) \Gamma_h(z)\, dM,$$

Upper dashed line $w = 1$

The lower one takes into account the destruction of the halo during the structure formation

PBHs mergers in clusters dominate compared to early binaries

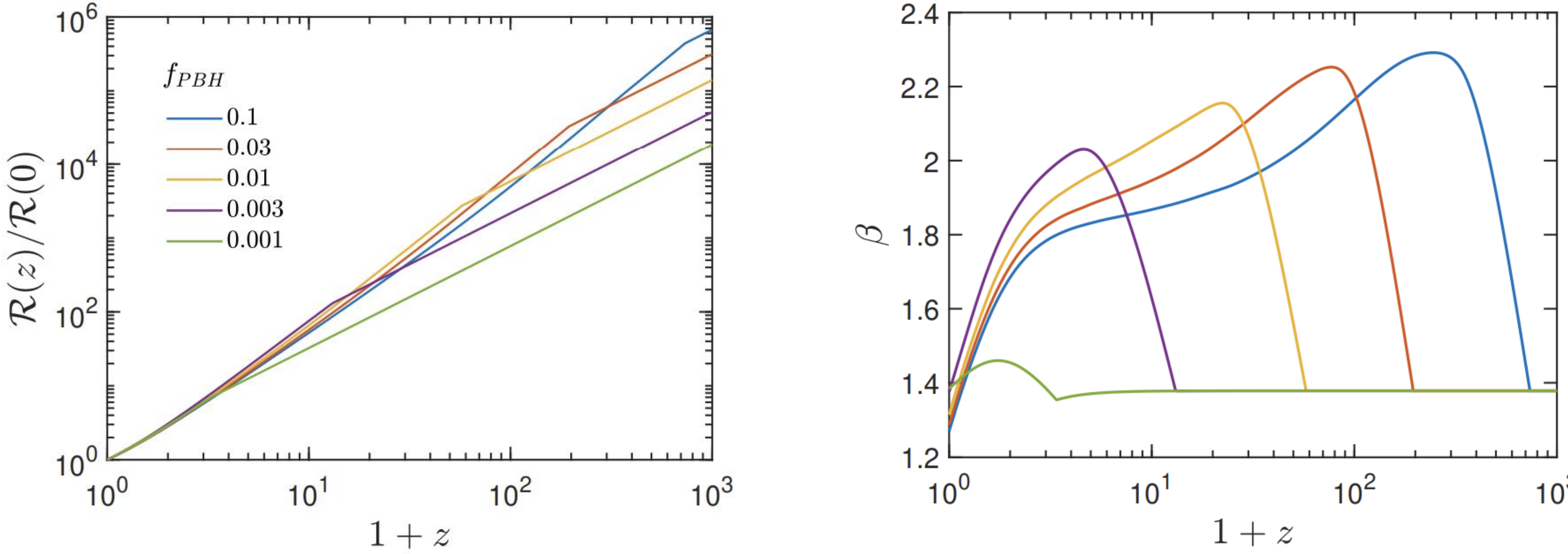
The suppression of the early binary mergers inevitably leads to an increase in the merger rate of late binaries!

What about high redshifts?

LVK probably will not answer the question about the nature of merging black holes

But third-generation ground-based detectors will be able to, because they will observe black hole mergers up to redshifts z < 100 *(Searching for Primordial Black Holes with the Einstein Telescope: impact of design and systematics, 2304.03160.)*

Mergers of stellar black holes are not expected at z > 30 *(Koushiappas and Loeb 2017)*



Left: merger rate as a function of z. Right: log slope $\mathcal{R} \propto (1+z)^{\beta}$. One can see a transition to an era when clustering has not yet begun.

Conclusion

- $f = \Omega_{PBH}/\Omega_{DM} = 0.01 - 0.1$ PBHs will actively clustering and may be abundant in modern era.
- The merger rate of early binaries is suppressed in clusters, but this also leads to an increase in the mergers of late binaries
- Black hole mergers at high redshifts will indicate the existence of PBH

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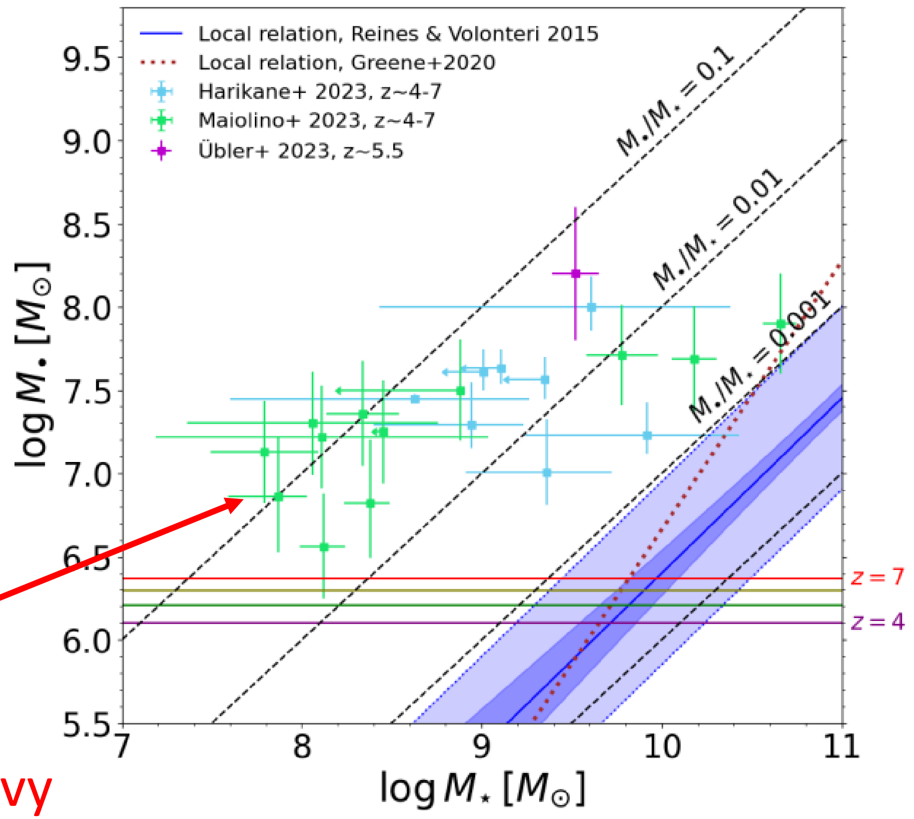
Indications of the PBH existence

Even if not all dark matter consists of PBHs, they may still be astrophysical significant.

- Supermassive black holes and galaxies in the early universe (*Accelerating Early Massive Galaxy Formation with Primordial Black Holes, 2208.13178; Carr and Silk, Primordial Black Holes as Generators of Cosmic Structures, 1801.00672*)
- SGW from the formation and mergers of the PBH (*NANOGrav Hints to Primordial Black Holes as Dark Matter, 2009.08268; Do pulsar timing arrays observe merging primordial black holes? 2306.17836.*) Future LISA observations
- Black holes mergers of tens solar masses – LIGO-Virgo-KAGRA
- This list goes on and on…

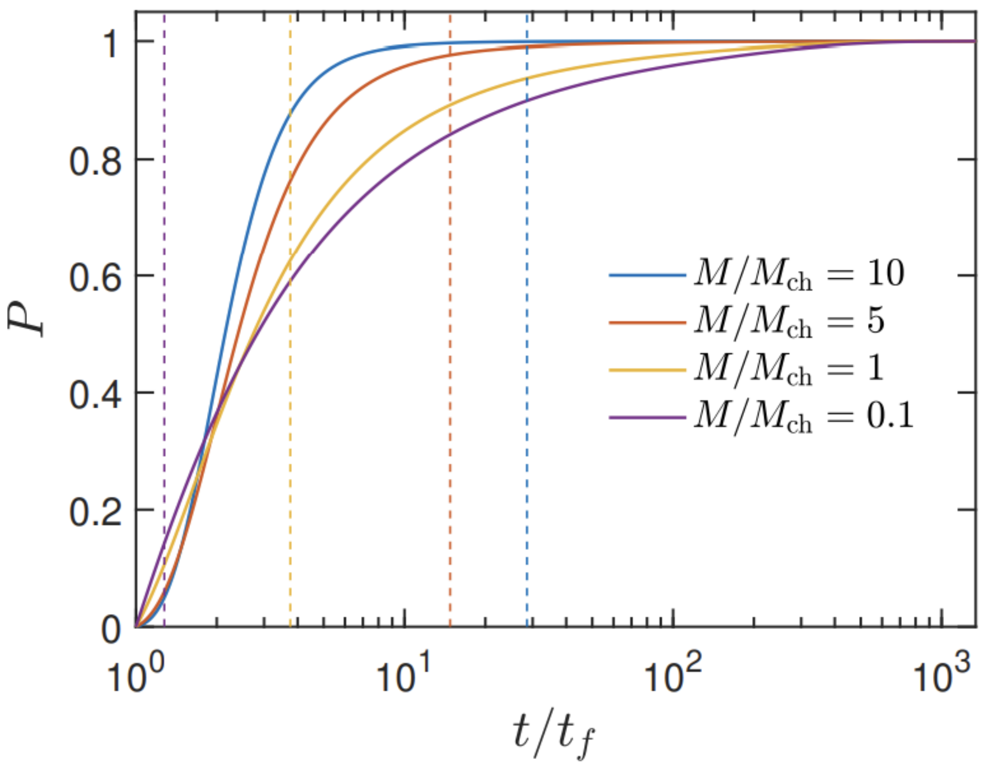


Black holes in early galaxies are too heavy

Black hole mass vs stellar mass in early galaxies (Pacucci et al., 2308.12331)

Cluster survival

The hierarchical structure formation proceeds «more slowly» than the internal evolution of the halo



The probability that a halo of mass M will be absorbed in a halo of mass $2M$ as a function of time. Vertical lines – the moment of dense PBH cluster formation.

The probability that a halo of mass M_1 will be in a halo of mass M_2 by time t_2 *(Lacey and Cole 1993)*

$$P(S_2, t < t_2 | S_1, t_1) = \frac{1}{2}\left[1 - \mathrm{erf}(\mathrm{A})\right] + \frac{\delta_1 - 2\delta_2}{2\delta_1} \exp\left(\frac{2\delta_2(\delta_1 - \delta_2)}{S1}\right)\left[1 - \mathrm{erf}(\mathrm{B})\right]$$

$$A = \frac{S_1\delta_2 - S_2\delta_1}{\sqrt{2S_1S_2(S_1 - S_2)}} \qquad B = \frac{S_2(\delta_1 - 2\delta_2) + S_1\delta_2}{\sqrt{2S_1S_2(S_1 - S_2)}}$$

~10% halos “live” for tens of Hubble times

The fraction of "surviving" clusters

When absorbed into the composition of large halos (host halos), the cluster will experience dynamical friction and settle in the center of the host over time

$$t_{\rm df} = \frac{400\,{\rm Gyr}}{\ln\Lambda}\left(\frac{R}{1\,{\rm kpc}}\right)^2\left(\frac{\sigma_{\rm h}}{10\,{\rm km\,s^{-1}}}\right)\left(\frac{10^4\,M_\odot}{M}\right), \quad (20)$$

The later the cluster is absorbed, the less effective the dynamical friction. If the cluster "survives" and is absorbed in approximately the modern era, then dynamic friction = 0

Dynamical friction time in large halos formed at z

$$t_{\rm df} = 15\,\frac{e^{\xi(4)}}{e^{\xi(z)}}\left(\frac{1+z}{5}\right)^{-3/2}\left(\frac{10^4\,M_\odot}{M}\right)\,{\rm Gyr}$$

$$\xi(z) = \left(5\delta_{\rm c}(1+z)\right)^{0.8},$$

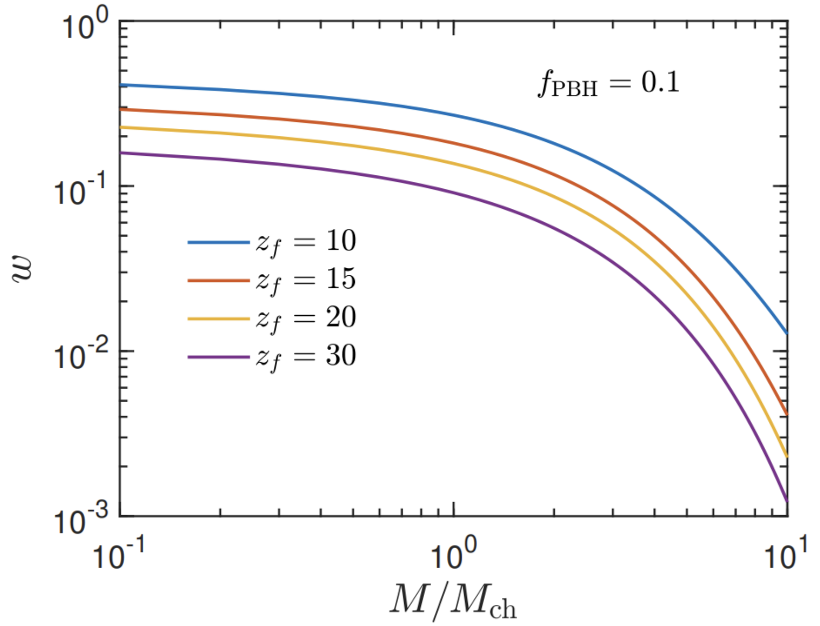


Figure 4. The fraction of surviving clusters by the modern epoch $z = 0$ depending on their mass. Different lines correspond to different moments of formation z_f showed on the legend.

Some of the forming clusters survive to this day

Mergers at large z

Future detectors are unlikely to be able to reconstruct the evolution of the merger rate well enough.

Therefore, one should also look at the integrated merger rate

$$N_{\rm events} = \int_{z_{\rm min}}^{z_{\rm max}} dz \, \frac{\mathcal{R}(z)}{1+z} \frac{dV_c}{dz} \qquad \frac{dV_c}{dz} = 4\pi r^2(z) \frac{c}{H(z)}$$

$$r(z) = \int_0^z dz' \frac{c}{H(z')}.$$

without taking into account the detector's efficiency.

With large amounts of PBH in the DM, there will be many merger events

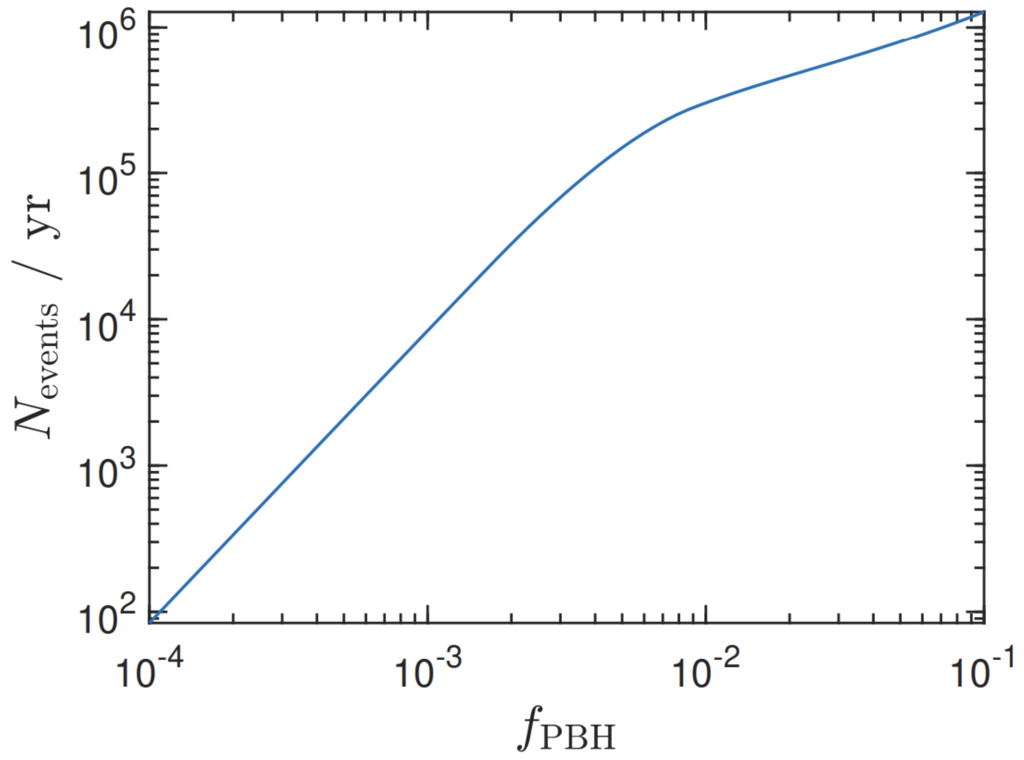


Figure 9. Number of PBH mergers per year, at redshifts $5 < z < 50$ depending on $f_{\rm PBH}$