Ther Role of Quarks in Nuclear Structure: α-clusterization and Nucleosythesis

Genis Musulmanbekov
JINR, Dubna
genis@jinr.ru

ICPPA-2024,MEPhI, 21.10.24 – 25.10.24

Content

- Motivation
- The model: Strongly Correlated Quark Model (SCQM)
  - Nucleon Structure
  - Nuclear Structure: SCQM+FCC
- Nucleosynthesis
  - Hoyle State
  - Hoyle-like 5 α-clusters state
- Summary

Motivation

Nucleosythesis of medium nuclei

- Hydrogen burning: $A \leq 4$
- ^{12}C – the key element for nucleosynthesis

Hoyle State

- Helium burning:

$$\alpha + \alpha \leftrightarrow {}^{8}Be^*$$

$$\alpha + {}^{8}Be^* \rightarrow {}^{12}C^*$$

$${}^{12}C^* \rightarrow \alpha + {}^{8}Be^*$$ The most probable

$${}^{12}C^* \rightarrow {}^{12}C + \gamma$$???

Conventional models

– Shell Model.

– Liquid Drop Model

– Cluster models

SCQM

⇩

FCC –Face-Centered Cubic

SCQM – Strongly Correlated Quark Model of Nucleon Structure

G. Musulmanbekov, "Quarks as Vortices in Vacuum"
in book *Frontiers of Fundamental Physics,*
Kluwer Acad./Plenum Pub., 2001, p. 109-120.

G. Musulmanbekov, "Hadron Modifications in a Dense Baryonic Matter"
PEPAN Lett., Vol., № 5, 2021, p. 548-558

G. Musulmanbekov From quark and nucleon correlations to discrete symmetry and clustering in nuclei in Exotic Nuclei, Singapore: World Scientic, 2016, p. 58{66; http://arxiv.org/abs/1708.04437v2

QCD – fundamental theory of strong interactions

- **Constituents of hadrons** – **quarks** of different flavors carrying spin, charge, color.
  - **flavors: u, d, s, c, b, t**
  - **spin: ½**
  - **charge: ⅓ , ⅔**
  - **color: SU(3)$_{\mathbf{Color}}$ -** $R, G, B, \bar{R}, \bar{G}, \bar{B}$
- **Fields – gluons –** perform interactions between quarks.
- **Nucleons –** 3–quark (**u/d**), color-singlet systems
- **Mesons –** quark-antiquark systems

QCD (cont.)

QCD is non-abelian theory

Hadronic processes with high Q^2

pQCD: $\alpha_S < 1$, $m_q \rightarrow 0$, chiral symmetry

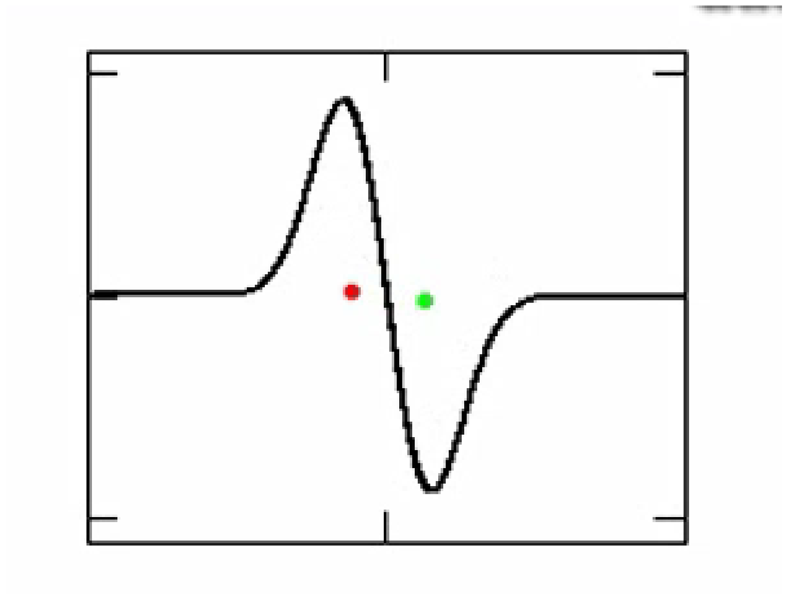
Low energy hadron and nuclear physics

non-pQCD: $\alpha_S > 1$, $m_q \neq 0$, chiral symmetry breaking

- Low energy approx. of QCD, effective theories., ...
- QCD–inspired phenomenology
  - NR constituent quark models
  - Bag models
  - Chiral quark models
  - Soliton models

quark – antiquark pair

φ(x,t)



Quarks – Solitons

SCQM ≡ Breather Solution of Sine-Gordon equation

$$\partial_\mu \partial^\mu \phi(x,t) + \sin\phi(x,t) = 0$$

Breather – oscillating soliton-antisoliton pair, the periodic solution of SG:

$$\phi(x,t)_{sas} = 4\tan^{-1}\left[\frac{\sin\left(ut/\sqrt{1+u^2}\right)}{u\cosh\left(x/\sqrt{1+u^2}\right)}\right]$$

$$\phi(x,t)_{sas} = \frac{\partial \phi(x,t)_{sa}}{\partial x}$$

is identical to our quark-antiquark system.

The SCQM

Hamiltonian of the quark – antiquark system

$$H = \frac{m_{\bar{q}}}{(1-\beta^2)^{1/2}} + \frac{m_q}{(1-\beta^2)^{1/2}} + V_{\bar{q}q}(2x)$$

$m_{\bar{q}}, m_q$ - current masses of quarks,

$\beta = \beta(x)$ – velocity of the quark (antiquark),

$V_{\bar{q}q}$ - quark–antiquark potential.

$$H = \left[\frac{m_{\bar{q}}}{(1-\beta^2)^{1/2}} + U(x)\right] + \left[\frac{m_q}{(1-\beta^2)^{1/2}} + U(x)\right] = H_{\bar{q}} + H_q$$

$U(x) = \frac{1}{2} V_{\bar{q}q}(2x)$ is the potential energy of a single quark/antiquark.

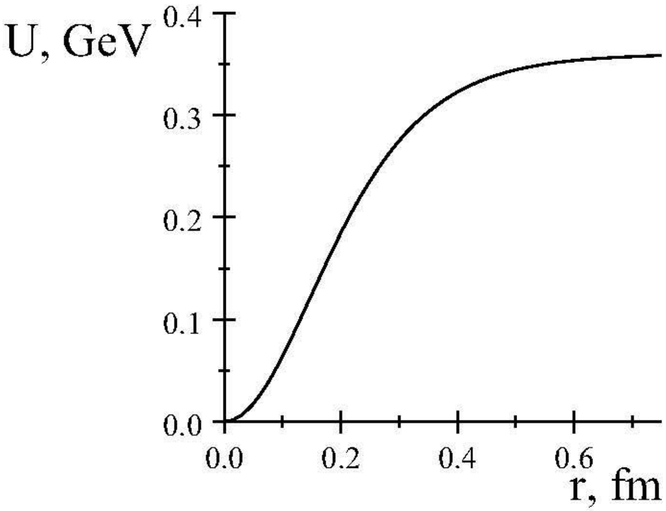
$$U(x) = \frac{1}{2} V_{\bar{q}q}(2x) = m \tanh^2(ax)$$

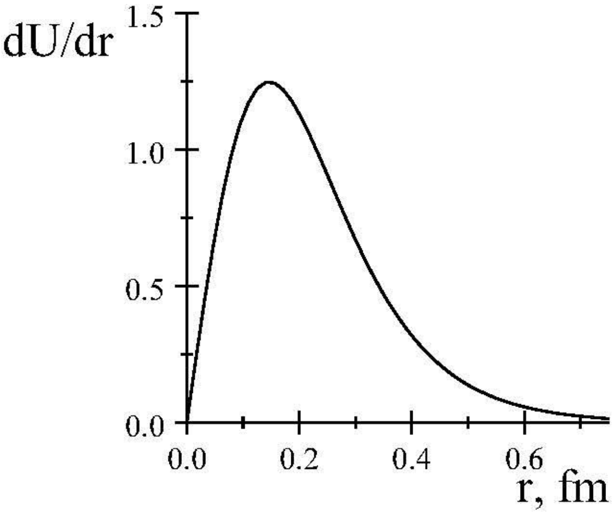
11

Quark Potential

Force
of quark-antiquark interaction

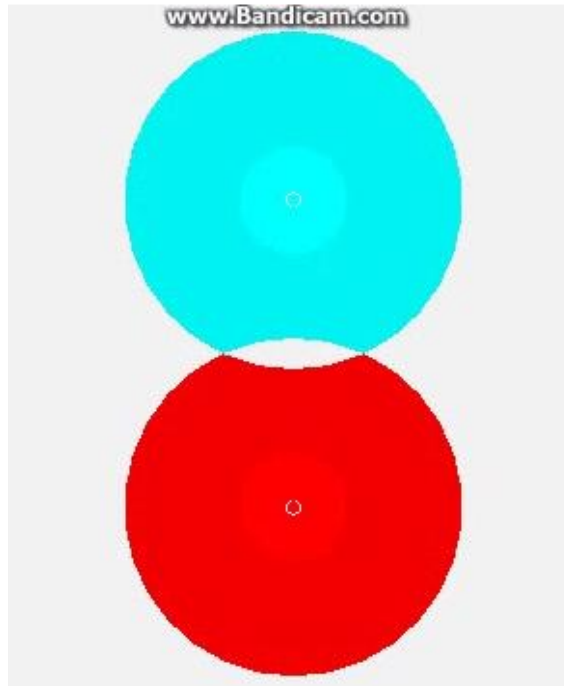
$U_q = 0.36\tanh^2(ax)$





quark–antiquark pair

meson



QCD: Exchange by gluons $\sqrt{\frac{1}{2}}(R\bar{R}+B\bar{B})$

SCQM: Overlap of color fields

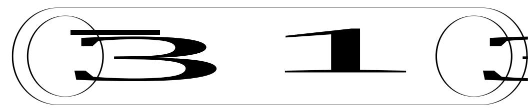
13

Generalization to the 3 – quark system (baryons)

$SU(3)_{Color}$

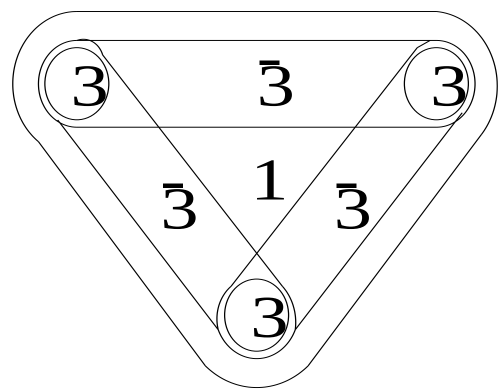
$q \Rightarrow SU(3) \Leftrightarrow RGB \qquad q \Rightarrow SU(3) \Leftrightarrow CMY$

$\bar{q}q \Rightarrow$



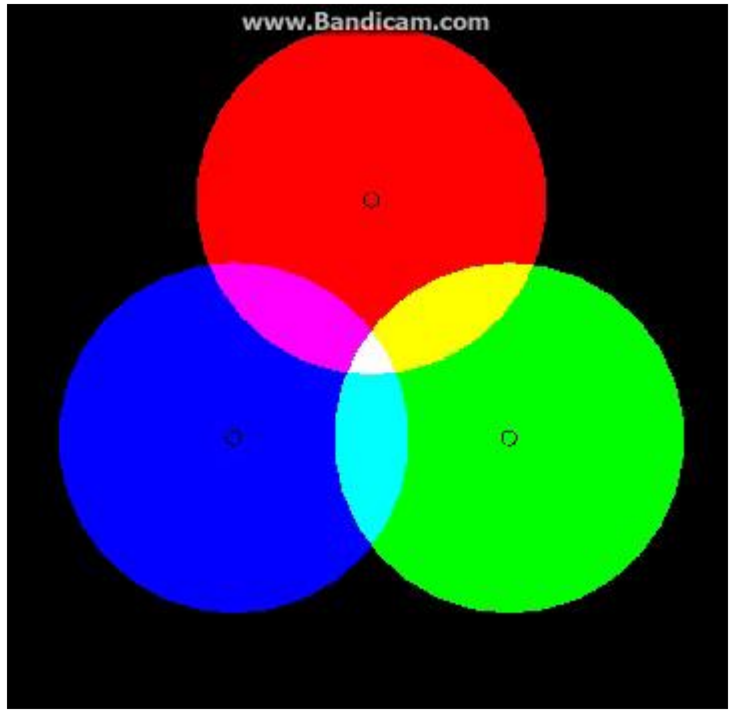
[illegible] $= \bar{q} \to qq$

$qqq \Rightarrow$



14

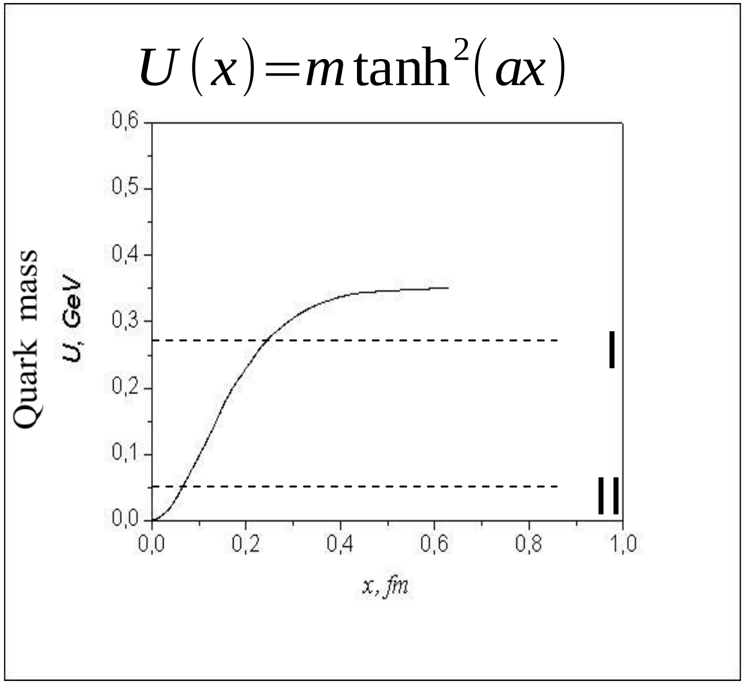
Nucleon
as 3 oscillating color quarks



"The wave packet solution of time-dependent Schrodinger equation for harmonic oscillator moves in exactly the same way as corresponding classical oscillator"
E. Schrodinger, 1926

15

Dynamic Breaking-Restoration of Chiral Symmetry



U(x) > I – constituent quarks

U(x) < II – current (relativistic) quarks

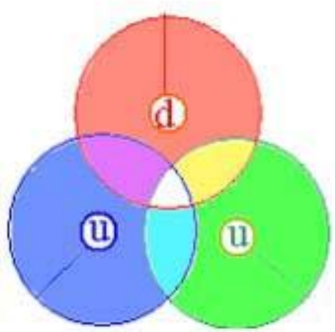
Interplay between constituent and current quark states

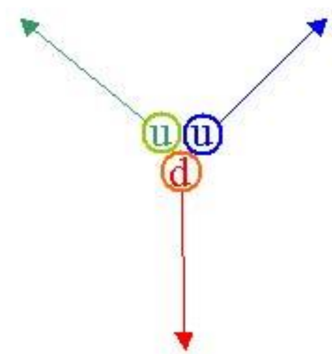
Chiral Symmetry Breaking ⟺ Restoration

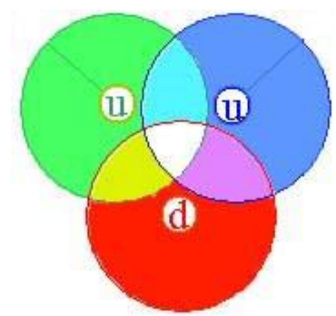
t = 0
x = xmax

t = T/4
x = 0

t = T/2
x = - xmax





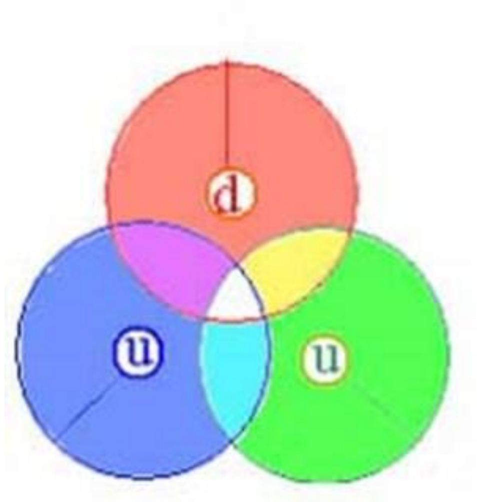


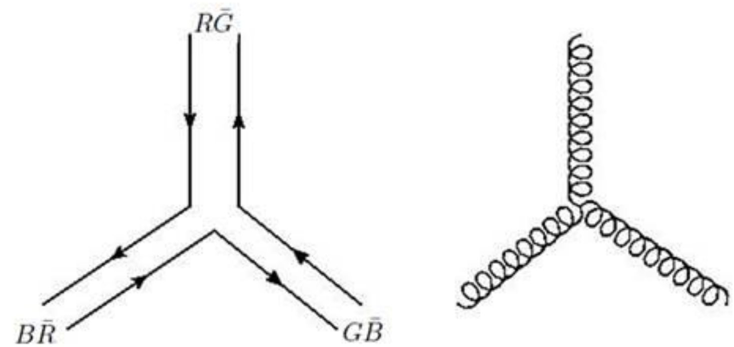
Constituent quaks | current quarks | Constituent quarks

Asymptotic freedom

During the valence quarks oscillations:

SCQM vs QCD





Parameters of SCQM for the Nucleon

1.Mass of Consituent Quark

$$M_Q(x_{max}) = \frac{1}{3}\left(\frac{m_\Delta + m_N}{2}\right) \text{[illegible]}$$

2.Amplitude of VQs oscillations : $x_{max}=0.64\ fm$,

3.Constituent quark dimensions (parameters of gaussian distribution): $\sigma_{x,y}=0.24\ fm$, $\sigma_z=0.12\ fm$

Parameters 2 and 3 are derived from comparison of Inelastic Overlap Function (IOF) and σ_{tot} in $\bar{p}\,p$ and pp – collisions.

Nucleons are nonspherical, triangular shaped!

They are three-colored objects!

19

Quark Arrangements inside Nuclei

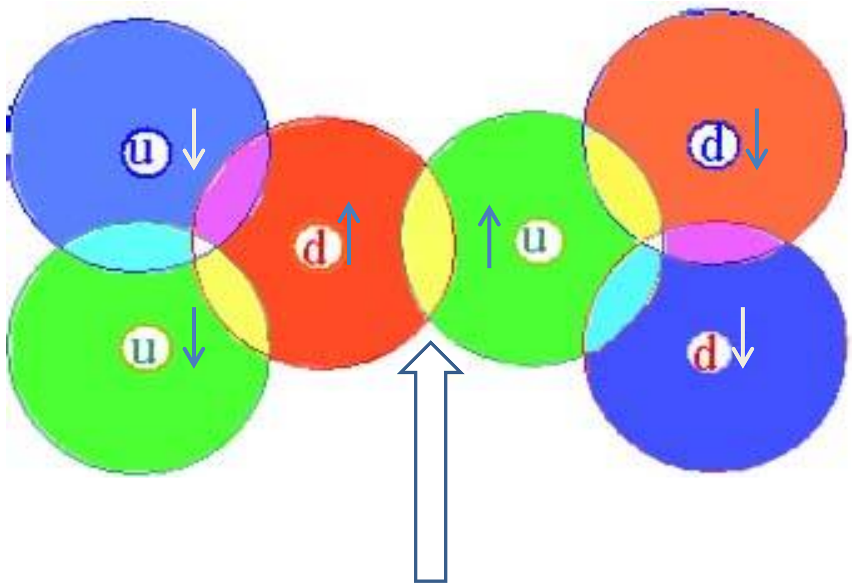
Strongly Correlated Quark Model

↓

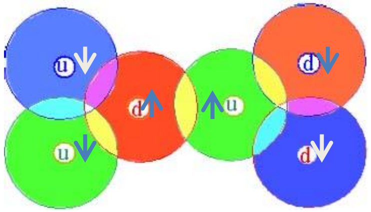
Lattice-like arrangement of Nuclear Structure

20

Two Nucleon System in SCQM



Interaction between nucleons is due to **overlap** of their quark color fields



Antisymmetrization

We need to define **isospins, spins and colors at junctions**

^{4}He: 4 nucleons = 12 quarks in s-state

Antisymmetrization

$$SU(12) \longrightarrow SU(2)_{isospin} \otimes SU(2)_{spin} \otimes SU(3)_{color}$$

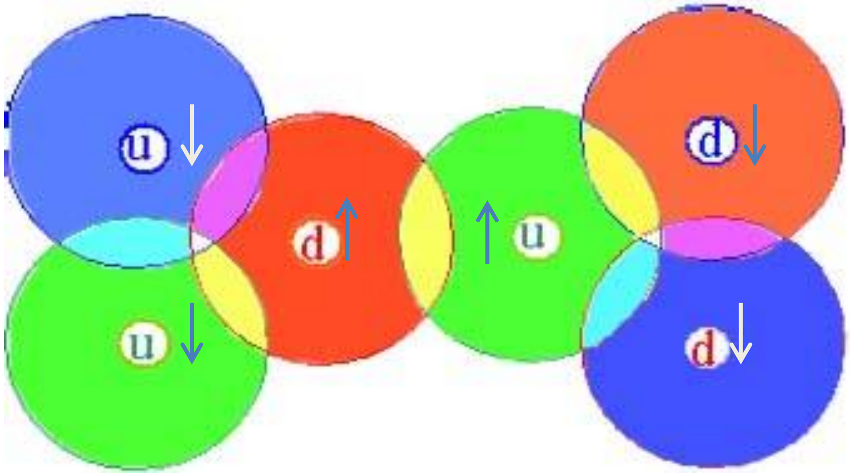
But ~ 90% of 3-quark clusters are colored states (*Matveev, Sorba, 1978*)
We select colorless 3-quark clusters by combinatorics imposing the following requirements to isospins, spins and colors at junctions:

$SU(2)_{isospin}$ – of different flavors (assumed)

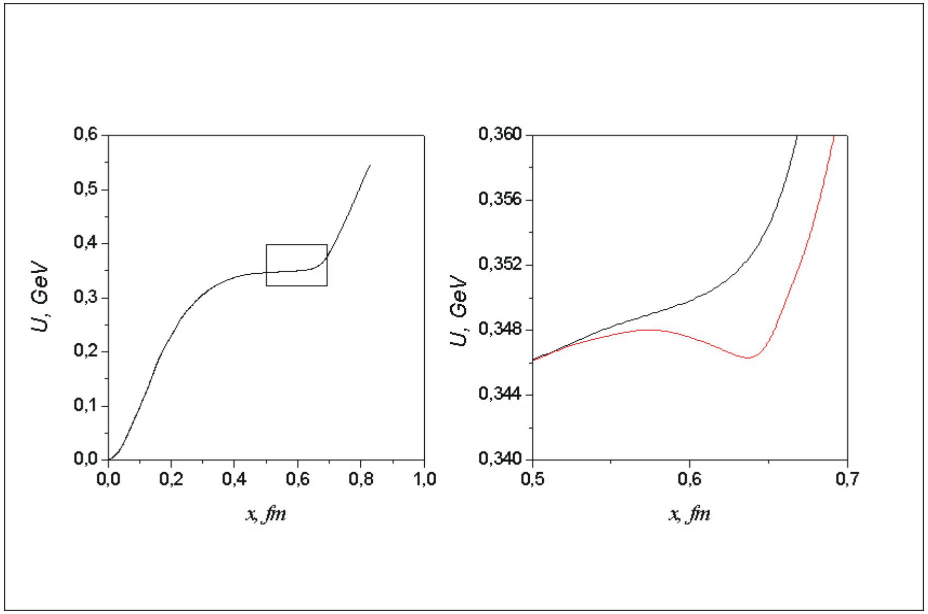
$SU(2)_{spin}$ – of parallel spins (calculated)

$SU(3)_{color}$ – of different colors (assumed)

Two Nucleon System in SCQM

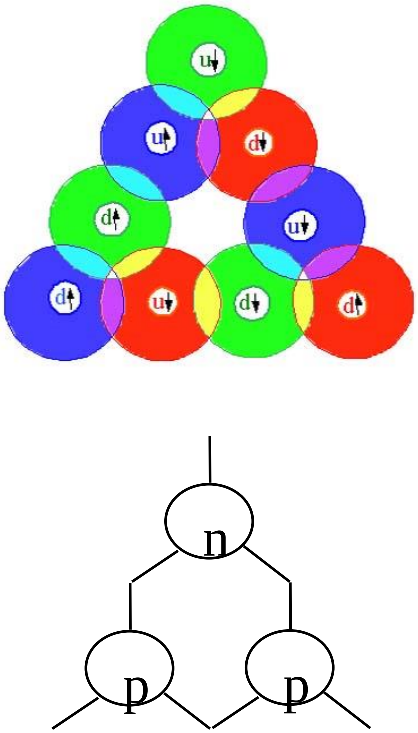


Quark Potential Inside Nuclei



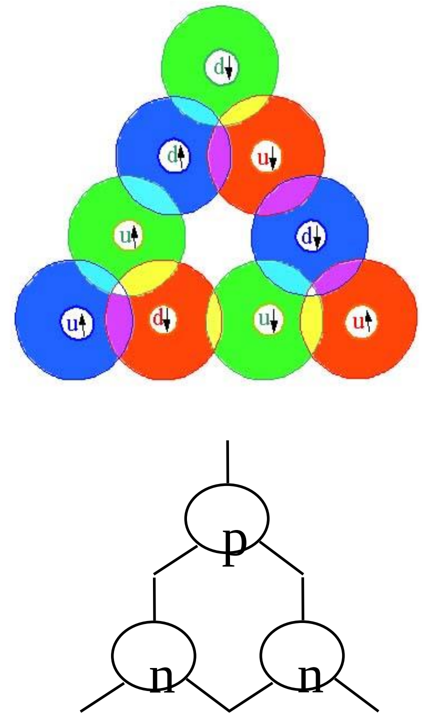
Building blocks in Shell Structure

^{3}H



^{3}He – block

^{3}He



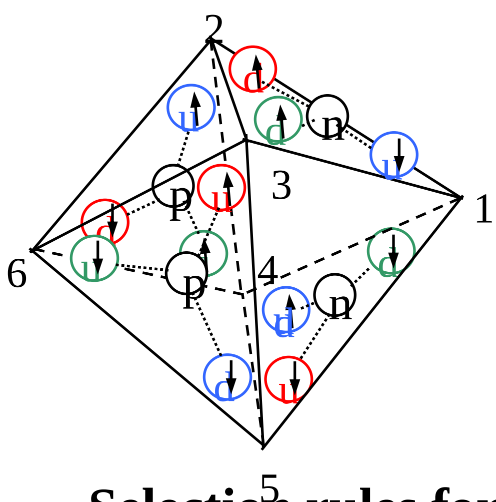
^{3}H – block

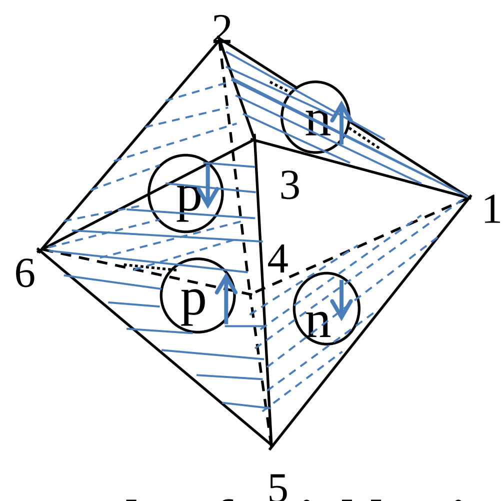
The closed shell n = 0, nucleus ^{4}He

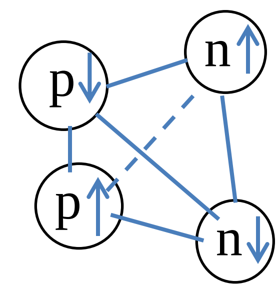
Antisymmetisation of
12 quarks in SU(12) state
$SU(2)_I \times SU(2)_S \times SU(3)_C$

Totally antisymmetrized
4 nucleons in **s**-state

Shell Closure







Selection rules for binding two quarks of neighboring nucleons at a junction:

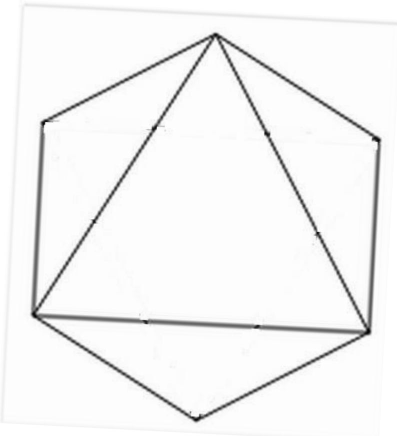
- $SU(2)_{Isospin}$ – of different flavors
- $SU(3)_{Color}$ – of different colors
- $SU(2)_{Spin}$ – of parallel spins

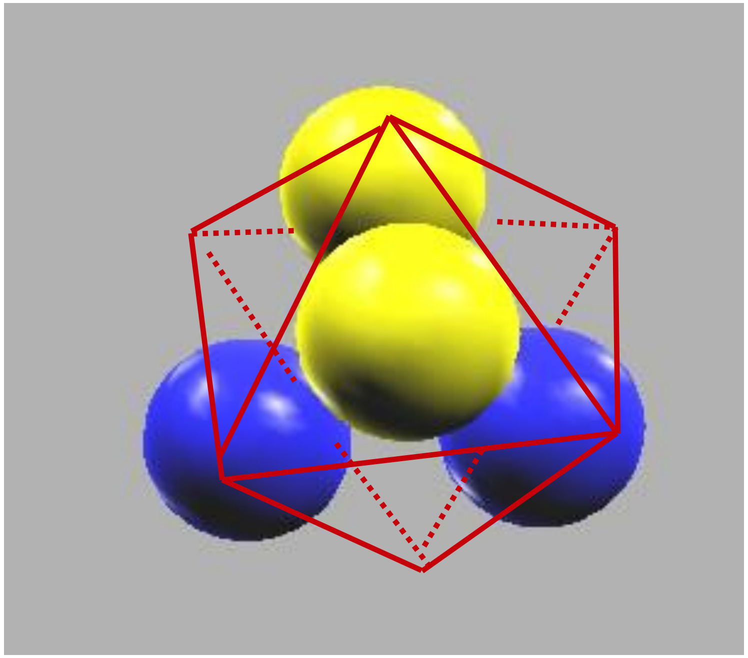
Experimental Binding Energy of Stable Nuclei and Quark Loops in SCQM

Nucleus	E_B MeV/junct. Exp.	Number of quark loops	Free quark ends	Nuclear forces
d	2.05	**0**	4	2-body
^3H	2.83	**1**	3	3-body
^3He	2.57	**1**	3	3-body
^4He	7.07	**4**	0	4-body

The more quark loops, the stronger the binding energy!

The closed shell n = 0, nucleus ^{4}He

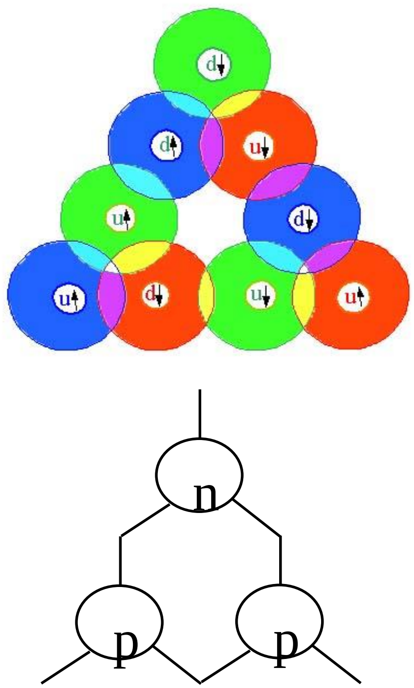




Yellow – protons are on opposite faces of upper piramid
Blue – neutrons are on another faces of below lower piramid

Building blocks in Shell Structure

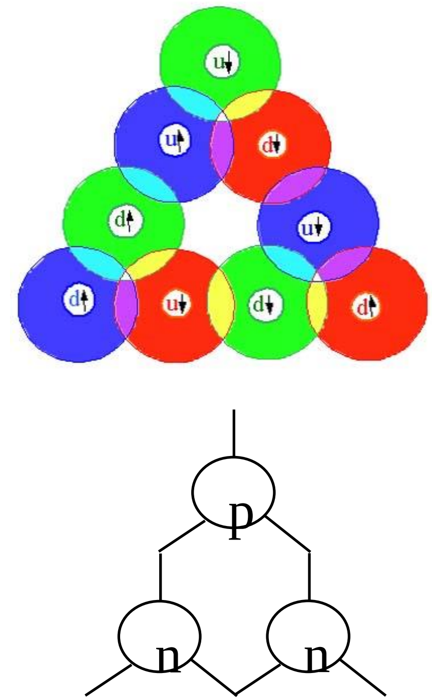
^{3}He



^{3}He – block

Forms Proton Halo

^{3}H

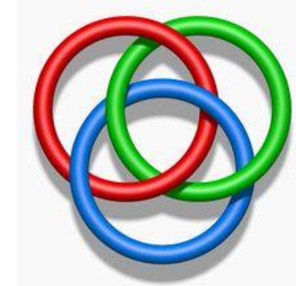


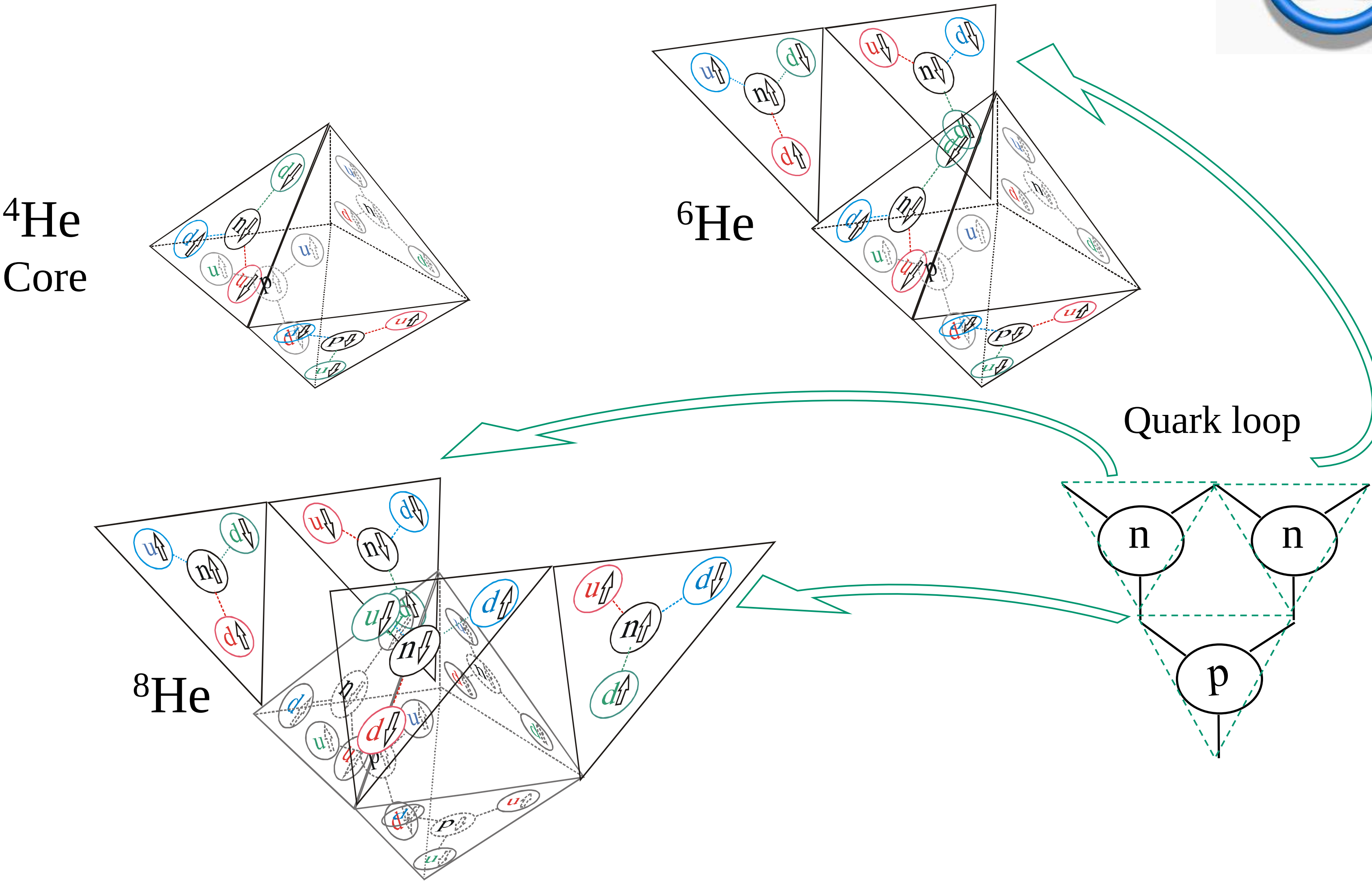
^{3}H – block

Forms Neutron Halo

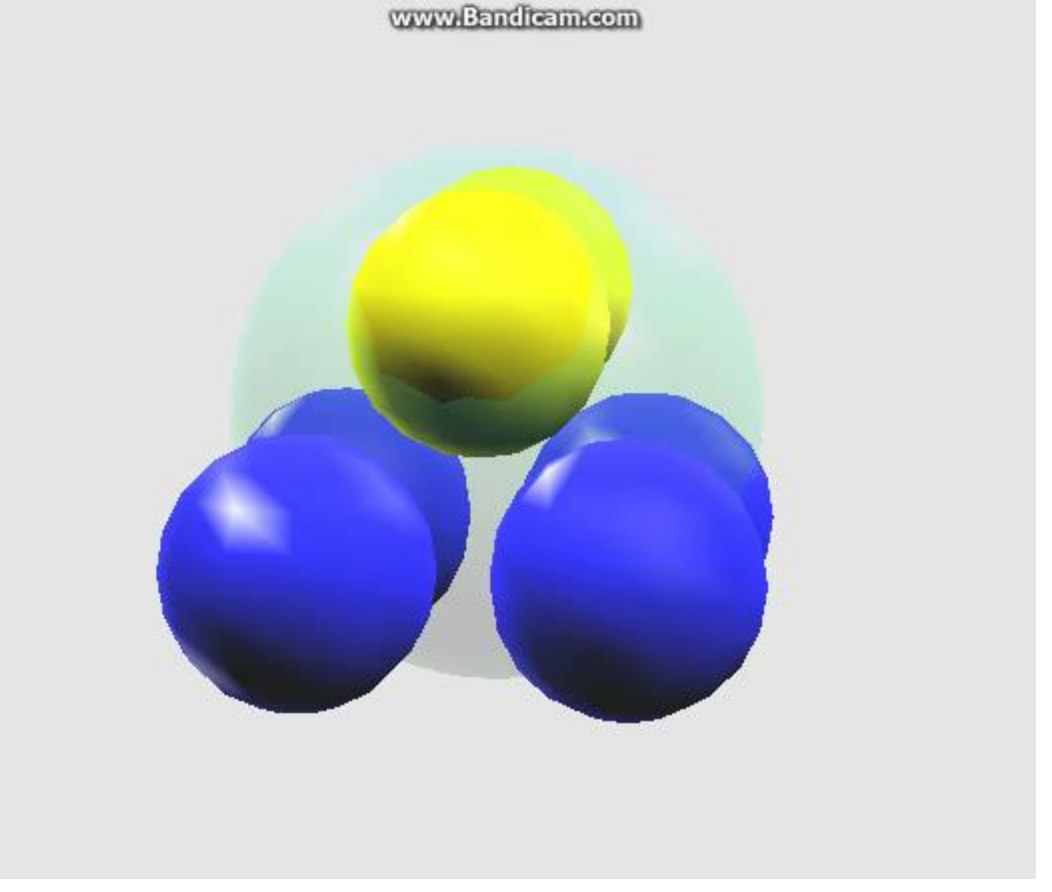
29

Helium Isotopes
Borromean Nuclei

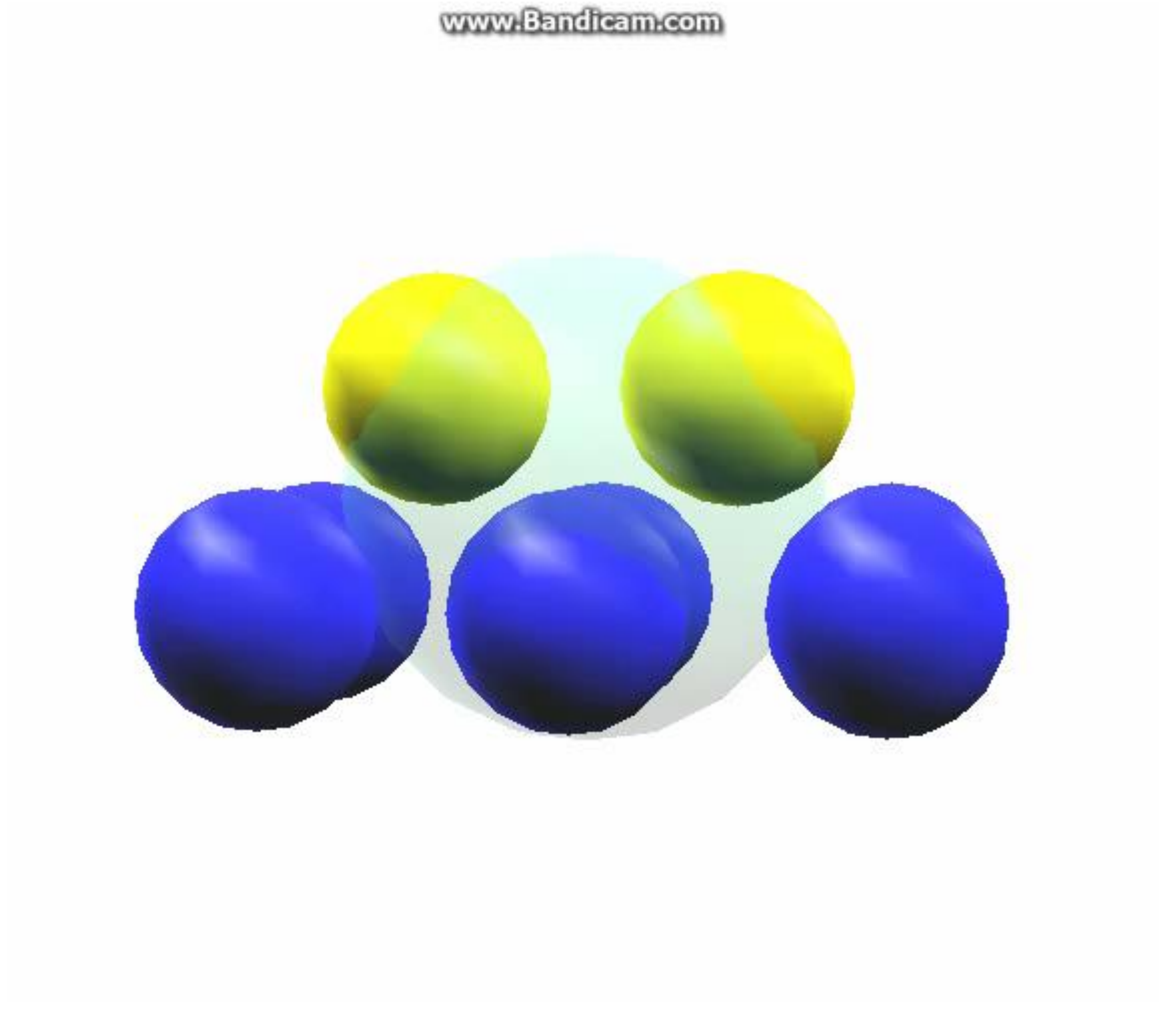




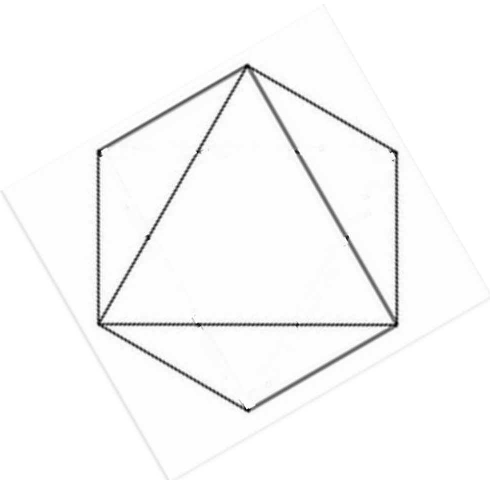
^{6}He, borromean

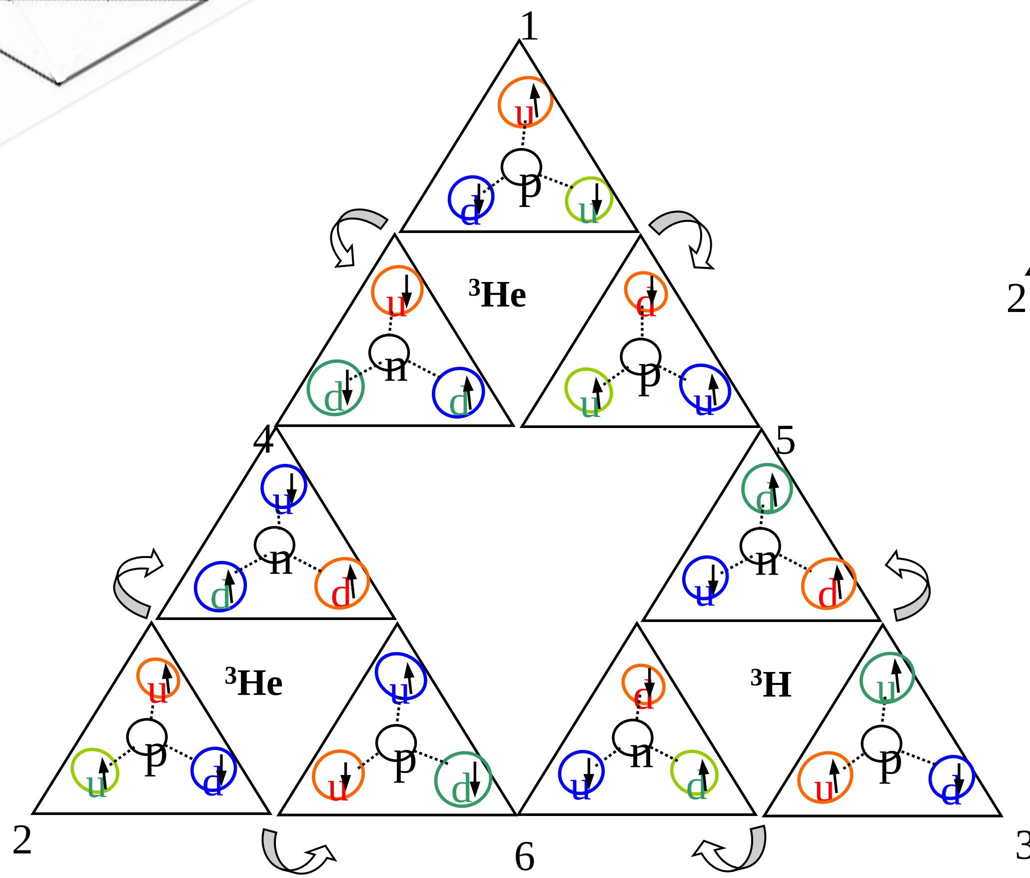


^{8}He, borromean

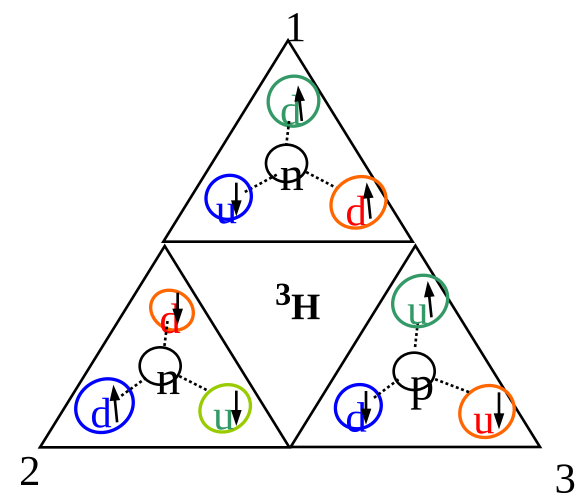


Next closed shell n = 1, ^{16}O





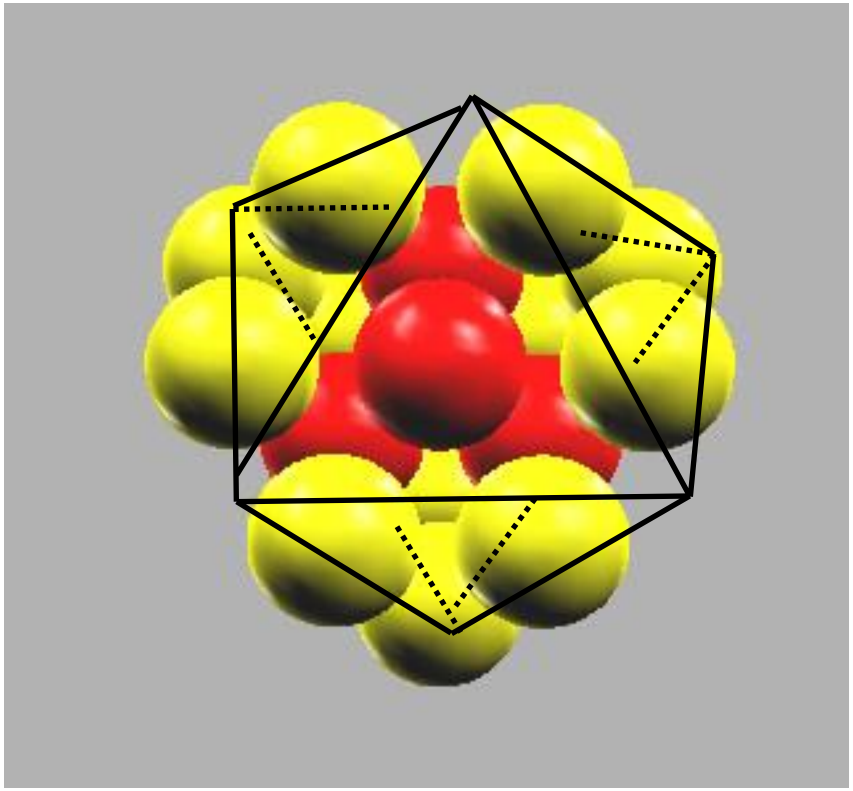
Face of ^{16}O octahedron



In analogy with ^{4}He

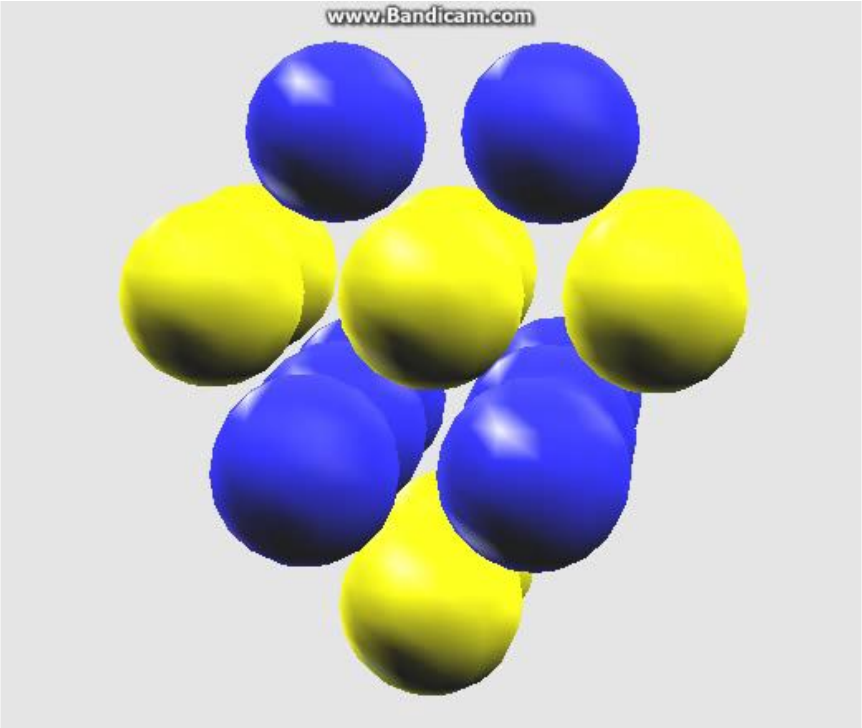
^{3}He and ^{3}H as proton and neutron in ^{4}He

^{16}O



RED – s-shell
YELLOW – p-shell

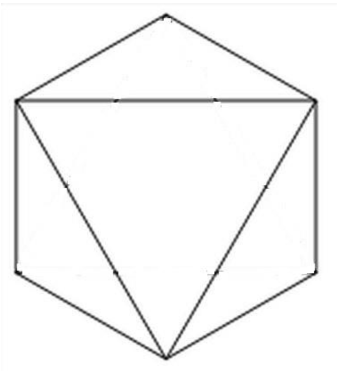
^{16}O



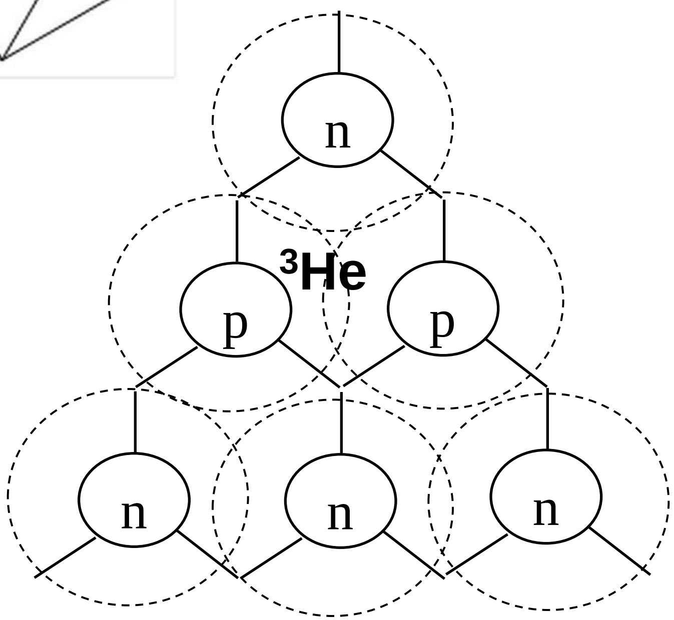
Yellow – protons
Blue – neutrons

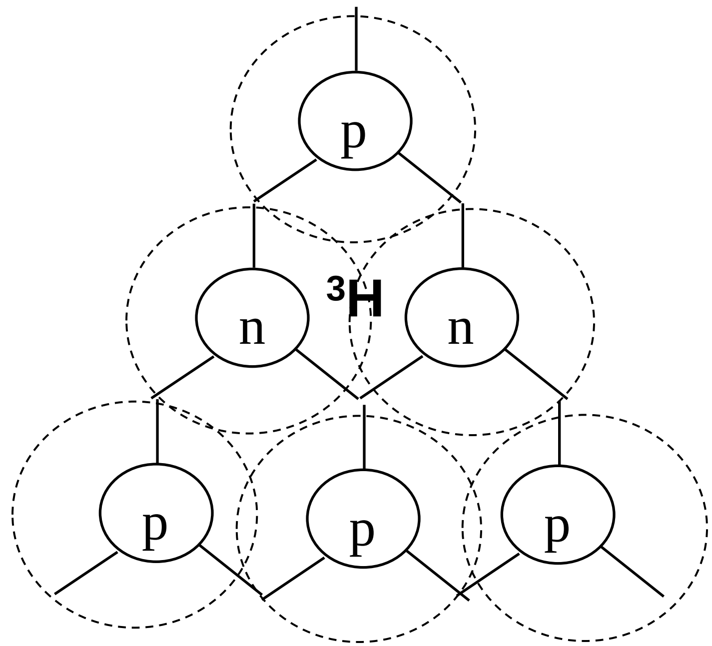
The closed shell n = 2, ^{40}Ca

Shell Closure



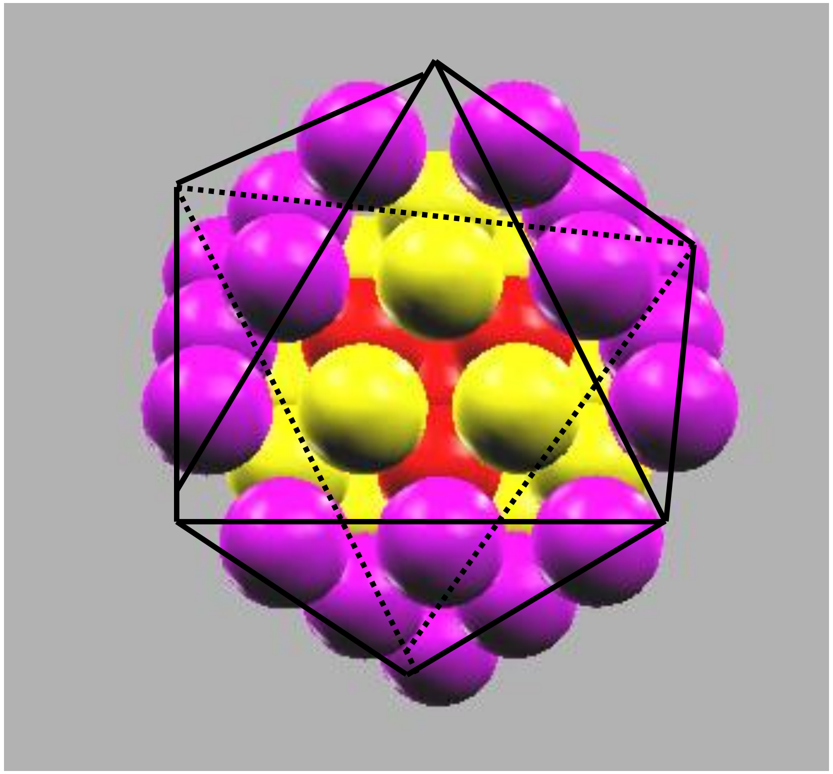
Faces of ^{40}Ca octahedron





36

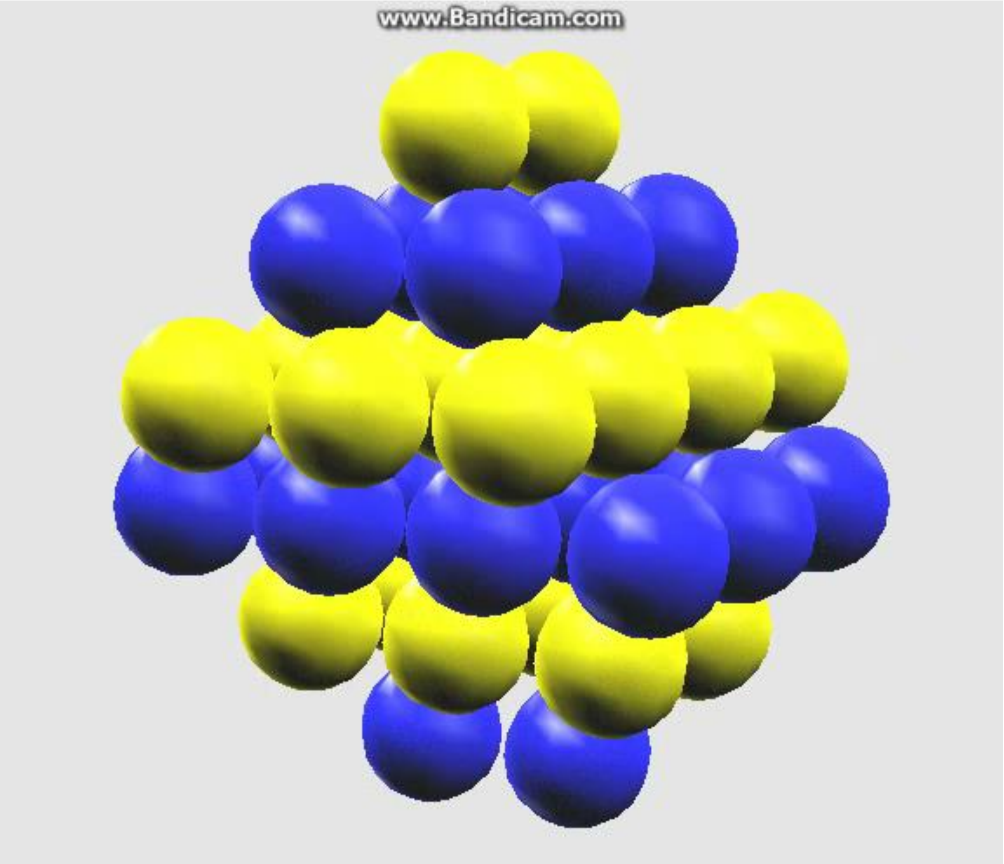
^{40}Ca



s-shell – red
p-shell – yellow
d-shell -- magenta

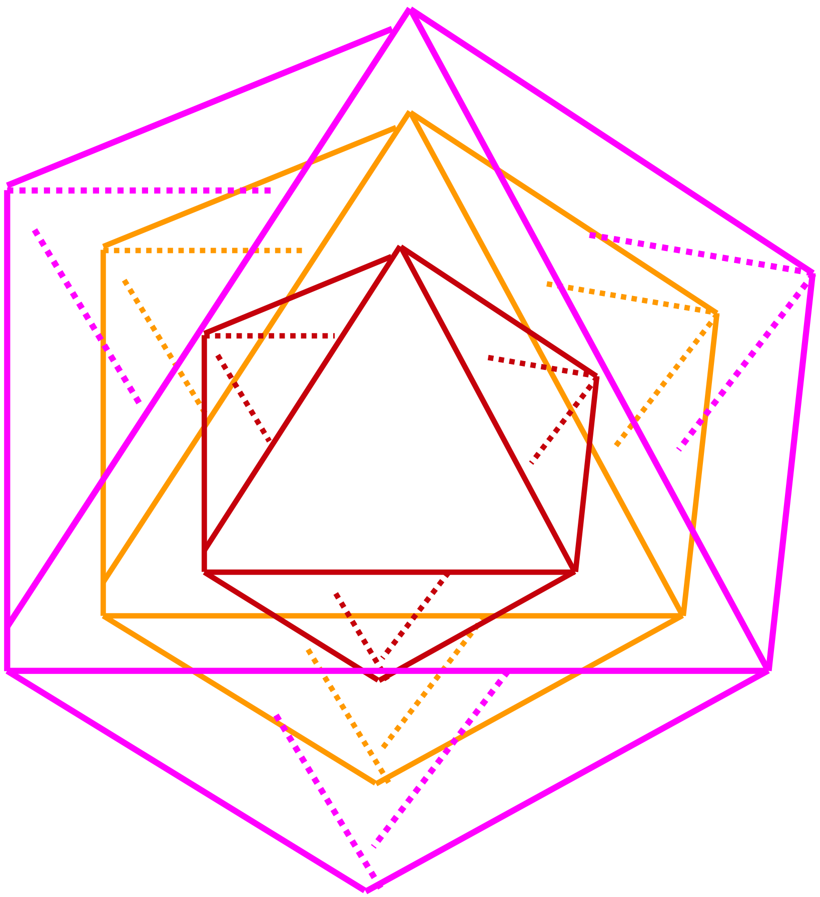
37

^{40}Ca



Yellow – protons
Blue – neutrons

^{40}Ca



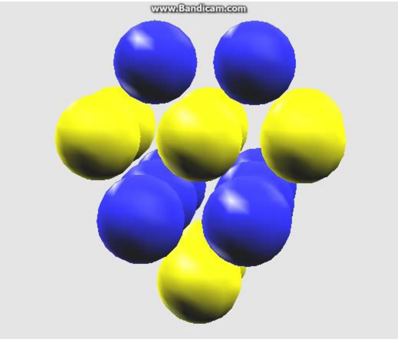
3 Nested Octahedra – s, p, d -shells

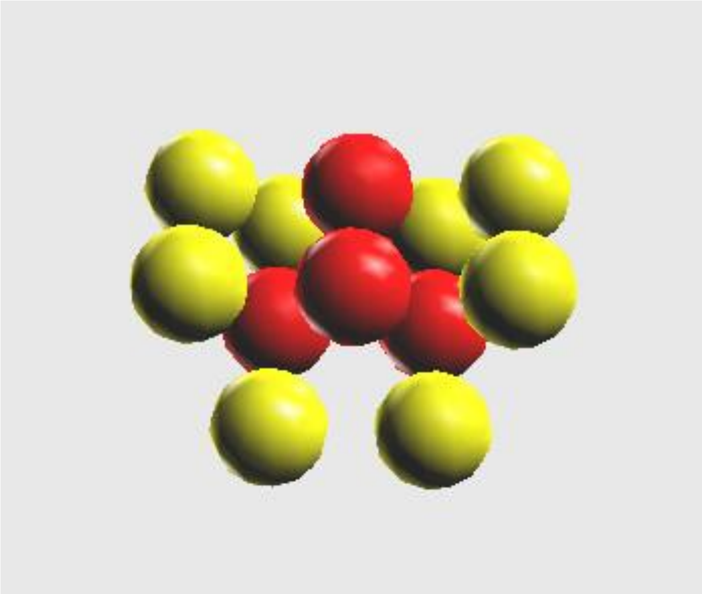
SCQM to FCC symmetry

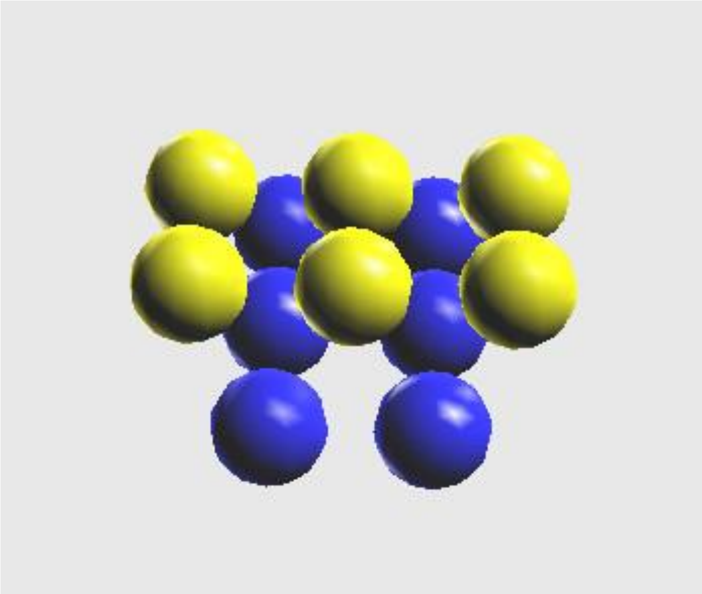
- Nuclear shells correspond to faces of nested octahedra
- Nucleons are arranged in alternating isospin and spin layers
- Protons and neutrons are **strongly correlated**
- **It turned out that** nucleons occupy the nodes of **Face Centered Cubic Lattice (FCC)**
- All bound nuclei are composed of **virtual ^{3}H-** and **^{4}He-**like **clusters**

^{12}C

6 protons, 6 neutrons







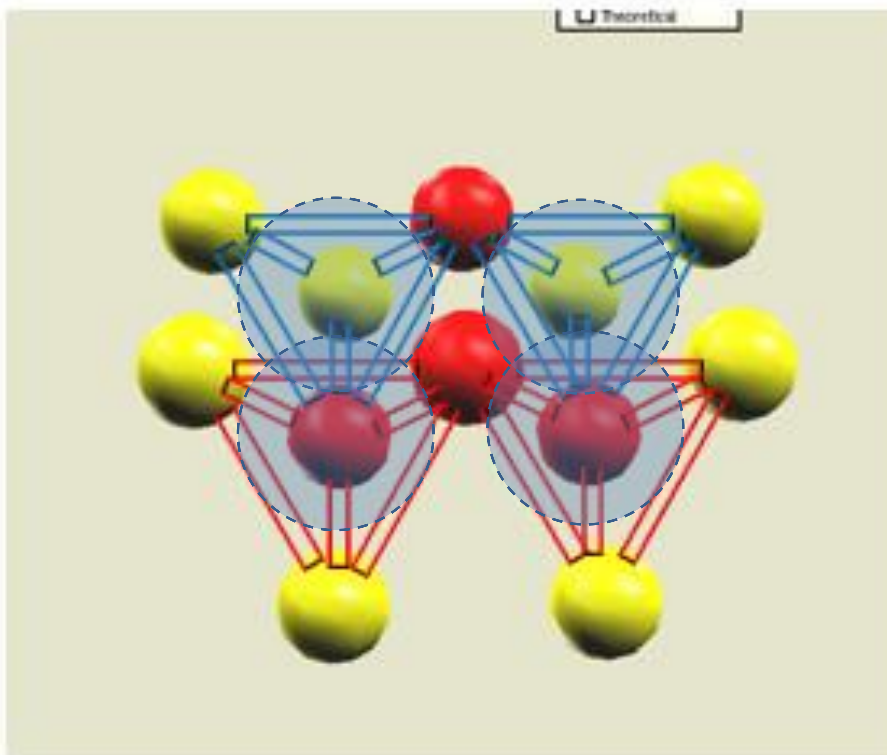
Problem for SM:

Why ^{12}C is so stable?

41

Virtual α-clusters

^{12}C - 4 virtual α-clusters

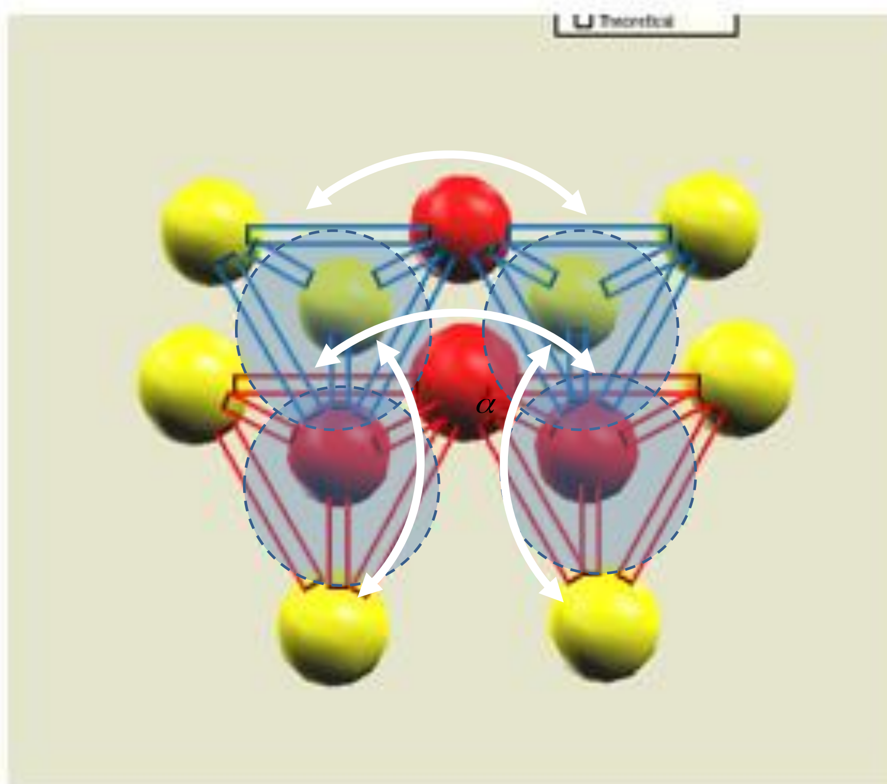


- 4 nucleons of ***s***-shell (red) form with
  6 nucleons of ***p***-shell (yellow) 4 virtual α-clusters.
- ***s***-shell nucleons are exchange particles

42

^{12}C

Crosswise bindings of 4 virtual α-clusters
by exchange (red) nucleons of s-shell

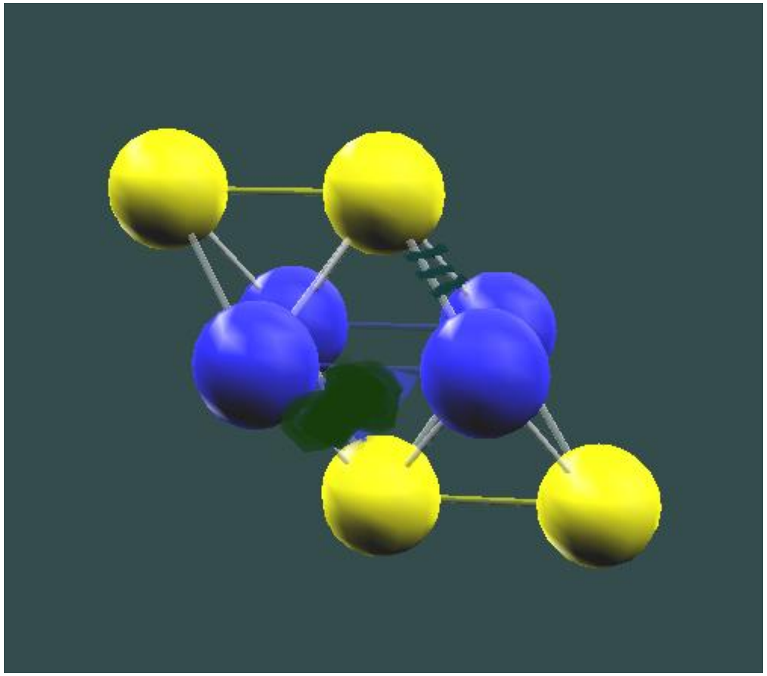


- exchange nucleons acquire larger binding energy as belonging simultaneously to 2 alpha clusters
- **s**-shell core is rearranged and disappears

43

Hoyle State

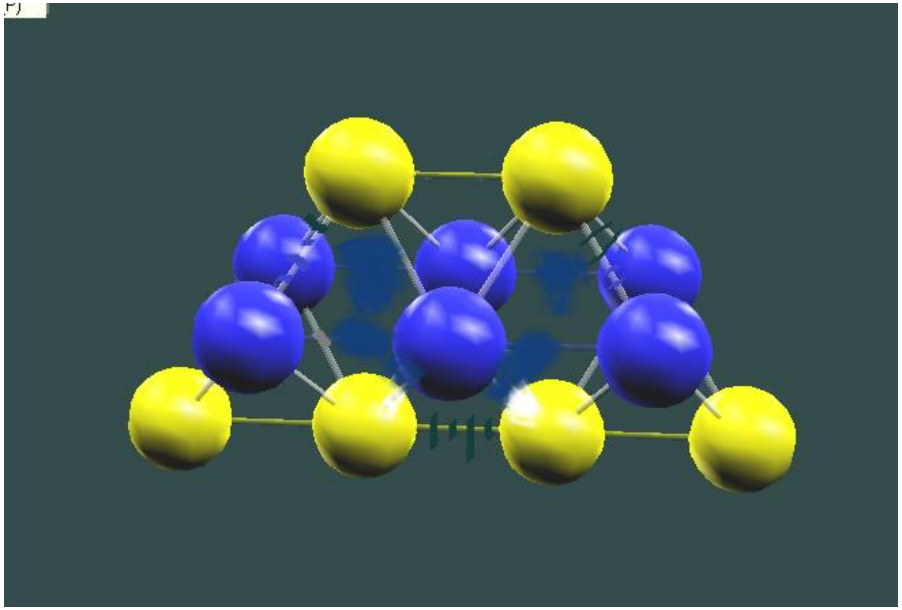
^{8}Be - 2 Loosely bound α- clusters



Neutrons of the right α-cluster are bound with protons of central α-cluster (like in ^{6}He)

Hoyle State

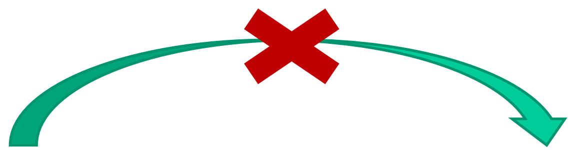
$^{12}C^*$- 3 Loosely bound α- clusters



Neutrons of left and right α-clusters are bound with protons of central α-cluster (like in ^8He), and their 2 nearest protons are bound together. Bonds between α-clusters are depicted by dashed lines.

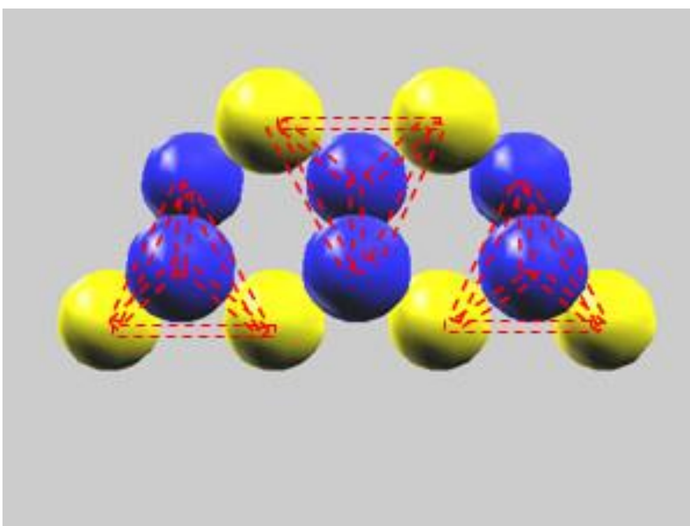
Hoyle State to ^{12}C

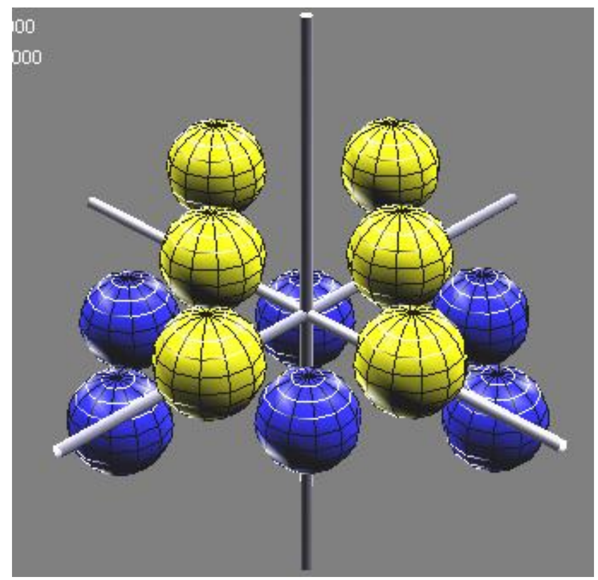
Transition from Hoyle state to ^{12}C g.s.
by γ emission is
IMPOSSIBLE



Hoyle Stale

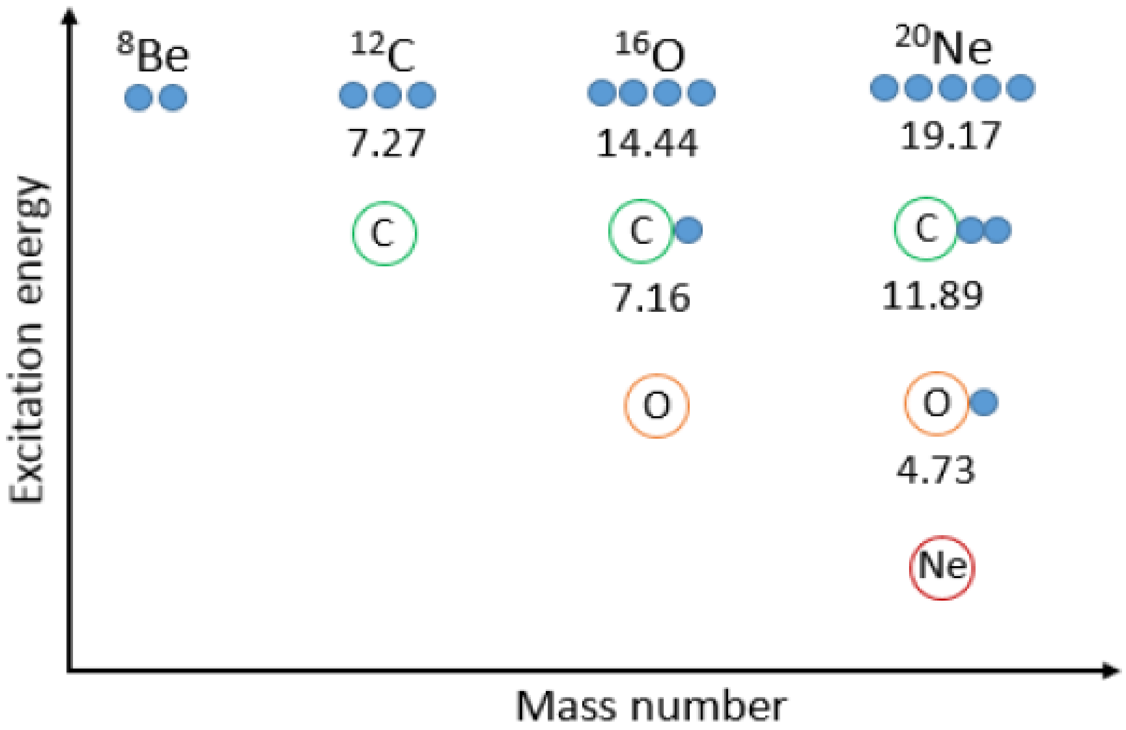
^{12}C g.s





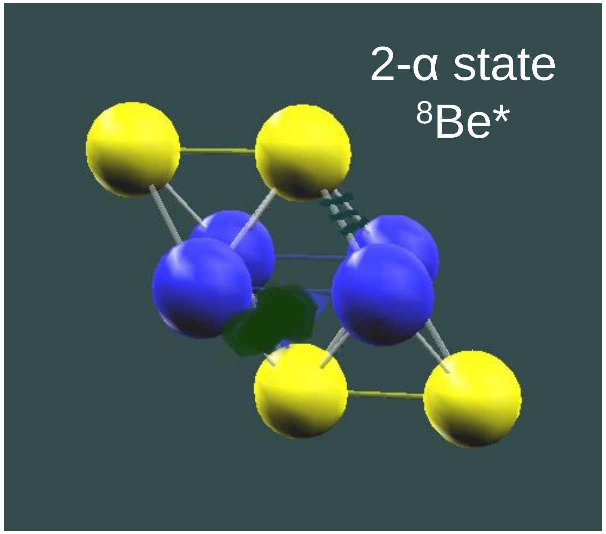
Hoyle-like States

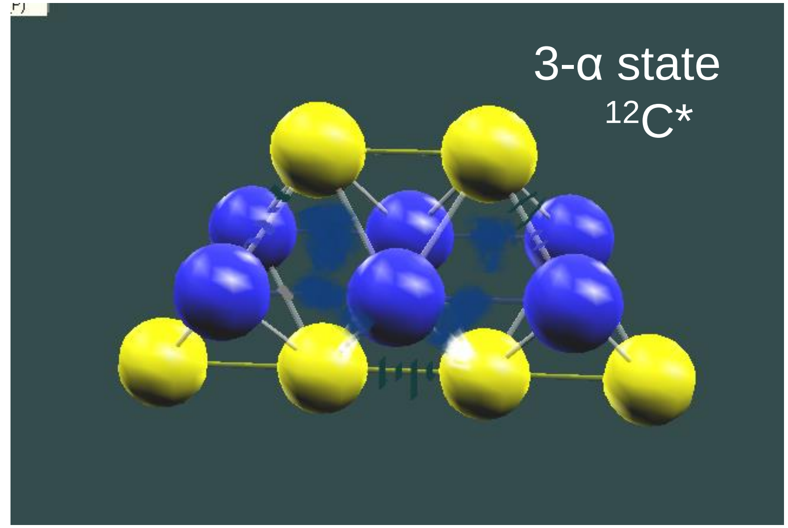
Ikeda diagram

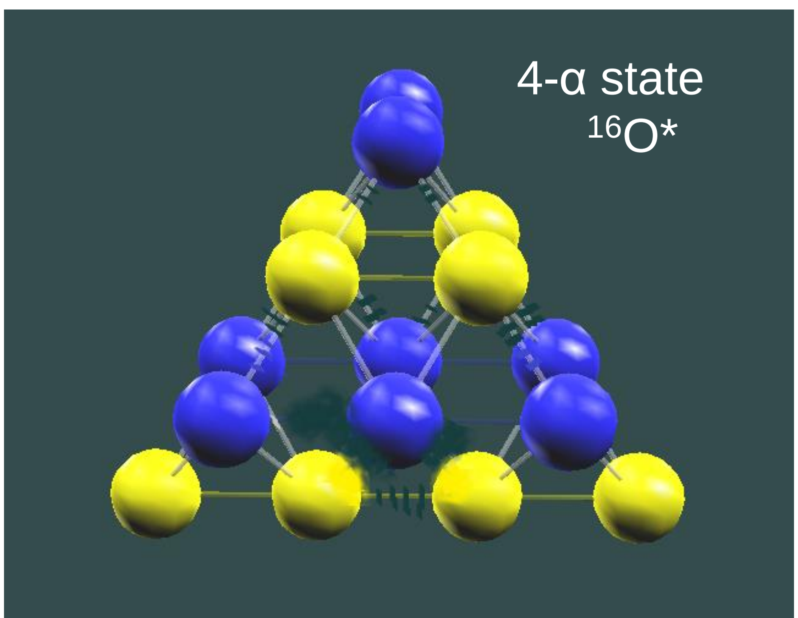


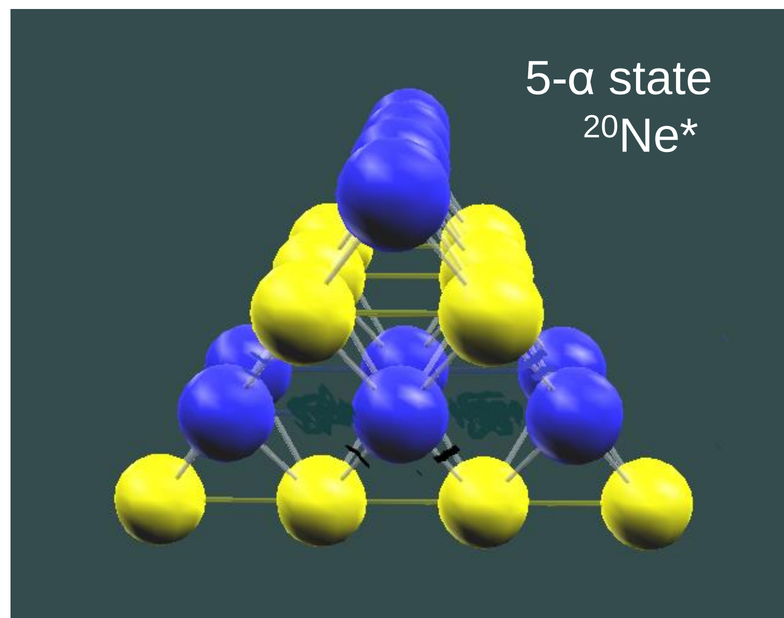
Nucleosynthesis in SCQM

^{4}He burning

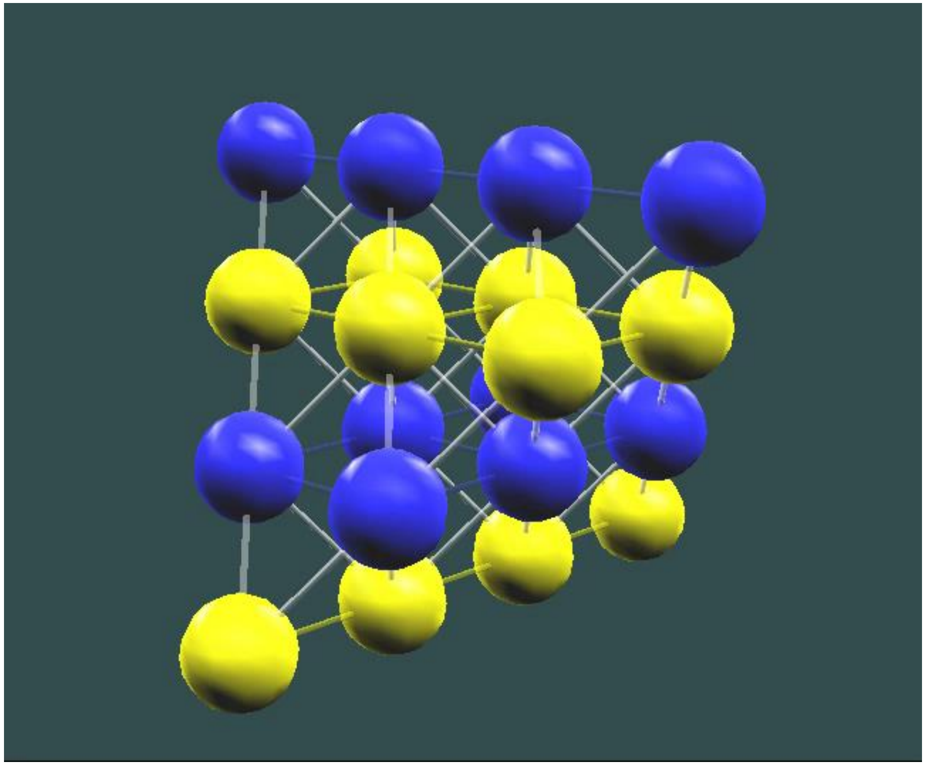






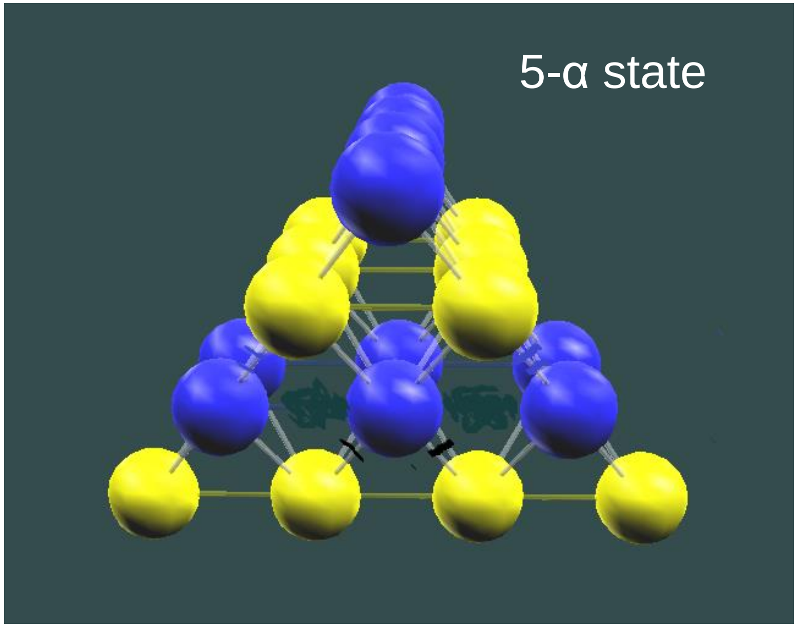


Nucleosynthesis in SCQM
^{4}He burning
^{20}Ne*- Key Nucleus for nucleosynthesis

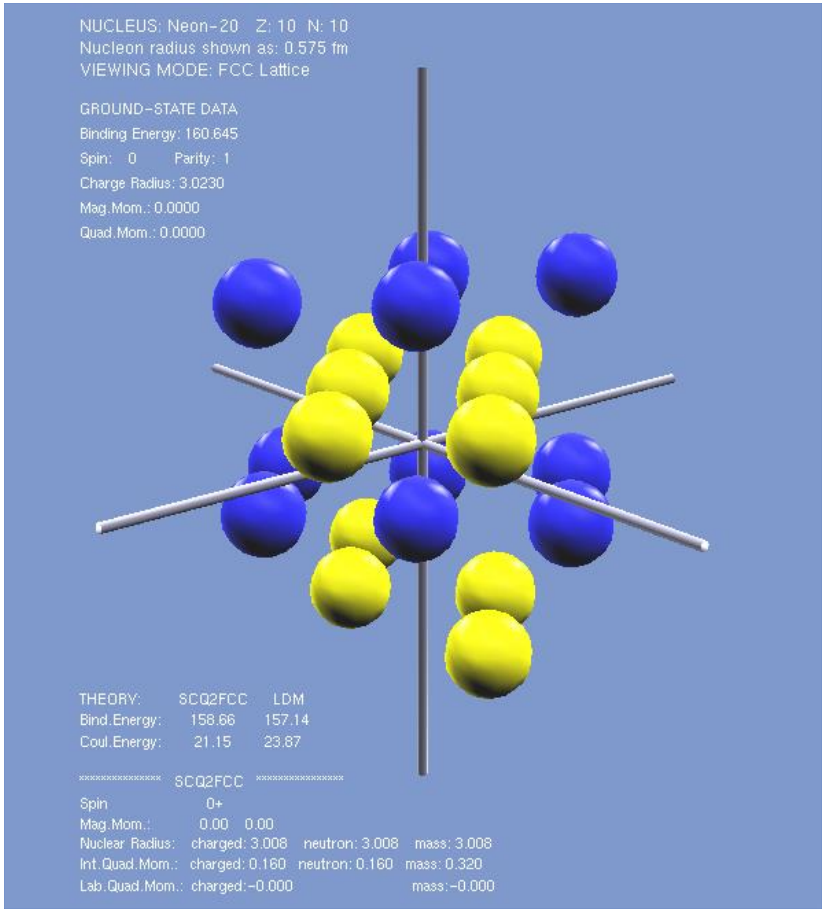


Nucleosynthesis in SCQM
^{4}He burning

^{20}Ne* → ^{20}Ne + 19.7 MeV

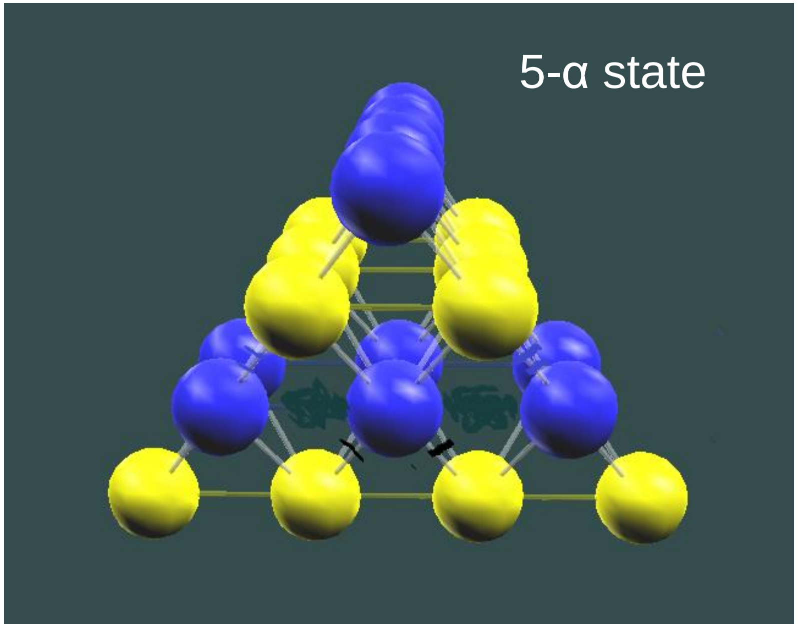


^{20}Ne g.s

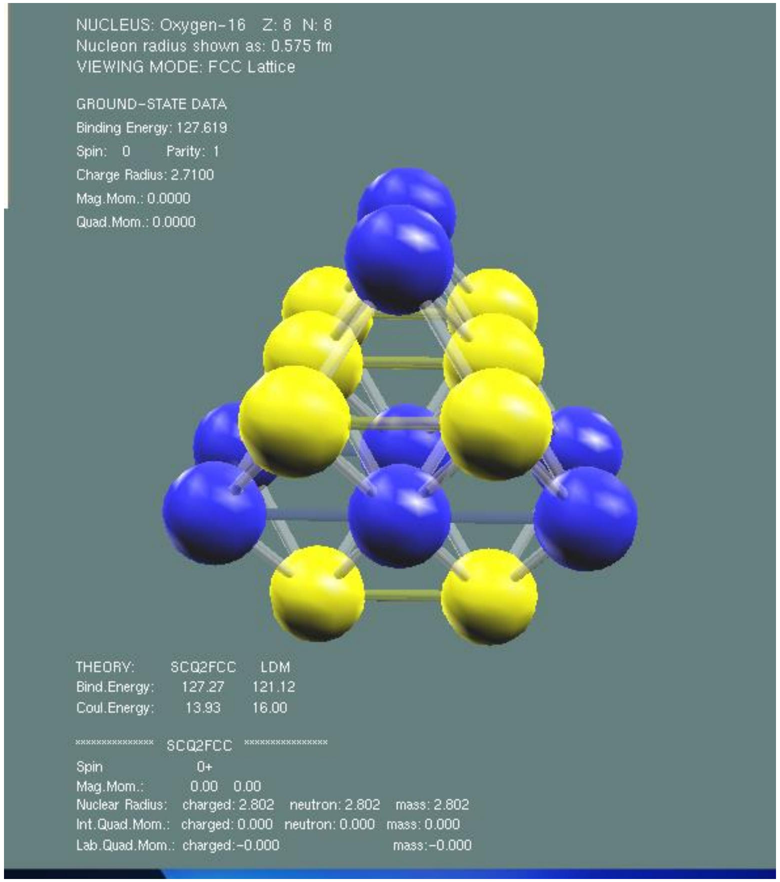


Nucleosynthesis in SCQM
^{4}He burning

$^{20}Ne^* \rightarrow {}^{16}O + 2p + 2n$



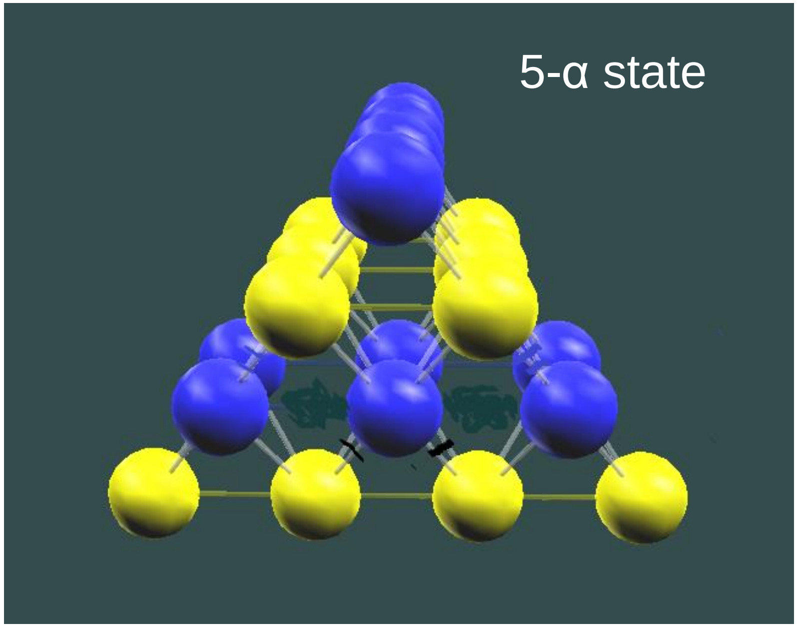
^{16}O g.s

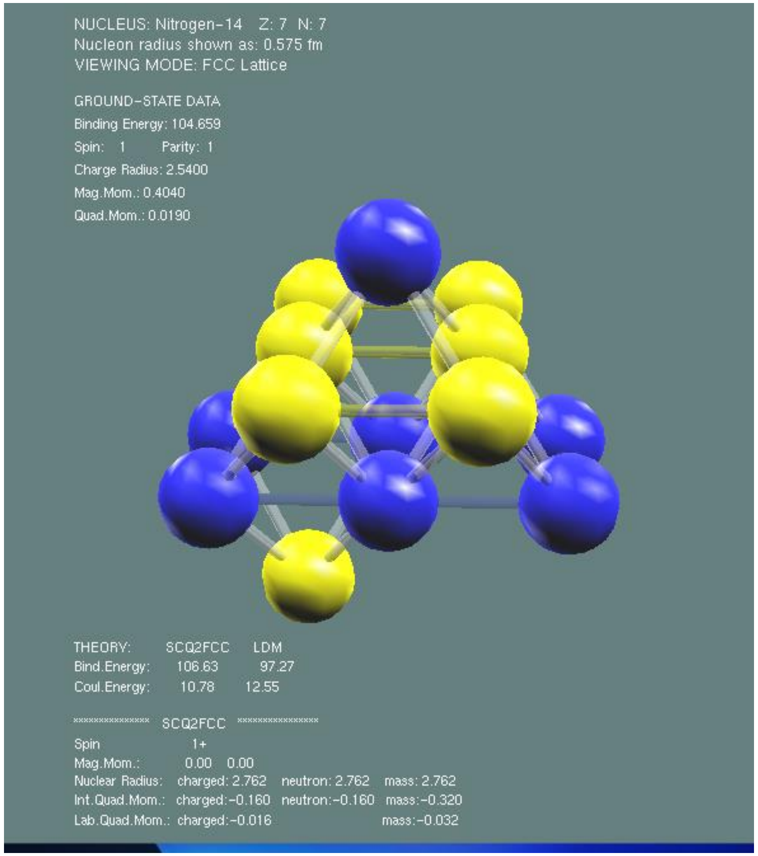


Nucleosynthesis in SCQM
^{4}He burning

$^{20}Ne^{*} \rightarrow {}^{14}N + 3p + 3n$

^{14}N g.s

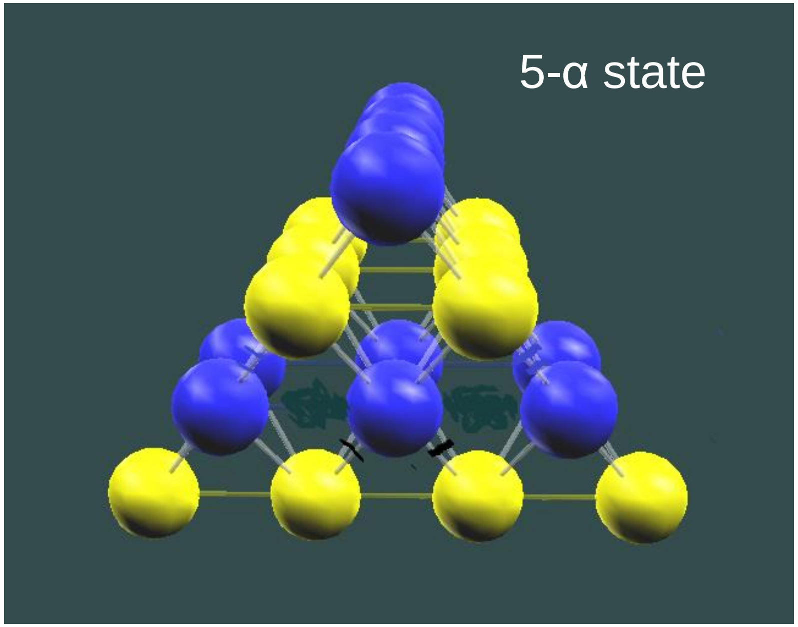


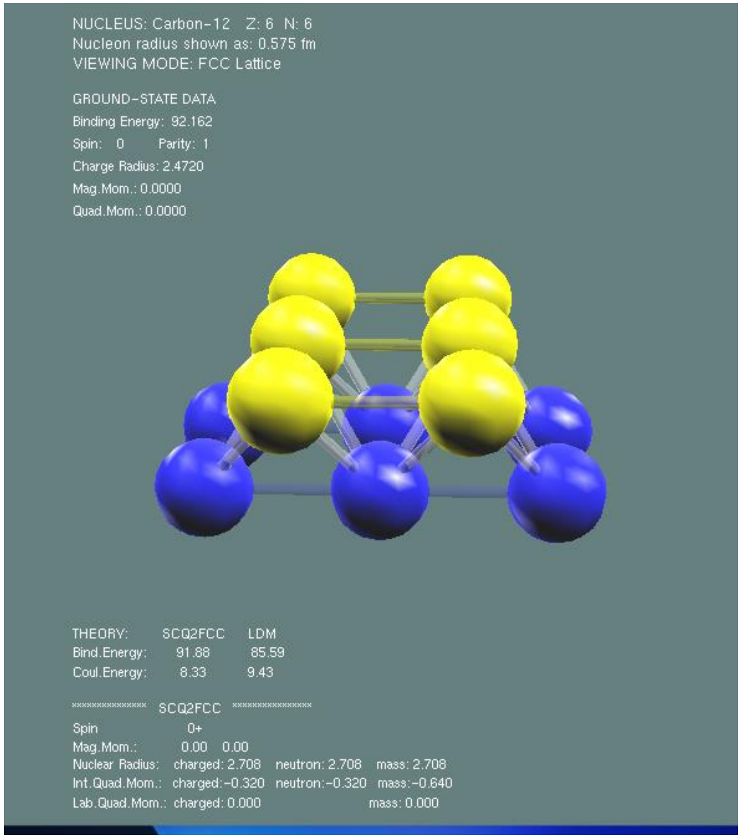


Nucleosynthesis in SCQM
^{4}He burning

$^{20}Ne^{*} \rightarrow {}^{12}C + 4p + 4n$

^{12}C g.s

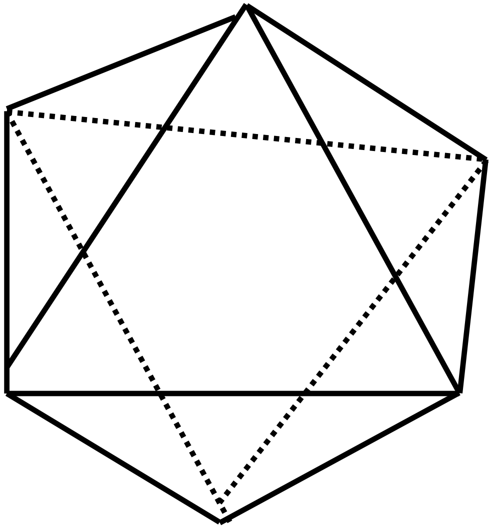




Conclusions

- Hoyle State $^{12}C^*$ does not lead to ^{12}C g.s.
- ^{12}C g.s is not the Key nucleus for the Nucleosynthesis
- Hoyle-like 5-α state is the Key state for the Nucleosynthesis

Thank you for your attention!



Back Slides

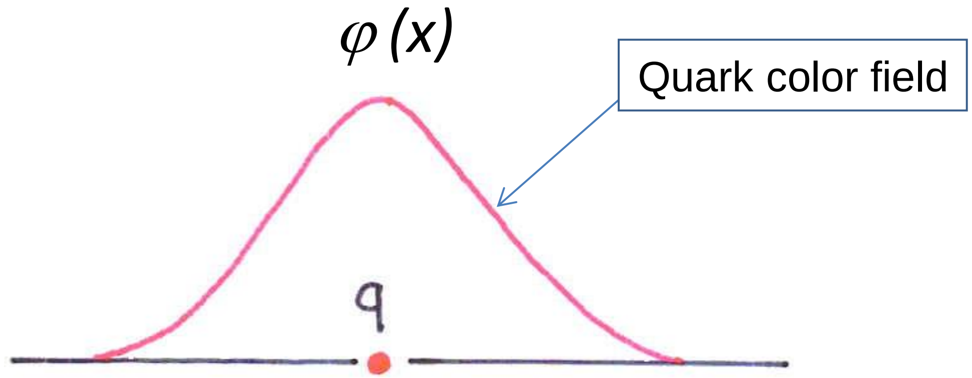
56

"Elementary particles are no more than holes in vacuum."

Henry Poincare

SCQM

Single Colored Quark inside Vacuum



Vacuum polarization by color field of a single quark

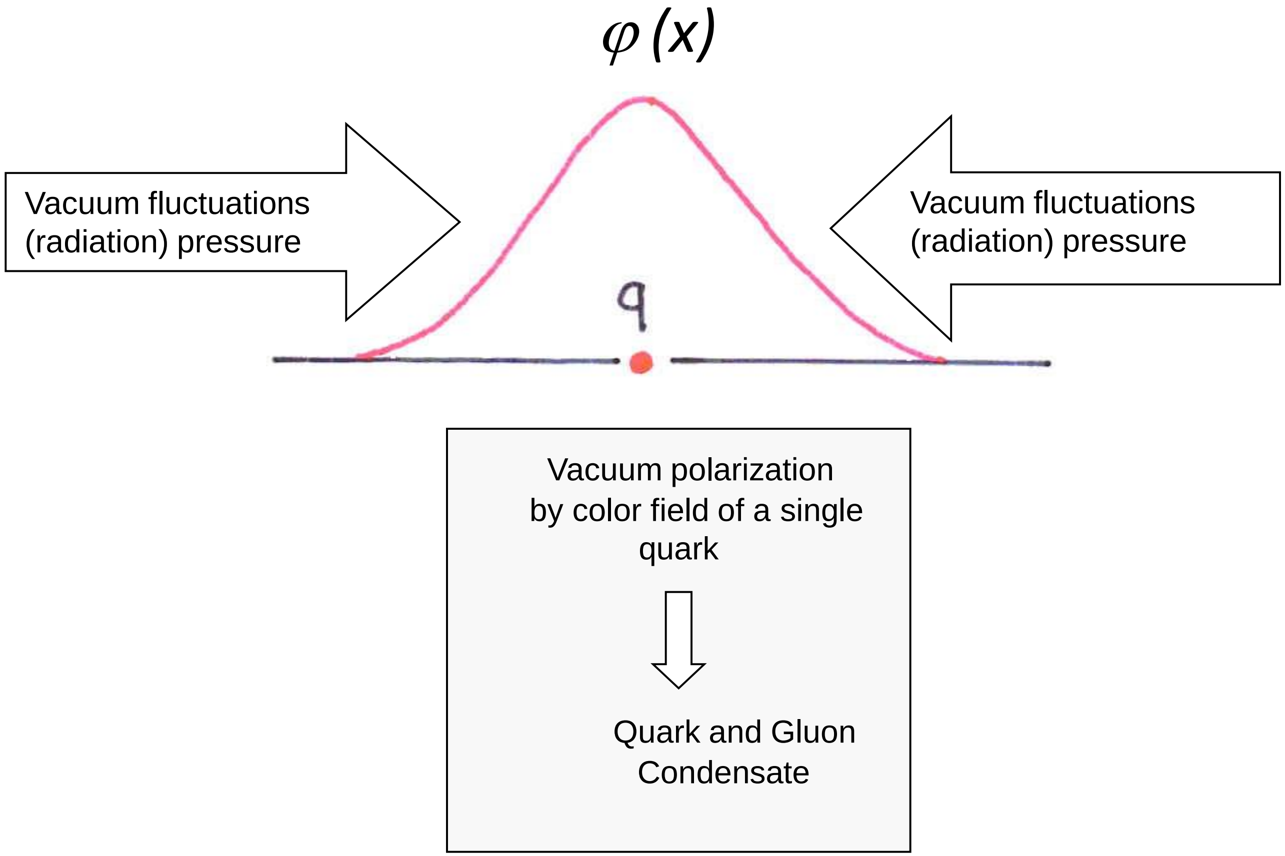
⇩

Quark and Gluon Condensate

57

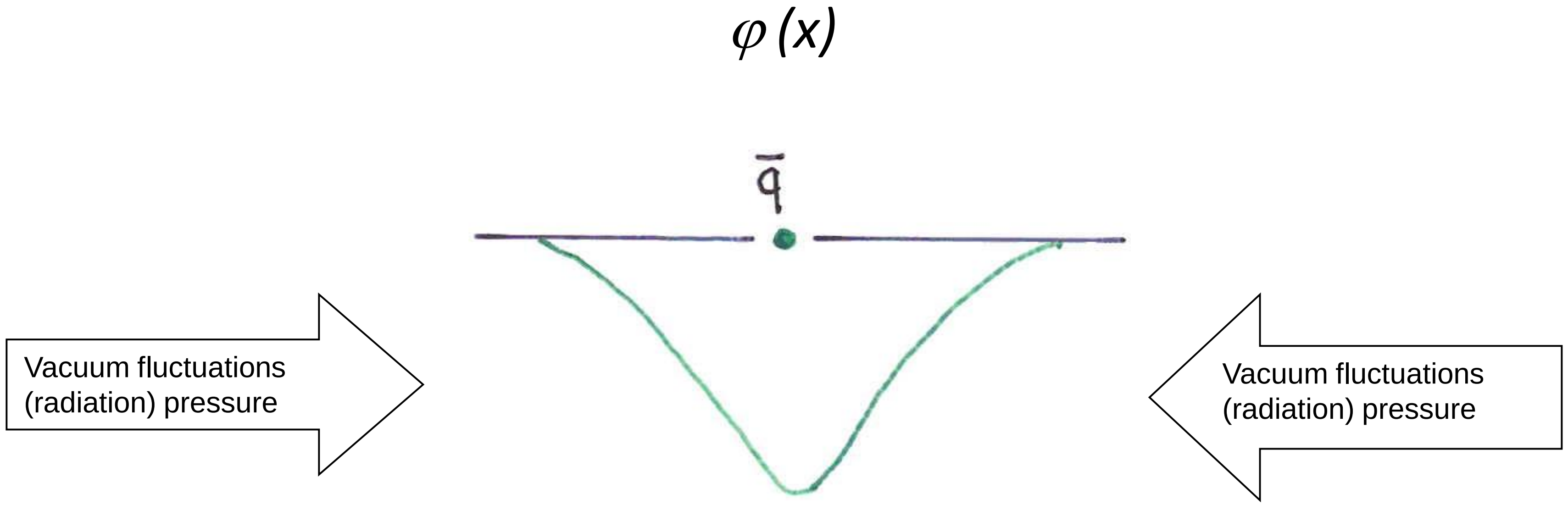
Strongly Correlated Quark Model

(SCQM)



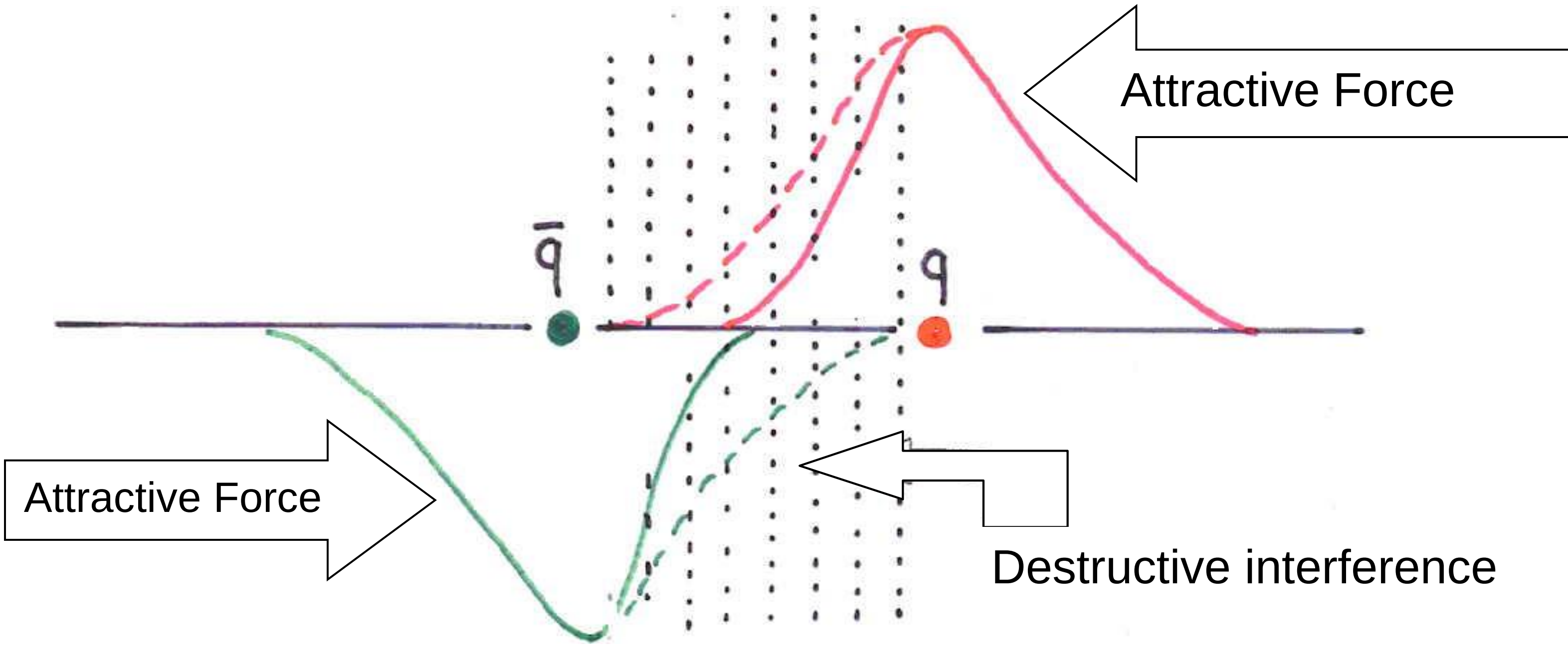
Strongly Correlated Quark Model

(SCQM)



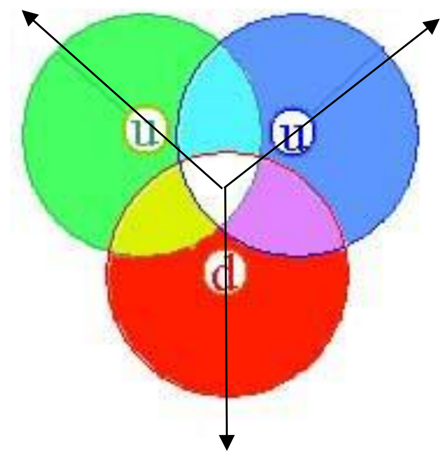
Strongly Correlated Quark Model

(SCQM)



Overlap of opposite color fields → attraction force between quark and antiquark
“Color Casimir” effect

Nucleon



Nucleon wave function composed of color quarks

$$|\psi\rangle \rightarrow \frac{1}{\sqrt{6}} \sum_{ijk} \epsilon_{ijk} |c_i\rangle |c_j\rangle |c_k\rangle$$

Where $|c_i\rangle$ are orthonormal states with i,j,k → R,G,B

61

g1

SCQM $\Longrightarrow$ The Local Gauge Invariance Principle

Destructive Interference of color fields ≡ Phase rotation of the quark w.f. in color space:

$$\psi(x)_{Color} \rightarrow e^{ig\theta(x)}\psi(x)$$

Phase rotation in color space $\Longrightarrow$ quark dressing (undressing) ≡ the gauge transformation

$$A^{\mu}(x) \rightarrow A^{\mu}(x) + \partial^{\mu}\theta(x)$$

Therefore, during quark oscillation its

color charge

momentum

mass

are continuously varying function of time.

g1

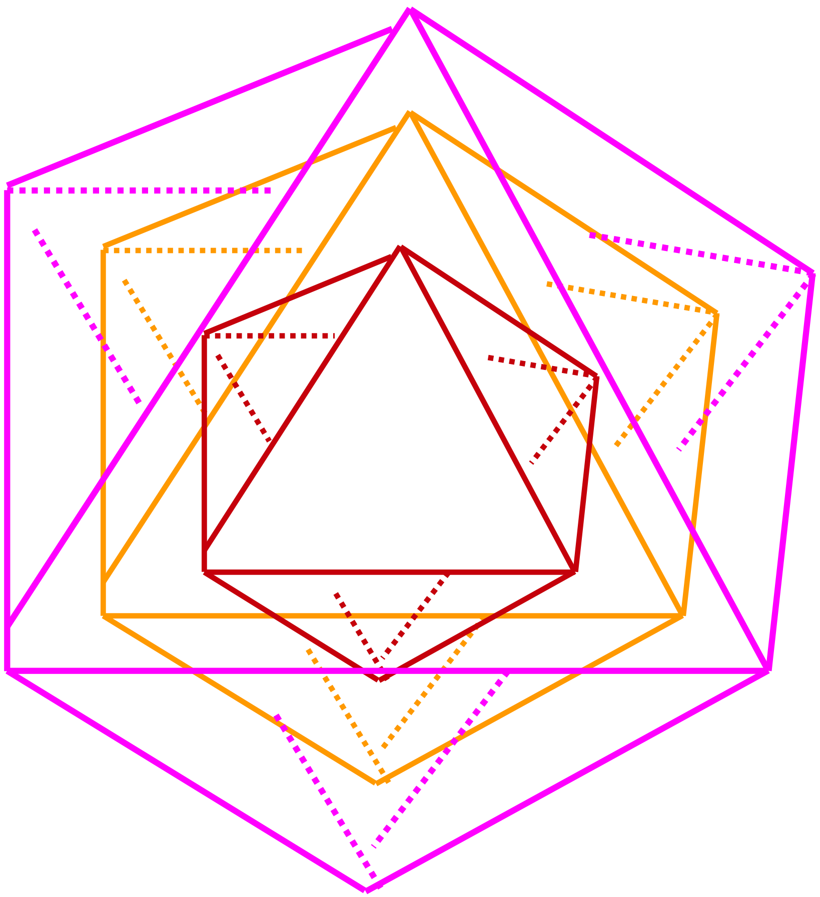
Relation SCQM to QCD

We reduce interaction of color quarks via **non-Abelian** fields to its **E-M** analog:

$$A_a^\mu(x) \rightarrow A^\mu(x)$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - \lambda f^{abc} A_b^\mu A_c^\nu \rightarrow F_{ch}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

^{40}Ca



3 Nested Octahedra – s, p, d -shells

FCC-SCQM vs SM

Source of spin-orbital coupling in FCC-SCQM

Increasing number of exchange nucleons, belonging to adjacent virtual alpha clusters with increasing J-value of sub-shells.

↓

Lowering of levels with higher J

Splitting of nuclear levels

FCC-SCQM vs SM

What about magic numbers?

SM

Describes observed magic numbers of protons and neutrons

2, 8, 20, 28, 50, 82, 126

FCC-SCQM

Closed Shells – Octahedra with filled faces

2, 8, 20, 40, 70, 112, … as given by HO potential

FCC-SCQM vs SM

What about magic numbers?

SM: 2, 8, 20, 28, 50, 82, 126

FCC-SCQM: 2, 8, 20, 40, 70, 112, …

But, **in FCC-SCQM the more preferable to start filling the next shell by the subshell with highest J** (from the base of octahedron)**.**

If these subsells are filled, we get the following magic numbers:

2, 6, 8, 14, 20, 28, 40, 50, 70, 82, 112, 126, …

Red numbers arise from adding to filled faces (shell) of octahedra the subshells with highest value J.

However, takes place only if both protons and neutrons fill this subshells forming virtual alpha clusters.

The role of Quarks in FCC

- Color fields of Quarks, responsible for strong interactions, arrange nuclear nucleons in FCC Lattice structure.
- Strong interactions are **tensorial**
- Quark loops form **virtual** 3- and 4-nucleon clusters inside bound nuclei
- Evidence of quark loops is **big separation energy** in even-even nuclei
- Halo nuclei are formed by core and virtual 3-nucleon clusters (^{3}H-type)
- Ground state nuclei are formed by virtual ^{3}H- and ^{4}He-type clusters.
- There are no real ^{4}He cluster in ground state nuclei

68

Summary (cont.)

Quantization

Rigid body quantization

As a rigid body Nuclei can possess:

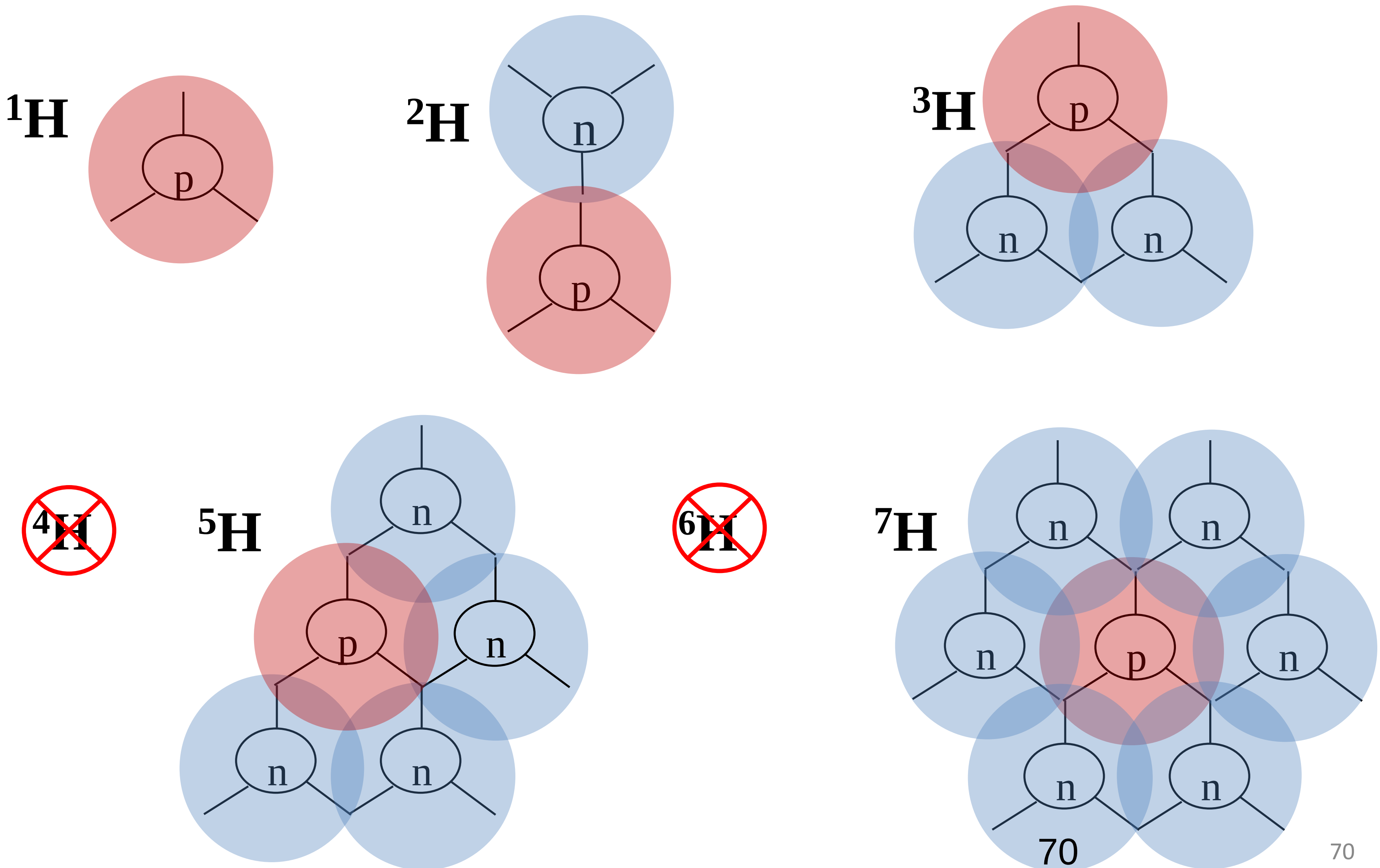
–particle – hole excitations

–collective modes of excitations

- Shape vibrations and fluctuations
- Rotations
- Isospin vibrations
- Sissor fluctuations

69

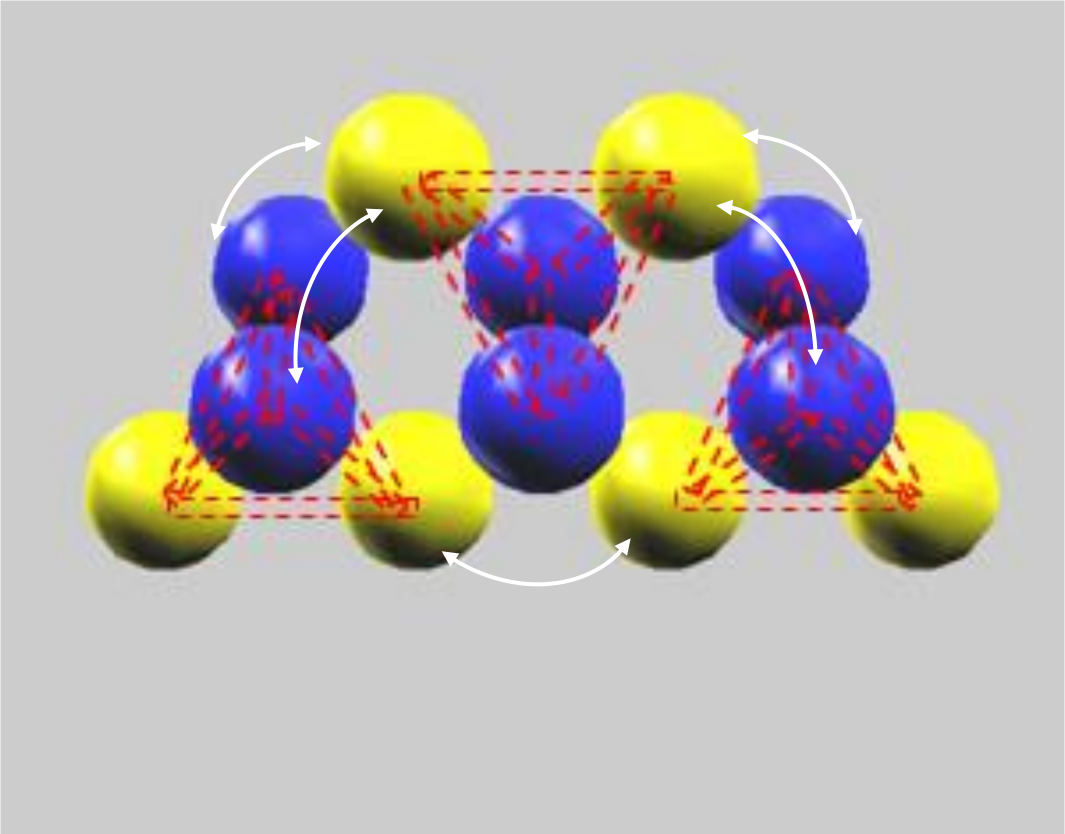
Bound Hydrogen Isotopes



^{12}C Hoyle state

Borromean nucleus

Loosely bound 3 real α- cluster nucleus

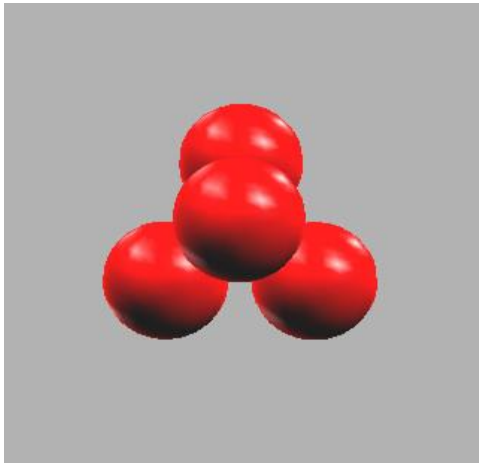
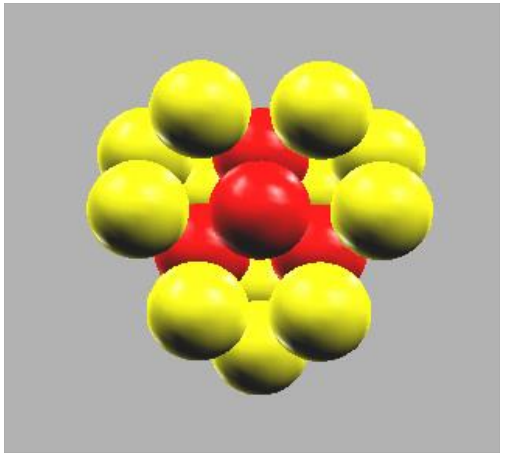
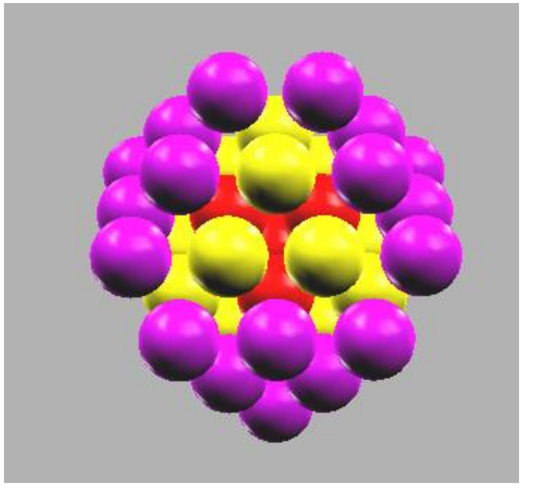


Frames of α-clusters are depicted as tethrahadrons. Neutrons of left and right α-clusters are bound with protons of central α-cluster (like in ^{8}He), and their 2 nearest protons are bound together.

FCC-SCQM vs SM
Spin-orbital coupling

In SCQM Increasing number of exchange nucleons leads to

Lowering of levels with higher **J**

J= 1/2	J = 1/2, 3/2	J = 1/2, 3/2, 5/2
		
s: n=0, l=0 1 alpha	**s**: n=1, l=1 6 virtual alpha	**s**: n=2, l=0, 2 22 virtual alpha

FCC-SCQM vs SM

What about spin-orbital coupling (SOC)?

SOC

- Splitting of nuclear levels
- Lowering of levels with higher **J**
- Description of observed magic numbers of protons and neutrons

2, 8, 20, 28, 50, 82, 126

Is it possible get the same numbers in FCC-SCQM?

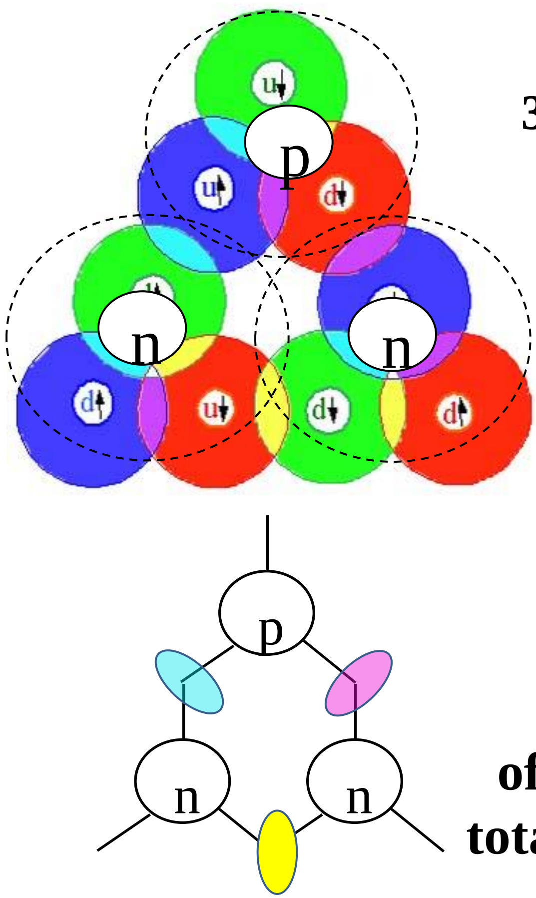
YES !

Summary

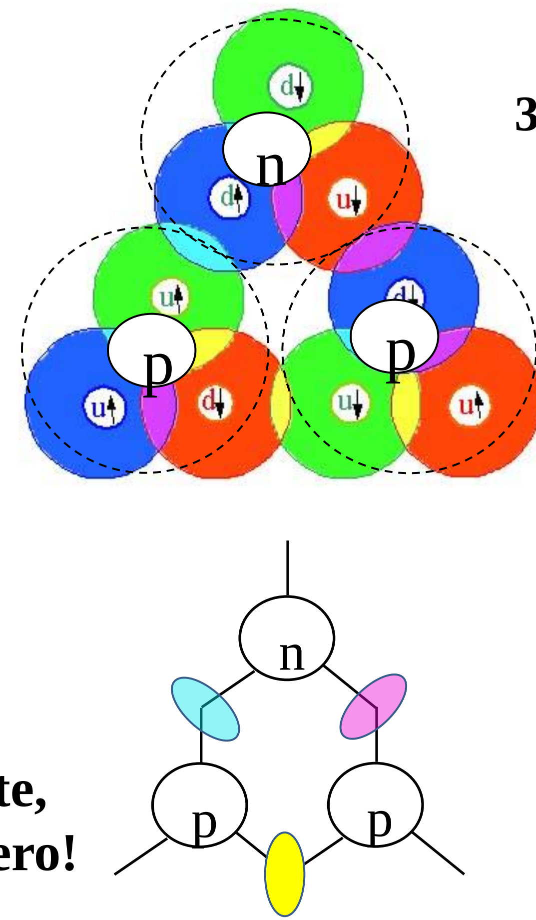
- Quarks play an explicit role in formation of the nuclear structure.
- Quark loops are building blocks of nuclear binding.
- Quarks and nucleons (protons and neutrons) inside nuclei are strongly correlated.
- 'Halo' nuclei **– fruits of quark-loop bindings**
- Effect of quark looping: $E_{sep} < E_{bound}/A$

74

Three Nucleon Systems in SCQM



^3H



^3He

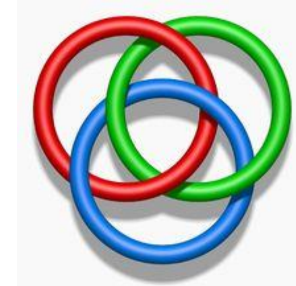
Summary color of 3 junctions is white, total color charge = zero!

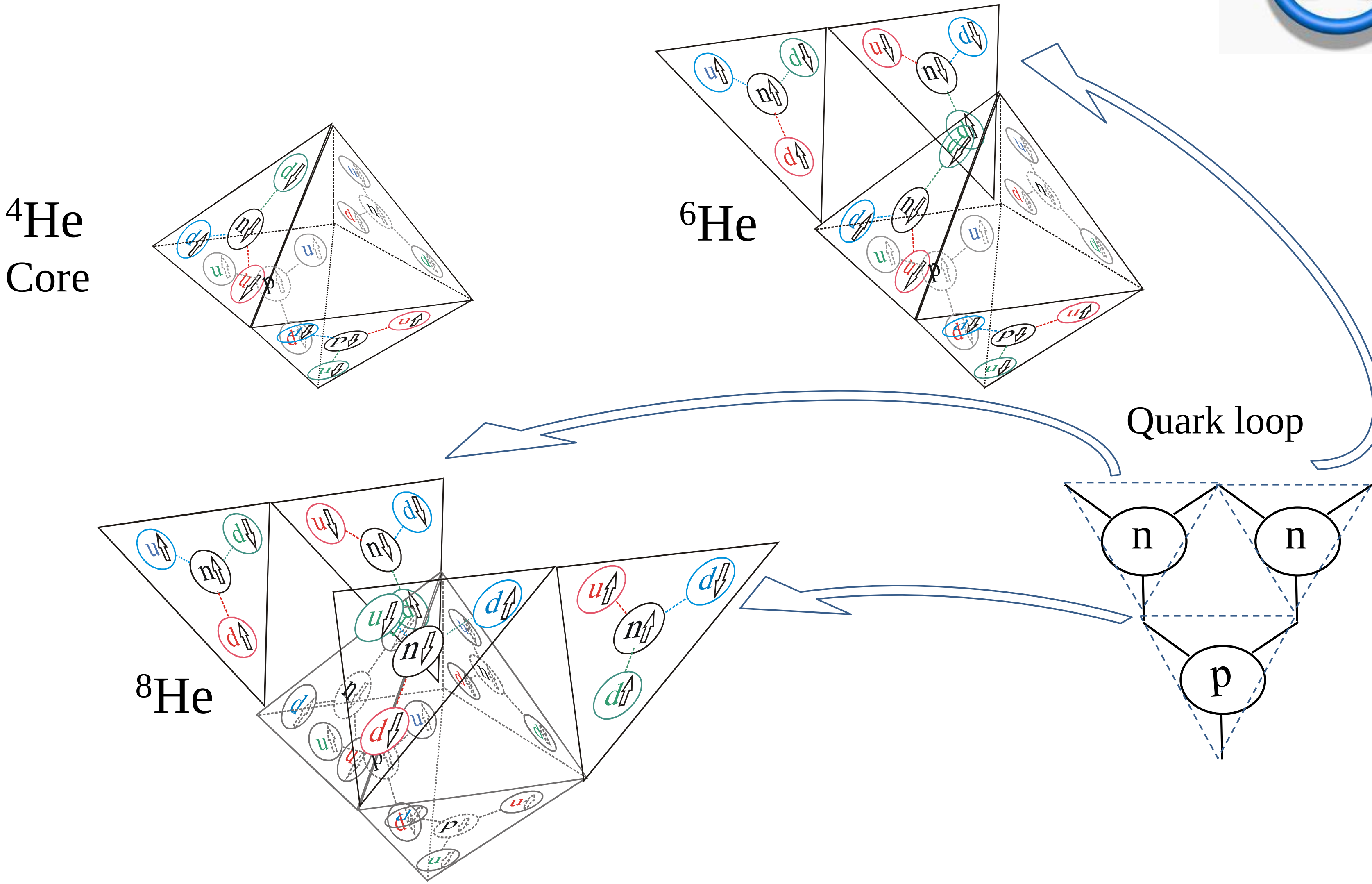
$$= \bar{q} \to qq$$

Quark loop formed by 3 nucleons → 3–body force

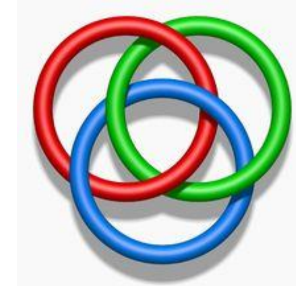
75

Helium Isotopes
Borromean Nuclei

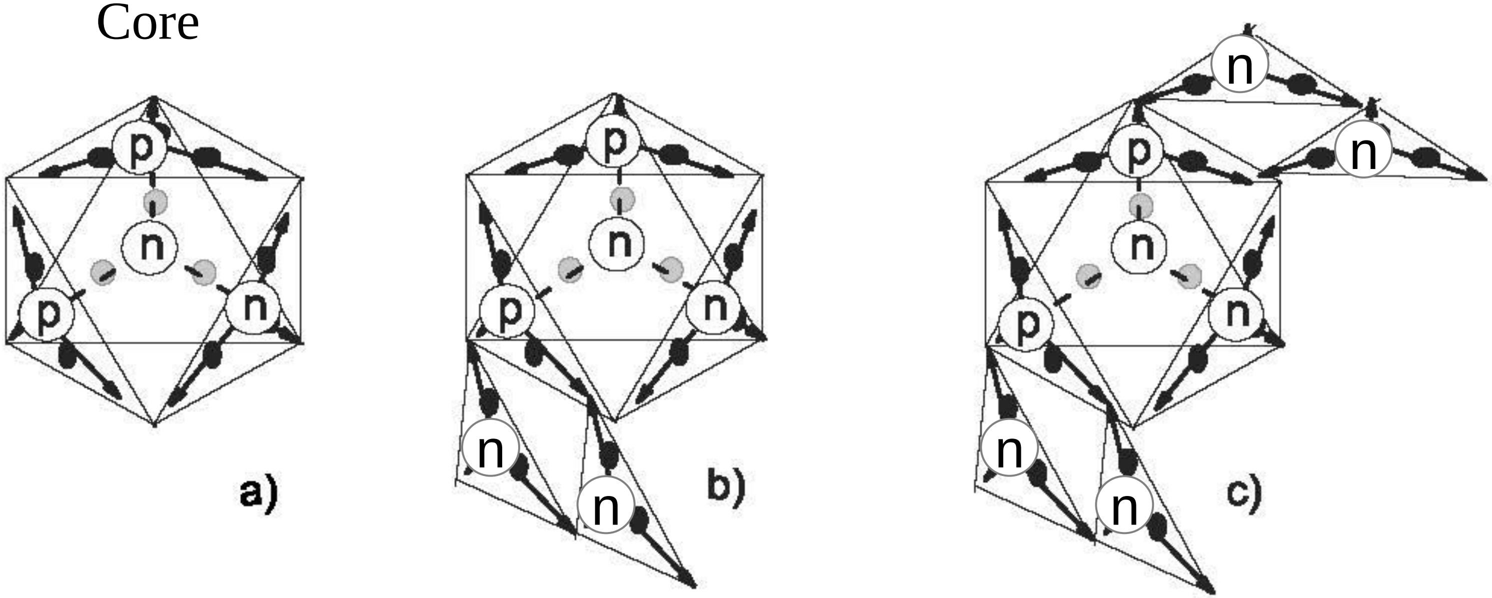




Helium Isotopes
Borromean Nuclei



^{4}He Core | ^{6}He | ^{8}He



R = 1.57 fm
R_{exp} = 1.6 fm

R = 2.2 fm
R_{exp} = 2.45 fm

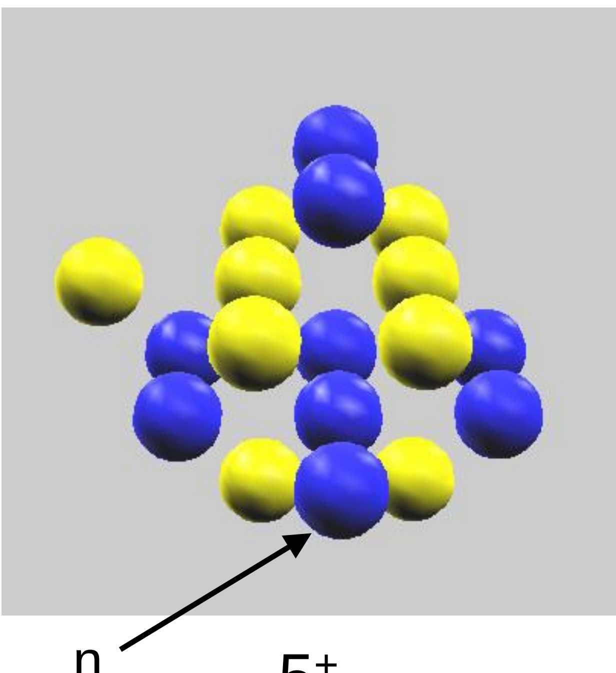
R = 2.4 fm
R_{exp} = 2.53 fm

Exp. *G.Alkhazov et al, PEPAN, 53 (2022) 661*

Fluorine Isomers

$^{18}F_m$

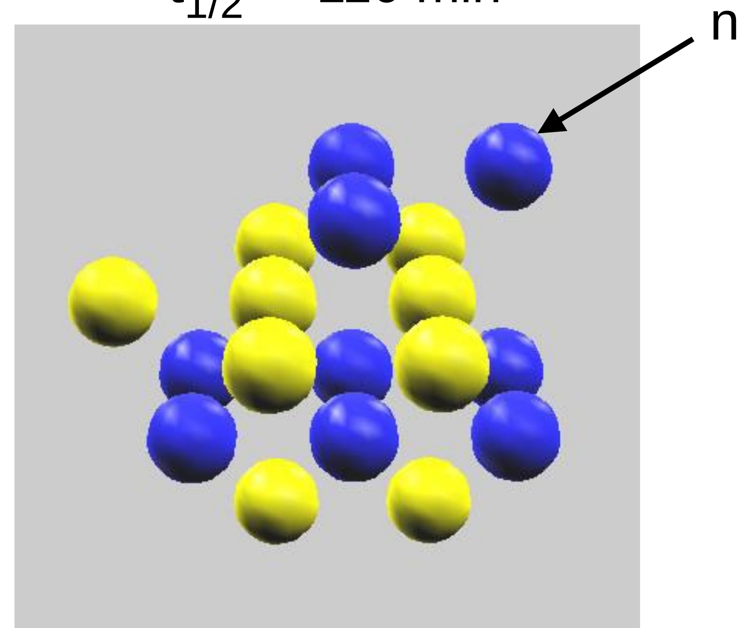
$t_{1/2}$ ~ 200 nc



5$^+$

^{18}F

$t_{1/2}$ ~ 110 min



1$^+$

protons – yellow
neutrons - blue

78